RT-MAE 9922

TWO EXTENDED PARTIALLY CORRELATED MODELS

by

Nikolai Kolev

Palavras-Chave: Generalization of the binomial distribution; Partially correlated models; Extended partially correlated models; Overdispersion; Zero-inflated distibutions.
Classificação AMS: 60E05.
(AMS Classification)

- Agosto de 1999 -

TWO EXTENDED PARTIALLY CORRELATED MODELS

Nikolai Kolev
Department of Statistics - IME
São Paulo University
C.P. 66281, 05315-970 São Paulo, Brazil

Abstract

In this paper the extended partially correlated binomial and Poisson distributions are introduced. The generalization of the binomial and partially correlated models studied by Fu and Sproule (1995) and Luceño (1995) are obtained as particular cases, respectively.

Key words and Phrases: *Generalization of the binomial distribution; Partially correlated models; Extended partially correlated models; Overdispersion; Zero-inflated distributions.*

AMS (MOS) subject classification: 60E05.

1 Introduction

Two genuine probability distributions for counts are the Poisson and the binomial distributions. Depending on the type of data they are used as a first step in the analysis usually. Real count data often exhibit overdispersion. When the the overdispersion is present the binomial or the Poisson model are not valid, so the successful modelling of count data requires non-standard richer probability models. Such extended models are studied by many authors and overview of the previous research and related applications are given, for example, in Collett (1991), Hinde and Demétrio (1998), Luceño (1995), Winkelmann (1997).

In this paper we introduce the extended correlated binomial and the extended correlated Poisson distributions which are over-/under- dispersed according to the Poisson and binomial distributions, respectively. We follow the idea of Fu and Sproule (1995) and the the approach developed by Luceño (1995).

The paper is organized as follows. In Section 2 we give two equivalent representations of the General binomial distribution and Correlated Poisson distribution, studied by Fu and Sproule (1995) and Luceño (1995), respectively. In Section 3 we define the Correlated general binomial and Correlated general Poisson distributions and in Section 4 we study the corresponding Extended partially correlated distributions. At the end several conclusions are summarized.

2 Two Relationships

In this section we show that the General binomial distribution studied by Fu and Sproule (1995) can be represented as a linear combination of the usual binomial distribution. Also, an equivalence between the Correlated Poisson distribution introduced by Luceño (1995) and the zero-inflated Poisson distribution is demonstrated.

2.1 General Binomial Distribution

Fu and Sproule (1995) present a new departure in the generalization of the binomial distribution by adopting the assumption that the underlying Bernoulli trials take on the values α or β, $\alpha < \beta$, instead of the conventional values 0 or 1, while retaining the assumptions that the probability of success is the constant $p \in (0,1)$ and the Bernoulli trials are independent.

Let ν_k, $k = 1, 2, \ldots, n$ be independent binary variables taking values 1 and 0 with probabilities p and $1-p$, correspondingly. The random variable (r.v.)

$$V = \nu_1 + \nu_2 + \cdots + \nu_n$$

has the binomial distribution with parameters n and p, to be denoted $V \sim Bi(n,p)$.

Consider the r.v.'s

$$Z_k = \alpha + (\beta - \alpha)\nu_k \tag{1}$$

for $\alpha < \beta$ and $k = 1, 2, \ldots, n$. Then

$$Z_k = \begin{cases} \beta & \text{with probability } p, \\ \alpha & \text{with probability } 1-p \end{cases}$$

are independent r.v.'s and their sum

$$Z = Z_1 + Z_2 + \cdots + Z_n$$

was called by Fu and Sproule (1995) the General binomial distribution with parameters n, p, α and β. We will denote this by $Z \sim GBi(n, p, \alpha, \beta)$.

From (1) immediately follows that

$$Z = n\alpha + (\beta - \alpha)V \tag{2}$$

for $V \sim Bi(n, p)$. Thus, under the given notations we obtain the following simple

Proposition 2.1. *The linear combination (2) for $V \sim Bi(n,p)$, represents the General binomial distributed r.v. $Z \sim GBi(n, p, \alpha, \beta)$.*

Remark 2.1. Let us note that the r.v.'s Z_k defined by (1) can be represented as

$$Z_k = \frac{\beta + \alpha}{2} + \frac{\beta - \alpha}{2}\varepsilon_k,$$

where

$$\varepsilon_k = \begin{cases} 1 & \text{with probability } p, \\ -1 & \text{with probability } 1-p \end{cases}$$

are independent binary variables, $k = 1, 2, \ldots, n$. It is clear that for the discussed binary variables the following equivalence relations are fulfilled

$$\varepsilon_k = \{1, -1\} \iff \nu_k = \{1, 0\} \iff Z_k = \{\beta, \alpha\}.$$

2.2 Correlated Binomial and Correlated Poisson Distributions

The second distribution of interest in this section is the *Correlated binomial distribution* with parameters n, p and ρ, implicitly introduced by Tallis (1962), and later rediscovered and studied by Luceño (1995) and Luceño and Caballos (1995). A r.v. W following this distribution counts the number of successes in a sample of n subjects that give equicorrelated binary responses with correlation coefficient ρ, probability of success p, under the condition that its probability mass function must depend linearly on ρ. The r.v. W can be represented as

$$W = w_1 + w_2 + \cdots + w_n,$$

where w_k, $k = 1, 2, \ldots, n$, are equicorrelated binary variables taking values 1 and 0 with probabilities p and $1-p$, correspondingly, having mean and variance

$$E(w_k) = p \quad \text{and} \quad Var(w_k) = p(1-p)$$

The covariance is given by

$$Cov(w_k, w_m) = \rho p(1-p) \quad \text{for} \quad k \neq m.$$

We will use the notation $W \sim CBi(n, p, \rho)$. The probability mass function of the r.v. W can be written in terms of the mixture

$$P(W = w) = (1-\rho)P(V = w) + \rho P(J = j(w)), \tag{3}$$

where $V \sim Bi(n, p)$, J being rescaled binomial r.v. taking values 0 and n with $j(w) = 0, n$ for $w = 0, 1, \ldots, n$.

The probability generating function (PGF) of the $CBi(n, p, \rho)$ distribution is given by

$$P_W(t) = \rho(1 - p + pt^n) + (1-\rho)(1 - p + pt)^n, \quad |t| \leq 1. \tag{4}$$

Remark 2.2. Let us note, that (4) is a mixture of the PGF's of binomial r.v. and two degenerated at 0 and n r.v.'s, since the sum of the corresponding coefficients is equal to one.

This means that the PGF $P_W(t)$ is decomposed (with corresponding probabilities) by the PGF of the binomial r.v. and two degenerated r.v.'s.

Now, putting in (4) $n \longrightarrow \infty$ and $p \longrightarrow 0$ such that $np = \lambda = const$, Luceño (1995) obtains the PGF of the *Correlated Poisson distribution*, given by

$$P(t) = \rho + (1-\rho)exp\{\lambda(t-1)\}. \tag{5}$$

In order to show the second relationship, we need to introduce briefly the so-called *zero-inflated distributions*, see Johnson et *al.* (1992), p. 312. Let X be an arbitrary nonnegative integer-valued r.v. with probability mass function

$$P(X=j) = p_j, \quad j = 0,1,\cdots, \quad \sum_{j=0}^{\infty} p_j = 1$$

and let $G_X(t) = E(t^X)$ be its PGF. The probability mass function of the corresponding zero-inflated r.v. Y can be written as

$$\begin{aligned} P(Y=0) &= \rho + (1-\rho)p_0, \\ P(Y=j) &= (1-\rho)p_j, \quad j = 1,2,\cdots. \end{aligned}$$

The parameter ρ in last relations may take negative values provided that $P(Y=0) > 0$, or equivalently $0 > \rho > -p_0(1-p_0)^{-1}$. This case corresponds to the negatively correlated binary variables w_k, $k = 1,2,\ldots,n$.

The inflated distributions are appropriate alternatives for modelling the clustered samples, for example when the population consists of two subpopulations, one containing only zeros while in other, counts from a discrete distribution may be observed.

The PGF of the zero-inflated r.v. Y, associated with the r.v. X is given by

$$G_Y(t) = \rho + (1-\rho)G_X(t). \tag{6}$$

Remark 2.3. For $\rho > 0$ from (6) one may obtain that the following inequality is fulfilled

$$\frac{Var(Y)}{E(Y)} > \frac{Var(X)}{E(X)}$$

and therefore the zero-inflated distributions lead to possible models for explaining overdispersion. If $\rho \in (-\frac{p_0}{1-p_0}, 0)$ the inverse inequality for the variance mean ratios is valid and we have a model for underdispersion in this case.

Now it is easy to see that if $X \sim Po(\lambda)$, i.e. if $P_X(t) = exp\{\lambda(t-1)\}$ in (6), then the PGF of the zero-inflated Poisson distribution coincides with the PGF of the Correlated Poisson distribution given by (5). Thus we proved the next

Proposition 2.2. *The zero-inflated Poisson distribution and the Correlated Poisson distribution are identical.*

Remark 2.4. From Proposition 2.2 we may interpret the parameter ρ in the zero-inflated Poisson distribution as the correlation coefficient between binary responses in the Correlated binomial model, when $\rho \in (-\frac{e^{-\lambda}}{1-e^{-\lambda}}, 1)$.

3 Correlated General Binomial and Correlated General Poisson Distributions

Let us define the r.v.'s

$$X_k = \alpha + (\beta - \alpha)w_k$$

for $\alpha < \beta$, where w_k are the defined in Section 2 equicorrelated binary variables, $k = 1, 2, \ldots, n$. Then

$$X_k = \begin{cases} \beta & \text{with probability } p, \\ \alpha & \text{with probability } 1-p \end{cases} \tag{7}$$

with mean and variance

$$E(X_k) = \alpha + (\beta - \alpha)p \quad \text{and} \quad Var(X_k) = p(1-p)(\beta - \alpha)^2.$$

The covariance and the correlation coefficient are given by

$$Cov(X_k, X_m) = \rho p(1-p)(\beta - \alpha)^2 \quad \text{and} \quad Corr(X_k, X_m) = \rho \quad \text{for} \quad k \neq m. \tag{8}$$

Therefore X_k's are equicorrelated binary variables taking values β and α with probabilities p and $1-p$, respectively.

Definition 3.1. We call the distribution of r.v.'s X_k defined by (7) the *Correlated general Bernoulli distribution* with parameters p, α, β and ρ.

Definition 3.2. The sum

$$Y_n = X_1 + X_2 + \cdots + X_n$$

represents a r.v. with a distribution which we will refer as the *Correlated general binomial* with parameters n, p, α, β and ρ, to be denoted $CGBi(n, p, \alpha, \beta, \rho)$.

Remark 3.1. If X_k's are independent, the General Bernoulli distribution introduced by Sproule (1992) is obtained, i.e. if $\rho = 0$ in (8), $X_k =^d Z_k$ where the r.v.'s Z_k are defined by (1). In this case the r.v. Y_n is General binomial distributed according Fu and Sproule (1995).

Further, our considerations will be related with the r.v.

$$Y_n - n\alpha = \sum_{k=1}^{n}(X_k - \alpha). \tag{9}$$

According Definition 3.2, $Y_n - n\alpha \sim CGBi(n.p, 0, \beta - \alpha, \rho)$. In order to reduce the parameter space, we will use in the following exposition the notation $CGBi(n, p, \beta - \alpha, \rho)$

Remark 3.2. The quantity

$$\frac{Y_n - n\alpha}{\beta - \alpha}$$

gives the number of trials for which $X_k = \beta$, $k = 1, 2, \ldots, n$.

Following Luceño (1995), by imposing that the probability function of a r.v. $Y_n - n\alpha$ defined by (9) depends linearly on ρ (compare with the equation (3)), it is easy to show that the following relation is fulfilled

$$P(Y_n - n\alpha = y) = (1-\rho)P(T = y) + \rho P(U = u(y)) \tag{10}$$

for $y = 0, \beta - \alpha, \dots, n(\beta - \alpha)$ and $u(y) = 0, n(\beta - \alpha)$, where $T \sim GBi(n, p, 0, \beta - \alpha)$, with

$$E(T) = np(\beta - \alpha) \quad \text{and} \quad Var(T) = np(1-p)(\beta - \alpha)^2,$$

$U \sim GBi(1, p, 0, \beta - \alpha)$ being a rescaled General binomial r.v. taking values 0 and $n(\beta - \alpha)$ and having mean and variance

$$E(U) = np(\beta - \alpha) \quad \text{and} \quad Var(U) = n^2 p(1-p)(\beta - \alpha)^2.$$

Hence,

$$E(Y_n - n\alpha) = np(\beta - \alpha) \quad \text{and} \quad Var(Y_n - n\alpha) = np(1-p)[1 + \rho(n-1)](\beta - \alpha)^2.$$

Remark 3.3. Two particular cases of (10) are of interest:

- if $\alpha = 0$ and $\beta = 1$, the Correlated binomial model studied by Luceño (1995) is obtained;
- if $\rho = 0$, we have the Generalization of the binomial distribution introduced by Fu and Sproule (1995).

For $|t| \le 1$, from (10) we have the following PGF

$$P_{Y_n - n\alpha}(t) = \rho[1 - p + pt^{(\beta-\alpha)n}] + (1-\rho)[1 - p + pt^{\beta-\alpha}]^n. \tag{11}$$

If we substitute $\alpha = 0$ and $\beta = 1$ in (11), the PGF of the $CBi(n, p, \rho)$ distribution, given by (4) is obtained.

Now, let $n \longrightarrow \infty$ and $p \longrightarrow 0$ such that $np = \lambda = const$. After simple transformations we have the following limiting PGF

$$P_Q(t) = \rho + (1-\rho)exp\{\lambda(t^{\beta-\alpha} - 1)\}. \tag{12}$$

Definition 3.3. The expression (12) is the PGF of r.v. Q with distribution, which we call the *Correlated general Poisson distribution* with parameters λ, ρ and $\beta - \alpha$, to be denoted by $Q \sim CGPo(\lambda, \rho, \beta - \alpha)$.

Remark 3.4. From (12) the PGF of

- the usual Poisson distribution is obtained for $\beta = 1$, $\alpha = 0$ and $\rho = 0$;
- the Correlated Poisson distribution introduced by Luceño (1995) is obtained for $\beta = 1$ and $\alpha = 0$, compare with relation (5).

4 Two Extended Partially Correlated Models

Although the $CGBi(n, p, \beta - \alpha, \rho)$ model produces overdispersion for $\rho > 0$, it assumes implicitly that each and every one of n subjects included in the sample belongs to one sole equicorrelated clump which, in general, is an unreasonable assumption. Some other unattractive properties of the model (which can be avoided by partially correlated models), are similar to those, discussed by Luceño and Caballos (1995), p. 1642, for the $CBi(n, p, \rho)$ model.

Following the notations used by Luceño (1995), assume that the events counted during the observation interval occur in independent clusters of size L and each item included in a cluster is randomly assigned to a clump according to a multinomial distribution with K equiprobable outcomes. Accordingly, the sizes of clumps form a random vector $\mathbf{n} = (n_1, \ldots, n_K)$ that may be considered as a latent variable following multinomial distribution $Mn(L, K^{-1}, \ldots, K^{-1})$, i.e.

$$\mathbf{n} = (n_1, \ldots, n_K) \sim Mn(L, K^{-1}, \ldots, K^{-1})$$

with mean and variance

$$E(n_i) = LK^{-1} \quad \text{and} \quad Var(n_i) = L(K-1)K^{-2} \tag{13}$$

under restriction $n_1 + n_2 + \cdots + n_K = L$.

For given $\mathbf{n}$, assume that the number of events

$$(Y_{n_1} - n_1\alpha = y_1,\ Y_{n_2} - n_2\alpha = y_2, \ldots,\ Y_{n_K} - n_K\alpha = y_K),$$

$y_i = 0,\ \beta - \alpha, \ldots,\ n_i(\beta - \alpha)$, provided by the clumps are independent r.v.'s and $Y_{n_i} - n_i\alpha \sim CGBi(n_i, p, \beta - \alpha, \rho)$, $i = 1, \ldots, K$, with conditional mean and conditional variance given by

$$E\left(Y_{n_i} - n_i\alpha | n_i\right) = n_i p(\beta - \alpha) \tag{14}$$

and

$$Var\,(Y_{n_i} - n_i\alpha | n_i) = n_i p(1-p)[1+\rho(n_i - 1)](\beta - \alpha)^2. \tag{15}$$

Definition 4.1. The total number of events

$$\sum_{i=1}^{K}(Y_{n_i} - n_i\alpha) = \sum_{i=1}^{K} Y_{n_i} - L\alpha$$

in each cluster, follows the *Extended partially correlated binomial distribution* with parameters L, p, $\beta - \alpha$, ρ and K, to be denoted $EPCBi(L, p, \beta - \alpha, \rho, K)$.

The PGF of the defined r.v. is given by

$$G(t) = E_{\mathbf{n}}\left[P_{Y_{n_i} - n_i\alpha}(t)\right], \tag{16}$$

where the expectation is taken with respect to the random vector $\mathbf{n}$ and $P_{Y_{n_i} - n_i\alpha}(t)$ is the PGF given by (11).

The mean and the variance of the $EPCBi(L, p, \beta - \alpha, \rho, K)$ distribution can be obtained after some algebra by using expressions (13), (14), (15), the properties of the conditional expectation and conditional variance and are given by

$$E\left(\sum_{i=1}^{K} Y_{n_i} - L\alpha\right) = pL(\beta - \alpha)$$

and

$$Var\left(\sum_{i=1}^{K} Y_{n_i} - L\alpha\right) = p(1-p)L[1 + \rho(L-1)K^{-1}](\beta - \alpha)^2,$$

respectively.

Let us find the limit of the PGF $[G(t)]^N$, where $G(t)$ is given by (16), N is the number of the independent clusters occurring during an observation interval, for $N \longrightarrow \infty$ and $p \longrightarrow 0$ while $NLp = \lambda = const.$ The limiting PGF in this case is given by the following expression

$$P_F(t) = exp\left\{\lambda(1-\rho)(t^{\beta-\alpha} - 1) + \lambda\rho\sum_{l=1}^{L}\delta_l[t^{(\beta-\alpha)l} - 1]\right\}, \tag{17}$$

where

$$\delta_l = \binom{L}{l}\left(1-K^{-1}\right)^{L-l}K^{1-l}L^{-1}, \quad l = 1,2,\ldots,L.$$

Definition 4.2. We call the r.v. F with a PGF given by (17) *Extended partially correlated Poisson distributed* with parameters L, λ, $\beta-\alpha$, ρ and K, to be denoted by $F \sim EPCPo(L,\lambda,\beta-\alpha,\rho,K)$.

The mean and the variance of the defined distribution are given by

$$E(F) = \lambda(\beta-\alpha) \quad \text{and} \quad Var(F) = \lambda[1+\rho(L-1)K^{-1}](\beta-\alpha)^2.$$

Remark 4.1. For $\beta = 1$ and $\alpha = 0$ from (16) and (17) as particular cases can be obtained the Generalized partially correlated binomial and Poisson models introduced by Luceño (1995). Let us denote them by $GPCBi(L,p,\rho,K)$ and $GPCPo(L,\lambda,\rho,K)$, respectively.

Let $F \sim EPCPo(L,\lambda,\beta-\alpha,\rho,K)$ and $R \sim GPCPo(L,\lambda,\rho,K)$. Comparing their PGF's one can conclude that

$$F = (\beta-\alpha)R$$

and therefore

$$P(F=f) = P\left(R = \frac{f}{\beta-\alpha}\right), \quad \text{for} \quad f = 0,\ \beta-\alpha,\ 2(\beta-\alpha),\ldots$$

with

$$P(R=0) = exp\left\{-\lambda\left[1-\rho+\rho\sum_{l=1}^{L}\delta_l\right]\right\}$$

and

$$P(R=r) = \lambda r^{-1}\left[(1-\rho)P(R=r-1)+\rho\sum_{l=1}^{min(r,L)} l\delta_l P(R=r-l)\right], \quad r = 1,2,\ldots,$$

where the last expressions for the probabilities are obtained by Luceño (1995). Hence, the exact distribution of the r.v. $F \sim EPCPo(L,\lambda,\beta-\alpha,\rho,K)$ is the same, as given by Luceño

(1995) for the r.v. $R \sim GPCPo(L, \lambda, \rho, K)$, provided that the possible values of the r.v. F are $f = 0,\ \beta - \alpha,\ 2(\beta - \alpha), \dots$.

A similar conclusion is valid in the binomial case: a $EPCBi(L, p, \beta - \alpha, \rho, K)$ distributed r.v. takes values $0,\ \beta - \alpha, \dots,\ L(\beta - \alpha)$ and its probability mass function coincides with the probability mass function of a $GPCBi(L, p, \rho, K)$ distributed r.v. as given by Luceño and Caballos (1995).

Remark 4.2. The $CGBi(n, p, \beta - \alpha, \rho)$ and $CGPo(\lambda, \beta - \alpha, \rho)$ models studied in Section 3, can be obtained from their partially correlated analogues, if the number of clusters is $N = 1$, i.e. by substituting $K = 1$ and $L = n$ in the corresponding formulas.

5 Conclusions

Several extensions of the binomial distribution discussed by Fu and Sproule (1995) and Luceño and Caballos (1995) have been generalized to the Extended partially correlated binomial distribution. As a limiting result, the Extended partially correlated Poisson distribution have been obtained. Presented results for the Extended partially correlated Poisson distribution may be used in a similar manner to extend the Generalized Erlang distribution studied by Luceño (1996). The introduced distributions are candidates for modelling of under- or overdispersion, when the count data are heterogeneous and grouped in clusters.

In this article we do not consider the tests for detecting extra-binomial and extra-Poisson variation, which may be described by the extended correlated binomial and Poisson models, correspondingly. Work on some score-tests for discovering possible extra-variation is currently in progress.

6 Acknowledgment

I would like to thank Dr. Leda Minkova for her kind comments on earlier version of this paper.

REFERENCES

Collett, D. (1991). *Modelling of Binary Data*, London: Chapman and Hall.

Hinde, J. and Demétrio, C. (1998). Overdispersion: models and estimation. *Computational Statistics & Data Analysis* **27**, 151-170.

Fu, J. and Sproule, R. (1995). A generalization of the binomial distribution, *Communications in Statistics: Theory and Methods* **24 (10)**, 2645-2658.

Johnson, N.L., Kotz, S., and Kemp, A.W. (1992). *Univariate Discrete Distributions.* Wiley Series in Probability and Mathematical Statistics. 2nd edition.

Luceño, A. (1995). A family of partially correlated Poisson models for overdispersion. *Computational Statistics & Data Analysis* **20**, 511-520.

Luceño, A. (1996). A generalized Erlang distribution showing overdispersion. *Statistics & Probability Letters* **28**, 375-386.

Luceño, A. and Caballos, F. (1995). Describing extra-binomial variation with partially correlated models. *Communications in Statistics: Theory and Methods* **24 (6)**, 1637-1653.

Sproule, R. (1992). The general Bernoulli probability density function and the properties of its central moments. *The Statistician* **41**, 155-159.

Tallis, G.M. (1962). The use of generalized multinomial distribution in the estimation of correlation in discrete data, *Journal of the Royal Statistical Society*, Ser. B **24**, 530-534.

Winkelmann, R. (1997). Econometric Analysis of Count Data, Berlin: Springer.

ÚLTIMOS RELATÓRIOS TÉCNICOS PUBLICADOS

9901 - **MONTOYA-DELGADO, L.E., IRONY, T.Z., WHITTLE, M.R., PEREIRA, C.A.B.** Unconditional Exact Test of Hardy-Weinberg Law. 1999. 19p.(RT-MAE-9901)

9902 - **MINKOVA, L.** Modelling of financial markets in high level of inflation. 1999. 8p. (RT-MAE-9902)

9903 - **GUERBEROFF, G.R.** Remark on the critical temperature of the hopfield model. 1999. 10p. (RT-MAE-9903)

9904 - **BAKEVA, V., KOLEV, N.** The optimal blocking time of the unreliable Geo/G_D/1 Queuing System. 1999. 12p. (RT-MAE-9904)

9905 - **KOLEV, N., MINKOVA, L.** Quotas on runs of successes and failures in a multi-state markov chain. 1999. 14p. (RT-MAE-9905)

9906 - **KOLEV, N., MINKOVA, L.** Run and frequency quotas in a multi-state Markov chain. 1999. 11p. (RT-MAE-9906)

9907 - **CHIANN, C., MORETTIN, P.A.** Estimation of time varying linear systems. 1999. 39p. (RT-MAE-9907)

9908 - **FONTES, L.R.G., NEVES, E.J., SIDORAVICIUS, V.** Limit velocity for a driven particle in a random medium with mass aggregation. 1999. 14p. (RT-MAE-9908)

9909 - **PEREIRA, C.A.B., STERN, J.M.** Evidence and credibility: Full bayesian significance test for precise hypotheses, 1999. 13p. (RT-MAE-9909)

9910 - **GALEA, M., PAULA, G.A., URIBE-OPAZO, M.** Assessing local influence in univariate elliptical linear regression models, 1999, 16p. (RT-MAE-9910)

9911 - **CARDOSO NETO, J., PAULA, G.A.** Wald one-sided test using generalized estimating equations approach, 1999, 18p. (RT-MAE-9911)

9912 - **CORGINI, M., SANKOVICH, D.P., TANAKA, N.I.** The imperfect boson gas: some mathematical approaches, 1999. 25p. (RT-MAE-9912)

9913 - **BOLFARINE, H., CASTRO, M.** Ancova models with measurement errors, 1999. 13p. (RT-MAE-9913)

9914 - **MACHADO, F.P., MENSHIKOV, M.V., POPOV, S.YU.** Recurrence and transience of multi type branching random walks, 1999. 20p. (RT-MAE-9914)

9915 - **FERNÁNDEZ, R., GALVES, A.** Identifying features in the presence of competing evidence. The case of first-language acquisition, 1999. 11p. (RT-MAE-9915)

9916 - **AUBIN, E.C.Q., CORDEIRO, G.M.** Bartlett-corrected tests for normal linear models when the error covariance matrix is nonscalar. 1999. 22p. (RT-MAE-9916)

9917 - **CRIBARI-NETO, F., FERRARI, S.L.P., CORDEIRO, G.M.** An improved heteroskedasticity-consistent covariance matrix estimator. 1999. 10p. (RT-MAE-9917)

9918 - **MINKOVA, L.** Further results on the family of inflated-parameter power series distributions. 1999. 13p. (RT-MAE-9918)

9919 - **KOLEV, N.** Run and frequency quotas under markovian fashion and their application in risk analysis. 1999. 15p. (RT-MAE-9919)

9920 - **KOLEV, N.** Two characterizations of the geometric distribution related to β-transformation. 1999. 11p. (RT-MAE-9920)

9921 - **KOLEV, N.** Correlated INAR(1) Process. 1999. 12p. (RT-MAE-9921)

The complete list of "Relatórios do Departamento de Estatística", IME-USP, will be sent upon request.

Departamento de Estatística
IME-USP
Caixa Postal 66.281
05315-970 - São Paulo, Brasil