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Predicting the existence of a 2.9 GeV $Df_0(980)$ molecular state

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A D-like meson resonance with mass around 2.9 GeV has been found in the $DK\bar{K}$ system using two independent and different model calculations based on (1) QCD sum rules and (2) solution of Fadeev equations with input interactions obtained from effective field theories built by considering both chiral and heavy quark symmetries. The QCD sum rules have been used to study the $D_{s^*0}(2317)\bar{K}$ and $Df_0(980)$ molecular currents. A resonance of mass 2.926 GeV is found with the $Df_0(980)$ current. Although a state in the $D_{s^*0}(2317)\bar{K}$ current is also obtained, with mass around 2.9 GeV, the coupling of this state is found to be two times weaker than the one formed in $Df_0(980)$. On the other hand, few-body equations are solved for the $DK\bar{K}$ system and its coupled channels with the input t matrices obtained by solving Bethe-Salpeter equations for the DK, $D\bar{K}$, and $K\bar{K}$ subsystems. In this study a D-like meson with mass 2.890 GeV and full width $\sim$55 MeV is found to get dynamically generated when $DK\bar{K}$ gets reorganized as $Df_0(980)$. However, no clear signal appears for the $D_{s^*0}(2317)\bar{K}$ configuration. The striking similarity between the results obtained in the two different models strongly indicates the existence of a $Df_0(980)$ molecule with mass nearly 2.9 GeV.

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I. INTRODUCTION

In the past years, the development of high-energy facilities has led to the discovery of a number of open and hidden charm resonances by collaborations like *BABAR*, Belle, and BES [1–5], which, in turn, has motivated many theoretical studies to understand the properties and nature of heavy-flavor hadrons. Some of the heavily discussed states are $D_{s^*0}(2317)$, $X(3872)$, and $Z^+(4430)$, whose properties have been studied within different models, assuming different configurations like diquarks, tetraquarks, hybrids, hadron molecules, etc. (for a review, see Refs. [6–9]).

The understanding of the nature of the different mesons and baryons of the hadron spectra, in general, is a long-standing puzzle in theoretical nuclear physics. QCD is the accepted fundamental theory describing the strong interactions in terms of the quarks and gluons which constitute the hadronic matter. However, while at high energies the theory becomes perturbative and has been successfully tested by the experiment, the situation is very different at low energies, where due to the confinement of the quarks, the theory is no longer perturbative, and nonperturbative methods are needed to extract information about the properties of the hadrons.

To face this challenging issue, different techniques have been developed. One of them is lattice QCD, which in the last few years has emerged as an important tool to extract information about hadronic observables, like mass, phase shifts, etc. However, due to the large number of degrees of freedom present in QCD (quarks and gluons of different flavors and colors), numerical calculations involving large numbers of lattice points and small lattice spacing are very time consuming for natural values of the mass of the quarks. Although a lot of progress has been done in this area, there are still some problems when addressing excited states that have decay channels [10–14].

Another alternative for studying hadrons within the spirit of QCD is the method of QCD sum rules (QCDSR) (see Refs. [15–18] for a pedagogical information on this topic). In this formalism the hadrons are described in terms of their interpolating quark currents, with which a correlation function is built. One begins evaluating this correlation function at short distances, where the quark-gluon dynamics is essentially perturbative, and then nonperturbative corrections are added to it. This method has been widely used to understand the mass, coupling, decay width, etc., of many hadron states.

Yet another way to elucidate the nature and properties of mesons and baryons is based on the use of effective field theories built by taking into account unitarity, chiral symmetry, and its spontaneous breaking. In this case, the hadrons are the degrees of freedom of the theory instead of the quarks which constitute them. In the last 20 years, there has been lot of activity in this field, and many resonances have been found to have important meson-meson or meson-baryon components in their wave functions. Some of the states most widely discussed are the $\Lambda(1405)$, generated as a consequence of the interaction of the coupled channel system $\bar{K}N$ and $\pi\Sigma$ [19–23], and the $f_0(980)$ resonance, formed in the $K\bar{K}$ and $\pi\pi$ system [24–26]. Recently, this theory has been generalized to study the properties of hadronic systems in a finite volume, and its value in the determination of related physical observables, using the

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energy levels obtained in the finite volume, and thus as a prospective tool for lattice QCD calculations has been shown [27–31].

The above-mentioned methods are in continuous development since the experimental access to higher and higher energies is becoming plausible and, consequently, more and more new states with heavy quarks are being found. The present time is thus ideal to study heavy hadron physics since model predictions can be immediately tested, which eventually helps in understanding the structure of hadrons. With this idea we present a study of the $DK\bar{K}$ system, which we find particularly interesting since the DK and $K\bar{K}$ interactions are attractive in nature. In this manuscript, we have studied this system using two methods: QCDSR and few-body equations. In the former case, we investigate the $Df_0(980)$ and $D_{s^*0}(2317)\bar{K}$ configurations, while in the latter we solve the Faddeev equations for the three-hadron system, where $f_0(980)$ and $D_{s^*0}(2317)$ are dynamically generated in the corresponding subsystems. As we shall see, we find a resonance with similar characteristics in both models.

In the following, we first discuss the calculations based on QCD sum rules and the results found in it. Subsequently, we tackle with the formalism to solve the Faddeev equations and discuss the results obtained with it. Finally, we draw some conclusions.

II. QCD SUM RULES

We start our study based on the QCDSR by writing the interpolating molecular currents for the Df_0 and $D_{s^*0}\bar{K}$ systems as

$$j^{Df_0} = i(\bar{q}_a\gamma_5 c_a)(\bar{s}_b s_b) \tag{1}$$

$$j^{D_{s^*0}\bar{K}} = i(\bar{s}_a c_a)(\bar{q}_b\gamma_5 s_b), \tag{2}$$

where a and b are color indices, and q represents a light quark (u or d). Using these currents, we write the two-point correlation function

$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle 0 \mid T[j(x)j^\dagger(0)] \mid 0\rangle, \tag{3}$$

which can be written in terms of the quark propagators by contracting all the quark antiquark pairs (for more details see, for example, Ref. [6]).

This function is of a dual nature: it represents a quark-antiquark fluctuation at short distances (or large negative q^2) and can be treated in perturbative QCD, while at large distances it can be related to hadronic observables. The sum rule calculations are based on the assumption that in some range of q^2 both descriptions are equivalent. One, thus, proceeds by calculating Eq. (3) for both cases and by eventually equating them to obtain information on the properties of the hadrons.

From the QCD side, for large momentum transfers, Eq. (3) can be calculated, in the first approximation, by assuming the involved propagators as those of free quarks. However, since we are finally interested in studying the properties of hadrons, the relevant energies are lower, where the distance between the quarks gets longer and quark-gluon interactions, quark-antiquark pair creation becomes important. It is, thus, required to include the effect of the presence of the gluons and quarks in the QCD vacuum. For practical calculations, then, one resorts to the Wilson operator product expansion (OPE) method, where the correlation function is expanded in a series of local operators

$$\Pi^{\mathrm{OPE}} = \sum_n C_n(Q^2)O_n. \tag{4}$$

In Eq. (4) the set $\{O^n\}$ contains all local gauge-invariant operators expressible in terms of the gluon fields and the fields of light quarks. The coefficients $C_n(Q^2)(Q^2 = -q^2)$, by construction, include only the short-distance domain and can, therefore, be evaluated perturbatively. Nonperturbative long-distance effects are contained only in the local operators.

In the expansion of Eq. (4), the operators are ordered according to their dimension n, where $n = 0$ corresponds to the unit operator, i.e., perturbative contribution, and the rest of the operators are related to the QCD vacuum fields in terms of condensates. For normal quark-antiquark states, the contributions of condensates with dimensions higher than four are suppressed by large powers of $\Lambda^2_{\mathrm{QCD}}/Q^2$, with $1/\Lambda_{\mathrm{QCD}}$ being the typical long-distance scale. However, for molecular states, condensates with higher dimensions can play an important role. This is taken into account by writing Eq. (4) in terms of the spectral density using the dispersion relation

$$\Pi^{\mathrm{OPE}}(q^2) = \int_{m_c^2}^{\infty} ds \frac{\rho^{\mathrm{OPE}}(s)}{s - q^2} + \text{Subtraction terms}. \tag{5}$$

We work at leading order in α_s, and we consider condensates up to dimension seven, as shown in Fig. 1.

Therefore, ρ^{OPE} can be written as

$$\rho^{\mathrm{OPE}}(q^2) = \rho^{\mathrm{pert}} + \rho^{m_s} + \rho^{\langle\bar{q}q\rangle} + \rho^{\langle g^2G^2\rangle} + \rho^{m_s\langle\bar{q}q\rangle} + \rho^{\langle\bar{q}g\sigma\cdot Gq\rangle} + \rho^{m_s\langle\bar{q}g\sigma\cdot Gq\rangle} + \rho^{\langle\bar{q}q\rangle^2} + \rho^{m_s\langle\bar{q}q\rangle^2}, \tag{6}$$

where m_s represents the mass of the strange quark. The spectral density ρ^{OPE} is related to the imaginary part of the correlation function as $\pi\rho^{\mathrm{OPE}}(s) = \mathrm{Im}[\Pi^{\mathrm{OPE}}(s)]$.

To calculate the different terms in Eq. (6), for the $D_{s^*0}\bar{K}$ and Df_0 currents, we use the momentum-space expression for the heavy quark propagator and the coordinate-space expression for the light quark propagator. The Schwinger parameters are used to evaluate the heavy quark part of the correlator and to perform the d^4x integration in Eq. (3). Finally, we get integrals in the Schwinger parameters. The result of these integrals are given in terms of logarithmic functions, from where we extract the spectral densities and the limits of the integration.

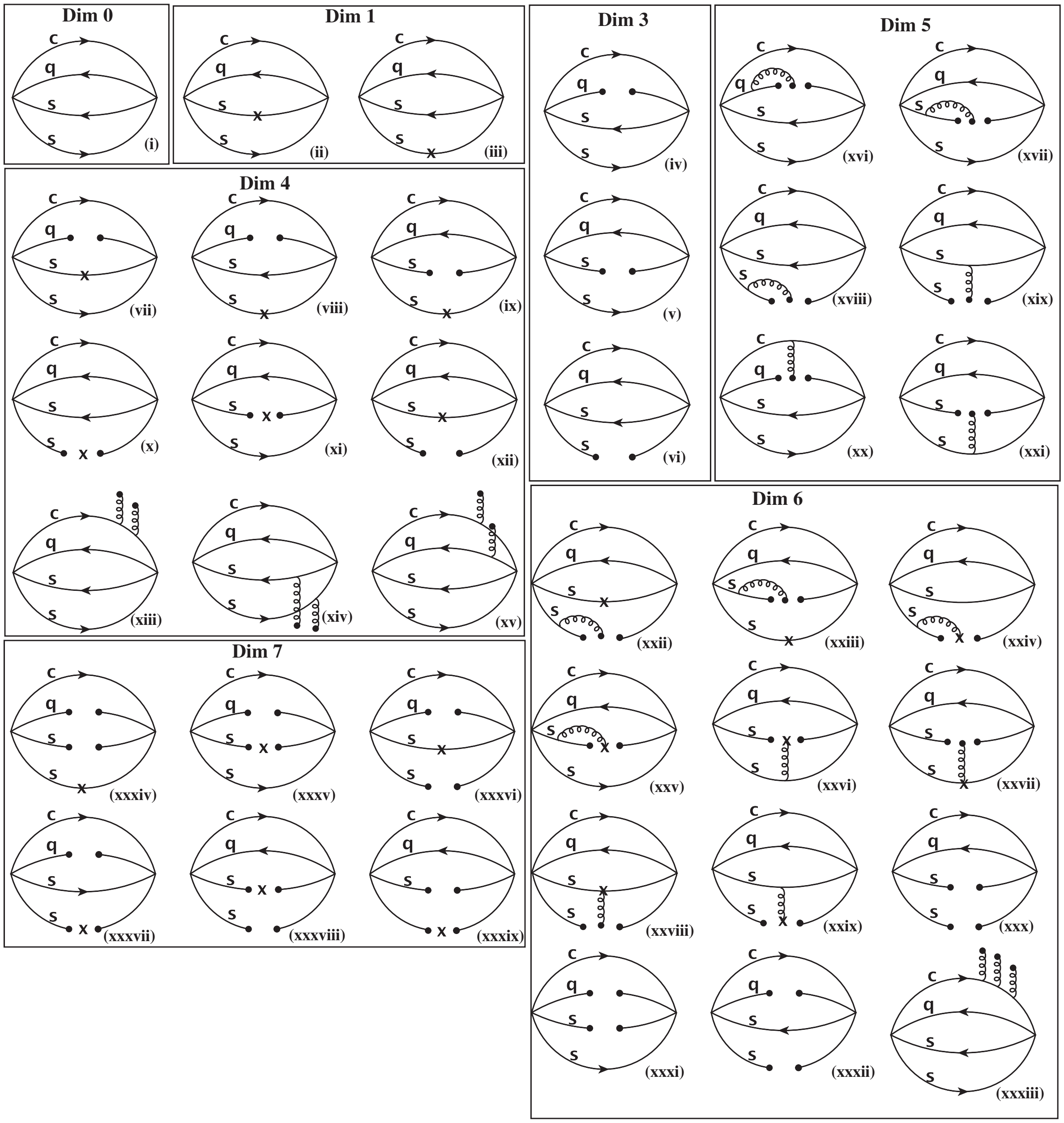

FIG. 1. Diagrams that contribute to the OPE side of the sum rule.

Carrying out the calculations for the different diagrams shown in Fig. 1 leads us to the following expressions, where m_c is the mass of the charm quark.

(1) The perturbative or dimension 0 contribution is found to be

$$\rho^{\rm pert}_{D_{s^*0}\bar{K}}(q^2) = \rho^{\rm pert}_{Df_0}(q^2) = -\int_0^{\alpha_{\rm max}} d\alpha \frac{((\alpha-1)q^2+m_c^2)^4\alpha^3}{2^{12}\pi^6(\alpha-1)^3}. \quad (7)$$

(2) For the terms of dimension one, which are proportional to m_s, we get

$$\rho^{m_s}_{D_{s^*0}\bar{K}}(q^2) = -\int_0^{\alpha_{\rm max}} d\alpha \frac{m_c((\alpha-1)q^2+m_c^2)^3\alpha^3 m_s}{2^{10}\pi^6(\alpha-1)^3},$$
$$\rho^{m_s}_{Df_0}(q^2) = 0. \quad (8)$$

(3) The calculation of the diagrams with one quark condensate gives

$$\rho^{\langle\bar{q}q\rangle}_{D_{s^*0}\bar{K}}(q^2) = \int_0^{\alpha_{\rm max}} d\alpha \frac{3m_c((\alpha-1)q^2+m_c^2)^2\alpha^2\langle\bar{s}s\rangle}{2^8\pi^4(\alpha-1)^2},$$
$$\rho^{\langle\bar{q}q\rangle}_{Df_0}(q^2) = -\int_0^{\alpha_{\rm max}} d\alpha \frac{3m_c((\alpha-1)q^2+m_c^2)^2\alpha^2\langle\bar{q}q\rangle}{2^8\pi^4(\alpha-1)^2}. \quad (9)$$

(4) Both $\rho^{m_s\langle\bar{q}q\rangle}$ and $\rho^{\langle g^2G^2\rangle}$ contribute to dimension four and the expressions for the corresponding spectral densities are

$$\rho^{m_s\langle\bar{q}q\rangle}_{D_{s^*0}\bar{K}}(q^2) = \int_0^{\alpha_{\max}} d\alpha \frac{3((\alpha-1)q^2+m_c^2)^2\alpha m_s}{2^7\pi^4(\alpha-1)}[\langle\bar{q}q\rangle - \langle\bar{s}s\rangle],$$
$$\rho^{m_s\langle\bar{q}q\rangle}_{Df_0}(q^2) = -\int_0^{\alpha_{\max}} d\alpha \frac{9((\alpha-1)q^2+m_c^2)^2\alpha\langle\bar{s}s\rangle m_s}{2^7\pi^4(\alpha-1)}, \tag{10}$$

$$\rho^{\langle g^2G^2\rangle}_{D_{s^*0}\bar{K}}(q^2) = \rho^{\langle g^2G^2\rangle}_{Df_0}(q^2) = \int_0^{\alpha_{\max}} d\alpha \frac{3\langle g^2G^2\rangle}{2^{12}\pi^6}\left(\frac{(2-\alpha)(m_c^2+(\alpha-1)q^2)}{2(1-\alpha)} + \frac{m_c^2\alpha^2}{9(1-\alpha)^2}\right)\frac{\alpha(m_c^2+(\alpha-1)q^2)}{1-\alpha}. \tag{11}$$

(5) Considering the mixed condensates, we get

$$\rho^{\langle\bar{q}g\sigma\cdot Gq\rangle}_{D_{s^*0}\bar{K}}(q^2) = \int_0^{\alpha_{\max}} d\alpha \frac{3m_c\langle\bar{s}g\sigma\cdot Gs\rangle\alpha(1-2\alpha)(m_c^2+(\alpha-1)q^2)}{2^8\pi^4(\alpha-1)^2},$$
$$\rho^{\langle\bar{q}g\sigma\cdot Gq\rangle}_{Df_0}(q^2) = -\int_0^{\alpha_{\max}} d\alpha \frac{3m_c\langle\bar{q}g\sigma\cdot Gq\rangle\alpha(1-2\alpha)(m_c^2+(\alpha-1)q^2)}{2^8\pi^4(\alpha-1)^2}. \tag{12}$$

(6) Going to the dimension six operator, we get the following contributions for the terms proportional to $m_s\langle\bar{q}g\sigma\cdot Gq\rangle$:

$$\rho^{m_s\langle\bar{q}g\sigma\cdot Gq\rangle}_{D_{s^*0}\bar{K}}(q^2) = \int_0^{\alpha_{\max}} d\alpha \frac{m_s}{2^8\pi^4}[m_c^2 - q^2(1-\alpha)]\left(\langle\bar{q}g\sigma\cdot Gq\rangle(6\ln(\alpha)-3) - \langle\bar{s}g\sigma\cdot Gs\rangle\left(\frac{1+2\alpha}{1-\alpha}\right)\right),$$
$$\rho^{m_s\langle\bar{q}g\sigma\cdot Gq\rangle}_{Df_0}(q^2) = \int_0^{\alpha_{\max}} d\alpha \frac{m_s\langle\bar{s}g\sigma\cdot Gs\rangle}{2^7\pi^4}[m_c^2 - q^2(1-\alpha)](1-6\ln(\alpha)), \tag{13}$$

four-quark condensates

$$\rho^{\langle\bar{q}q\rangle^2}_{D_{s^*0}\bar{K}}(q^2) = -\int_0^{\alpha_{\max}} d\alpha \frac{((\alpha-1)q^2+m_c^2)\langle\bar{q}q\rangle\langle\bar{s}s\rangle}{2^4\pi^2},$$
$$\rho^{\langle\bar{q}q\rangle^2}_{Df_0}(q^2) = \int_0^{\alpha_{\max}} d\alpha \frac{((\alpha-1)q^2+m_c^2)\langle\bar{s}s\rangle^2}{2^4\pi^2}, \tag{14}$$

and three-gluon condensates

$$\rho^{\langle g^3G^3\rangle}_{D_{s^*0}\bar{K}}(q^2) = \rho^{\langle g^3G^3\rangle}_{Df_0}(q^2) = -\int_0^{\alpha_{\max}} d\alpha \frac{((\alpha-1)q^2+3m_c^2)\alpha^3\langle g^3G^3\rangle}{3\times 2^{14}\pi^6(\alpha-1)^3}. \tag{15}$$

In the case of the (dimension six) four-quark condensate, we have used the factorization assumption. Therefore, its vacuum saturation value is given by:

$$\langle\bar{q}q\bar{q}q\rangle = \langle\bar{q}q\rangle^2. \tag{16}$$

(7) Finally, for dimension seven, we get

$$\rho^{m_s\langle\bar{q}q\rangle^2}_{D_{s^*0}\bar{K}}(q^2) = \int_0^{\alpha_{\max}} d\alpha \frac{m_c m_s}{2^3\pi^2}\left(\frac{\langle\bar{s}s\rangle^2}{2^2} - \langle\bar{q}q\rangle\langle\bar{s}s\rangle\right),$$
$$\rho^{m_s\langle\bar{q}q\rangle^2}_{Df_0}(q^2) = -\int_0^{\alpha_{\max}} d\alpha \frac{3m_c\langle\bar{q}q\rangle\langle\bar{s}s\rangle m_s}{2^4\pi^2}. \tag{17}$$

The integration limit in Eqs. (7)–(17) is $\alpha_{\max} = 1 - \frac{m_c^2}{q^2}$. For numerical calculations we need the values of the different condensates and quark masses. We have used here the same values for these inputs as those used in QCDSR calculations for other exotic molecular states [6,32–34], which are given in Table I. For the $\langle g^3G^3\rangle$ condensate, we have used the new numerical value estimated in Ref. [35].

We now calculate the correlation function from the hadronic or phenomenological point of view. In this case, the currents $j^\dagger$ and j are interpreted as the creation and annihilation operator of the hadrons which have the quantum numbers of the current j. For this $\Pi(q^2)$ is written by inserting a complete set of states with the same quantum numbers as those of the currents under consideration,

$$\Pi^{\text{phenom}}(q^2) = i\int d^4x e^{iq\cdot x}\int \frac{d^3p}{2p^0(2\pi)^3} \times \sum_{k=0}^{\infty}\langle 0\,|\,j(x)\,|\,m_k\vec{p}\rangle\langle m_k\vec{p}\,|\,j^\dagger(0)\,|\,0\rangle. \tag{18}$$

TABLE I. Values of the different known parameters required for numerical calculations of the correlation function given by Eq. (3) (see Refs. [6,32–35]).

Parameters	Values
m_s	0.10 ± 0.022 GeV
m_c	1.23 ± 0.05 GeV
$\langle\bar{q}q\rangle$	$-(0.23 \pm 0.03)^3$ GeV^3
$\langle\bar{s}s\rangle$	$0.8\ \langle\bar{q}q\rangle$
$\langle g^2G^2\rangle$	(0.88 ± 0.25) GeV^4
$\langle g^3G^3\rangle$	(0.58 ± 0.18) GeV^6
$\langle\bar{q}\sigma\cdot Gq\rangle$	$0.8\langle\bar{q}q\rangle$ GeV^2

Thus, the correlation function contains the information on all the hadrons of a given set of quantum numbers including the one we are interested in, which is the low-mass, relatively narrow, hadron of the series. One proceeds in such a situation by assuming that the spectral density of hadrons, for a fixed set of quantum numbers, can be expressed as a sum of a narrow, sharp state (which we are interested in) and a smooth continuum

$$\rho^{\rm phenom}(s) = \lambda^2\delta(s-m^2) + \rho_{\rm continuum}(s), \qquad (19)$$

where $\rho_{\rm continuum}$ is assumed to vanish below a certain value of s, s_0, which corresponds to the continuum threshold. Above this threshold, it is assumed to be given by the result obtained with the OPE. Therefore, one uses the ansatz [36] $\rho_{\rm continuum}(s) = \rho^{\rm OPE}(s)\Theta(s-s_0)$.

The delta function in Eq. (19) implies that the width of the particle is assumed to be zero. In principle, the introduction of a finite width in the above calculation could change the final result obtained for the mass and, more importantly, it could be another important source of error in the final result for the mass. However, our experience with this type of calculation suggests that the introduction of a width is not a very important source of errors. Indeed, in Ref. [37] (see also the discussion in Ref. [6]), a careful discussion of this effect was presented with the conclusion that for the X(3872), Z(4430), and Z(4250) the uncertainty in the width, when properly taken into account, generates at most a 5% error in the final mass of the state. Moreover, in Ref. [38] a careful study of the role played by the particle width was performed. The semileptonic decay $D\to\kappa l\nu$ was calculated with QCDSR. From experiment we know that $m_\kappa = 0.797$ GeV and the width is $\Gamma_\kappa = 0.410$ GeV. With this extremely large value of the width, we would expect that the zero width approximation for the κ would change the result dramatically. However, as shown in the quoted article, the zero width approximation yields a total D semileptonic decay rate that is only about 20% larger. Given the huge size of the kappa width (half of its mass!), the above mentioned estimate could be considered an upper limit of the error introduced by neglecting the particle width. In view of these examples and bearing in mind the exploratory nature of the present work, we will postpone the inclusion of the width for a future study. However, we are aware and must remind the reader that the estimated error in our results could be slightly larger.

In Eq. (19), λ is the coupling of the current j with the low-lying hadron with mass, $\langle 0|j|m\rangle = \lambda$.

The spectral density given by Eq. (19) is related to the correlation function of Eq. (18) as

$$\Pi^{\rm phenom}(q^2) = \frac{\lambda^2}{m^2-q^2} + \int_{s_0}^{\infty} ds\frac{\rho^{\rm OPE}(s)}{s-q^2}. \qquad (20)$$

To carry out the calculations, s_0 is taken as a parameter of the method but its value is not completely arbitrary: it is related to the onset of the continuum in the current j under consideration and is taken to be roughly 0.5 GeV above the mass of the hadron we are interested in Refs. [6,18]. In this work, we are looking for a resonance with a possible $Df_0(980)$ or $D_{s^*0}(2317)\bar{K}$ moleculelike structure. Since such resonances are weakly bound, they are expected to get generated close to the threshold of the constituent mesons. Thus, $\sqrt{s_0}$ in the present case can be $\sim$3.4 GeV.

The correlation function calculated using QCD suffers from divergent contributions coming from long-range interactions, while the one calculated phenomenologically contains contribution from the continuum. This situation can be improved by taking the Borel transform of both Eqs. (5) and (20), which kills the problematic terms of both sides, and which is defined as

$$\mathcal{B}_{M^2}[\Pi(q^2)] = \lim_{\substack{-q^2,n\to\infty \\ -q^2/n=M^2}} \frac{(-q^2)^{n+1}}{n!}\left(\frac{d}{dq^2}\right)^n \Pi(q^2). \qquad (21)$$

After taking the Borel transform, we equate the resulting expressions of the correlation functions on the basis of its dual nature and get

$$\lambda^2 e^{-m^2/M^2} + \int_{s_0}^{\infty} ds\rho^{\rm OPE}(s)e^{-s/M^2} = \int_{m_c^2}^{\infty} ds\rho^{\rm OPE}(s)e^{-s/M^2}, \qquad (22)$$

which can be rearranged as

$$\lambda^2 e^{-m^2/M^2} = \int_{m_c^2}^{s_0} ds\rho^{\rm OPE}(s)e^{-s/M^2}, \qquad (23)$$

where M represents the Borel mass parameter. Calculating the derivative of Eq. (23) with respect to M^2 and dividing the resulting expression by Eq. (23), we obtain the mass sum rule

$$m^2 = \frac{\int_{m_c^2}^{s_0} ds s\rho^{\rm OPE}(s)e^{-s/M^2}}{\int_{m_c^2}^{s_0} ds\rho^{\rm OPE}(s)e^{-s/M^2}}. \qquad (24)$$

Having the mass one can evaluate the current-state coupling constant through Eq. (23),

$$\lambda^2 = \frac{\int_{m_c^2}^{s_0} ds s\rho^{\rm OPE}(s)e^{-s/M^2}}{e^{-m^2/M^2}}. \qquad (25)$$

The reliability of the results obtained within QCD sum rules depends on the definition of a valid Borel window. This range of the Borel mass is obtained by making the following constraints:

(i) The maximum value of the Borel mass, $M_{\rm max}$, where the results should be reliable, is fixed by ensuring that the pole term (low mass hadron) gives the dominant contribution to the calculations. However, $M_{\rm max}$ is a function of s_0. As mentioned earlier, a reasonable value of $\sqrt{s_0}$ in the present calculation can be 3.4 GeV. We show the contributions of the pole and continuum terms weighted by their sum [6,32,33] for the $Df_0(980)$ and $D_{s^*0}(2317)\bar{K}$ systems, obtained with $\sqrt{s_0} = 3.4$ GeV, in Fig. 2, which shows that $M^2_{\rm Max} \sim 2.06$ GeV^2 in the former case and 1.79 GeV^2 in the latter, respectively.

(ii) The second constraint is to look for that Borel mass range where a convergence in the OPE series is found. For this we calculate the perturbative contribution and add to it the diagrams with higher dimensions step by step. In other words, we calculate the right-hand side of Eq. (23) by first using Eq. (7) for $\rho^{\rm OPE}$, then by using the sum of Eqs. (7) and (8), which means including the diagrams up to dimension one. Next we do the calculations up to the subsequent higher dimension by taking a sum of Eqs. (7)–(9), etc, until going to diagrams with dimension seven [given by Eqs. (17)]. For a convenient comparison, the result obtained in each case is weighted (divided) by the one obtained by using the whole series of Eq. (6) for the spectral density. In Fig. 3, we show the results of such an analysis of OPE convergence for the $Df_0(980)$ (left panel) as well as $D_{s^*0}(2317)\bar{K}$ (right panel) systems.

The final condition, which is imposed to identify the minimum value for the Borel mass, is that the contribution defined by

$$\left| \frac{\sum_{\rm dim=1}^{\rm N_{max}-1}(\int_{m_c^2}^{s_0} ds \rho^{\rm OPE}_{\rm dim}(s) e^{-s/M^2}) - \sum_{\rm dim=1}^{\rm N_{max}}(\int_{m_c^2}^{s_0} ds \rho^{\rm OPE}_{\rm dim}(s) e^{-s/M^2})}{\sum_{\rm dim=1}^{\rm N_{max}}(\int_{m_c^2}^{s_0} ds \rho^{\rm OPE}_{\rm dim}(s) e^{-s/M^2})} \right| \qquad (26)$$

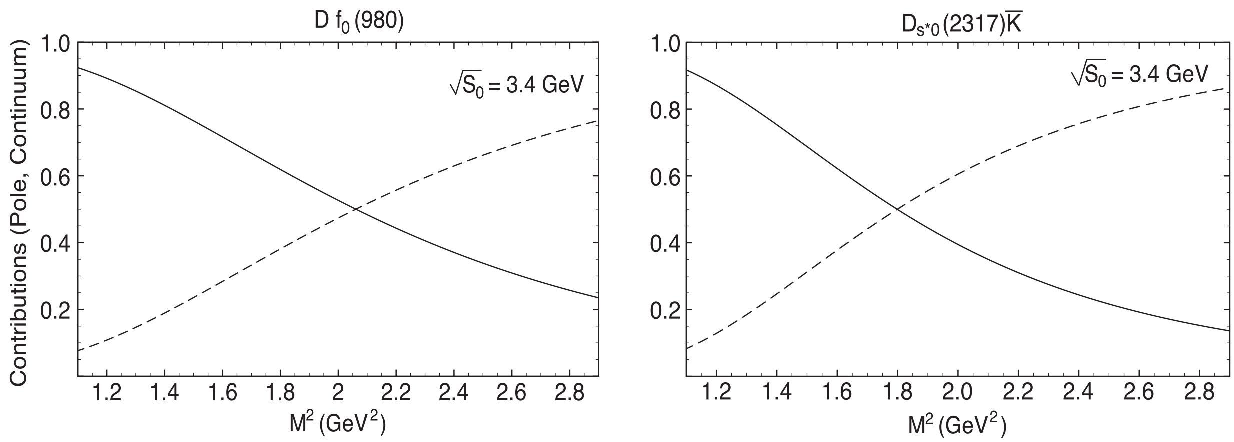

FIG. 2. The contributions of the pole (solid line) and continuum (dashed line) weighted by (divided by) their sum for the $Df_0(980)$ (left panel) and $D_{s^*0}(2317)\bar{K}$ (right panel) systems.

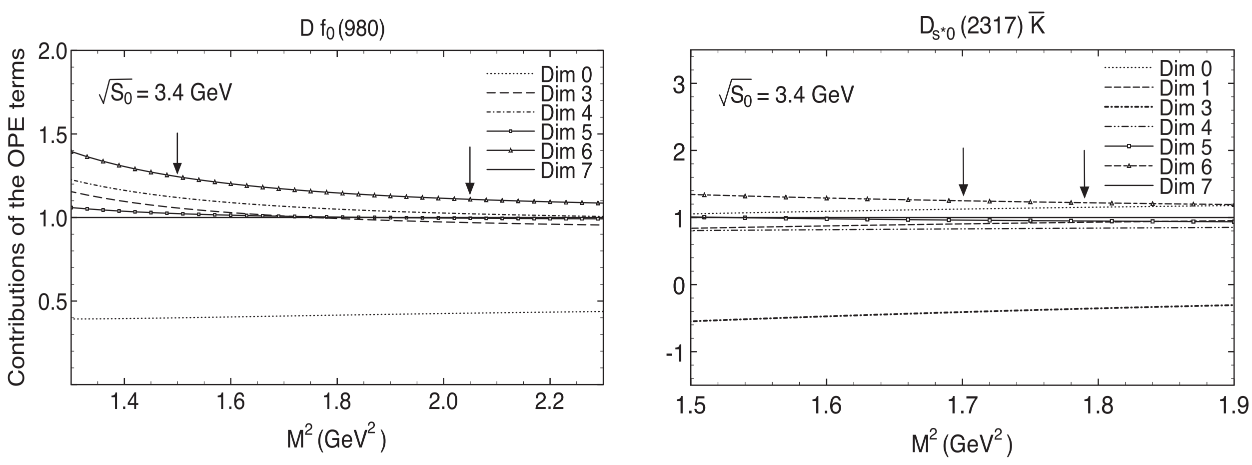

FIG. 3. Relative contributions of the different OPE terms as a function of the squared Borel Mass, for the $Df_0(980)$ (left panel) as well as $D_{s^*0}(2317)\bar{K}$ (right panel) systems. A value of $\sqrt{s_0} = 3.4$ GeV is used in these calculations. The arrows in the figures indicate the valid Borel window, which is determined by using the conditions discussed in the text.

is less than 0.25. In the equation written above, $N_{\max}$ refers to the maximum dimension of the condensates taken into account in the calculation, which is seven in the present case. With this, the OPE convergence is ensured in the Borel mass range where the results can be taken as reliable ones. It can be seen from Fig. 3 that a good convergence of the OPE series is found for $M^2_{\rm Min} = 1.5\ {\rm GeV}^2$ and $1.7\ {\rm GeV}^2$, for the $Df_0(980)$ (left panel) and $D_{s^*0}(2317)\bar{K}$ (right panel) cases respectively.

The valid Borel windows established using both criteria discussed above are indicated with arrows for the $Df_0(980)$ and $D_{s^*0}(2317)\bar{K}$ systems in Fig. 3, for $\sqrt{s_0} = 3.4$ GeV.

Having fixed these conditions, we show, in Fig. 4, the results of the calculation of the mass [using Eq. (24)] of the states described using $Df_0(980)$ and $D_{s^*0}(2317)\bar{K}$ moleculelike currents. It can be seen that a mass of 2.852 ± 0.008 GeV is found in the case of $D_{s^*0}(2317)\bar{K}$, while it is 2.921 ± 0.021 GeV in the $Df_0(980)$ system (within the Borel window, indicated by the arrows in Fig. 4). However, these calculations have been done using the value of 3.4 GeV for $\sqrt{s_0}$, which is a parameter. We now vary the continuum threshold in the range $3.3 \le \sqrt{s_0} \le 3.6$ GeV to check the sensitivity of our results to this parameter. We also take into account the fact that there exists uncertainty in the knowledge of the values of the different condensates and quark masses listed in Table I. Considering all these uncertainties we finally get

$$m_{D_{s^*0}\bar{K}} = (2.913 \pm 0.140)\ {\rm GeV} \tag{27}$$

for the $D_{s^*0}(2317)\bar{K}$ molecular current, while for the $Df_0(980)$ molecular current we get

$$m_{Df_0} = (2.926 \pm 0.237)\ {\rm GeV}. \tag{28}$$

The above results have been determined by averaging the mass over the corresponding Borel windows and by calculating the standard deviation to estimate the error.

Next, following the same procedure, we have calculated the coupling λ for the two configurations studied and found that the current-state coupling of the state described by the $D_{s^*0}(2317)\bar{K}$ current is around two times weaker than the one found for the $Df_0(980)$ current,

$$\lambda_{D_{s^*0}\bar{K}} = (5.8 \pm 1.2) \times 10^{-3}\ {\rm GeV}^5 \quad \text{and} \quad \lambda_{Df_0} = (9.4 \pm 3.3) \times 10^{-3}\ {\rm GeV}^5. \tag{29}$$

At first sight these couplings may look compatible within error bars. However, this is not the case. Our error analysis shows that, for a given set of parameters, λ_{Df_0} turns out to be $1.4\lambda_{D_{s^*0}\bar{K}} - 2\lambda_{D_{s^*0}\bar{K}},$ and this situation repeats for all other set of inputs. We can interpret this result as an indication that a $Df_0(980)$ state is better represented by the respective molecular current than the $D_{s^*0}(2317)\bar{K}$ state.

III. THREE-HADRON APPROACH

Let us now discuss the study of the $DK\bar{K}$ system within a very different approach that is based on effective field theories, treating hadrons as the degrees of freedom instead of quarks, and examine if the findings obtained in such a calculation are compatible with the ones found with QCD sum rules.

In the unitary chiral models [19–26,39,40], the $f_0(980)$ resonance is described as a molecular hadron state generated in the interaction of the $K\bar{K}$ and $\pi\pi$ coupled channels [24,25]. Similarly, the $D_{s^*0}(2317)$ state can be interpreted as a DK bound state formed in the DK, $D_s\eta$ coupled channel system [41–45]. In these models, Lagrangians based on symmetries, like chiral [46–48] and heavy quark symmetries [49–51], are used to determine the lowest order amplitude describing the transition between the different coupled channels. These amplitudes are further unitarized by using them as driving terms in the Bethe-Salpeter equation, and the scattering matrix t for the system is

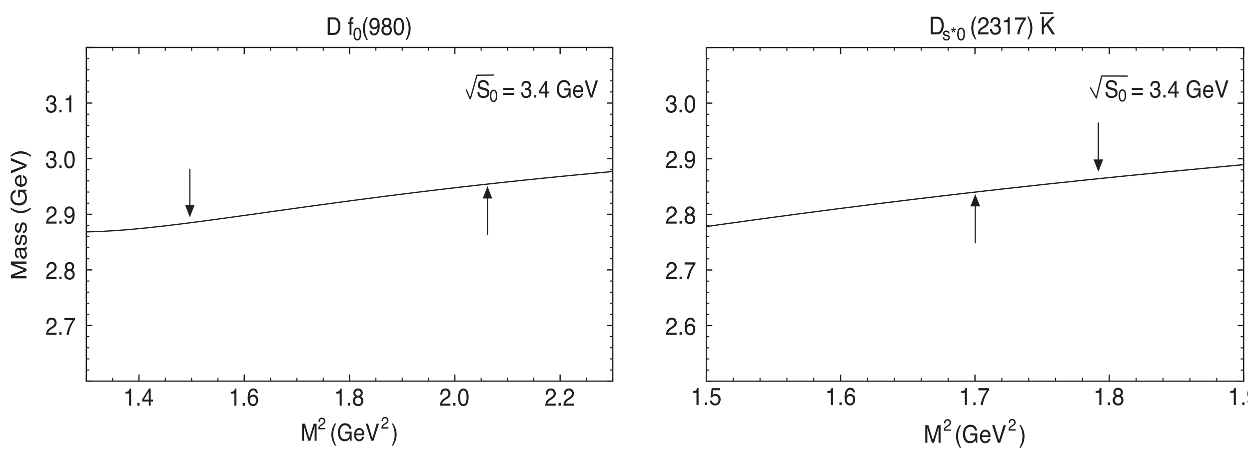

FIG. 4. Mass of a moleculelike resonance obtained by using QCD sum rules to study the $Df_0(980)$ (left panel) and $D_{s^*0}(2317)\bar{K}$ (right panel) systems. The arrows indicate the range where the reliability of the results is ensured.

obtained. Recently, these models based on effective field theories, chiral symmetry, and unitarity in coupled channels have been further extended to investigate the interaction of three-hadron systems formed by different mesons and baryons, like $\pi\bar{K}N$, $NK\bar{K}$, $J/\psi K\bar{K}$, $\phi K\bar{K}$, etc., and generation of several hadron states like $\Sigma(1660)$, $\phi(2170)$, $Y(4260)$, $N^*(1710)$ has been found [52–55]. Analogously to the two-body studies where the scattering matrix is obtained by solving the Bethe-Salpeter equation taking as kernel the lowest-order chiral amplitude, in the case of the approach of Refs. [52–55], the Faddeev equations [56] are solved, having as driving term the chiral two-body scattering matrices for the different pairs of the system. In this way, the input two-body t matrices in the Faddeev equation contain the information related to the generation of the corresponding two-body resonances.

In line with the above mentioned works, a different strategy to the one discussed in the previous section to study the $Df_0(980)$ and $D_s\bar{K}$ systems would be to consider $f_0(980)$ and $D_{s^*0}(2317)$ as molecular resonances formed, respectively, in the $K\bar{K}$ and DK systems, together with their respective coupled channels, and study the three-body system $DK\bar{K}$ following the approach of Refs. [52–55]. To do this, we consider ten coupled channels for total charge zero and charm $C=1$: $D^0K^+K^-$, $D^0K^0\bar{K}^0$, $D^0\pi^+\pi^-$, $D^0\pi^-\pi^+$, $D^0\pi^0\pi^0$, $D^0\pi^0\eta$, $D^+K^0K^-$, $D^+\pi^-\pi^0$, $D^+\pi^-\eta$, $D^+\pi^0\pi^-$.

As mentioned above, to solve the Faddeev equations for the $DK\bar{K}$ system and coupled channels, we first need to determine the two-body scattering matrices t for the different pairs of the system. This is done by solving the Bethe-Salpeter equation through its on-shell factorization form [20,24–26,39],

$$t=(1-V\mathcal{G})^{-1}V. \tag{30}$$

The kernel V in Eq. (30) corresponds to the lowest-order two-body amplitude obtained from a suitable Lagrangian, and $\mathcal{G}$ represents the loop function of two hadrons.

In the case of the $DK\bar{K}$ system and coupled channels, we have two different types of interactions: one involving two light pseudoscalars, like $K\bar{K}$, $\pi\pi$, and the other between a heavy and a light pseudoscalar meson, like DK, $D\pi$.

For the description of the $K\bar{K}$ system, we follow Refs. [24,25] and solve Eq. (30), considering $K\bar{K}$, $\pi\pi$, and $\pi\eta$ as coupled channels. The kernel V is obtained from the lowest-order chiral Lagrangian for the process $PP\to PP$, with P representing a light pseudoscalar (i.e., π, K, η),

$$\mathcal{L}_{PP}=\frac{1}{12f^2}\,\mathrm{Tr}\{(\partial_\mu PP-P\partial_\mu P)^2+MP^4\}. \tag{31}$$

In Eq. (31), f is the pion decay constant, Tr{...} indicates the trace in the flavor space of the SU(3) matrices appearing in P, which is a matrix containing the different Goldstone bosons, and M is a mass matrix,

$$P=\begin{pmatrix}\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & \pi^+ & K^+\\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0+\frac{1}{\sqrt{6}}\eta & K^0\\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta\end{pmatrix}, \tag{32}$$

$$M=\begin{pmatrix}m_\pi^2 & 0 & 0\\ 0 & m_\pi^2 & 0\\ 0 & 0 & 2m_K^2-m_\pi^2\end{pmatrix}. \tag{33}$$

The V matrix obtained using the Lagrangian of Eq. (31) is a function of the Mandelstam variables s, t, and u. This matrix is further projected on s wave, and the resulting expressions can be found in Ref. [24].

The loop function $\mathcal{G}$ in Eq. (30) is calculated using the dimensional regularization scheme of Ref. [25]. In the present case, i.e., for a two-pseudoscalar system,

$$\begin{aligned}\mathcal{G}_r=\frac{1}{16\pi^2}\bigg\{&a_r(\mu)+\ln\frac{m_{1r}^2}{\mu^2}+\frac{m_{2r}^2-m_{1r}^2+E^2}{2E^2}\ln\frac{m_{2r}^2}{m_{1r}^2}\\&+\frac{q_r}{E}\Big[\ln(E^2-(m_{1r}^2-m_{2r}^2)+2q_rE)\\&+\ln(E^2+(m_{1r}^2-m_{2r}^2)+2q_rE)\\&-\ln(-E^2+(m_{1r}^2-m_{2r}^2)+2q_rE)\\&-\ln(-E^2-(m_{1r}^2-m_{2r}^2)+2q_rE)\Big]\bigg\}.\end{aligned} \tag{34}$$

In Eq. (34), E is the total energy of the two-body system, m_{1r}, m_{2r}, and q_r correspond, respectively, to the masses and the center of mass momentum of the two pseudoscalars present in the rth channel, μ is a regularization scale, and $a_r(\mu)$ a subtraction constant. Following Ref. [25], we have taken $\mu=1224$ MeV and a value for $a_r(\mu)\sim-1$ (note that there is only one independent parameter here since a change in μ can be reabsorbed in a_r). In this way we can reproduce the observed two-body phase shifts and inelasticities for the different coupled channels as done in Refs. [24,25]. The resulting scattering matrix t exhibits poles on the unphysical sheet which are related to the resonances $\sigma(600)$, $f_0(980)$, and $a_0(980)$.

In the case of the subsystem constituted by a heavy and a light pseudoscalar meson, like DK, $D\pi$, since the heavy mesons contain both light and heavy quarks, one expects both the chiral symmetry of the light quarks and the symmetry of the heavy quarks to be considered. Having this in mind, to determine the scattering matrix of a heavy meson H with a light pseudoscalar P, we follow Refs. [42,45], where the leading order Lagrangian describing this interaction is given by the kinetic and mass term of the heavy mesons (chirally coupled to pions),

$$\mathcal{L}=D_\mu HD^\mu H^\dagger-\mathring{M}_H^2HH^\dagger, \tag{35}$$

with $H=(\,D^0\quad D^+\quad D_s^+\,)$ collecting the heavy mesons, whose mass in the chiral limit is $\mathring{M}_H$, P is given by Eq. (32), and D_μ is the covariant derivative [51]

$$D_\mu H^\dagger = (\partial_\mu + \Gamma_\mu)H^\dagger, \qquad D_\mu H = H(\overleftarrow{\partial}_\mu + \Gamma_\mu^\dagger),$$
$$\Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger), \qquad u^2 = e^{i\sqrt{2}P/f}. \quad (36)$$

For the process which concerns us, i.e., $HP \to HP$, Eq. (35) becomes

$$\mathcal{L}_{HP} = \frac{1}{4f^2}\{\partial^\mu H[P, \partial_\mu P]H^\dagger - H[P, \partial_\mu P]\partial^\mu H^\dagger\}, \quad (37)$$

and the lowest-order amplitude obtained from this Lagrangian in terms of the Mandelstam variables reads as

$$V_{ij} = -\frac{C_{ij}}{4f^2}(s-u). \quad (38)$$

In Eq. (38) i and j represent the initial and final channels, respectively, and the C_{ij} coefficients have been earlier calculated and can be found in Refs. [42,45]. This potential is further projected on s wave.

As in Refs. [42,45], we consider the coupled channels DK, $D_s\eta$ and $D_s\pi$ for strangeness $+1$ and $D\pi$, $D\eta$, $D_s\bar{K}$ for strangeness 0. The loop function $\mathcal{G}$ of Eq. (30) is determined using Eq. (34) with $\mu = 1000$ MeV and $a = -1.846$ [44,45], obtaining in this way a pole in the DK system for total isospin 0 at 2318 MeV, which corresponds to the state $D_{s^*0}(2317)$, and a pole at 2446-i43 MeV in the $D\pi$ system in isospin 0, associated with the resonance $D_0^*(2400)$.

Once the two-body scattering matrices are calculated, we can proceed with the determination of the three-body T matrix for the $DK\bar{K}$ system. To do that we use the approach of Refs. [52–55], in which the Faddeev partitions, T^1, T^2, and T^3, are written as

$$T^i = t^i\delta^3(\vec{k}_i' - \vec{k}_i) + \sum_{j\neq i=1}^{3} T_R^{ij}, \qquad i = 1,2,3, \quad (39)$$

with $\vec{k}_i$ ($\vec{k}_i'$) being the initial (final) momentum of the particle i and t^i the two-body t matrix which describes the interaction of the (jk) pair of the system, $j \neq k \neq i = 1, 2, 3$. The total three-body T matrix is obtained by summing the T^i partitions,

$$T = T^1 + T^2 + T^3 = \sum_{i=1}^{3} t^i\delta^3(\vec{k}_i' - \vec{k}_i) + T_R, \quad (40)$$

where we define

$$T_R \equiv \sum_{i=1}^{3}\sum_{j\neq i=1}^{3} T_R^{ij}. \quad (41)$$

The T_R^{ij} partitions in Eq. (39) satisfy the following set of coupled equations

$$T_R^{ij} = t^i g^{ij} t^j + t^i[G^{iji}T_R^{ji} + G^{ijk}T_R^{jk}],$$
$$i \neq j, \qquad j \neq k = 1,2,3, \quad (42)$$

where g^{ij} corresponds to the three-body Green's function of the system and its elements are defined as

$$g^{ij}(\vec{k}_i', \vec{k}_j) = \left(\frac{N_k}{2E_k(\vec{k}_i' + \vec{k}_j)}\right) \times \frac{1}{\sqrt{s} - E_i(\vec{k}_i') - E_j(\vec{k}_j) - E_k(\vec{k}_i' + \vec{k}_j) + i\epsilon}, \quad (43)$$

with $N_k = 1$ for mesons and E_l, $l = 1, 2, 3$, is the energy of the particle l.

The G^{ijk} matrix in Eq. (42) represents a loop function of three particles, and it is written as

$$G^{ijk} = \int \frac{d^3k''}{(2\pi)^3}\tilde{g}^{ij} \cdot F^{ijk} \quad (44)$$

with the elements of $\tilde{g}^{ij}$ being

$$\tilde{g}^{ij}(\vec{k}'', s_{lm}) = \frac{N_l}{2E_l(\vec{k}'')}\frac{N_m}{2E_m(\vec{k}'')} \times \frac{1}{\sqrt{s_{lm}} - E_l(\vec{k}'') - E_m(\vec{k}'') + i\epsilon}, \qquad i \neq l \neq m, \quad (45)$$

and the matrix F^{ijk}, with explicit variable dependence, is given by

$$F^{ijk}(\vec{k}'', \vec{k}_j', \vec{k}_k, s_{ru}^{k''}) = t^j(s_{ru}^{k''})g^{jk}(\vec{k}'', \vec{k}_k)[g^{jk}(\vec{k}_j', \vec{k}_k)]^{-1} \times [t^j(s_{ru})]^{-1}, \quad j \neq r \neq u = 1,2,3. \quad (46)$$

In Eq. (45), $\sqrt{s_{lm}}$ is the invariant mass of the (lm) pair and can be calculated in terms of the external variables. The upper index k'' for the invariant mass $s_{ru}^{k''}$ of Eq. (46) indicates its dependence on the loop variable (see Ref. [53] for more details).

The T_R^{ij} partitions given in Eq. (42) are functions of the total three-body energy, $\sqrt{s}$, and the invariant mass of the particles 2 and 3, $\sqrt{s_{23}}$. The other invariant masses, $\sqrt{s_{12}}$ and $\sqrt{s_{31}}$, can be obtained in terms of $\sqrt{s}$ and $\sqrt{s_{23}}$, as it was shown in Refs. [53,54]. In this model, peaks obtained in the modulus squared of the three-body T matrix are related to dynamically generated resonances. Finally, it should be mentioned that the first term in Eq. (40) cannot give rise to any state generated due to the three-body dynamics and, hence, we can just study the properties of the T_R matrix defined in Eq. (41).

Further, we work in the charge basis, then, to associate the peaks found in the three-body T-matrix with physical states we need to project T on an isospin basis. We do this by defining a basis where the states are labeled in terms of the total isospin I of the three-body system and the isospin of one of the two-body subsystems, which in the present case is taken as the isospin of the $K\bar{K}$ subsystem or the system made by particles 2 and 3, I_{23}, and evaluate the transition amplitude $\langle I, I_{23}|T_R|I, I_{23}\rangle$. The isospin I_{23} can be either 0 or 1, thus, the total isospin I can be 1/2, or 3/2.

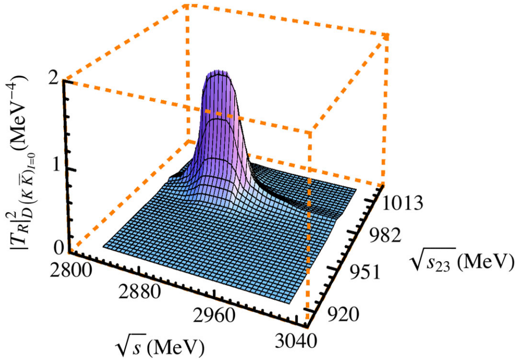

FIG. 5 (color online). Squared amplitude for the $DK\bar{K}$ channel for total isospin $I = 1/2$ with the $K\bar{K}$ subsystem in isospin zero.

For the cases involving the states $|I = 1/2, I_{23} = 1\rangle$, $|I = 3/2, I_{23} = 1\rangle$, we find no structure that could be related to a resonance or a bound state. Thus, in the following, we discuss the case $I = 1/2$ with $I_{23} = 0$, where we do find a resonance.

In Fig. 5 we show the results obtained for the modulus squared of the scattering amplitude of the $DK\bar{K}$ channel for total isospin 1/2 with the $K\bar{K}$ system in total isospin 0. A peak around 2890 MeV with a width of 55 MeV is found when the $K\bar{K}$ system is in the isospin zero configuration with an invariant mass of around 985 MeV, thus, forming the $f_0(980)$ resonance.

If instead of using the isospin base $|I, I_{23}\rangle$ we use $|I, I_{12}\rangle$, with I_{12} the isospin of the (12) subsystem, i.e., the DK system, no clear signal for the peak shown in Fig. 5 is observed when the DK system is in isospin zero.

These results are similar to the ones obtained in the previous section using QCD sum rules, in which a state of mass around ~2900 MeV is found to couple more to the $Df_0(980)$ current than to the one corresponding to $D_{s^*0}(2317)\bar{K}$.

IV. SUMMARY

We have studied the $DK\bar{K}$ system using two different methods: one based on QCD sum rules and the other on solving few-body equations. In the former case, the $Df_0(980)$ and $D_{s^*0}(2317)\bar{K}$ configurations of the $DK\bar{K}$ system have been investigated and a state with a mass around 2.9 GeV has been found, which couples more to a Df_0 molecular current. In the latter, the Faddeev equations have been solved with input two-body t matrices that generate the $f_0(980)$ and $D_{s^*0}(2317)$, respectively, in the $K\bar{K}$ and DK systems and related coupled channels. As a result, a state with a mass close to 2.9 GeV and a width of 55 MeV was found when the $K\bar{K}$ subsystem generates the $f_0(980)$ resonance. The findings obtained within these two different methods are quite similar, hinting towards the existence of a $Df_0(980)$ molecular state with a mass close to 2.9 GeV. A state with this mass has not been discovered experimentally so far, and the heaviest known D meson is the $D(2750)$, whose mass is around 150 MeV below the one found in this manuscript. We strongly encourage the search of a state decaying into $DK\bar{K}$ with the characteristics of the one found here.

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