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Distance Tests Under Nonregular Conditions: Applications to the Comparative Calibration Model

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Abstract

In this paper, we present a modification of the general distance test statistics under nonregular conditions with application to an special error-in-variables model. Specifically, we consider the comparative calibration model, a special case of the multivariate regression model. We compare, using artificial data, the power function of the distance test for restricted hypotheses with power functions of other general tests that were developed without the assumptions of errors in variables and restricted hypotheses occurring simultaneously. It is known that, when the Jacobian matrix of the restriction function describing the hypotheses has full rank, the asymptotic null distribution of the distance test statistic is a mixture of chi-square distributions. This assertion is not necessarily true when there exist singular points in the null hypothesis. We suggest a modification to the distance test statistic which ensures convergence to a mixture of chi-square distributions. A real data set is analyzed according to the proposed methods.

Key Words: comparative calibration; distance statistics; restricted hypotheses.

1 Introduction

We start by giving a description of the distance test, according to Kodde and Palm (1986). Consider a statistical model with sample distributions in the family $\{P_{\boldsymbol{\theta}}, \boldsymbol{\theta} \in \Theta\}$, where $\Theta \subset \Re^s$ is an open set. Let $\mathbf{h} = (\mathbf{h}_1', \mathbf{h}_2')' : \Theta \to \Re^p \times \Re^{q-p}$ a continuously differentiable function, where $q \leq s$. Suppose we wish to test the null hypothesis

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$H : \mathrm{h}(\boldsymbol{\theta}) = 0$ against the alternative $K : \mathrm{h}_2(\boldsymbol{\theta}) \geq 0$. The alternative K means that all $q-p$ coordinates of $\mathrm{h}_2(\boldsymbol{\theta})$ are non negative. Denoting convergence in distribution by "$\xrightarrow{D}$", let $\{\hat{\boldsymbol{\theta}}_n\}$ be a sequence of estimators of $\boldsymbol{\theta}$ which satisfies

$$n^{1/2}(\hat{\boldsymbol{\theta}}_n - \boldsymbol{\theta}) \xrightarrow{D} N(0, \boldsymbol{\Omega}(\boldsymbol{\theta})), \tag{1}$$

for all $\boldsymbol{\theta} \in \Theta$, where $\boldsymbol{\Omega}(\boldsymbol{\theta})$ is a positive definite matrix and $\boldsymbol{\Omega}(\cdot)$ is a continuous function defined in Θ. In general, $\hat{\boldsymbol{\theta}}_n$ is the maximum likelihood estimator based on n observations and $\boldsymbol{\Omega}(\boldsymbol{\theta})$ is the inverse of the Fisher information matrix. From (1), it follows that

$$n^{1/2}(\mathrm{h}(\hat{\boldsymbol{\theta}}_n) - \mathrm{h}(\boldsymbol{\theta})) \xrightarrow{D} N(\mathbf{0}, \boldsymbol{\Sigma}(\boldsymbol{\theta})) \tag{2}$$

for all $\boldsymbol{\theta} \in \Theta$, where $\boldsymbol{\Sigma}(\boldsymbol{\theta}) = J(\boldsymbol{\theta})\boldsymbol{\Omega}(\boldsymbol{\theta})J(\boldsymbol{\theta})'$ and $J(\boldsymbol{\theta})$ is the Jacobian matrix of h at $\boldsymbol{\theta}$. We assume that this matrix has full rank for all $\boldsymbol{\theta} \in \Theta$. Thus, $\boldsymbol{\Sigma}(\boldsymbol{\theta})$ has an inverse for all $\boldsymbol{\theta} \in \Theta$. Let $\hat{\boldsymbol{\phi}}_n = n^{1/2}\mathrm{h}(\hat{\boldsymbol{\theta}}_n)$. The *distance test* statistic for H against K is defined as

$$D_n = \hat{\boldsymbol{\phi}}_n'[\boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}}_n)]^{-1}\hat{\boldsymbol{\phi}}_n - \min\{(\hat{\boldsymbol{\phi}}_n - \mathrm{x})'[\boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}}_n)]^{-1}(\hat{\boldsymbol{\phi}}_n - \mathrm{x});\, \mathrm{x}_2 \geq 0\}, \tag{3}$$

where $\mathrm{x} = (\mathrm{x}_1', \mathrm{x}_2')'$, $\mathrm{x}_1 : p \times 1$. The point in $\Re^q$ which yields the minimum above is the *projection of* $\hat{\boldsymbol{\phi}}_n$ *onto* $\{\mathrm{x}; \mathrm{x}_2 \geq 0\}$, *defined in the metric of* $\boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}}_n)$. The relevant theory and algorithms to solve this quadratic program can be found in, for example, Bazaraa and Shetty (1979) and Luenberger (1984). To solve this problem we use the LCP subroutine in the SAS/IML software.

If $\mathrm{h}(\boldsymbol{\theta}) = \mathbf{0}$, then the limit distribution of D_n is given by

$$\lim_{n\to\infty} P_{\boldsymbol{\theta}}(D_n \geq c) = \sum_{k=0}^{q-p} P(\chi^2_{p+k} \geq c)w(q-p, k, \boldsymbol{\Sigma}_{22}(\boldsymbol{\theta})), \tag{4}$$

where $\boldsymbol{\Sigma}_{22}(\boldsymbol{\theta})$ is the $(q-p) \times (q-p)$ lower right block of $\boldsymbol{\Sigma}(\boldsymbol{\theta})$ and $w(q-p, k, \boldsymbol{\Sigma}_{22}(\boldsymbol{\theta}))$ is the probability that the projection of a vector distributed as $N_{q-p}(0, \boldsymbol{\Sigma}_{22}(\boldsymbol{\theta}))$ onto $\{\mathrm{x}; \mathrm{x}_2 \geq 0\}$ (defined in the metric of $\boldsymbol{\Sigma}_{22}(\boldsymbol{\theta})$) has exactly k positive coordinates. In other words, the limit distribution of D_n is , under H, a mixture of chi-square distributions. For the cases in which $q-p \leq 4$, there are closed-form expressions for the weights w. They can be found, for instance, in Wolak (1987). In situations where $q-p > 4$, computational procedures are available, see Bohrer and Chow (1978) and Sun (1988a, 1988b).

Let

$$\psi(c, \boldsymbol{\Sigma}) = \sum_{k=0}^{q-p} P(\chi^2_{p+k} \geq c)w(q-p, k, \boldsymbol{\Sigma}_{22}),$$

where $\boldsymbol{\Sigma} : q \times q$ is positive definite. *The distance test with asymptotic level* ξ is defined as: reject H if and only if $D_n > c_\xi(\hat{\boldsymbol{\theta}}_n)$, where $c_\xi(\hat{\boldsymbol{\theta}}_n)$ is determined by solving the equation

$$\psi(c_\xi(\hat{\boldsymbol{\theta}}_n), \boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}}_n)) = \xi. \tag{5}$$

Suppose that $J(\boldsymbol{\theta})$ has not full rank for some $\boldsymbol{\theta}$ in the null hypothesis (that is, there exists $\boldsymbol{\theta} \in \Theta$ which is a *singular point in* H). In section 3 we present simulation studies showing evidence that the limit distribution of D_n is not a mixture of chi-square distributions as in (4). Clearly, the distance test is not applicable to this case. Gaffke et al. (1999) obtain the null distribution of D_n when $p = q$ (in this case the distance test statistic and the usual Wald statistic are the same) and $J(\boldsymbol{\theta}) = \mathbf{0}$. They also show that, in a particular situation using the normal linear regression model, the test is conservative. To overcome the singularity problem, Lütkepohl and Burda (1997) propose modifications which ensure a chi-square limit null distribution for the Wald statistic. Here, we consider an extension of their idea in order to obtain an modification of the distance test statistic. The limit null distribution of this new test statistic is a mixture of chi-square distributions. We consider an application to the comparative calibration model. The methodology is applied to a real data set.

Section 2 considers the modified distance test statistic for testing restricted hypothesis when the Jacobian matrix of the restriction function does not have full rank for some points in the null hypothesis. It is shown that the asymptotic null distribution of the modified version is a mixture of chisquare distributions. The methodology described in Section 2 is applied to the comparative calibration model in Section 3. Simulation studies are presented which indicates that the usual distance and Wald statistics present problems in the significance level at singular points in the null hypothesis. Furthermore, power comparisons show that the modified version is almost as efficient as the unmodified version at nonsingular points. Comparisons with a modified Wald statistics are also reported showing similar results. An application to a real data set related to the comparisons of three different instruments for comparing adolescent testicular volumes is reported.

2 The Modified Distance Test

The basic idea is to add independent random noise to the vector $\hat{\boldsymbol{\phi}}_n$. Let $\{R(\boldsymbol{\theta}), \boldsymbol{\theta} \in \Theta\}$ be a stochastic process independent of $\{\hat{\boldsymbol{\theta}}_n\}$, chosen such that $R(\boldsymbol{\theta}) \sim N(0, \boldsymbol{\Sigma}_R(\boldsymbol{\theta}))$ and $\boldsymbol{\Sigma}_\gamma(\boldsymbol{\theta}) = \boldsymbol{\Sigma}_R(\boldsymbol{\theta}) + \boldsymbol{\Sigma}(\boldsymbol{\theta})$ is positive definite, for all $\boldsymbol{\theta} \in \Theta$. We assume that the function $\boldsymbol{\Sigma}_\gamma(\cdot)$ is continuous. Let

$$\hat{\boldsymbol{\gamma}}_n = \hat{\boldsymbol{\phi}}_n + R(\hat{\boldsymbol{\theta}}_n). \tag{6}$$

We define the *modified distance test statistic* (D^M) exactly as in (3), but with $\hat{\boldsymbol{\gamma}}_n$ replacing $\hat{\boldsymbol{\phi}}_n$ and $\boldsymbol{\Sigma}_\gamma(\hat{\boldsymbol{\theta}}_n)$ replacing $\boldsymbol{\Sigma}(\hat{\boldsymbol{\theta}}_n)$, that is,

$$D^M = \hat{\boldsymbol{\gamma}}_n' \left[\boldsymbol{\Sigma}_\gamma(\hat{\boldsymbol{\theta}}_n)\right]^{-1} \hat{\boldsymbol{\gamma}}_n - \min\left\{(\hat{\boldsymbol{\gamma}}_n - \mathbf{x})' \left[\boldsymbol{\Sigma}_\gamma(\hat{\boldsymbol{\theta}}_n)\right]^{-1} (\hat{\boldsymbol{\gamma}}_n - \mathbf{x}); \mathbf{x}_2 \geq 0\right\}.$$

For $\boldsymbol{\theta}$ in H we have that the limit distribution of D^M is a mixture of chi-square distributions. The proof of this result is a consequence of the next result.

Lemma 2.1. *Let Φ be a continuous function with domain S, a subset of the Euclidean space, where $\Phi(t)$ is a positive semidefinite matrix for all $t \in S$. Let $\{W_t, t \in S\}$ a stochastic process such that $W_t \sim N(0, \Phi(t))$ for all $t \in S$, and $\{T_n\}$ a sequence of random vectors in S independent of $\{W_t\}$ and such that $T_n \to t_0$ almost surely, $t_0 \in S$. Then,*
(i) W_{T_n} and T_n are independent for all n;
(ii) The limit distribution of $\{W_{T_n}\}$ is $N(0, \Phi(t_0))$.

Proof. (i) E denotes expectations. We have that, for events A and B,

$$P(W_{T_n} \in A, T_n \in B) = E\left[P(W_{T_n} \in A, T_n \in B \mid T_n)\right].$$

Using the fact that W_{t_n} e T_n are independent, we obtain

$$\begin{aligned} P(W_{T_n} \in A, T_n \in B \mid T_n = t_n) &= P(W_{T_n} \in A \mid T_n = t_n) I_{(t_n \in B)} \\ &= P(W_{T_n} \in A) I_{(t_n \in B)}. \end{aligned}$$

Taking expectations, we obtain (i).
(ii) Let $\psi_{W_{T_n}}$ be the characteristic function of W_{T_n}. We must prove that, for each $\mathbf{u}$,

$$\psi_{W_{T_n}}(\mathbf{u}) \to \exp\{-\frac{1}{2}\mathbf{u}'\Phi(t_0)\mathbf{u}\}$$

when $n \to \infty$. We have that

$$\psi_{W_{T_n}}(\mathbf{u}) = E\left\{E\left[\exp(i\mathbf{u}'W_{T_n}) \mid T_n\right]\right\}.$$

Using the independence between W_{T_n} and T_n,

$$\begin{aligned} E\left[\exp(i\mathbf{u}'W_{T_n}) \mid T_n = t_n\right] &= E\left[\exp(i\mathbf{u}'W_{T_n}) \mid T_n = t_n\right] = E\left[\exp(i\mathbf{u}'W_{t_n})\right] \\ &= \exp\{-\frac{1}{2}\mathbf{u}'\Phi(t_n)\mathbf{u}\}, \end{aligned}$$

which implies that

$$E\left[\exp(i\mathbf{u}'W_{T_n}) \mid T_n\right] = \exp\{-\frac{1}{2}\mathbf{u}'\Phi(T_n)\mathbf{u}\}.$$

Thus,

$$E\left[\exp(i\mathbf{u}'W_{T_n}) \mid T_n\right] \longrightarrow \exp\{-\frac{1}{2}\mathbf{u}'\Phi(t_0)\mathbf{u}\}$$

almost surely. Since $\mid \exp\{-\frac{1}{2}\mathbf{u}'\Phi(T_n)\mathbf{u}\} \mid \leq 1$ for all $\mathbf{u}$ and for all n, the assertion follows from the dominated convergence theorem.

Theorem 2.1. *Let $\boldsymbol{\theta} \in H$. Then, the limit distribution of D^M is given in (4), with $\Sigma_\gamma(\boldsymbol{\theta})$ replacing $\Sigma(\boldsymbol{\theta})$.*

Proof: By Lemma 2.1, we have that $\hat{\gamma}_n$ converges in distribution to the $N(\mathbf{0}, \Sigma_\gamma(\boldsymbol{\theta}))$. The remaining of the proof parallels the proof presented in Kodde and Palm (1986) to the case where there are no singularity points in the null hypothesis.

The *modified distance test* is defined exactly in the same way as the distance test, with $\Sigma_\gamma(\hat{\boldsymbol{\theta}}_n)$ replacing $\Sigma(\hat{\boldsymbol{\theta}}_n)$ in the equation (5). We observe that, for an actual implementation of the test, is necessary the determination of the function $\Sigma_R(\cdot)$. This function must be specified considering two main goals: (i) to attain the convergence presented in Theorem 2.1 for all $\boldsymbol{\theta} \in H$ and (ii) to preserve the power properties of the distance test for alternatives far from (singularity) points $\boldsymbol{\theta}$ such that the determinant of $\Sigma(\boldsymbol{\theta})$ is zero. Because the choice of $\Sigma_R(\cdot)$ depends on the particular form of Σ, we must consider each situation of model/hypotheses formulation in its own way.

3 The Comparative Calibration Model

We show now an application of the theory presented in Section 2 to the comparative calibration model, where singularity points appear in some hypotheses of interest in a natural way.

Consider a random sample with n experimental units from some population. Suppose that an unidimensional numerical feature of this population is observed for each unit using m measuring instruments (devices). For some specified unit, each instrument measures the feature only once. It is assumed that there is an instrument that makes unbiased measurements (the exact definition of bias will be given below) and, for this reason, it is referred to as the *standard instrument.* The other instruments can be biased and we must make corrections, that is, calibrate them, in order to overcome this problem.

Let x_i be the true (unobserved) value of the characteristic corresponding to unit i. Moreover, X_i denotes the measurement value obtained by the standard instrument, $\mathbf{Y_i} = (Y_{i1}, \ldots, Y_{ir})'$ denotes the vector with measurements obtained by the remaining instruments ($m = r+1$) and $\mathbf{Z}_i = (X_i, \mathbf{Y}_i')'$ denotes the vector containing the observations corresponding to unit i, $i = 1, \ldots, n$. We assume that the functional relation between $\mathbf{Z}_i$ and x_i is given by

$$(X_i, \mathbf{Y}_i')' = (x_i, \boldsymbol{\alpha}' + \boldsymbol{\beta}' x_i)' + (\delta_i, \mathbf{e}_i')', \ \ i = 1 \ldots, n, \tag{7}$$

where $\boldsymbol{\alpha} : r \times 1$ and $\boldsymbol{\beta} : r \times 1$ are vectors of parameters to be estimated and $\boldsymbol{\epsilon}_i = (\delta_i, \mathbf{e}_i')'$ is the vector of observation errors. We suppose that

$$(x_i, \boldsymbol{\epsilon}_i')', \ \ i = 1, \ldots, n \tag{8}$$

are independent and identically distributed as $N\left[(\mu_x, \mathbf{0}')', \text{block diag}\{\sigma_x^2, \sigma^2\boldsymbol{\Omega}\}\right]$, where $\sigma_x^2 > 0$, $\sigma^2 > 0$, $\boldsymbol{\Omega}$ is a known positive definite matrix and blockdiag$\{\sigma_x^2, \sigma^2\boldsymbol{\Omega}\}$ means that x_i and $\boldsymbol{\epsilon}_i$ are independent with variance/covariance matrix given by σ_x^2 and $\sigma^2\boldsymbol{\Omega}$, respectively. Writing $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_r)'$ and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_r)'$, the coordinates α_j e β_j

are called *additive bias e multiplicative bias*, respectively, associated with the instrument j, $j = 1, \ldots, r$. The model defined in this way is called *comparative calibration model.* This model is a particular case of the multivariate linear regression model with error in variables, as considered in Fuller (1987), Barnett (1969) and Bolfarine and Galea-Rojas (1995). For a description, which includes maximum likelihood estimation, consistency and asymptotic normality of the maximum likelihood estimates and a goodness of fit test, see the Appendix. A detailed proof of the main results are presented in Cabral (2000). We use these results in the presentation that follows.

For $\beta_j \neq 0$, we define a *calibrated measurement in unit i obtained with instrument j* as

$$(Y_{ij} - \alpha_j)/\beta_j = x_i + e_{ij}/\beta_j, \tag{9}$$

$i = 1, \ldots, n$, $j = 1, \ldots, r$. In practice, calibrated measurements are obtained replacing the unknown parameters in the left side of (9) by their estimates. Perfectly calibrated instruments as shown in (9) give unbiased measurements . Thus, a criteria to compare instruments is obtained by comparing the variances of the reciprocal of the errors of the calibrated instruments, $\delta_i, e_{i1}/\beta_1, \ldots, e_{ir}/\beta_r$, that is, the *instruments precisions*, which are given by

$$1/(\sigma^2\Omega_{11}), \beta_1^2/(\sigma^2\Omega_{22}), \ldots, \beta_r^2/(\sigma^2\Omega_{mm}),$$

respectively.

In our study and for illustrative purposes, we consider $r = 2$, $\Omega = I$ and the hypotheses

$$H_\alpha : |\alpha_1| = |\alpha_2|, \quad K_\alpha : |\alpha_1| > |\alpha_2|. \tag{10}$$

H_α means that the instruments 1 and 2 have, in absolute value, the same additive bias. The corresponding hypotheses to the parameter $\boldsymbol{\beta}$ is given by $H_\beta : |\beta_1 - 1| = \beta_2 - 1|$ and $K_\beta : |\beta_1 - 1| > |\beta_2 - 1|$. Extensions for the case $r > 2$ can be done in a similar fashion. Let $\boldsymbol{\theta} = (\mu_x, \boldsymbol{\alpha}', \boldsymbol{\beta}', \sigma_x^2, \sigma^2)'$ be the model parameter vector and $\hat{\boldsymbol{\theta}} = (\hat{\mu}_x, \hat{\boldsymbol{\alpha}}', \hat{\boldsymbol{\beta}}', \hat{\sigma}_x^2, \hat{\sigma}^2)'$ the corresponding maximum likelihood estimator, which are given in Theorem A.1 in the Appendix. Using the notation considered in Section 1, we have that the function describing the hypothesis H_α is given by $h(\boldsymbol{\theta}) = \alpha_1^2 - \alpha_2^2$. We have that, for $\boldsymbol{\theta}$ in H_α,

$$\hat{\phi} = n^{1/2}(\hat{\alpha}_1^2 - \hat{\alpha}_2^2) \xrightarrow{D} N(0, \sigma_\alpha^2(\boldsymbol{\theta})), \tag{11}$$

where

$$\sigma_\alpha^2(\boldsymbol{\theta}) = 4\left(\sigma^2 + \frac{\mu_x^2\sigma^2}{\sigma_x^2} + \frac{\mu_x^2\sigma^4}{\sigma_x^4}\right)\left[(\alpha_1\beta_1 - \alpha_2\beta_2)^2 + \alpha_1^2 + \alpha_2^2\right]. \tag{12}$$

This result follows from Theorem A.2 in the Appendix. As well established, the Wald statistic W is based on the statistics $W = \hat{\phi}^2/\sigma_\alpha^2(\hat{\boldsymbol{\theta}})$. We have that $\sigma_\alpha^2(\boldsymbol{\theta}) > 0$ if and only if $\boldsymbol{\alpha} \neq \mathbf{0}$. The distance test statistic for testing H_α against K_α,

$$D = \frac{\hat{\phi}^2}{\sigma_\alpha^2(\hat{\boldsymbol{\theta}})} I_{(0,\infty)}(\hat{\phi}),$$

has limit distribution given by

$$F(c) = \left(\frac{1}{2} + \frac{1}{2}P(\chi_1^2 \leq c)\right) I_{[0,\infty)}(c),$$

when $\boldsymbol{\theta} \in H_\alpha$ and $\boldsymbol{\alpha} \neq \mathbf{0}$. We will see, in experiments with simulated data, that this is not necessarily true when $\boldsymbol{\alpha} = \mathbf{0}$. The approximation to the F distribution may be poor even if $\boldsymbol{\alpha} \neq \mathbf{0}$. This situation occurs, for instance, when $\boldsymbol{\theta} \in H_\alpha$ and α_1 and α_2 are close to zero.

Let $R(\boldsymbol{\theta}) \sim N(0, \sigma_R^2(\boldsymbol{\theta}))$ be independent of $\hat{\boldsymbol{\theta}}$. Defining

$$\hat{\gamma} = n^{1/2}(\hat{\alpha}_1^2 - \hat{\alpha}_2^2) + R(\hat{\boldsymbol{\theta}}),$$

we have that

$$D^M = \frac{\hat{\gamma}^2}{\sigma_\alpha^2(\hat{\boldsymbol{\theta}}) + \sigma_R^2(\hat{\boldsymbol{\theta}})} I_{(0,\infty)}(\hat{\gamma})$$

is the modified distance test statistic for H_α against K_α. For implementing this test, we suggest the following form to $\sigma_R^2(\boldsymbol{\theta})$:

$$\sigma_R^2(\boldsymbol{\theta}) = (\sigma_*^2 - \sigma_\alpha^2(\boldsymbol{\theta})) I_{(0,\infty)}(\sigma_*^2 - \sigma_\alpha^2(\boldsymbol{\theta})), \tag{13}$$

where $\sigma_*^2 > 0$. The idea is not to consider the correction $R(\hat{\boldsymbol{\theta}})$ for sufficiently large $\sigma_\alpha^2(\hat{\boldsymbol{\theta}})$.

Analogously, we can define the distance statistic for testing

$$H_\beta : |\beta_1 - 1| = |\beta_2 - 1| \quad \text{against} \quad K_\beta : |\beta_1 - 1| > |\beta_2 - 1|.$$

Under H_β, it follows that

$$n^{1/2}[(\hat{\beta}_1 - 1)^2 - (\hat{\beta}_2 - 1)^2] \xrightarrow{D} N(0, \sigma_\beta^2(\boldsymbol{\theta})),$$

where

$$\sigma_\beta^2(\boldsymbol{\theta}) = 4\left(\frac{\sigma^2}{\sigma_x^2} + \frac{\sigma^4}{\sigma_x^4(1+\boldsymbol{\beta}'\boldsymbol{\beta})}\right)\{[\beta_1(\beta_1-1) - \beta_2(\beta_2-1)]^2 + (\beta_1-1)^2 + (\beta_2-1)^2\}. \tag{14}$$

In this case, the modified distance statistic is given by

$$D^M = \frac{\hat{\gamma}^2}{\sigma_\beta^2(\hat{\boldsymbol{\theta}}) + \sigma_R^2(\hat{\boldsymbol{\theta}})} I_{(0,\infty)}(\hat{\gamma}),$$

with $\sigma_R^2(\hat{\boldsymbol{\theta}})$ as defined in (13).

A Simulation Study. Now we present simulation studies[1] to investigate the properties of the testing statistics considered above and also of a modified Wald statistics

[1]For these studies we used the IML/SAS procedure.

Table 1: Rejection rates for the Modified Wald and Modified Distance tests (H_α : $|\alpha_1| = |\alpha_2|$, $K_\alpha : |\alpha_1| > |\alpha_2|$; $\sigma_x^2 = 0.1$, $\sigma^2 = 0.01$, $\sigma_R^2 = 0.3$, $n = 100$, level = 5%)

α_1	Test	
	Mod. Wald	Mod. Distance
0.05	0.0420	0.0475
0.10	0.0470	0.0675
0.15	0.0500	0.0810
0.20	0.0595	0.1070
0.25	0.0925	0.1605
0.30	0.1650	0.2750
0.35	0.2400	0.3760
0.40	0.3425	0.4890

defined next. Each entry in Tables 1, 2 and 3, represents the rejection rate of H_α in 2000 samples of the comparative calibration model, simulated according to the specified parameters, for the Wald test (W), the distance test (D), a modified Wald test (W^M) suggested by Lütkepohl and Burda (1997) and the modified distance test (D^M). The modified Wald test statistic is given by

$$W^M = \frac{\hat{\gamma}^2}{\sigma_\alpha^2(\hat{\boldsymbol{\theta}}) + \sigma_R^2(\hat{\boldsymbol{\theta}})}.$$

Using the proof of the Theorem 2.1, we can show that, under H_α, the distribution of this statistic is chi-square with one degree of freedom. We took always $\boldsymbol{\Omega} = I$, $\alpha_2 = 0.05$, $\mu_x = 2$, $\beta_1 = 1.2$ and $\beta_2 = 1$. We used $\sigma_R^2 = 0.3$ for all $\boldsymbol{\theta}$ to construct the Table 1, and (13) to the others. These tables seem to indicate that, even for large samples ($n = 500$), the Wald test and the distance test statistics do not have an approximate chi-square distribution with one degree of freedom and the distribution F given above, respectively, when $\boldsymbol{\theta} \in H_\alpha$ and α_1 and α_2 are small. We can observe that the rejection rates under H_α are closer to the nominal levels (5% and 10%) when using the modified tests. Using (13) we note that, for large values of α_1, there is practically no power loss when we compare the modified distance test with the distance test. This assertion can be verified observing Tables 2 and 3. Nevertheless, it is clear that there are losses when we take σ_R^2 being the constant 0.3, as can be seen by comparing results in Table 1 with the third and fourth columns (corresponding to the 5% level) in Table 2.

We also present, in Table 4, a comparison between rejection rates of the Wald and Distance tests for the hypotheses

$$H : \beta_1^2 = \beta_2^2 = 1 \text{ against } K : 1 \geq \beta_1^2 \geq \beta_2^2. \tag{14}$$

Since $\boldsymbol{\Omega} = I$, if H is true, then all the instruments have the same precision. The hypothesis K establishes an order of precisions of the instruments, the instrument 1 is at least as precise as instrument 2 and is less or as precise as the standard

Table 2: Rejection rates for the Wald , Distance, Modified Wald and Modified Distance tests ($H_\alpha : | \alpha_1 |=| \alpha_2 |$, $K_\alpha : | \alpha_1 |>| \alpha_2 |$; $\sigma_x^2 = 0.1$, $\sigma^2 = 0.01$, $n = 100$)

Level	α_1	Wald	Distance		Mod. Wald			Mod. Dist.		
				σ_*^2	0.1	0.2	0.3	0.1	0.2	0.3
5%	0.05	0.0005	0.0020		0.0210	0.0320	0.0390	0.0205	0.0315	0.0375
	0.10	0.0010	0.0050		0.0135	0.0260	0.0395	0.0225	0.0445	0.0530
	0.15	0.0115	0.0325		0.0200	0.0380	0.0440	0.0500	0.0725	0.0795
	0.20	0.0315	0.1030		0.0375	0.0495	0.0625	0.1105	0.1325	0.1405
	0.25	0.1040	0.2180		0.1060	0.1100	0.1205	0.2205	0.2380	0.2460
	0.30	0.1900	0.3570		0.1910	0.1930	0.2015	0.3580	0.3615	0.3710
	0.35	0.3575	0.5295		0.3575	0.3580	0.3625	0.5295	0.5315	0.5355
	0.40	0.5050	0.6990		0.5050	0.5060	0.5080	0.6990	0.6995	0.6990
10%	0.05	0.0040	0.0125		0.0505	0.0760	0.0870	0.0685	0.0910	0.0950
	0.10	0.0110	0.0430		0.0485	0.0725	0.0875	0.0905	0.1170	0.1215
	0.15	0.0370	0.1255		0.0605	0.0995	0.1155	0.1605	0.1970	0.1910
	0.20	0.0930	0.2305		0.1085	0.1375	0.1550	0.2500	0.2885	0.2785
	0.25	0.2030	0.4050		0.2065	0.2280	0.2505	0.4135	0.4320	0.4100
	0.30	0.3445	0.5665		0.3470	0.3540	0.3615	0.5705	0.5780	0.5530
	0.35	0.5410	0.7275		0.5415	0.5445	0.5515	0.7285	0.7305	0.7260
	0.40	0.6920	0.8375		0.6920	0.6920	0.6915	0.8375	0.8400	0.8345

Table 3: Rejection rates for the Wald , Distance, Modified Wald and Modified Distance ($H_\alpha : | \alpha_1 |=| \alpha_2 |$, $K_\alpha : | \alpha_1 |>| \alpha_2 |$; $\sigma_x^2 = 0.1$, $\sigma^2 = 0.01$, $n = 500$)

Level	α_1	Wald	Distance		Mod. Wald			Mod. Dist.		
				σ_*^2	0.1	0.2	0.3	0.1	0.2	0.3
5%	0.05	0.0025	0.0035		0.0460	0.0495	0.0515	0.0535	0.0535	0.0535
	0.10	0.0255	0.0945		0.0790	0.0720	0.0690	0.1400	0.1085	0.0990
	0.15	0.2490	0.4425		0.3010	0.1760	0.1315	0.4440	0.2850	0.2150
	0.20	0.6665	0.8365		0.6795	0.4910	0.3455	0.8190	0.6115	0.4690
	0.25	0.9030	0.9595		0.9035	0.8300	0.6890	0.9590	0.8880	0.7835
	0.30	0.9835	0.9960		0.9835	0.9780	0.9340	0.9960	0.9925	0.9640
	0.35	0.9975	1.0000		0.9975	0.9970	0.9915	1.0000	0.9995	0.9985
	0.40	0.9995	1.0000		0.9995	0.9995	0.9995	1.0000	1.0000	1.0000
10%	0.05	0.0070	0.0165		0.0935	0.0960	0.1005	0.1005	0.1005	0.0990
	0.10	0.0795	0.2255		0.1480	0.1245	0.1165	0.2330	0.1880	0.1660
	0.15	0.4540	0.6775		0.4480	0.2765	0.2105	0.5900	0.4010	0.3275
	0.20	0.8230	0.9340		0.8065	0.6135	0.4760	0.8960	0.7300	0.6175
	0.25	0.9675	0.9915		0.9670	0.9135	0.8055	0.9885	0.9545	0.8905
	0.30	0.9955	0.9985		0.9955	0.9860	0.9645	0.9985	0.9955	0.9800
	0.35	0.9990	1.0000		0.9990	0.9980	0.9970	1.0000	0.9995	0.9990
	0.40	1.0000	1.0000		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 4: Rejection rates for the Wald and Distance tests ($H : \beta_1^2 = \beta_2^2 = 1, \;\; K : 1 \geq \beta_1^2 \geq \beta_2^2$)

n	Level	β_2	Wald	Distance	n	Level	β_2	Wald	Distance
100	5%	1.00	0.0575	0.0645	1000	5%	1.00	0.0525	0.0530
		0.99	0.0615	0.1015			0.99	0.1045	0.1780
		0.98	0.0905	0.1480			0.98	0.3090	0.4435
		0.97	0.1220	0.1945			0.97	0.5705	0.7270
		0.96	0.1775	0.2780			0.96	0.8420	0.9195
		0.95	0.2305	0.3365			0.95	0.9675	0.9850
		0.94	0.3210	0.4575			0.94	0.9950	0.9975
		0.93	0.4045	0.5455			0.93	0.9995	1.0000
	10%	1.00	0.0990	0.1140		10%	1.00	0.1010	0.1130
		0.99	0.1170	0.1725			0.99	0.1880	0.2995
		0.98	0.1410	0.2205			0.98	0.4085	0.5735
		0.97	0.1910	0.2990			0.97	0.7050	0.8215
		0.96	0.2665	0.3940			0.96	0.8850	0.9490
		0.95	0.3370	0.4720			0.95	0.9775	0.9935
		0.94	0.4035	0.5660			0.94	0.9985	1.0000
		0.93	0.5055	0.6485			0.93	0.9995	1.0000

instrument. There may exist problems concerning to the approximation to the null limit distribution in a neighborhood of some $\boldsymbol{\theta}$ with $\beta_1 = 0$ or $\beta_2 = 0$, but we believe that small precisions like these are not common in practice. The function describing the hypotheses is given by $\mathbf{h}(\boldsymbol{\theta}) = (1 - \beta_1^2, \beta_1^2 - \beta_2^2)'$, and the variance/covariance matrix of the sequence $\{n^{1/2}(\hat{\beta}_1 - \beta_1, \hat{\beta}_2 - \beta_2)'\}$ follows from Theorem A.2 in the Appendix. These elements are sufficient to construct the distance test statistic. We took $\beta_1 = 1$, $\alpha_1 = 0.4$, $\alpha_2 = 0.01$, $\mu_x = 2$, $\sigma_x^2 = 0.1$, $\sigma^2 = 0.01$.

4 An Application

It is well accepted that measurement of the testis is a more readily available method of estimating spermatogenesis. Doubt remains about the best method for measuring testicular volume. Table 5 presents measurements of the left testicular volumes of 40 adolescents which were previously considered in Chopkevitch et al. (1996). The measurements were made using five different methods but, in this example, we consider only three methods: Ultrasound (assumed to be the standard or reference "instrument"), a graphical method suggested by the authors ("instrument" 1) and Ring Orchidometer ("instrument" 2).

The following analysis is made using the comparative calibration model with $\Omega = I$, after taking a cubic root transformation of the volume, in order to better approach normality. We performed the goodness-of-fit test for the model using the statistics L_R reported in the Appendix, obtaining p-value 0.0721. The estimates for the parameters and for the asymptotic variances of some parameters of interest are shown in Table 6.

First, we perform a test for the hypotheses in (14), since it is natural suppose that ultrasound is more precise than the other methods. Performing the distance test we obtain the p-value 0.0283, indicating the rejection of H. However, the p-value obtained when performing the Wald test is 0.0911, and we do not reject H at the 5% level.

Concerning the methods 1 and 2, the estimates in Table 6 indicate that the additive bias for method 2 is, in absolute value, larger than the additive bias for method 1. The estimates for the multiplicative bias are close to each other. Consider testing simultaneously the hypotheses in (10) and $H : \beta_1 = \beta_2$. Now we perform a sequential test for them, using D^M for H_α and the Wald test for H. An adequate procedure is suggested in Holm (1979). Its application is extremely simple and, for the two hypotheses case, can be described as follows: first, consider the ordered p-values obtained by performing each test, say, $p^{(1)} \leq p^{(2)}$. Denote the respective hypotheses as $H^{(1)}$ and $H^{(2)}$. Let $\xi \in (0,1)$. Consider the following steps
Step 1: If $p^{(1)} > \xi/2$ then accept $H^{(1)}$ and $H^{(2)}$ and stop; otherwise, reject $H^{(1)}$ and continue; Step 2: If $p^{(2)} > \xi$ then accept $H^{(2)}$; otherwise, reject $H^{(2)}$.
The procedure guarantees a multiple significance level ξ.

We obtain $\sigma^2_\alpha(\hat{\boldsymbol{\theta}}) = 0.2506$ and p-values 0.0268, for the modified distance test (using $\sigma^2_* = 0.1$), and 0.3650 for the Wald test. If we use a multiple significance level of at least 6%, we decide to reject H_α and accept H, indicating that, although the methods are equally precise, the graphical method is less biased than the Ring Orchidometer. For the sake of comparison, we performed a Wald test for the intersection of H_α and H (the hypothesis $\mid \alpha_1 \mid = \mid \alpha_2 \mid, \beta_1 = \beta_2$), that is, equal biases, obtaining the p-value 0.1547, indicating its acceptance.

Appendix

Consider the measurement error model defined in (7)-(8), so that

$$\mathbf{Z}_i \sim N(\boldsymbol{\mu}(\boldsymbol{\theta}), \boldsymbol{\Sigma}(\boldsymbol{\theta})), \tag{A.1}$$

where

$$\boldsymbol{\mu}(\boldsymbol{\theta}) = \begin{pmatrix} \mu_x \\ \boldsymbol{\alpha} + \boldsymbol{\beta}\mu_x \end{pmatrix},$$

$$\boldsymbol{\Sigma}(\boldsymbol{\theta}) = \boldsymbol{\Sigma}_z(\boldsymbol{\theta}) + \sigma^2\boldsymbol{\Omega},$$

with

$$\boldsymbol{\Sigma}_z(\boldsymbol{\theta}) = \sigma^2_x \begin{pmatrix} 1 & \boldsymbol{\beta}' \\ \boldsymbol{\beta} & \boldsymbol{\beta}\boldsymbol{\beta}' \end{pmatrix}$$

and $\boldsymbol{\Omega}$ is a known positive definite covariance matrix. Thus, the unknown parameter vector is given by $\boldsymbol{\theta} = (\mu_x, \boldsymbol{\alpha}', \boldsymbol{\beta}', \sigma^2_x, \sigma^2)'$, with $\sigma^2_x > 0$ and $\sigma^2 > 0$. Let

$$\mathbf{S}_Z = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{Z}_i - \bar{\mathbf{Z}})(\mathbf{Z}_i - \bar{\mathbf{Z}})',$$

Table 5: Testicular Volume in Adolescents (ml)

Adolescent	Measurement Method		
Number	Ultrasound (standard)	Graphical (1)	Ring Orchidometer (2)
1	4.8	5.0	8.0
2	5.1	5.0	8.0
3	10.3	7.5	12.0
4	2.4	3.5	4.5
5	4.4	5.0	6.0
6	5.7	5.0	8.0
7	4.3	3.5	6.0
8	5.9	5.0	8.0
9	8.0	5.0	10.0
10	9.5	10.0	11.0
11	16.3	20.0	15.0
12	7.0	5.0	9.0
13	16.4	15.0	20.0
14	2.3	2.0	3.0
15	15.3	15.0	20.0
16	22.8	12.5	20.0
17	9.7	5.0	11.0
18	3.7	3.5	6.0
19	12.3	10.0	12.0
20	9.5	7.5	10.0
21	9.2	10.0	15.0
22	21.7	10.0	25.0
23	3.6	2.0	6.0
24	3.3	2.0	4.0
25	9.6	7.5	10.0
26	8.0	10.0	12.0
27	6.5	5.0	10.0
28	6.1	5.0	7.0
29	5.5	5.0	10.0
30	17.4	15.0	22.5
31	20.7	20.0	25.0
32	13.8	10.0	15.0
33	9.9	10.0	15.0
34	6.8	7.5	10.0
35	10.6	10.0	15.0
36	9.5	10.0	11.0
37	20.7	20.0	22.5
38	8.1	5.0	12.0
39	8.4	7.5	11.0
40	2.8	3.5	6.0

Table 6: Estimates for the parameters and for asymptotic variances of some parameters of interest, testicular volume data

Parameter	Estimate	Variance estimate
α_1	0.0123	0.0110
α_2	0.3909	0.0105
β_1	0.9448	0.0026
β_2	0.8976	0.0024
μ_x	2.0352	
σ_x^2	0.1591	
σ^2	0.0085	

be the sample variance/covariance matrix. There exist matrices $\mathbf{\Delta} = \text{diag}\{\lambda_1, \ldots, \lambda_m\}$ and $\mathbf{T} = [\mathbf{T}_1|\ldots|\mathbf{T}_m]$ such that

$$\begin{aligned} \mathbf{T}'\mathbf{S}_Z\mathbf{T} &= \mathbf{\Delta} \\ \mathbf{T}'\Omega\mathbf{T} &= \mathbf{I}, \end{aligned}$$

where λ_i, $i = 1, \ldots, m$, are known as the *eigenvalues of* $\mathbf{S}_Z$ *in the metric of* Ω and $\mathbf{T}_i$, $i = 1, \ldots, m$, are the corresponding eigenvectors.

Theorems A.1 and A.2 present the maximum likelihood estimators and their asymptotic behavior under the model specified in (A.1).

Theorem A.1. *Consider the regression model with measurement errors defined in (A.1). The (consistent) maximum likelihood estimators of the elements of* $\boldsymbol{\theta}$ *are given by*

$$\hat{\mu}_x = \bar{X},$$

$$\hat{\sigma}^2 = \frac{1}{r}\sum_{i=2}^{m}\hat{\lambda}_i,$$

$$\hat{\mathbf{\Sigma}}_z = \begin{pmatrix} \hat{\sigma}_x^2 & \hat{\mathbf{\Sigma}}_{xy} \\ \hat{\mathbf{\Sigma}}_{yx} & \hat{\mathbf{\Sigma}}_y \end{pmatrix} = (\hat{\lambda}_1 - \hat{\sigma}^2)\mathbf{\Omega}\mathbf{T}_1\mathbf{T}_1'\mathbf{\Omega},$$

$$\hat{\boldsymbol{\beta}} = \frac{1}{\hat{\sigma}_x^2}\hat{\mathbf{\Sigma}}_{yx}$$

and

$$\hat{\boldsymbol{\alpha}} = \bar{\mathbf{Y}} - \hat{\boldsymbol{\beta}}\bar{X}.$$

Theorem A.2. *Consider the maximum likelihood estimators given in Theorem A.1. Then, as* $n \to \infty$,

$$\begin{aligned} n^{1/2}(\hat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}) &\xrightarrow{D} N(\mathbf{0}, \mathbf{\Sigma}_\alpha), \\ n^{1/2}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) &\xrightarrow{D} N(\mathbf{0}, \mathbf{\Sigma}_\beta), \end{aligned}$$

where

$$\mathbf{\Sigma}_\alpha = \mathbf{\Sigma}_\nu\left(1 + \frac{\mu_x^2}{\sigma_x^2} + \frac{\mu_x^2\Sigma_\rho}{\sigma_x^4}\right),$$

and

$$\mathbf{\Sigma}_\beta = \frac{1}{\sigma_x^2}\mathbf{\Sigma}_\nu + \frac{1}{\sigma_x^4}\Sigma_\rho\mathbf{\Sigma}_\nu,$$

with

$$\Sigma_\rho = [\frac{1}{\sigma^2}(1, \boldsymbol{\beta}')\mathbf{\Omega}^{-1}(1, \boldsymbol{\beta}')']^{-1} \quad \text{and} \quad \mathbf{\Sigma}_\nu = \sigma^2(-\boldsymbol{\beta}, \mathbf{I}_r)\mathbf{\Omega}(-\boldsymbol{\beta}, \mathbf{I}_r)'.$$

Consider now testing the hypotheses $H : (\boldsymbol{\mu}(\boldsymbol{\theta}), \mathbf{\Sigma}(\boldsymbol{\theta}))$, with $\boldsymbol{\mu}(\boldsymbol{\theta})$ and $\mathbf{\Sigma}(\boldsymbol{\theta})$ in (A.1) versus $K : (\boldsymbol{\mu}, \mathbf{\Sigma}) \in \Re^p \times \mathcal{M}_p$ where $\mathcal{M}_p$ denotes the space of all p-dimensional symmetric matrices. The likelihood ratio statistics for testing H versus K is given by

$$L_R = -n \sum_{i=2}^{m} \log \hat{\lambda}_i + nr \log \hat{\sigma}^2.$$

For large n, L_R is distributed approximately according to the chisquare distribution with $r(r+1)/2 - 1$ degrees of freedom and H is rejected for large values of L_R. Detailed proofs of the above results can be found in Cabral (2000).

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