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BIAS CORRECTION FOR ESTIMATORS OF THE RESIDUAL VARIANCE IN THE ARMA(1,1) MODEL

by

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Bias Correction for Estimators of the Residual Variance in the ARMA(1,1) Model

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Abstract

We consider the ARMA(1,1) model and deal with the estimation of the residual variance. Results are known for the maximum likelihood(ML) estimators under normality, both for known and unknowm mean, in which case the asymptotic bias depends on the number of parameters(including the mean) and on the true residual variance, but not on the values of the remaining parameters. For moment and least squares estimators the situation is different: The asymptotic bias depends on the values of the parameters, besides the true variance. Some simulation results are also presented.

Key words: Time series, autoregressive moving average models,residual variance, bias, maximum likelihood,method of moments, least squares.

1. Introduction

We consider the ARMA(1,1) model defined by

$$(X_t - \mu) + \beta(X_{t-1} - \mu) = a_t + \alpha a_{t-1}\ , \quad t = 0, \pm 1, \dots\ , \tag{1.1}$$

where a_t is a white noise sequence with mean zero and finite variance σ_a^2, and μ, α, β are real parameters. For stationarity we require that $|\beta| < 1$, which also gives invertibility of (1.1) into an infinite moving average(the model is causal). If $|\alpha| < 1$, then (1.1) is invertible into an infinite autoregression.

The autocovariance sequence is $\gamma_s = E(X_t - \mu)(X_{t+|s|} - \mu)$, $s = 0, \pm 1, \dots$, and satisfies the equations

$$\gamma_0 = \frac{1 + \alpha^2 - 2\alpha\beta}{1 - \beta^2}\sigma_a^2\ , \quad \gamma_1 = \frac{(1 - \alpha\beta)(\alpha - \beta)}{(1 - \beta^2)}\sigma_a^2\ , \tag{1.2}$$

$$\gamma_j = -\beta\gamma_{j-1}\ , j \geq 2\ .$$

The autocorrelation sequence is $\rho_s = \gamma_s/\gamma_0, s = 0, \pm 1, \dots$ and from (1.2) is given by

$$\rho_j = \begin{cases} 1, & \text{if } j = 0 \\ \frac{(1-\alpha\beta)(\alpha-\beta)}{1+\alpha^2-2\alpha\beta}(-\beta)^{j-1}, & \text{if } j \geq 1. \end{cases} \tag{1.3}$$

The autocovariance and autocorrelation sequences are two of the basic tools in the time domain analysis of the model (1.1).The autocovariance satisfies the inversion formula

$$\gamma_s = \int_{-\pi}^{\pi} e^{i\lambda s} f(\lambda) d\lambda\ ,$$

where

$$f(\lambda) = \frac{\sigma_a^2}{2\pi}\frac{|1 + \alpha e^{i\lambda}|^2}{|1 + \beta e^{i\lambda}|^2}\ , \quad -\pi \leq \lambda \leq \pi \tag{1.4}$$

is the spectral density of the process.

In this paper we consider the estimation of the parameter σ_a^2. This is important because estimates of the residual variance enter, for example, into confidence sets for parameters, in the estimation of the spectrum and in the expression of the estimated error of prediction.

Estimates of σ_a^2 come from the methods of moments(MM), least squares(LS) and maximum likelihood(ML) under normality, and also from frequency domain arguments.

The main purposes of this paper are to review the literature on the subject and present new material on the large sample bias of estimators for the residual variance for model (1.1). Results for ML estimators and general ARMA(p,q) models are available in the literature. For pure autoregressive and moving average models see Mentz,Morettin and Toloi(1995a,1995b).

2. Review of the Literature

The object of the inference will be the autocovariances and autocorrelations introduced in (1.2) and (1.3), respectively, and of course the parameters α, β and σ_a^2. For the first two sequences, large sample expectations, variances, covariances and distributions are available, for several standard definitions of the sample quantities. This point will be briefly considered in Section 3.

Tanaka(1984) suggests a technique for obtaining the Edgeworth type asymptotic expansions associated with ML estimates in ARMA models. He obtains biases up to order $1/T$ for AR(1),AR(2),MA(1),MA(2) and ARMA(1,1) models with and without constant terms.Biases for the residual variance are also considered.

Cordeiro and Klein(1994) present a general procedure to obtain the biases of ML estimates in ARMA models. It turns out that the formula is difficult to obtain for models other than the lower order ones, but numerically it is easy to be implemented.

De Gooijer and Pukkila(1994) present a technique for obtaining expressions for the approximate expectations of estimates in ARMA models. They derive first and second order approximations, based on Taylor series expansions of the log-likelihood in terms of the expected values of the sample covariances or in terms of the expected values of the periodogram ordinates.

Good references for techniques and results for asymptotic analysis in time series are Anderson(1971) and Fuller(1976).

3. Estimation of Covariances and Correlations

Let $X_1, \ldots, X_T$ be a sample from (1.1); we consider estimating γ_j by

$$c_j = \frac{1}{T} \sum_{T=1}^{T-j} (X_t - \bar{X})(X_{t+j} - \bar{X}) , \tag{3.1}$$

for $j = 0, \ldots, T-1$, $c_{-j} = c_j$, and $\bar{X}$ is the usual sample mean. Other estimators are considered in the literature, for example by changing in (3.1) the denominator or the range of the sums(see for example Anderson,1971,ch.8).We use (3.1) because for $T > 1$ the autocovariance matrix with elements $c_{|i-j|}$ is positive definite, a fact we shall use below.

With these estimators we form estimators of the autocorrelations, $r_j = c_j/c_0$, $j = 1, \ldots, T-1$.

Large sample moments of these estimators are available in the literature. They are derived under the assumption that the process follows a general linear model, of which

(1.1) is a special case. Some useful results are(Fuller,1976):

For $h \geq q \geq 0$,

$$E(c_h - \gamma_h) = \frac{-h}{T}\gamma_h - \frac{T-h}{T}Var(\bar{X}) + O(T^{-2}) , \tag{3.2}$$

$$Cov(c_h, c_q) = \frac{T-h}{T^2}\sum_p (\gamma_p\gamma_{p-h+q} + \gamma_{p+q}\gamma_{p-h}) + O(T^{-2}) , \tag{3.3}$$

$$E(r_h) = \frac{T-h}{T}\rho_h - \frac{1}{\gamma_0}(1-\rho_h)Var(\bar{X})$$

$$+\frac{1}{\gamma_0^2}[\rho_h Var(c_0) - Cov(c_h, c_0)] + O(T^{-2}) , \tag{3.4}$$

$$Cov(r_h, r_q) = \frac{1}{T}\sum_p (\rho_p\rho_{p-h+q} + \rho_{p+q}\rho_{p-h} - 2\rho_q\rho_p\rho_{p-h}$$

$$-2\rho_h\rho_p\rho_{p-q} + 2\rho_h\rho_q\rho_p^2) + O(T^{-2}) , \tag{3.5}$$

$$E(c_0 - \gamma_0) \simeq -Var(\bar{X}) = -\frac{2\pi f(0)}{T} = \frac{-\sigma_a^2}{T}\frac{(1+\alpha)^2}{(1+\beta)^2} \tag{3.6}$$

for the ARMA(1,1) model.

Here and elsewhere in the paper we assume that the a_t are normally distributed, with mean zero and variance σ_a^2.

4. Moment Estimators

Box and Jenkins(1976,p.201) give a general procedure for obtaining initial estimates of the parametrs of an ARMA(p,q) model, which can be viewed as moment type estimates. A related reference is Brockwell and Davis(1991,p.250).

From (1.2) we find that an estimate for σ_a^2 is given by

$$\hat{\sigma}_{MM}^2 = \frac{1-\hat{\beta}^2}{1+\hat{\alpha}^2 - 2\hat{\alpha}\hat{\beta}}c_0 \tag{4.1}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the moment estimators of α and β , respectively, defined implicitly by the sample analog of (1.3), for $j = 1, 2$, where ρ_1 and ρ_2 are estimated by the sample autocorrelations r_1 and r_2.

In deriving the asymptotic bias of $\hat{\sigma}_{MM}^2$, the asymptotic biases and variances of the moment estimates of α and β will be needed.

From(1.3) we obtain

$$\rho_1 = \frac{(1-\alpha\beta)(\alpha-\beta)}{1+\alpha^2-2\alpha\beta}, \quad \rho_2 = -\beta\rho_1 , \tag{4.2}$$

from which we get $\alpha = h(\rho_1, \rho_2)$ and $\beta = k(\rho_1, \rho_2)$.

Asymptotic bias of the moment estimators $\hat{\alpha}$ and $\hat{\beta}$ are obtained as follows. Let

$$\dot{\alpha}_i = \frac{\partial h}{\partial \rho_i}, \quad \ddot{\alpha}_{ij} = \frac{\partial^2 h}{\partial\rho_i\partial\rho_j}, \quad \dot{\beta}_i = \frac{\partial k}{\partial \rho_i}, \quad \ddot{\beta}_{ij} = \frac{\partial^2 k}{\partial\rho_i\partial\rho_j}, \tag{4.3}$$

for $i, j = 1, 2$. Here $\frac{\partial h}{\partial \rho_i}$ is a shorthand notation for $\frac{\partial h(r_1, r_2)}{\partial r_i}|_\theta$, where θ is the vector of parameters. The same is true for the other derivatives and this simplification will be used consistently in the paper.

Then using a Taylor expansion up to $O(T^{-1})$ and taking expected values we obtain

$$E(\hat{\alpha}-\alpha) = \sum_{i=1}^{2} \dot{\alpha}_i E(r_i - \rho_i) + \frac{1}{2}\sum_{i=1}^{2} \ddot{\alpha}_{ii} E(r_i-\rho_i)^2 \ddot{\alpha}_{12} E(r_1-\rho_1)(r_2-\rho_2) + o(T^{-1}), \tag{4.4}$$

$$E(\hat{\beta}-\beta) = \sum_{i=1}^{2} \dot{\beta}_i E(r_i - \rho_i) + \frac{1}{2}\sum_{i=1}^{2} \ddot{\beta}_{ii} E(r_i-\rho_i)^2$$

$$+\ddot{\beta}_{12} E(r_1-\rho_1)(r_2-\rho_2) + o(T^{-1}) \tag{4.5}$$

From (4.3) and (4.4) we have

$$E(\hat{\alpha}-\alpha)(\hat{\beta}-\beta) = \sum_{i=1}^{2} \dot{\alpha}_i\dot{\beta}_i E(r_i-\rho_i)^2 + (\dot{\alpha}_2\dot{\beta}_1 + \dot{\alpha}_1\dot{\beta}_2) E(r_1-\rho_1)(r_2-\rho_2) + o(T^{-1}) , \tag{4.6}$$

$$E(\hat{\alpha}-\alpha)^2 = \sum_{i=1}^{2} \dot{\alpha}_i^2 E(r_i-\rho_i)^2 + 2\dot{\alpha}_1\dot{\alpha}_2 E(r_1-\rho_1)(r_2-\rho_2) + o(T^{-1}) , \tag{4.7}$$

$$E(\hat{\beta}-\beta)^2 = \sum_{i=1}^{2} \dot{\beta}_i^2 E(r_i-\rho_i)^2 + 2\dot{\beta}_1\dot{\beta}_2 E(r_1-\rho_1)(r_2-\rho_2) + o(T^{-1}). \tag{4.8}$$

We now consider finding the asymptotic bias of $\hat{\sigma}^2_{MM}$ .Using a Taylor expansion up to second order and taking expectations we obtain

$$E(\hat{\sigma}^2_{MM} - \sigma^2_a) = \frac{\partial\sigma^2_a}{\partial\gamma_0} E(c_0 - \gamma_0) + \frac{\partial\sigma^2_a}{\partial\alpha} E(\hat{\alpha}-\alpha)$$

$$+\frac{\partial\sigma^2_a}{\partial\beta} E(\hat{\beta}-\beta) + \frac{1}{2}\frac{\partial^2\sigma^2_a}{\partial\alpha^2} E(\hat{\alpha}-\alpha)^2 + \frac{1}{2}\frac{\partial^2\sigma^2_a}{\partial\beta^2} E(\hat{\beta}-\beta)^2 \tag{4.9}$$

$$+\frac{\partial^2\sigma_a^2}{\partial\gamma_0\partial\alpha}E(c_0-\gamma_0)(\hat\alpha-\alpha)+\frac{\partial^2\sigma_a^2}{\partial\gamma_0\partial\beta}E(c_0-\gamma_0)(\hat\beta-\beta)$$

$$+\frac{\partial^2\sigma_a^2}{\partial\alpha\partial\beta}E(\hat\alpha-\alpha)(\hat\beta-\beta)+o(T^{-1})\,.$$

In the Appendix we show that (4.9) leads to

$$E(\hat\sigma_{MM}^2-\sigma_a^2)=\frac{-\sigma_a^2}{T}\frac{M(\alpha,\beta)}{(1-\alpha^2)^3(\beta^2-1)(\alpha\beta-1)^2}+o(T^{-1})\,, \tag{4.10}$$

with

$$M(\alpha,\beta)=-3+5\alpha^2-2\alpha^3-17\alpha^4+14\alpha^6-2\alpha^7-6\alpha^8-\alpha^{10}+2\alpha\beta+2\alpha^2\beta+2\alpha^3\beta$$

$$-2\alpha^4\beta+12\alpha^5\beta-2\alpha^6\beta-6\alpha^7\beta+2\alpha^8\beta+6\alpha^9\beta+5\beta^2-2\alpha^2\beta^2$$

$$+3\alpha^4\beta^2-11\alpha^6\beta^2+4\alpha^8\beta^2+\alpha^{10}\beta^2-10\alpha\beta^3-2\alpha^2\beta^3+6\alpha^3\beta^3$$

$$+2\alpha^4\beta^3-4\alpha^5\beta^3+2\alpha^6\beta^3-2\alpha^7\beta^3-2\alpha^8\beta^3-6\alpha^9\beta^3$$

$$+5\alpha^2\beta^4+2\alpha^3\beta^4-6\alpha^4\beta^4-4\alpha^5\beta^4+5\alpha^6\beta^4+2\alpha^7\beta^4+4\alpha^8\beta^4\,.$$

In the special case of $\alpha=0$ this expression reduces to

$$\frac{-\sigma_a^2}{T}\frac{3-5\beta^2}{1-\beta^2}+o(T^{-1})\,, \tag{4.11}$$

which can be compared with the bias of the residual variance estimator for the AR(1) model, given by Mentz et al(1995a) as

$$\frac{-\sigma_a^2}{T}\frac{2-4\beta^2}{1-\beta^2}+o(T^{-1})\,. \tag{4.12}$$

Expressions (4.11) and (4.12) are compared in Figure 1.
In the special case of $\beta=0$, (4.10) reduces to

$$\frac{-\sigma_a^2}{T}\frac{(3-5\alpha^2+2\alpha^3+17\alpha^4-4\alpha^5-14\alpha^6+2\alpha^7+6\alpha^8+\alpha^{10})}{(1-\alpha^2)^3}+o(T^{-1})\,, \tag{4.13}$$

which can be compared with the bias of the residual variance estimator for the MA(1) model, given by Mentz et al(1995b) as

$$-\frac{\sigma_a^2}{T}\frac{2-6\alpha^2-2\alpha^3+15\alpha^4+4\alpha^5-4\alpha^6-2\alpha^7+\alpha^8}{(1-\alpha^2)^3}+o(T^{-1})\,. \tag{4.14}$$

Expressions (4.13) and (4.14) are compared in Figure 2.

5. Least Squares Estimators

One procedure that is often used in practice is to minimize the sum

$$S(\alpha,\beta)=\sum_{j=1}^{T}(X_j-\hat{X}_j)^2/r_{j-1} \tag{5.1}$$

with respect to α and β, where $r_n = v_n/\sigma_a^2$, $v_n = E(X_{n+1}-\hat{X}_{n+1})^2$ is the mean square error of prediction and the predictors $\hat{X}_j$ can be computed recursively through the Innovations Algorithm, for example(see Brockwell and Davis,1991a). The estimators obtained in this way will be referred to as the least squares estimators(LSE) $\hat{\alpha}_{LS}$ and $\hat{\beta}_{LS}$, of α and β , respectively. For the minimization of $S(\alpha,\beta)$ it is necessary to restrict α so that $|\alpha|<1$, which means that the model is invertible.

To develop the theory we use the idea of Durbin(1959), approximating the a_t in (1.1) by a "long autoregression"

$$a_t^* = \sum_{s=0}^{B_T}\delta_s^*(X_{t-s}-\mu)\,, \tag{5.2}$$

where $B_T\to\infty$ as $T\to\infty$ but in such a way that $B_T/T\to 0$ as $T\to\infty$. In fact, we use (5.2) together with the substitution of μ by $\bar{X}$.

The corresponding LSE of the residual variance will be taken as

$$\hat{\sigma}_{LS}^2=\frac{1}{T-3}\sum_{t=1}^{T}\hat{a}_t^2\,, \tag{5.3}$$

taking into account the estimation of the parameters α,β and σ_a^2.

Defining $u_t = a_t+\alpha a_{t-1}$ we have

$$a_t=\sum_{j=0}^{\infty}(-\alpha)^j u_{t-j}=\sum_{j=0}^{\infty}(-\alpha)^j[X_{t-j}-\mu+\beta(X_{t-j-1}-\mu)]$$

$$=(X_t-\mu)+(\beta-\alpha)\sum_{j=1}^{\infty}(-\alpha)^{j-1}(X_{t-j}-\mu)$$

that can be approximated by

$$a_t^* = (X_t - \mu) + (\beta - \alpha)\sum_{j=1}^{B_T}(-\alpha)^{j-1}(X_{t-j} - \mu) . \tag{5.4}$$

Thus the LSE of σ_a^2 using the long AR approximation is

$$\hat{\sigma}_{LS}^2 = \frac{1}{T - B_T - 3}\sum_{t=B_T+1}^{T}[\hat{a}_t^*]^2$$

$$= \frac{1}{T - B_T - 3}\sum_{t=B_T+1}^{T}[(X_t - \bar{X}) + (\hat{\beta} - \hat{\alpha})\sum_{j=1}^{B_T}(-\hat{\alpha})^{j-1}(X_{t-j} - \bar{X})]^2 \tag{5.5}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the least squares estimators. By comparing the sums on t with (3.1) we justify the approximation

$$\hat{\sigma}_{LS}^2 \simeq c_0[1 + \frac{(\hat{\beta} - \hat{\alpha})^2}{1 - \hat{\alpha}^2}] + 2(\hat{\beta} - \hat{\alpha})[1 - \frac{\hat{\alpha}(\hat{\beta} - \hat{\alpha})}{1 - \hat{\alpha}^2}]\sum_{j=1}^{\infty}(-\hat{\alpha})^{j-1}c_j . \tag{5.6}$$

Then, by (1.2) we have

$$\hat{\sigma}_{LS}^2 - \sigma_a^2 \simeq c_0[1 + \frac{(\hat{\beta} - \hat{\alpha})^2}{1 - \hat{\alpha}^2}] - \gamma_0[1 + \frac{(\beta - \alpha)^2}{1 - \alpha^2}]$$

$$+2(\hat{\beta} - \hat{\alpha})[1 - \frac{\hat{\alpha}(\hat{\beta} - \hat{\alpha})}{1 - \hat{\alpha}^2}]\sum_{j=1}^{\infty}(-\alpha)^{j-1}c_j - 2(\beta - \alpha)[1 - \frac{\alpha(\beta - \alpha)}{1 - \alpha^2}]\sum_{j=1}^{\infty}(-\alpha)^{j-1}\gamma_j . \tag{5.7}$$

If we take this expression as a function of $\hat{\alpha}, \hat{\beta}, c_0$ and the other c_j 's, we derive a Taylor expansion in terms up to second order and obtain the asymptotic bias as

$$E(\hat{\sigma}_{LS}^2 - \sigma_a^2) = -\frac{\sigma_a^2}{T}\frac{1 - 3\alpha^2 + 4\alpha\beta - 3\beta^2 + \alpha^2\beta^2}{(\alpha^2 - 1)(\beta^2 - 1)} + \sum_{i=1}^{7}A_i + o(T^{-1}) , \tag{5.8}$$

where

$$A_1 = \frac{-2(\alpha\beta - 1)(\alpha - \beta)}{(\alpha^2 - 1)^2}E(c_0 - \gamma_0)(\hat{\alpha} - \alpha) ,$$

$$A_2 = \frac{2(\beta - \alpha)}{1 - \alpha^2}E(c_0 - \gamma_0)(\hat{\beta} - \beta) ,$$

$$A_3 = \frac{2}{\alpha\beta - 1}\sigma_a^2 E(\hat{\alpha} - \alpha)(\hat{\beta} - \beta) ,$$

$$A_4 = -\frac{\sigma_u^2}{\alpha^2 - 1} E(\hat{\alpha} - \alpha)^2, \quad A_5 = -\frac{\sigma_u^2}{\beta^2 - 1} E(\hat{\beta} - \beta)^2,$$

$$A_6 = \sum_{j=1}^{\infty} E(\hat{\alpha} - \alpha)(c_j - \gamma_j)K_j, \quad A_7 = \sum_{j=1}^{\infty} E(\hat{\beta} - \beta)(c_j - \gamma_j)L_j. \qquad (5.9)$$

and K_j, L_j are given in the Appendix, together with the details of the derivation of (5.8).

The first term in the right-hand side of (5.8) is the sum of the contributions coming from terms involving the (asymptotic) biases of the covariance estimators, that is, approximations to $E(c_j - \gamma_j), j = 0, 1, \ldots$. The second term includes two parts: (1) The contributions from the variances and covariances of $\hat{\alpha}$ and $\hat{\beta}$, namely approximations to $E(\hat{\alpha}-\alpha)^2, E(\hat{\beta}-\beta)^2$ and $E(\hat{\alpha}-\alpha)(\hat{\beta}-\beta)$; these can be taken as equivalent(asymptotically) to those of the maximum likelihood estimators, as given, for example, in Brockwell and Davis(1991a,Section 8.8), or else calculated numerically as will be indicated below; (2) the contributions from the asymptotic covariances between the c_j 's and $\hat{\alpha}$ and $\hat{\beta}$; for these there are no closed-form approximations available, and hence we resort to a numerical procedure, namely the bootstrap(Efron and Tibshirani,1993).

The bootstrap procedure may be described as follows:

(a) An ARMA(1,1) model with $T+1$ observations is generated;

(b) The parameters α and β are estimated by LS, yielding $\hat{\alpha}_{LS}$ and $\hat{\beta}_{LS}$;

(c) The residuals of the fitted model in (b) are computed, namely

$$\hat{a}_1 = 0, \quad \hat{a}_t = X_t + \hat{\beta}_{LS}X_{t-1} - \hat{\alpha}_{LS}\hat{a}_{t-1}, \quad t = 2, \ldots, T+1,$$

mean-corrected, yielding the 'bootstrap residuals' $\hat{a}_2^*, \ldots, \hat{a}_{T+1}^*$;

(d) These bootstrap residuals are used to generate B bootstrap replicates, all with size T, through a simple random sampling scheme with replacement;

(e) Through model (1.1) and each bootstrap replicate, bootstrap samples are produced by

$$X_{t,i}^* = -\hat{\beta}_{LS}X_{t-1,i}^* + \hat{\alpha}_{LS}\hat{a}_{t-1,i}^* + \hat{a}_{t,i}^*, \quad t = 1, \ldots, T, \quad i = 1, \ldots, B \qquad (5.10)$$

where $X_1^* = X_1$ and $\hat{a}_1^* = 0$;

(f) For each series generated according to (5.10), compute

$$\hat{\alpha}_{LS,i}^*, \hat{\beta}_{LS,i}^*, c_{0,i}^*, \ldots, c_{10,i}^*, \quad i = 1, \ldots, B$$

The values obtained in (f) are then used to compute bootstrap estimates of all expectations appearing in (5.9).

To be able to compare with the MM and ML procedures, computations were done with:

$$\begin{array}{l} \text{(i) Model I: } \alpha = 0.9 \quad \text{and} \quad \beta = 0.3; \\ \text{(ii) Model II: } \alpha = -0.8 \quad \text{and} \quad \beta = -0.6; \\ \text{(iii) Model III: } \alpha = 0.4 \quad \text{and} \quad \beta = -0.7, \end{array} \tag{5.11}$$

for $T = 50$ and $T = 100$.

6. Maximum Likelihood Estimators

We now consider MLE under the assumption that the underlying process is Gaussian. A convenient way to treat the problem is by using the prediction error decomposition of the likelihood. This avoids the direct calculation of the determinant and inverse of the covariance function of $\mathbf{X} = (X_1, \ldots, X_T)'$ in

$$L(\mu, \Sigma) = (2\pi)^{-T/2}|\Sigma|^{-1/2} \exp\{-\frac{1}{2}(\mathbf{X} - \mu)'\Sigma^{-1}(\mathbf{X} - \mu)\} . \tag{6.1}$$

The prediction error decomposition gives

$$L(\beta, \alpha, \sigma_a^2) = (2\pi\sigma_a^2)^{-T/2}(r_0 r_1 \cdots r_{T-1}) \exp\{-\frac{1}{2}\sigma_a^2 S(\alpha, \beta)\} , \tag{6.2}$$

where $S(\alpha, \beta)$ is given by (5.1).The MLE of the residual variance is then

$$\hat{\sigma}^2_{ML} = \frac{S(\hat{\alpha}, \hat{\beta})}{T} , \tag{6.3}$$

where $\hat{\alpha}, \hat{\beta}$ are the values of α, β which minimize the reduced likelihood

$$\ell(\alpha, \beta) = \frac{\ell n S(\alpha, \beta)}{T} + \frac{1}{T}\sum_{j=1}^{T} \ell n(r_{j-1}) . \tag{6.4}$$

In our simulations the program PEST of the ITSM package(Brockwell and Davis, 1991b), which uses the innovations algoritm, gives the MLE of the variance.

Tanaka(1984) and Cordeiro and Klein(1994) derived formulas for the biases of MLE of the coefficients and residual variance of ARMA models. In particular for the ARMA(1,1) case we have that

$$E(\hat{\sigma}^2_{ML} - \sigma_a^2) = \begin{cases} -\frac{3\sigma_a^2}{T} + O(T^{-2}), & \text{if } \mu \text{ unknown;} \\ -\frac{2\sigma_a^2}{T} + O(T^{-2}), & \text{if } \mu = 0. \end{cases} \tag{6.5}$$

In the case of invertible models ($|\alpha| < 1$), the minimization of $S(\alpha, \beta)$ and $\ell(\alpha, \beta)$ are equivalent and then MLE and LSE will have the same asymptotic properties.

7. Simulations

In (5.11) we have the models that were generated in order to verify empirically the conclusions of the theoretical results presented in this paper. In all cases the a_t are i.i.d normal random variables, with mean zero and variance 1. We assume that the mean of the process is $\mu = 0$. Four sample sizes were considered:$T = 50, 100, 200, 400$.For each sample size 100 replicates were taken for each model.

For the computations we have used the ITSM package(Brockwell and Davis, 1991b). The moment estimators are computed via the innovations algorithm, the least squares estimators are computed minimizing (5.1) and exact maximum likelihood estimators are computed via the prediction error decomposition in conjunction with the innovations algorithm.

Table 7.1 reports the findings. In each cell we present the estimated bias(EST.BIAS) obtained by averaging over the 100 replications, the standard error(ST.ERROR) of the estimated bias, computed as $s/10$, where

$$s^2 = \sum_{i=1}^{100} (\hat{b}_i - \bar{b})^2/100 ,$$

with $\hat{b}_i = \hat{\sigma}_i^2 - 1$,$\bar{b} = \sum_{i=1}^{100} \hat{b}_i/100$ and $\hat{\sigma}_i^2$ is the variance estimated for each method, and finally the asymptotic bias(ASYM.BIAS) given by the theoretical formulas(MM and ML methods).

For the least squares estimators we computed an "approximated asymptotic bias" through the bootstrap procedure(as described in section 5.2), using 10 covariances in the calculations of A_6 and A_7 in (5.9). These computations were done for $T = 50$ and $T = 100$ only and are summarized in Table 7.2.

To facilitate the interpretation of Table 7.1 we computed the intervals $\bar{b} \pm \frac{2s}{10}$ and marked with $*$ those estimates for which the interval does not include the corresponding asymptotic bias. The results of this analysis can be summarized as follows:

Parameters	Analysis of Results
$\alpha = 0.9, \beta = 0.3$	$*$ at MM, $T = 50, 100, 200, 400$
$\alpha = -0.8, \beta = -0.6$	$*$ at MM, $T = 50, 100, 200, 300, 400$
	$*$ at LS, $T = 50$
$\alpha = 0.4, \beta = -0.7$	$*$ at MM and ML, $T = 50$

Some conclusions are:

1. Moment estimators tend to have larger biases than least squares and maximum likelihood estimators, in agreement with what is expected.

2. For moment estimators, when the MA parameter is close to the invertibility region($\alpha =$ 0.9 and $\alpha = -0.8$) the estimated bias can differ considerably from the theoretical value, even for large sample sizes.
3. For given sample size, the variability of the estimated biases, as measured by the estimated standard error, tend to have similar values for the LS and ML estimators.
4. The more frequent cases of lack of fit occured when the method of moments was used.
5. The values of the "asymptotic biases" obtained through the bootstrap procedure for the LS estimators seem reasonable in most of the cases.

8. Concluding Remarks

In this paper we considered the estimation of the residual variance in the ARMA(1,1) model. This variance is a nuisance parameter and its estimation is important because estimators enter into prediction errors, confidence intervals, tests of hypotheses, spectral estimators and other inferencial procedures.

In spite of the indicated usefulness, not many results are available about properties of estimators of the residual variance in ARMA models, except for maximum likelihood estimators under normality.

We considered estimation by three standard methods, namely moments,least squares and maximum likelihood under normality. In the analytical part of our work we concentrated in the study of the asymptotic biases of the estimators by using Taylor-type expansions and asymptotic results in the literature for means, variances, autocovariances and autocorrelations of linear processes.

For the ARMA(1,1) model, Figure 3 shows the behavior of (T.ASYM.BIAS) for the MM estimator for $|\alpha| = 0.9, 0.6, 0.3$ and $-0.9 \leq \beta \leq 0.9$, which can be compared with the corresponding value for the ML estimator, namely, -3 , for all values of α and β. This figure shows that:

(i) For $|\alpha| = 0.9$ and $-0.9 \leq \beta \leq 0.9$ the value of T.ASYM.BIAS is large and negative, indicating an underestimation of σ_a^2;

(ii) For $|\alpha| = 0.6$ and $-0.7 \leq \beta \leq 0.7$, the above quantity assumes values which are smaller(around -9), indicating also an underestimation of σ_a^2 , but with a smaller bias;

(iii) For $|\alpha| = 0.3$ and $-0.65 \leq \beta \leq 0.65$, T.ASYM.BIAS assumes negative values, but very small and often smaller than the corresponding ML ones.

Summarizing, for values of $|\alpha|$ not very close to one, we have reasonable values for the asymptotic biases, when compared with the ML estimators biases, for $-0.65 \leq \beta \leq 0.65$ (not too close to the nonstationarity region). Figure 4 shows the values of T.ASYM.BIAS for $-0.9 \leq \alpha \leq 0.9$ and $-0.9 \leq \beta \leq 0.9$, confirming the above conclusions.

Our analysis for the LS estimator is less complete. We were able to provide an asymptotically closed-form representation in (5.6), and some empirical examples show that the behavior of the estimators is reasonable.

The simulations confirm expected results, that ML and LS estimators perform better than MM or that better fits are obtained for large sample sizes. The fit of the simulated results is much better for ML and LS than for MM procedures.

In conclusion, correction for biases when using ML estimators is simple, since the correction does not depend on the values of the model parameters. For LS and MM estimators the correction will include a more complex function of all parameters, which, in practice will have to be estimated. We should also expect considerable biases near the admissibility regions for MM estimators.

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Appendix

A1. Proof of (4.10)

Expression (4.1) gives $\hat{\sigma}_a^2$ as a function of $c_0, \hat{\alpha}$ and $\hat{\beta}$, and is the sample analog of σ_a^2 defined in (1.2). We now consider (4.9): $E(c_0 - \gamma_0)$ is given by (3.6) and the expectations $E(\hat{\alpha} - \alpha)^r(\hat{\beta} - \beta)^s$, for $0 \leq r + s \leq 2$ are given by (4.4)-(4.8).

The necessary imputs to apply formulas (4.4)-(4.8) are:

(a) Calculate the first and second-order derivatives of α and β with respect to ρ_k; this can be done by (4.2) where $\rho_1 = f(\alpha, \beta), \rho_2 = g(\alpha, \beta)$, so $\alpha = h(\rho_1, \rho_2), \beta = k(\rho_1, \rho_2)$.

Denoting the various derivatives with subindices, the first and second-order derivatives of the inverse functions h and k are determined by the matrix relation $\mathbf{AB} = \mathbf{I}_6$ where

$$\mathbf{A} = \begin{pmatrix} f_1 & f_2 & f_{11} & f_{12} & f_{21} & f_{22} \\ g_1 & g_2 & g_{11} & g_{12} & g_{21} & g_{22} \\ 0 & 0 & f_1^2 & f_1 f_2 & f_2 f_1 & f_2^2 \\ 0 & 0 & f_1 g_1 & f_1 g_2 & f_2 g_1 & f_2 g_2 \\ 0 & 0 & g_1 f_1 & g_1 f_2 & g_2 f_1 & g_2 f_2 \\ 0 & 0 & g_1^2 & g_1 g_2 & g_2 g_1 & g_2^2 \end{pmatrix}$$

and

$$\mathbf{B} = \begin{pmatrix} h_1 & h_2 & h_{11} & h_{12} & h_{21} & h_{22} \\ k_1 & k_2 & k_{11} & k_{12} & k_{21} & k_{22} \\ 0 & 0 & h_1^2 & h_1 h_2 & h_2 h_1 & h_2^2 \\ 0 & 0 & h_1 k_1 & h_1 k_2 & h_2 k_1 & h_2 k_2 \\ 0 & 0 & k_1 h_1 & k_1 h_2 & k_2 h_1 & k_2 h_2 \\ 0 & 0 & k_1^2 & k_1 k_2 & k_2 k_1 & k_2^2 \end{pmatrix}.$$

The results can be checked by repeated application of the chain rule.

(b) From (3.4) and (3.5) evaluate $E(r_j - \rho_j), E(r_j - \rho_j)^2, j = 1, 2$ and $E(r_1 - \rho_1)(r_2 - \rho_2)$.

In (4.9) it remains to evaluate $E(c_0 - \gamma_0)(\hat{\alpha} - \alpha)$ and $E(c_0 - \gamma_0)(\hat{\beta} - \beta)$. From (4.2) we can expand $(\hat{\alpha} - \alpha)$ in a Taylor expansion up to order $O(T^{-1})$ and multiplying by $(c_0 - \gamma_0)$ we have that, to this same order,

$$(c_0 - \gamma_0)(\hat{\alpha} - \alpha) \simeq (c_0 - \gamma_0)(r_1 - \rho_1)\dot{\alpha}_1 + (c_0 - \gamma_0)(r_2 - \rho_2)\dot{\alpha}_2.$$

Also,

$$r_j - \rho_j \simeq -(c_0 - \gamma_0)\gamma_j/\gamma_0^2 + (c_0 - \gamma_0)^2\gamma_j/\gamma_0^3 + \gamma_0^{-1}(c_j - \gamma_j) - \gamma_0^{-2}(c_0 - \gamma_0)(c_j - \gamma_j)$$

and finally

$$E(c_0-\gamma_0)(\hat{\alpha}-\alpha)=[\frac{-\gamma_1}{\gamma_0^2}E(c_0-\gamma_0)^2+\frac{1}{\gamma_0}E(c_0-\gamma_0)(c_1-\gamma_1)]\dot{\alpha}_1$$

$$+[\frac{-\gamma_2}{\gamma_0^2}E(c_0-\gamma_0)^2+\frac{1}{\gamma_0}E(c_0-\gamma_0)(c_2-\gamma_2)]\dot{\alpha}_2+o(T^{-1})\,, \tag{A.1}$$

which can be evaluated using (3.3).

The same procedure is used to find an entirely similar expression for $E(c_0-\gamma_0)(\hat{\beta}-\beta)$, simply replacing in (A.1) $\dot{\alpha}_i$ by $\dot{\beta}_i, i=1,2$.

Finally, substituting all these expressions in (4.9) leads to (4.10).

A.2. Proof of (5.8)

We first note that if in (5.6) we write the corresponding parameters instead of the LS estimators, and use for the parametrics functions the relations (1.2), we obtain

$$\sigma_a^2=\gamma_0[1+\frac{(\beta-\alpha)^2}{1-\alpha^2}]+2(\beta-\alpha)[1-\frac{\alpha(\beta-\alpha)}{1-\alpha^2}]\sum_{j=1}^{\infty}(-\alpha)^{j-1}\gamma_j. \tag{A.2}$$

Hence, an expansion of (5.7) up to second order terms leads to

$$E(\hat{\sigma}_{LS}^2-\sigma_a^2)=AE(c_0-\gamma_0)+BE(\hat{\alpha}-\alpha)+CE(\hat{\beta}-\beta)$$

$$+\sum_{j=1}^{\infty}D_jE(c_j-\gamma_j)+ME(c_0-\gamma_0)(\hat{\alpha}-\alpha)+FE(\hat{\beta}-\beta)(c_0-\gamma_0)$$

$$+GE(\hat{\alpha}-\alpha)(\hat{\beta}-\beta)+\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}H_{jk}E(c_j-\gamma_j)(c_k-\gamma_k) \tag{A.3}$$

$$+\frac{I}{2}E(\hat{\alpha}-\alpha)^2+\frac{J}{2}E(\hat{\beta}-\beta)^2+\sum_{j=1}^{\infty}K_jE(\hat{\alpha}-\alpha)(c_j-\gamma_j)$$

$$+\sum_{j=1}^{\infty}L_jE(\hat{\beta}-\beta)(c_j-\gamma_j)+o(T^{-1})$$

where the needed derivatives are

$$A=\frac{\partial\sigma_a^2}{\partial\gamma_0}=\frac{1+\beta^2-2\alpha\beta}{1-\alpha^2},\quad B=\frac{\partial\sigma_a^2}{\partial\alpha}=C=\frac{\partial\sigma_a^2}{\partial\beta}=0\,.$$

$$D_j=\frac{\partial\sigma_a^2}{\partial\gamma_j}=\frac{2(\beta-\alpha)(1-\alpha\beta)}{1-\alpha^2}(-\alpha)^{j-1}\,.$$

$$M = \frac{\partial^2 \sigma_a^2}{\partial \gamma_0 \partial \alpha} = \frac{-2(\alpha\beta - 1)(\alpha - \beta)}{(\alpha^2 - 1)^2} ,$$

$$F = \frac{\partial^2 \sigma_a^2}{\partial \gamma_0 \partial \beta} = \frac{2(\beta - \alpha)}{1 - \alpha^2} ,$$

$$G = \frac{\partial^2 \sigma_a^2}{\partial \alpha \partial \beta} = \frac{-2\sigma_a^2}{1 - \alpha\beta} ,$$

$$H_{jk} = \frac{\partial^2 \sigma_a^2}{\partial \gamma_j \partial \gamma_k} = 0 \quad , \quad I = \frac{\partial^2 \sigma_a^2}{\partial \alpha^2} = \frac{-2\sigma_a^2}{\alpha^2 - 1} ,$$

$$J = \frac{\partial \sigma_a^2}{\partial^2 \beta^2} = \frac{-2\sigma_a^2}{\beta^2 - 1} ,$$

$$K_j = \frac{\partial^2 \sigma_a^2}{\partial \alpha \partial \gamma_j} = 2\alpha^{j-2}(-1)^j [2\alpha^3 + \beta - 4\alpha^2\beta - \alpha^4\beta + 2\alpha^3\beta^2$$

$$+ j(\alpha - \alpha^3 - \beta + \alpha^4\beta + \alpha\beta^2 - \alpha^3\beta^2](\alpha^2 - 1)^{-2} ,$$

$$L_j = \frac{\partial^2 \sigma_a^2}{\partial \beta \partial \gamma_j} = 2(-\alpha)^{j-1} \frac{1 + \alpha^2 - 2\alpha\beta}{1 - \alpha^2} .$$

Replacing in (A.3) we have that

$$E(\hat{\sigma}_{LS}^2 - \sigma_a^2) = \frac{1 + \beta^2 - 2\alpha\beta}{1 - \alpha^2} E(c_0 - \gamma_0) +$$

$$+ \sum_{j=1}^{\infty} \frac{2(\beta - \alpha)(1 - \alpha\beta)}{1 - \alpha^2} (-\alpha)^{j-1} E(c_j - \gamma_j) + \sum_{i=1}^{7} A_i + o(T^{-1}) , \qquad (A.4)$$

where $E(c_0 - \gamma_0)$ is given by (3.6) and (3.2) gives,

$$E(c_j - \gamma_j) = \frac{\sigma_a^2}{T} [\frac{(\alpha - \beta)(1 - \alpha\beta)(-j)(-\beta)^{j-1}}{1 - \beta^2} - \frac{(1 + \alpha)^2}{(1 + \beta)^2}] .$$

Finally we obtain (5.8) by replacing the expectations in (A.4).

Note: All the computations were done with the Mathematica program(see Wolfram,1988)

Table 5.1: Some bootstrap results for the LS method.

	T	$E(\hat{\alpha}-\alpha)^2$	$E(\hat{\beta}-\beta)^2$	$E(c_0-\sigma_0)(\hat{\alpha}-\alpha)$	$E(c_0-\sigma_0)(\hat{\beta}-\beta)$	$E(\hat{\alpha}-\alpha)(\hat{\beta}-\beta)$
Model I	50	0.037	0.056	0.006	-0.026	0.033
	100	0.013	0.026	0.006	-0.021	0.007
Model II	50	0.359	0.317	0.012	0.027	0.323
	100	0.071	0.092	-0.004	0.014	0.067
Model III	50	0.032	0.015	-0.011	-0.113	0.013
	100	0.013	0.007	0.001	-0.093	0.004

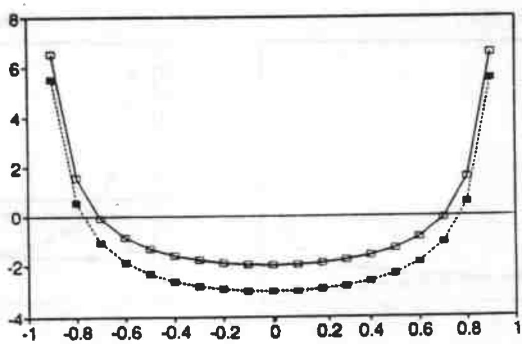
Table 7.1: Estimated bias of residual variance (with standard error) and asymptotic bias for moment, least square and maximum likelihood estimators in the ARMA (1,1) model.

T		$\alpha = 0.9$, $\beta = 0.3$			$\alpha = -0.8$, $\beta = -0.6$			$\alpha = 0.4$, $\beta = -0.7$		
		MM	LS	ML	MM	LS	ML	MM	LS	ML
50	Est. Bias	0.013 *	0.027	-0.030	0.014 *	0.018 *	-0.026	0.049 *	0.039	-0.001 *
	(St. Error)	(0.019)	(0.019)	(0.018)	(0.023)	(0.022)	(0.021)	(0.018)	(0.018)	(0.016)
	Asym. Bias	-16.704	0.031 +	-0.060	-1.906	0.190 +	-0.060	0.010	0.053 +	-0.060
100	Est. Bias	0.001 *	0.020	-0.011	0.021 *	0.028	0.006	0.011	0.016	-0.004
	(St. Error)	(0.014)	(0.014)	(0.013)	(0.017)	(0.017)	(0.016)	(0.013)	(0.013)	(0.013)
	Asym. Bias	-8.351	0.036 +	-0.030	-0.953	0.016 +	-0.030	0.005	0.004 +	-0.030
200	Est. Bias	-0.009 *	0.004	-0.009	0.005 *	0.014	0.003	0.000	0.006	-0.004
	(St. Error)	(0.011)	(0.009)	(0.009)	(0.015)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
	Asym. Bias	-4.170		-0.015	-0.476		-0.015	0.003		-0.015
400	Est. Bias	0.010 *	0.002	-0.005	0.028 *	0.009	0.004	-0.006	-0.003	-0.008
	(St. Error)	(0.009)	(0.006)	(0.006)	(0.019)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)
	Asym. Bias	-2.088		-0.008	-0.238		-0.008	0.001		-0.008

+ Values computed using bootstrap

Table 7.2: "Asymptotic bias" of residual variance for LS estimator using bootstrap, in the ARMA (1,1) model.

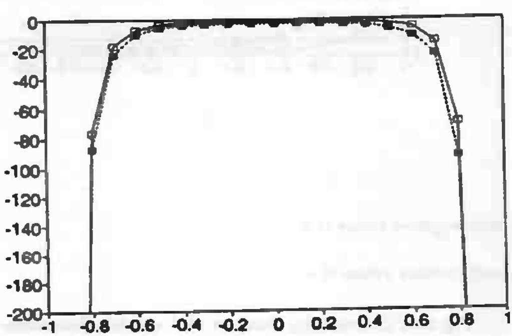
	T	Model I	Model II	Model III
First term of (5.8)	50	0.063	-0.013	0.093
	100	0.032	-0.007	0.046
Second term of (5.8)	50	-0.032	0.203	-0.040
	100	0.004	0.023	-0.042
"Asymptotic bias"	50	0.031	0.190	0.053
	100	0.036	0.016	0.004



■ : $ARMA(1,1)$ with $\alpha = 0$

□ : $AR(1)$

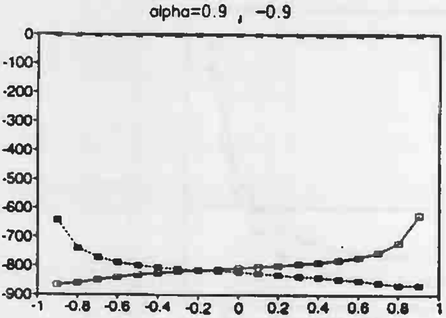
Fig.1: T.(Asymptotic Bias) for MM Estimators



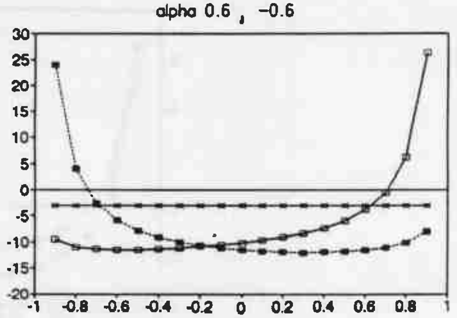
■ : $ARMA(1,1)$ with $\beta = 0$

□ : $MA(1)$

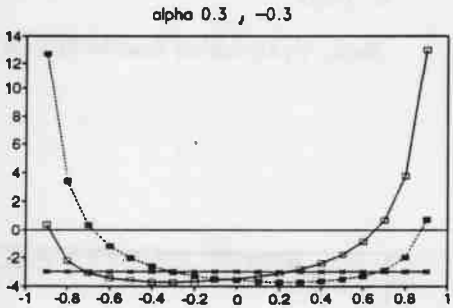
Fig.2: T.(Asymptotic Bias) for MM Estimators



(a)



(b)



(c)

∗ : ML Estimators

□ : MM Estimators with negative values of α

■ : MM Estimators with positive values of α

Fig.3: T.(Asymptotic Bias) for MM and ML Estimators for some values of α and $-0.9 \leq \beta \leq 0.9$.

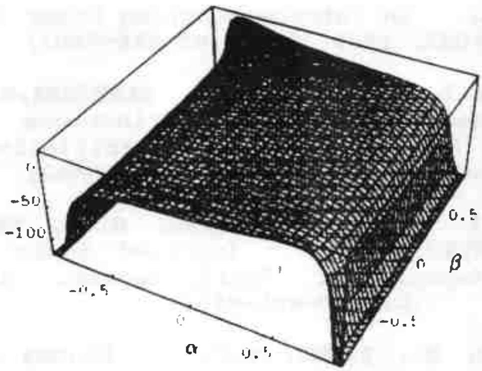


Fig.4: T.(Asymptotic Bias) of MM Estimators

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