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Mesh Algebras

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MESH ALGEBRAS

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ABSTRACT. Let k be an algebraically closed field and Δ be a finite directed translation quiver. The algebra $A = \frac{k\Delta}{\mu_\Delta}$, where μ_Δ is the ideal generated by all mesh relations of Δ, is called the mesh algebra of Δ. In this paper we shall characterize the (strongly) simply connected mesh algebras. As a consequence, we also describe the completely separated mesh algebras.

Let A be a basic finite dimensional algebra over an algebraically closed field k. It is a well-established fact that A is isomorphic to a quotient of a path algebra $k\Delta_A$, where Δ_A is a quiver, by an admissible ideal (see, for instance, [7]). This is particularly useful if one can read properties of A directly from its ordinary quiver Δ_A.

Let now Δ be a finite translation quiver and let A be the quotient of the path algebra of Δ by the ideal μ_Δ generated by the mesh relations (see below for definitions). We shall call such an algebra a mesh algebra. Examples of mesh algebras include the hereditary and the Auslander algebras [5]. See, for instance, [21] for details.

The notion of simple connectedness was introduced by Bongartz and Gabriel in [10]. This notion played an important role in the study of representation-finite algebras (see [8] and [11]). In order to explore the same ideas more generally, Assem and Skowroński extended in [4] the notion of simply connectedness to representation-infinite algebras. Later on, Skowroński introduced the concept of strongly simply connected algebras and since then, this notion plays an important role in the representation theory of algebras [20].

One of the main purpose of this work is to characterize the simply connected mesh algebras. We shall see, in particular, that a mesh algebra is simply connected if and only if it is strongly simply connected, extending a result proven for Auslander algebras [2]. One of our main results can be stated as follows. If Δ' is a subquiver of a translation quiver denote by $\overline{\mathcal{O}}(\Delta')$ its orbit-graph (see below for a definition).

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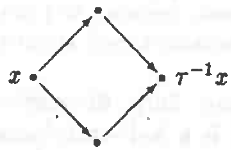
Theorem. *Let (Δ, τ) be a finite directed translation quiver and A be the corresponding mesh algebra. The following statements are equivalent:*

(a) *Δ is simply connected.*
(b) *A is simply connected.*
(c) *A is strongly simply connected.*
(d) *The orbit graph $\overline{\mathcal{O}}(\Delta)$ of Δ is a tree.*
(e) *The orbit graph $\overline{\mathcal{O}}(\Delta')$ of any subquiver $\Delta' \subset \Delta$ of type $\tilde{\mathbf{A}}_n$ is a tree.*
(f) *Each irreducible cycle is a contour and each irreducible contour is of type*



for some non-injective vertex x.

The proof of this result is given in sections 2 and 3 and uses recent results by Assem-Liu [3] and their notions of cycles and contours (see below). As a consequence, we also characterize the completely separated mesh algebras (Theorem 4.3). Recall that an algebra is completely separated provided it is Schurian and strongly simply connected. A subclass of mesh algebras defined using the so-called postprojective partition is studied in section 5. Also, we give necessary conditions for the first Hochschild cohomology of a mesh algebra to vanishes, extending a result by Happel [17] for Auslander algebras.

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1. Preliminaries

1.1. Along these notes, k will denote an algebraically closed field. All algebras are assumed to be basic, finite dimensional over k. For an algebra A, denote by *modA* the category of all finitely generated right A-modules and by *indA* its full subcategory with one representative of each isoclass of indecomposable A-module.

1.2. A *quiver* is given by two sets Δ_0 and Δ_1 whose elements are called *vertices* and *arrows*, respectively, together with two maps $s, e \colon \Delta_1 \longrightarrow \Delta_0$ which assign to each arrow $\alpha \in \Delta_1$ its start point $s(\alpha)$ and its end point $e(\alpha)$. A *path* γ in Δ is a concatenation of arrows $\alpha_n \cdots \alpha_1$ such that $e(\alpha_i) = s(\alpha_{i+1})$ for each $1 \leq i \leq n-1$. If $e(\alpha_n) = s(\alpha_1)$, we say that γ is an *oriented cycle*. An oriented cycle of length 1 (the *length of a path* is the number of arrows in it) is also called a *loop*. A *directed* quiver is a quiver without oriented cycles. A *walk* between vertices a and b is given by a sequence of arrows $\alpha_1, \cdots, \alpha_n$ such that $a \in \{s(\alpha_1), e(\alpha_1)\}$, $b \in \{s(\alpha_n), e(\alpha_n)\}$, and, for each i, $\{s(\alpha_i), e(\alpha_i)\} \cap \{s(\alpha_{i+1}), e(\alpha_{i+1})\} \neq \emptyset$. A quiver is called *connected* if there always exist a walk between any two given vertices on it. Let Δ be a quiver and Δ' be a subquiver of Δ (that is, $\Delta'_0 \subseteq \Delta_0$ and $\Delta'_1 \subseteq \Delta_1$) and let $a, b \in \Delta'_0$. Then, Δ' is said to be *full* if all arrows from a to b in Δ_1 also belong to Δ'_1, and it is said to be *convex* if all paths from a to b in Δ also belong to Δ'.

1.3. Let A be an algebra. By the assumptions above, we infer that A is isomorphic to a quotient of a path algebra $k\Delta_A$ by an admissible ideal. Recall that the path algebra $k\Delta$ of a quiver Δ is defined as follows. As a k-vector space, $k\Delta$ has a basis formed by all possible paths (including the so-called *trivial paths* associated to each vertex) and the multiplication is given naturally as concatenation of paths whenever possible. Denote by J the ideal of $k\Delta$ generated by all arrows of Δ. An *admissible ideal* I of $k\Delta$ is an ideal such that there exists an $n \geq 2$ such that $J^n \subset I \subset J^2$. Observe, then, that an algebra can also be considered as a k-category (see [10]).

1.4. Let Δ be a directed quiver and I an admissible ideal of $k\Delta$. A k-linear combination $\sigma = \sum_{j=1}^m \lambda_j w_j$ of paths w_j from x to y is called a *minimal relation* provided (i) $\sigma \in I$; (ii) $m \geq 2$; and (iii) for any nonempty proper subset J of $\{1, \ldots, m\}$, $\sum_{j \in J} \lambda_j w_j \notin I$. We shall now define a homotopy relation on the set of all walks of Δ. Observe that a walk can be seen as a formal concatenation of arrows and inverse of arrows. So, define the *homotopy relation* $\sim$ as the smallest equivalence relation satisfying :

(a) $\alpha\alpha^{-1} \sim \beta^{-1}\beta \sim$ trivial path at $x, \forall \alpha, \beta \in \Delta_1$ with $s(\beta) = x = e(\alpha)$.

(b) $\gamma_1 \sim \gamma_2$ if $\exists$ a minimal relation $\sum_{j=1}^m \lambda_j w_j$ such that $\gamma_1 = w_i$ and $\gamma_2 = w_j$ for some i, j.

(c) If $\gamma_1 \sim \gamma_2$ then $\gamma_1\gamma \sim \gamma_2\gamma$ and $\gamma\gamma_1 \sim \gamma\gamma_2$ whenever these make sense.

Fix $x \in \Delta_0$. So there is naturally a product on the set $\pi_1(\Delta, I, x)$ of the homotopy classes of walks starting and ending at x, which makes it a group. When Δ is connected, then the groups $\pi_1(\Delta, I, x)$ and $\pi_1(\Delta, I, y)$ are isomorphics for any $x, y \in \Delta_0$. In this case, it is called the *fundamental group of* (Δ, I) and it is denoted by $\pi_1(\Delta, I)$.

An algebra A is said to be *simply connected* provided for each presentation $A = \frac{k\Delta}{I}$ of A, its fundamental group is trivial. Also, A is called *strongly simply connected* provided any full convex subcategory of A is simply connected.

1.5. A locally finite quiver $\Delta = (\Delta_0, \Delta_1, s, e)$ is said to be a *translation quiver* provided:

(a) Δ has no loops and it is locally finite, that is, each vertex of Δ_0 is the start and the end point of at most finitely many arrows in Δ; and
(b) there exists a bijection $\tau\colon \Delta_0^p \longrightarrow \Delta_0^i$, where Δ_0^p and Δ_0^i are subsets of Δ_0 such that for each $x \in \Delta_0^p$ and each $y \in x^- = \{y \in \Delta_0 : \exists \text{ an arrow } y \longrightarrow x\}$, the number of arrows from y to x equals the number of arrows from τx to y.

Such a bijection τ is called a *translation* in Δ. A vertex x in $\Delta_0 \setminus \Delta_0^p$ (respectively, in $\Delta_0 \setminus \Delta_0^i$) will be called *projective* (respectively, *injective*) and we write $\tau x = 0$ (respectively, $\tau^{-1}x = 0$). The symbol 0 does not belong to Δ_0.

A *semitranslation* σ in Δ is defined as follows. For $x \in \Delta_0^p$, σ is a bijection between the arrows $y \longrightarrow x$ with $y \in x^-$ and the arrows $\tau x \longrightarrow y$. If Δ has no multiple arrows, then such a σ is uniquely determined. In case Δ has multiple arrows this is no longer true but for our purposes here (that is, to assign an algebra to a given translation quiver) the particular choice of a semitranslation will not affect our results. So, whenever we refer to a translation quiver, we shall assume that the semitranslation on it is fixed.

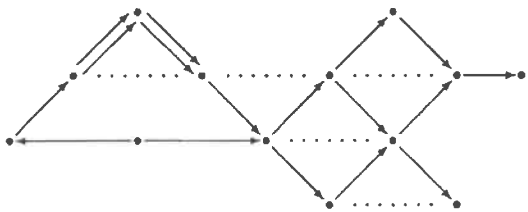
The Auslander-Reiten quiver of an algebra is a translation quiver, the translation being given by the so-called Auslander-Reiten translation DTr (see [7]).

1.6. Let Δ be a finite directed translation quiver. The *τ-orbit* of a vertex $x \in \Delta_0$ is the set $\mathbf{o}(x) = \{\tau^n x : n \in \mathbf{Z}\}$ (we shall denote $\tau^0 x = x$). By our assumptions on Δ, for each $x \in \Delta_0$, $\mathbf{o}(x)$ contains a unique projective vertex and a unique injective vertex.

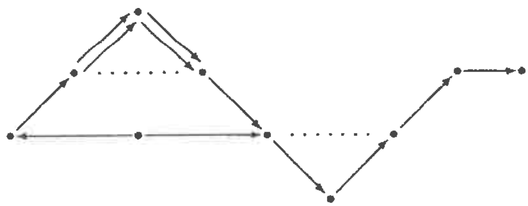
Let now $\Delta' \subset \Delta$ be a (full) subquiver of Δ. We shall define the *orbit graph* $\overline{\mathcal{O}}(\Delta')$ of Δ' as follows. Denote by $\mathbf{o}'(x) = \mathbf{o}(x) \cap \Delta'_0$. The vertices of $\overline{\mathcal{O}}(\Delta')$ are in a one-to-one correspondence with the sets $\mathbf{o}'(x)$. Suppose now that there are arrows between vertices u and v belonging to the τ-orbits $\mathbf{o}(x)$ and $\mathbf{o}(y)$, respectively. So, there are as many edges between the vertices $\mathbf{o}(x)$ and $\mathbf{o}(y)$ in $\overline{\mathcal{O}}(\Delta')$ as the number of arrows between u and v.

Observe that if $\Delta' = \Delta$, then the orbit-graph of Δ as defined above is just the underlying graph of the usual orbit-quiver (see [19]). Also, it should be clear that if $\Delta' \subset \Delta'' \subset \Delta$ are subquivers of Δ, then $\overline{\mathcal{O}}(\Delta')$ is a subgraph of $\overline{\mathcal{O}}(\Delta'')$.

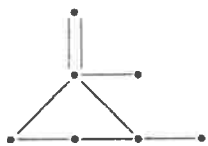
Example. Let Δ be the translation quiver



where the dotted lines indicate the translations and let Δ' be the following subquiver of Δ.



The orbit graph $\overline{\mathcal{O}}(\Delta')$ of Δ' is the following graph



1.7. Let now Δ be a finite directed translation quiver, and let $x \in \Delta_0$ be a non-projective vertex (and fix a semitranslation σ). The element $\Sigma\sigma(\alpha)\alpha$, where α runs over all arrows $\alpha\colon y \longrightarrow x$ with $y \in x^-$, is called a *mesh relation.* The *mesh algebra of* Δ is defined now to be the quotient of $k\Delta$ by the ideal μ_Δ generated by all possible mesh relations.

We further assume in our considerations that (Δ, τ) is finite and directed, in order to garantee that the corresponding mesh algebra is finite dimensional.

Observe that if one starts with the Auslander-Reiten quiver Γ_A of a representation-finite standard algebra A, then the mesh algebra of Γ_A is finite dimensional independently of Γ_A having oriented cycles or not. These later algebras are called *Auslander algebras* (see [7] for details).

1.8. We end this section by giving a necessary condition for a mesh algebra A to have its first Hochschild cohomology $\mathrm{H}^1(A)$ equals to zero. See [18] for a definition of the Hochschild cohomology groups $\mathrm{H}^i(A)$ of an algebra A, $i \geq 0$. Here we shall only need the following characterization of the first Hochschild cohomology $\mathrm{H}^1(A)$ of A. A *derivation in* A is a morphism $\delta \in \mathrm{Hom}_k(A, A)$ such that $\delta(ab) = \delta(a)b + a\delta(b)$ for each $a, b \in A$. An *inner derivation* is a derivation $\delta_x \colon A \longrightarrow A$ such that $\delta_x(a) = ax - xa$ for some $x \in A$. Denote by $\mathrm{Der}(A)$ and $\mathrm{Der}^0(A)$ the k-vector space of all derivations and the inner derivations, respectively.

Let now $\{e_1, \cdots, e_n\}$ be a complete system of primitive orthogonal idempotents of A and denote by $\mathrm{Der}^n(A) = \{\delta \in \mathrm{Der}(A) : \delta(e_i) = 0$ for each $i\}$ the subspace of the normalized derivations. Setting $\mathrm{Der}^{n,0}(A) = \mathrm{Der}^n(A) \cap \mathrm{Der}^0(A)$, we will have that

$$\mathrm{H}^1(A) = \frac{\mathrm{Der}^n(A)}{\mathrm{Der}^{n,0}(A)}$$

(see [16]).

1.9. We shall now establishes a necessary condition on a mesh algebra A to have $\mathrm{H}^1(A) = 0$. The proof is based in Happel's proof of a similar result for Auslander algebras [17]. Here, however, we shall prove a further condition, that is, that the associated quiver of such A is $\tilde{\mathbf{A}}_{1,p}$-free for $p \geq 2$.

Proposition. Let Δ be a connected finite translation quiver and $A = k\Delta/\mu_\Delta$ be the corresponding mesh algebra. Assume furthermore that the characteristic of k is zero. If $\mathrm{H}^1(A) = 0$, then Δ has neither oriented cycles nor subquivers of type $\tilde{\mathbf{A}}_{1,p}$, with $p \geq 2$.

Proof. For a path γ in Δ, denote its length by $l(\gamma)$. Define now a derivation $\delta \colon k\Delta \longrightarrow k\Delta$ given by $\delta(\gamma) = l(\gamma)\gamma$. Clearly, δ is a normalized derivation. Now, since $\delta(m_x) \in \mu_\Delta$ for each mesh relation m_x of Δ,

δ induces a normalized derivation $\overline{\delta}\colon \frac{k\Delta}{\mu_\Delta} \longrightarrow \frac{k\Delta}{\mu_\Delta}$. Since, by hypothesis, $\frac{\mathrm{Der}^n(A)}{\mathrm{Der}^{n,0}(A)} = 0$, there exists a $\lambda \in A$ such that $\overline{\delta} = \delta_\lambda$. An easy calculation yields to $\lambda = \sum_{i=1}^n \mu_i e_i + \lambda'$, where $\mu_i \in k$ and $\lambda' \in \bigoplus_i e_i(\mathrm{rad}A)e_i$. For an arrow $\alpha\colon e_i \longrightarrow e_j$ denote by $\overline{\alpha}$ its residual class in A. Hence,

$$\overline{\alpha} = \overline{\delta}(\overline{\alpha}) = \overline{\delta}_\lambda(\overline{\alpha}) = \mu_i\overline{\alpha} - \overline{\alpha}\mu_j + \overline{\lambda}$$

with $\overline{\lambda} \in \mathrm{rad}^2 A$. So $\mu_i - \mu_j = 1$.
Suppose now that Δ has a subquiver of type $\tilde{\mathbf{A}}_{1,p}$ as:

$$a_1 \xrightarrow{\alpha_1} a_2 \xrightarrow{\alpha_2} \cdots \longrightarrow a_r \xleftarrow{\alpha_r} a_{r+1} = a_1$$

$r \geq 2$. By the above, we have $\mu_i - \mu_{i+1} = 1$ for each $1 \leq i < r$ and $\mu_1 - \mu_r = 1$. Since the characteristic of k is zero we reach a contradiction. A similar argument proves that Δ is directed and the result is proven. □

Until recently it was an open problem to decide whether an algebra A with $\mathrm{H}^1(A) = 0$ could have cycles in its ordinary quiver. In [12], Buchweitz-Liu provided an example showing that this has not to be true in general. The above result shows, however, that the if A is a mesh algebra with $\mathrm{H}^1(A) = 0$, then the ordinary quiver of A has no cycles.

2. Irreducible contours in mesh algebras

2.1. Let Δ be a locally finite directed quiver. Following [3], we say that a pair of paths of positive length (p, q) from a vertex x to a vertex y is a *contour in* Δ, and we shall indicate it by C_{xy}. Also, a *cycle* in Δ is a subquiver C of Δ such that each of its vertices belongs to exactly 2 edges of C and there exists an enumeration $\{x_0, \cdots, x_n = x_0\}$ of the vertices of C such that there exists an edge between x_{i-1} and x_i for $1 \leq i \leq n$.

We say that a contour $C_{xy} = (p, q)$ is *interlaced* if the paths p and q have some common vertex other than x and y. Also, C_{xy} is *reducible* if there is a sequence of paths $p = p_0, \cdots, p_m = q$ in Δ from x to y such that, for each $1 \leq i \leq m$, the contour (p_{i-1}, p_i) is interlaced. If this does not happen, we say that C_{xy} is irreducible.

Remark. Observe that a contour $C_{xy} = (p, q)$ in which p is just an arrow from x to y is irreducible.

Let C be a cycle which is not a contour. We shall denote by $\eta(C)$ (respectively, by $\eta'(C)$) the number of sources (respectively, sinks) of C. By our definition of cycles, $\eta(C) = \eta'(C)$. Suppose now there is a path θ in Δ between two vertices x and y of C. Observe that C can be written then as a pair of walks w_1 and w_2 from x to y. We shall say that such path θ *reduces* the cycle C if $\theta^{-1}w_1$ and $\theta^{-1}w_2$ are both cycles and $\eta(\theta^{-1}w_i) < \eta(C)$, for $i = 1, 2$. In this case, we also say that C is *reducible* and otherwise *irreducible.*

For a quiver Δ as above, let I be an admissible ideal of $k\Delta$. We say that two paths p and q from x to y in Δ are *naturally homotopic* if there exists a sequence of paths $p = p_o, p_1, \cdots, p_m = q$ in Δ such that for each $1 \leq i \leq m$, p_i and p_{i+1} have subpaths q_i and q_{i+1} respectively which are involved in the same minimal relations of (Δ, I). A contour $C_{xy} = (p, q)$ is *naturally contractible* provided the paths p and q are naturally homotopic in (Δ, I).

In [3], Assem and Liu have proven the following result.

Theorem. *Let A be a connected triangular locally bounded k-category. The following conditions are equivalent:*

(a) *A is strongly simply connected.*

(b) *For any presentation $A \cong k\Delta_A/I_A$, each irreducible cycle in Δ_A is an irreducible contour, and each irreducible contour is naturally contractible.*

(c) *There exists a presentation $A \cong k\Delta_A/I_A$ such that each irreducible cycle in Δ_A is an irreducible contour, and each irreducible contour is naturally contractible.*

2.2. For the rest of this section we shall describe the irreducible contours and cycles in a mesh algebra. We start with the following lemma.

Lemma. *Let Δ be a locally finite directed quiver and ω be an irreducible cycle in Δ which is not an irreducible contour. Let x, c_1 and c_n be sources of ω and denote by $v\colon c_1 \longrightarrow c_2 \cdots c_{n-1} \longleftarrow c_n$ the subquiver of ω linking c_1 and c_n not passing through x. Then, for each $1 \leq i \leq n$, there are neither paths from x to c_i nor from c_i to x.*

Proof. Suppose there is an i with either a path from x to c_i or a path from c_i to x. Call ν and ξ the two distinct subquivers of ω linking x and c_i. Also, decompose the walk $v = v_1v_2$ into walks v_1 and v_2 in such a way that c_i is the (unique) common vertex of v_1 and v_2 and v_1 is a subpath of ν and v_2 is a subpath of ξ. Suppose first that there is a path θ from x to c_i. We will then have two cycles $\omega_1 = \theta^{-1}\nu$ and $\omega_2 = \theta^{-1}\xi$. We shall show that $\eta(\omega_i) < \eta(\omega)$ for $i = 1, 2$, leading to a contradiction to the fact that ω is irreducible. Indeed, if c_i is a sink in ω, we have $\eta(\omega) = 1 + \eta(v_1) + \eta(v_2)$, and so $\eta(\omega_i) = \eta(v_i) < \eta(\omega)$,

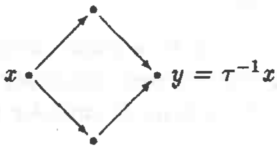
for $i = 1,2$. On the other hand, if c_i is not a sink in ω, we will have that $\eta(\omega) = \eta(v_1) + \eta(v_2)$, and since $\eta(v_i) \geq 1$, we infer that $\eta(\omega_2) = \eta(v_1) < \eta(\omega)$ for $i = 1,2$.
In case there is a path θ from c_i to x, the argument is very similar. Write again $\omega_1 = \theta^{-1}\nu$ and $\omega_2 = \theta^{-1}\xi$ the cycles induced by θ. If c_i is a source in ω, then $\eta(\omega) = \eta(v_1) + \eta(v_2) + 2$, and so $\eta(\omega_i) = \eta(v_i) + 1 < \eta(\omega)$ for $i = 1,2$, giving a contradiction to the fact that the cycle ω is irreducible. Also, if c_i is not a source in ω, then $\eta(\omega) = \eta(v_1) + \eta(v_2) + 1$ and so $\eta(\omega_i) = \eta(v_i) + 1 < \eta(\omega)$, since $\eta(v_i) \geq 1$, which is also a contradiction and the result is proven. □

2.3. **Proposition.** *Let Δ be a finite directed connected translation quiver. If $\overline{\mathcal{O}}(\Delta')$ is a tree for each subquiver $\Delta' \subset \Delta$ of type $\tilde{\mathrm{A}}_n$, then each irreducible cycle in Δ is an irreducible countor.*

Proof. Suppose there exists an irreducible cycle ω in Δ which is not an irreducible contour. Let x be a source of ω and let a and b sinks of ω such that there are paths p and q from x to a and to b, respectively. Since ω is not a contour, then $a \neq b$. Let now c_1 and c_n be sources of ω such that there are paths p_1 from c_1 to a and q_1 from c_n to b. Denote, also, by $v \colon c_1 \longrightarrow c_2 \cdots c_{n-1} \longleftarrow c_n$ the walk between c_1 and c_n.
We now observe that $\mathrm{o}(a) \neq \mathrm{o}(b)$. Otherwise, suppose $a = \tau^r b$ and without loss of generality assume that $r > 0$ ($a \neq b$ by hypothesis). So there is a path θ from a to b. It is not difficult to see that θ reduces ω, a contradiction. Write $\omega_1 = qp^{-1}$ and $\omega_2 = q_1 v p_1^{-1}$.
Claim. If the walk ω_2 passes through $y = \tau^s x$ for some $s \neq 0$, then $s < 0$, and y belongs either to p_1 or to q_1.
First observe that $y \neq c_i$ for each i by the above lemma. Suppose now $s > 0$. Then y cannot belong to p_1 because otherwise there would exist a path from c_1 to x, a contradiction to the above lemma. Similarly, y cannot belong to q_1. Therefore, $s < 0$ and y belongs either to p_1 or to q_1, and the claim is proven.
Write now $p_1 : c_1 = y_1 \longrightarrow \cdots \longrightarrow y_r = a$ and $q_1 \colon c_n = y'_1 \longrightarrow \cdots \longrightarrow y'_s = b$. Choose a vertez z as follows. If the path p_1 crosses the τ-orbit of x, choose j the minimum index such that y_j is in the τ-orbit of x and set $z = y_j$. If p_1 does not cross the τ-orbit of x, just set $z = a$. Similarly, choose a vertex z' as follows. If the path q_1 crosses the τ-orbit of x, choose l the minimum index such that y'_l is in the τ-orbit of x and set $z' = y'_l$. If q_1 does not cross the τ-orbit of x, just set $z' = b$. Clearly now, there are: (i) a path $\overline{p}$ from x to z (which could be even p if $z = a$); and (ii) a path $\overline{p}_1$ from c_1 to z; (iii) a path $\overline{q}$ from x to z' (which could be even q if $z' = b$); and (iv) a path $\overline{q}_1$ from c_n to z'. Finally, consider the cycle $\overline{p}_1 v \overline{q}_1^{-1} \overline{q} \overline{p}^{-1}$. Observe that the walks

$\overline{\omega}_1 = \overline{qp}^{-1}$ and $\overline{\omega}_2 = \overline{q}_1 v \overline{p}_1^{-1}$ induce two different walks in $\overline{\mathcal{O}}(\Delta)$ from $\mathrm{o}(z)$ to $\mathrm{o}(z')$ (one passes through the orbit of x and the other one does not). A contradiction to our hypothesis. □

2.4. **Proposition.** *Let Δ be a finite connected directed translation quiver and assume that $\overline{\mathcal{O}}(\Delta')$ is a tree for each subquiver $\Delta' \subset \Delta$ of type $\tilde{\mathrm{A}}_n$. If C_{xy} is an irreducible contour in Δ, then $y = \tau^{-1}x$ and the contour C_{xy} is of type*



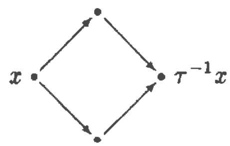
Proof. Write C_{xy} as (p, q). Since it is irreducible, then there exist arrows $\alpha_1 \colon x \longrightarrow x_1$, and $\alpha_2 \colon x \longrightarrow x_2$, with $x_1 \neq x_2$ and $p = p'\alpha_1$ and $q = q'\alpha_2$.
Claim. The contour C_{xy} passes through a vertex $z = \tau^s x$, with $s < 0$. Suppose this does not happen. Then the walks $\alpha_2\alpha_1^{-1}$ and $q'^{-1}p'$ from x_1 to x_2 induce two different walks in $\overline{\mathcal{O}}(\Delta)$ between $\mathrm{o}(x_1)$ and $\mathrm{o}(x_2)$ (they are different because $\alpha_2\alpha_1^{-1}$ crosses the τ-orbit of x and $q'^{-1}p'$ does not), a contradiction. So C_{xy} passes through a vertex $z = \tau^s x$ for some s. Since Δ does not have oriented cycles, we infer that $s < 0$ and the claim is proven.
In particular, x is not an injective vertex. We shall show now that $y = \tau^{-1}x$. Suppose $y \neq \tau^{-1}x$. So there exists a path v from $\tau^{-1}x$ to y (using the claim above). Write $\beta_i \colon x_i \longrightarrow \tau^{-1}x$, for $i = 1, 2$. Hence there is a sequence of interlaced paths $p, v\beta_1\alpha_1, v\beta_2\alpha_2, q$ from x to y which implies that (p, q) is not irreducible, a contradiction to our hypothesis, and so $y = \tau^{-1}x$.
Using the above notations, we shall show that $p = \beta_1\alpha_1$ and $q = \beta_2\alpha_2$. Suppose, for instance, that $p \neq \beta_1\alpha_1$. So, there exists $x_3 \neq x_1$ and an arrow $\beta_3 \colon x_3 \longrightarrow \tau^{-1}x$ such that $p = \beta_3 p'\alpha_1$, where p' is a path from x_1 to x_3. Clearly, also there is an arrow $\alpha_3 \colon x \longrightarrow x_3$. Observe that p' does not cross the τ-orbit of x, otherwise we would have an oriented cycle. Clearly, $\mathrm{o}(x_3) \neq \mathrm{o}(x_1)$, and so the walk $\alpha_3^{-1}p'\alpha_1$ of Δ induces a walk in $\mathcal{O}(\Delta)$ which is not a tree, a contradiction. Similarly, $q = \beta_2\alpha_2$, and the result is proven. □

3. Simply connected mesh algebras

3.1. In this section we shall proof the result which characterize the simply connected mesh algebras.

Theorem. *Let (Δ, τ) be a finite directed translation quiver and A be the corresponding mesh algebra. The following statements are equivalent:*

(a) *Δ is simply connected.*
(b) *A is simply connected.*
(c) *A is strongly simply connected.*
(d) *The orbit graph $\overline{\mathcal{O}}(\Delta)$ of Δ is a tree.*
(e) *The orbit graph $\overline{\mathcal{O}}(\Delta')$ of any subquiver $\Delta' \subset \Delta$ of type $\tilde{\mathbf{A}}_n$ is a tree.*
(f) *Each irreducible cycle is a contour and each irreducible contour is of type*

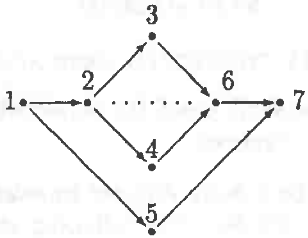


for some non-injective vertex x.

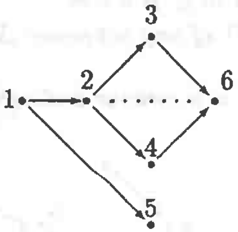
Proof. The implications (c) $\Rightarrow$ (b) $\Rightarrow$ (a) and (d) $\Rightarrow$ (e) should be clear. The implication (e) $\Rightarrow$ (f) follows from Propositions 2.3 and 2.4, while implication (f) $\Rightarrow$ (c) follows from Assem-Liu's result (2.1). To complete the proof, it suffices to show (a) $\Rightarrow$ (d).
(a) $\Leftrightarrow$ (d). Observe that Δ is simply connected if and only if the fundamental group $\pi_1(\Delta, x)$ is trivial for each $x \in \Delta_0$. This is clearly equivalent to $\pi_1(\bar{\mathcal{O}}(\Delta), x)$ being trivial for each $x \in \Delta_0$ (see [10]). Also, this happens if and only if $\bar{\mathcal{O}}(\Delta)$ is a tree. □

3.2. We shall now see some consequences of the above result. It is an open problem whether an Auslander algebra has postprojective component or not. For mesh algebras this is not true in general as shown by the following example.

Example. Let A be the mesh algebra of the following translation quiver



where the dotted line indicates the translation. Observe that A is the one-point extension $A = B[M]$ where $M = \mathrm{rad}P_7 = P_5 \oplus P_6$ and B is the (mesh) algebra given by the quiver



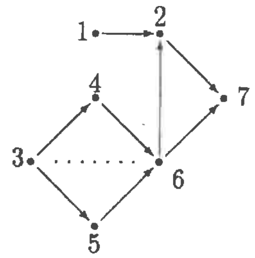
The module M is not directing because there exist a non-sectional path between P_5 and P_6. Therefore, by the criterion of Dräxler-de la Peña [15], A has no postprojective component.

3.3. By [9, 20], it follows that any strongly simply connected algebra has postprojective components. This together with our main result gives the following.

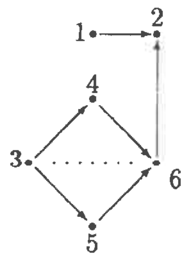
Corollary. *Let Δ be a directed finite translation quiver and $A = k\Delta/\mu_\Delta$ be the corresponding mesh algebra. If $\overline{\mathcal{O}}(\Delta)$ is a tree, then Γ_A has a postprojective component.*

3.4. Next example shows that the converse of the above result is not true.

Example. Let A be the mesh algebra of the following translation quiver



where the dotted line indicates the translation. Clearly, A does not satisfy the properties of our main theorem. Observe that A is the one-point extension $A = B[M]$, where $M = \mathrm{rad} P_7 = P_2 \oplus P_6$ and B is the (mesh) algebra given by the quiver



The B-module M is directing and belongs to the postprojective component of Γ_B. By [15], A has a postprojective component.

4. Completely separated mesh algebras

4.1. The purpose of this section is to characterize the completely separated mesh algebras, that is, mesh algebras which are Schurian and strongly simply connected. Recall that an algebra $A = k\Delta/I$, where Δ is a quiver and I is an admissible ideal of $k\Delta$, is called *Schurian* provided $dim_k \mathrm{Hom}_A(P_x(A), P_y(A)) \leq 1$ for all vertices $x, y \in \Delta_0$. We shall see that, for a mesh algebra, to be Schurian or completely separated is equivalent. We refer to [14] for a discussion on completely separated algebras.

4.2. In order to prove our main result, we shall need the following lemma.

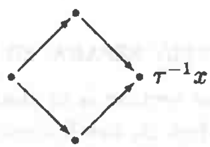
Lemma. *Let Δ be a finite quiver without oriented cycles and let $C_{xy} = (p_1, p_2)$ be a contour in Δ such that $p_1 = \alpha_1 p'_1$ and $p_2 = \alpha_2 p'_2$, where α_1 and α_2 are distinct arrows (ending at y). Then, there exists an irreducible contour $C_{x'y} = (q_1, q_2)$ with $q_1 = \alpha_3 q'_1$ and $q_2 = \alpha_4 q'_2$, where α_3 and α_4 are distinct arrows, and a path from x to x'.*

Proof. There is nothing to prove if C_{xy} is irreducible. Also, if $s(\alpha_1) = s(\alpha_2)$, then the contour (α_1, α_2) is as required. Suppose then that C_{xy} is reducible and that $s(\alpha_1) \neq s(\alpha_2)$. Therefore, there exist paths $\varphi_1 = p_1, \cdots, \varphi_s = p_2$, with $s > 1$, such that, for each $i = 1, \cdots, s-1$, φ_i and φ_{i+1} are interlaced. Since $\alpha_1 \neq \alpha_2$, there exists an j such that $\varphi_j = \alpha_1 \varphi'_j \varphi''_j$ and $\varphi_{j+1} = \alpha'_1 \varphi'_{j+1} \varphi''_{j+1}$ where $\alpha'_1 \in \Delta_1$, $\alpha_1 \neq \alpha'_1$, and φ_j and φ_{j+1} are interlaced at $s(\varphi'_{j+1}) = s(\varphi'_j) = \overline{x}$. If the contour $C_{\overline{x}y} = (\varphi'_j, \varphi'_{j+1})$ is irreducible we are done. Otherwise, repeat the above argument to this contour. Since Δ is finite and has no oriented cycles, this procedure has to stop at a contour as in the statement. □

4.3. We shall now prove our characterization of the mesh algebras which are completely separated.

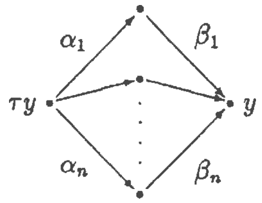
Theorem. *Let (Δ, τ) be a finite directed translation quiver and let $A = k\Delta/\mu_\Delta$ be the corresponding mesh algebra. The following conditions are equivalent:*

(a) *A is completely separated;*
(b) *A is Schurian;*
(c) *The following two conditions are satisfied:*
 (i) *Each irreducible contour in Δ is of type.*



for some non-injective vertex $x \in \Delta_0$; and
 (ii) *For each non-projective vertex y Δ_0, $|y^-| \leq 2$.*

Proof. (a) $\Rightarrow$ (b) Trivial.
(b) $\Rightarrow$ (c) Assume that A is Schurian. Clearly, then, all the irreducible contour should be of the type described in (i). Also, if there exists a non-projective vertex with $| y^- |= n > 2$ in Δ, then there exists a full subquiver of Δ of the following type:



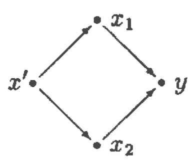
with $\sum_{i=1}^{n} \beta_i \alpha_i \in \mu_\Delta$. Hence $S_{\tau y}$ appear at least $n-1$ times as a composition factor of P_y and so $dim_k \mathrm{Hom}_A(P_{\tau y}, P_y) \geq n-1 \geq 2$, which is a contradiction to the fact that A is Schurian.
(c) $\Rightarrow$ (a) Suppose first that (Δ, τ) satisfies properties (i) and (ii). Observe that it follows from (i) that Δ has no multiple arrows.

Let $y \in \Delta_0$. Since Δ is directed, clearly it is enough to show that $dim_k \mathrm{Hom}_A(P_x, P_y) \leq 1$ for each proper predecessor x of y. We shall do it by induction on the number $n = n(y)$ of the proper predecessors of y. There is nothing to prove if $n = 0$.

Suppose now $n > 0$ and that $dim_k \mathrm{Hom}_A(P_x, P_y) \leq 1$ for each proper predecessor x of a vertex y' which has less than n proper predecessors.

Suppose that there exists a vertex x such that $dim_k \mathrm{Hom}_A(P_x, P_y) \geq 2$. Then there exist at least two non-zero paths φ and ξ from x to y with $\varphi + \xi \notin \mu_\Delta$.

Assume first that $x \in y^-$ and let α denote the arrow from x to y. Since Δ has neither oriented cycles nor multiple arrows, one of the above paths from x to y, let us say φ, does not pass through the arrow α. Consider the contour C_{xy} given by the paths φ and α. It is clearly irreducible (see (2.1)), which is a contradiction to condition (i). So, from now on, we can assume that x is a proper predecessor of y not belonging to y^-. Write $\varphi = \alpha_1 \varphi'$ where α_1 is an arrow from $x_1 \in y^-$ to y and φ' is a path from x to x_1. If the path ξ pass through α_1, say $\xi = \alpha_1 \overline{\xi}$, then there would exist two paths φ' and ξ' from x to x_1 such that $\varphi' + \overline{\xi} \notin \mu_\Delta$, a contradiction to the induction hypothesis. So, $\xi = \alpha_2 \xi'$, where $\alpha_2 \colon x_2 \longrightarrow y$ is an arrow and $x_1 \neq x_2$. Consider the contour $C_{xy} = (\varphi, \xi)$. If C_{xy} is irreducible, then by the hypothesis (i) and (ii), it follows that $\dim_k \mathrm{Hom}_A(P_x, P_y) \leq 1$. Suppose then that C_{xy} is reducible. By Lemma 4.2 and property (ii), there exist an irreducible contour $C_{x'y} = (\varphi_1, \xi_1)$ such that $\varphi_1 = \alpha_1 \varphi_1'$, $\xi_1 = \alpha_2 \xi_1'$ and a path θ from x to x'. By properties (i) and (ii), $C_{x'y}$ is of type



where $x' = \tau y$ and $y^- = \{x_1, x_2\}$. Observe that, by the induction hypothesis, we have that $\beta_1\theta + \varphi' \in \mu_\Delta$ and $\beta_2\theta + \xi' \in \mu_\Delta$. Since $\alpha_1\beta_1 + \alpha_2\beta_2 \in \mu_\Delta$, it follows that $\varphi + \xi \in \mu_\Delta$ and this implication is proven.

□

5. A CLASS OF MESH ALGEBRAS

5.1. For the rest of the paper we shall apply the characterization given in Theorem 3.1 to a special kind of mesh algebra. Let B be an algebra. Recall that a partition $\mathcal{P}_0, \dots, \mathcal{P}_n, \dots, \mathcal{P}_\infty$ of ind$B$ is called *postprojective* provided:

(a) $\text{ind}B = \bigcup_{i \leq \infty} \mathcal{P}_i$ and $\mathcal{P}_i \cap \mathcal{P}_j = \emptyset$, if $i \neq j$;

(b) for each $j < \infty$, $\mathcal{P}_j$ has the following properties:

- (b1) $\mathcal{P}_j$ is finite.
- (b2) for each $X \in \mathcal{C}_j = indB \backslash (\bigcup_{i<j} \mathcal{P}_i)$, there exists an epimorphism $Y \to X$ with $Y \in add\mathcal{P}_j$.
- (b3) No proper subcategory of $\mathcal{P}_i$ satisfies the condition (b2).

The existence and uniqueness of a postprojective partition of a given algebra was established by Auslander and Smalø in [6]. We refer to this paper for more details.

5.2. Let B be an algebra and denote by $\mathcal{P}_0, \dots, \mathcal{P}_n, \dots, \mathcal{P}_\infty$ its postprojective partition. Following [13], we say that B belongs to the class $\mathcal{H}_m$, for some $m \geq 0$, provided it satisfies the following conditions for $j = m$ but not for $j < m$: (i) add$(\mathcal{P}_0 \cup \cdots \cup \mathcal{P}_j)$ is closed under submodules; and (ii) for every irreducible map $X \longrightarrow Y$ with $Y \in \mathcal{P}_{j+1}$ and X indecomposable, $X \notin (\mathcal{P}_0 \cup \cdots \cup \mathcal{P}_{j-1})$.

Let $B \in \mathcal{H}_m$, for some $m \geq 0$. By [13](2.3), for each $n \geq m$, the subcategory add$(\mathcal{P}_0 \cup \cdots \cup \mathcal{P}_n)$ is also closed under submodules. Consider now, for each $n \geq m$, the algebra A_n defined as the endomorphism algebra of the module which is the sum of one copy of each module of $\mathcal{P}^n = \mathcal{P}_0 \cup \cdots \cup \mathcal{P}_n$.

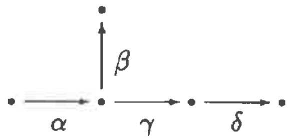
Lemma. *Let $B \in \mathcal{H}_m$, for some $m \geq 0$ and suppose Γ_B has a postprojective component. Then, the algebras A_n, with $n \geq m$, as defined above are mesh algebras.*

Proof. Let $n \geq m$. Observe that $\mathcal{P}^n$ generates a full convex subquiver Γ_n of Γ which is finite and directed. Since, by [19], a postprojective component is standard, we infer that the algebras A_n is the mesh algebra of (Γ_n, τ).

□

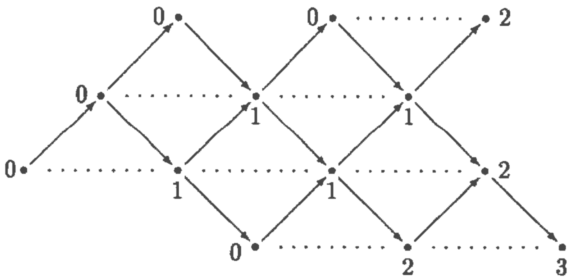
5.3. It has also been proven in [1] that if $B \in \mathcal{H}_1$, then either B is the radical square zero algebra with ordinary quiver $\tilde{A}_n$ linearly oriented or the Auslander-Reiten quiver Γ_B of B has a postprojective component containing all the projectives. If B is the square radical zero with ordinary quiver $\tilde{A}_n$ linearly oriented, then $\text{ind}B = \mathcal{P}_0 \cup \mathcal{P}_1$ and A_n is the Auslander algebra of B for each $n \geq 1$. Hence A_n is also a mesh algebra.

Example. Let B be the algebra given by the quiver

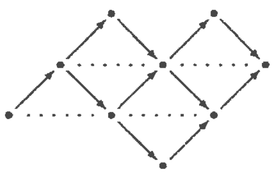


with $\alpha\beta = 0 = \alpha\gamma\delta$

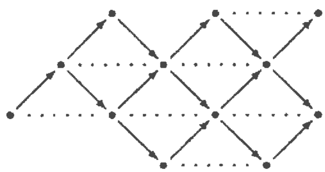
The Auslader Reiten quiver of B is



where the dotted lines indicates the Auslander-Reiten translation. The number next to a vertex indicates the postprojective level of the corresponding module. The algebra B is clearly in $\mathcal{H}_1$. The algebras A_1 e A_2 are given by the full translation subquivers of Γ_A



and



respectively, while A_3 is the Auslander algebra of B.

5.4. As a consequence of our main result we have the following result.

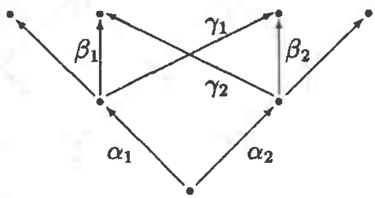
Proposition. *Let $B \in \mathcal{H}_m$, for some $m \geq 0$ and suppose Γ_B has a postprojective component. Then, the following statements are equivalent:*

(a) *B satisfies the separation condition.*
(b) *For each $n \geq m$, A_n is simply connected.*
(c) *For each $n \geq m$, A_n is strongly simply connected.*

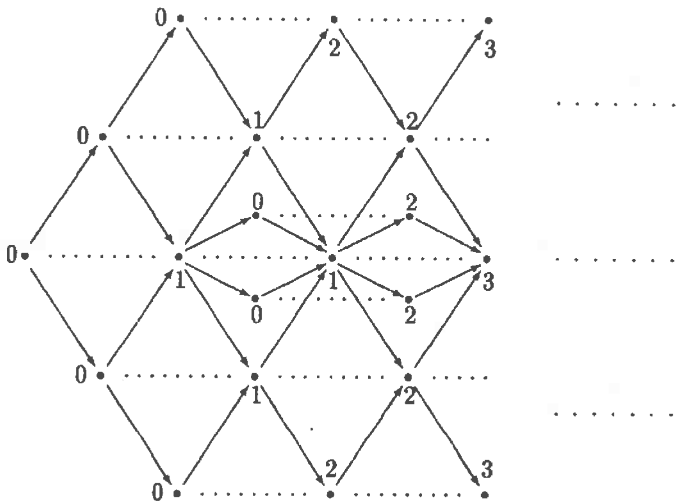
Proof. We first observe that B satisfies the separation condition if and only if the orbit graph of the postprojective component Γ of B is a tree (see [9]). Now, for $n \geq m$ denote by Γ_n the full convex subquiver of Γ generated by $\mathcal{P}^n$. Clearly, the orbit graph $\overline{\mathcal{O}}(\Gamma_n)$ equals the orbit graph of the postprojective component Γ. The result now follows easily using (3.1). □

5.5. We finish this paper with an example of an algebra in $\mathcal{H}_1$ which satisfies the separation condition but it is not strongly simply connected.

Example. Let A be the algebra given by the quiver:



with $\gamma_1\alpha_1 = \beta_2\alpha_2$ and $\gamma_2\alpha_2 = \beta_1\alpha_1$. The Auslander-Reiten quiver Γ_A of A has a postprojective component Γ containing all the projective modules as follows



where the number near a vertex indicates the postprojective level of the corresponding module. Clearly, $A \in \mathcal{H}_1$. It is not difficult to check that A satisfies the separation condition but it is not strongly simply connected.

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