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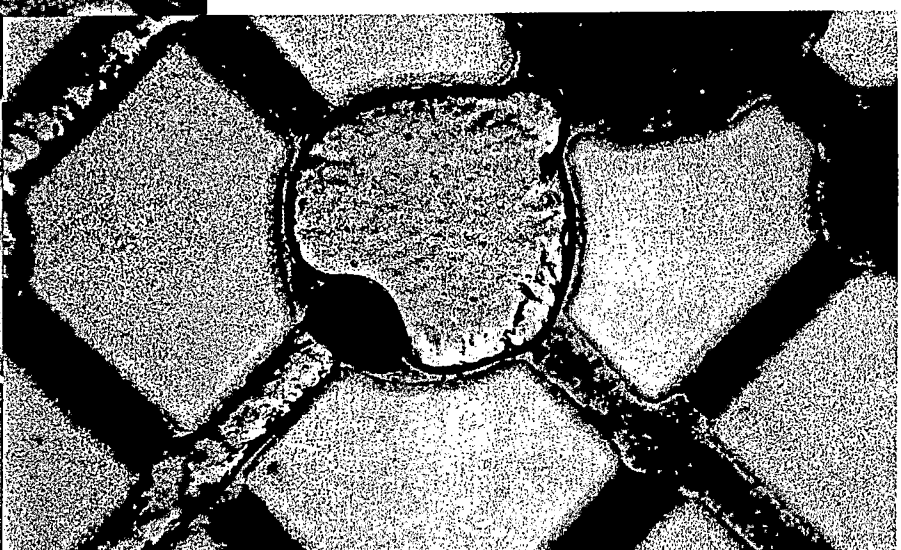
Computational Methods in Water Resources XII

Computational Methods in Contamination and Remediation of Water Resources

Editors
V.N. Burganos, G.P. Karatzas, A.C. Payatakes,
C.A. Brebbia, W.G. Gray, G.F. Pinder



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Computational Methods in Water Resources XII – Volume 1

Computational Methods in Contamination and Remediation of Water Resources

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EDITORS:

V. N. Burganos
ICE-HT/FORTH, Patras, Greece

G.P. Karatzas
University of Vermont, USA

A.C. Payatakes
University of Patras, Greece

C.A. Brebbia
Wessex Institue of Technology, UK

W.G. Gray
University of Notre Dame, USA

G.F. Pinder
University of Vermont, USA

Computational Mechanics Publications
Southampton, UK and Boston, USA

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Vasilis N. Burganos
Institute of Chemical Engineering and High Temperature Chemical Processes – Foundation for Research and Technology, Hellas
P.O. Box 1414
GR26500, Patras, Greece

George P. Karatas
University of Vermont
Department of Civil and Environmental Engineering
Burlington, VT 05405, USA

Alkiviades C. Payatakes
University of Patras
Department of Chemical Engineering
GR26500, Patras, Greece

Carlos A. Brebbia
Wessex Institute of Technology
Ashurst Lodge, Ashurst
Southampton SO40 7AA, UK

William G. Gray
University of Notre Dame
Department of Civil Engineering and Geological Sciences
Notre Dame, IN 46566-0767, USA

George F. Pinder
University of Vermont
Department of Civil and Environmental Engineering
Burlington, VT 05405, USA

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Ashurst Lodge, Ashurst, Southampton
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Tel: 44(0)1703 293223; Fax: 44(0)1703 292853
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Computational Mechanics Inc
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PREFACE

These volumes comprise the proceedings of the Twelfth International Conference on Computational Methods in Water Resources held in Crete, Greece, in June 1998. The Conference was organized by the Institute of Chemical Engineering and High Temperature Chemical Processes—Foundation for Research and Technology, Hellas. This series of conferences, formerly called Finite Elements in Water Resources, was initiated at Princeton University in 1976 and has taken place biennially since then alternating between America and Europe. Previous conferences, after Princeton, have taken place at Imperial College (1978), University of Mississippi (1980), University of Hannover (1982), University of Vermont (1984), Laboratorio Nacional de Engenharia Civil of Portugal (1986), Massachusetts Institute of Technology (1988), Giorgio Cini Foundation of Italy (1990), University of Colorado at Denver (1992), University of Heidelberg (1994) and Mexican Institute of Water Technology (1996).

These proceedings include 153 papers of the 157 presented orally at the Conference and contain the work of 325 researchers from 31 countries (105 from USA and Canada, 20 from Latin America, 162 from Europe, 28 from Asia, 2 from Australia, and 8 from Africa). The scope of the work has evolved considerably from the first few editions, and now it includes topics such as three-phase flow in porous media, pore-scale simulations, soil contamination and remediation, and hydrologic modeling.

The Organizing Committee gratefully acknowledges the participation of featured speakers M. Celia, P. Kitandis, A. Payatakes and Y. Yortsos, as well as invited speakers A. Aldama, G. Gambolati, C. Miller, U. Meissner and J. Sykes. Financial and administration services were provided by the Institute of Chemical Engineering and High Temperature Chemical Processes— Foundation for Research and Technology, Hellas. We also thank MITOS S.A., particularly S. Papadopoulos and M. Balothiaris, for their management of the local organizational details and logistics of the Conference. In closing, special thanks are due to the Conference Secretary, P. Vergi, for a very efficient secretarial support.

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The Editors
June 1998

The "exact" solution (6) was used to test the Bigflow and Geoprof numerical approaches. Some of the Bigflow code results are shown below.

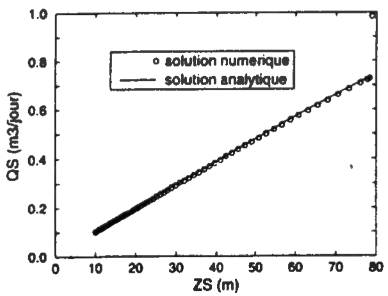

Figure 12: Head drop law (Q_S,Z_S) (numerical *versus* analytical).

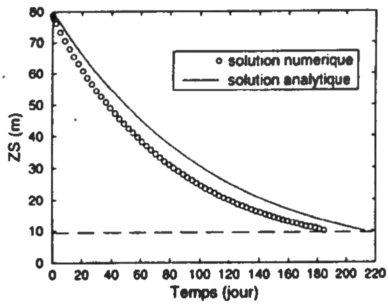

Figure 13: Water table level $Z_S(t)$ (numerical *versus* analytical).

The analytical and numerical solutions coïncide almost exactly as far as the head drop law is concerned ($Q_S(t)$ *vs.* $Z_S(t)$ in Fig.12). However, the simulated water table height $Z_S(t)$ is falling a bit too rapidly (Fig.13). Indeed, in the numerical experiment, the unsaturated soil column continues to drain some water *after* the passage of the free surface (defined as the atmospheric pressure surface). Detailed pressure profiles analyses have been developed (not shown here). There are only slight differences between Bigflow (finite volume) and Geoprof (finite element) pressure profiles: the discrepancies occur mostly in the vicinity of the water table.

Acknowledgments

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A comparison of various finite element procedures for watershed routing

J.L.Franco & F.H.Chaudhry

São Carlos School of Engineering, University of São Paulo – C.P. 359, 13560-970 – São Carlos, SP, Brazil, E-mail: fazal@sc.usp.br

Abstract

A number of studies on kinematic routing in watersheds have been reported in the literature demonstrating the usefulness of the finite element method. In these studies, different dependent variables are chosen for their approximate representation by basis functions. Further, different solution schemes in the time domain were used. Each decision as regards the approximate representation of functions and time integration may introduce varying effects on the results of the numerical integration. One-dimensional kinematic wave equation is solved in this paper by finite element method according to various solution strategies proposed in literature with reference to an example problem. The time distributions of overland flow obtained by different methods are compared against the analytical solution and the convenience of the procedures discussed.

1 Introduction

Computational methods are the backbone of the efforts for simulating the behavior of hydrologic systems particularly the watershed routing where finite element methods are specially indicated due to their natural malleability in describing irregular terrain surfaces. The use of this method has been the subject of many recent researches being approached in different ways in terms of discharge, velocities and flow depth. In general terms, finite element method discretizes a region in various elements. The variation of the flow variables over each element being approximately expressed by discrete interpolation functions. Their substitution in a functional and its minimization lead to a system of equations for values of flow variables at the nodes which can be solved easily by the methods of linear algebra.

From a study of the relevant literature on watershed routing, it is observed that different dependent variables are chosen for their approximate representation by the discrete basis functions. For example, while Goodrich et al.[1] approximated only the flow depth, Vieux et al.[2] and Taylor et al.[3] introduced basis function representation for depth and velocity, and Jayawardena and White [4] chose discharge and depth for this purpose. However, no discussion is available in literature as to the suitability of these combinations in terms of numerical efficiency. It is the purpose of this paper to compare the solutions obtained through various schemes for a simple overland flow problem for which no analytical solution is available and thereby indicate one that is more accurate.

2 Numerical Development

Watershed routing by one-dimensional kinematic wave model uses continuity equation and a stage-discharge relationship as follow:

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = q \tag{1}$$

$$Q = \alpha h^n \tag{2}$$

where Q is discharge, h is flow depth, q is the lateral inflow minus infiltration, t is time, x is the spatial coordinate, α and n are determined by the characteristic of the terrain such as roughness, slope, etc. In the development of the finite element method, one can choose to work with the approximating functions for various combinations of h, Q or v=Q/h. In the following, we reproduce three of these developments.

Case 1: Interpolating h only — Substituting eqn. (2) in eqn. (1):

$$nV\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} = q \tag{3}$$

If h is approximated by the following interpolation function over a one-dimensional linear element with two nodes i=1,2:

$$h \approx \sum_{i=1}^{2} h_i(t)N_i(x) = \mathbf{N}^T\mathbf{h} \tag{4}$$

then its substitution in eqn. (3) produces a residual error as:

$$R = nv\,\dot{\mathbf{N}}_x^T\mathbf{h} + \mathbf{N}^T\dot{\mathbf{h}} - q \tag{5}$$

Here $N_i(x)$ are the linear shape functions. Following Goodrich et al. [1], v can be attributed its average value over the element. The Galerkin formulation requirement $\int_D \mathbf{N}R dx = 0$ can be expressed as a sum of the integrals evaluated over individual elements of the flow domain. Considering a generic element (i,j) only, the integral leads to the matrix equation:

$$\mathbf{a}\dot{\mathbf{h}} + \mathbf{b}\mathbf{h} + \mathbf{c} = \mathbf{0} \tag{6}$$

$$\begin{aligned}
\mathbf{a} &= \int_e \mathbf{N}\mathbf{N}^T dx = \int_0^L \begin{bmatrix} N_i^2 & N_iN_j \\ N_iN_j & N_j^2 \end{bmatrix} dx = \frac{L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \\
\mathbf{b} &= nv_e \int_e \mathbf{N}\dot{\mathbf{N}}_x^T dx = nv\int_0^L \begin{bmatrix} -\dot{N}_i & \dot{N}_i \\ -\dot{N}_j & \dot{N}_j \end{bmatrix} dx = \frac{nv}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \\
\mathbf{c} &= -q\int_e \mathbf{N} dx = -q\int_0^L \begin{bmatrix} N_i \\ N_j \end{bmatrix} dx = \frac{-qL}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{aligned} \tag{7}$$

Assembling eqns. (6) for all the elements of the flow surface, one obtains the matrix equation for the unknown flow depths **H**,

$$\mathbf{A}\dot{\mathbf{H}} + \mathbf{B}\mathbf{H} + \mathbf{C} = \mathbf{0} \tag{8}$$

where **A**,**B** and **C** are the assembled matrices. This system of first order equations can be solved by a convenient finite difference method. In this formulation, v is the average velocity over the element in consideration.

Case 2: Interpolating h and v — Taylor et al. [3] and Vieux et al. [2] suggested using linear interpolation function for v as,

$$v \approx \sum_{i=1}^{2} v_i(t)N_i(x) = \mathbf{N}^T\mathbf{v} \tag{9}$$

in addition to the approximation for h in eqn. (4). This leads to the system of eqns. (8), matrix **B** now assembled from elemental matrices **b** given by:

$$\mathbf{b} = n\int_e \mathbf{N}\mathbf{N}^T \mathbf{v}\dot{\mathbf{N}}_x^T dx = \frac{n}{L}\int_0^L \begin{bmatrix} N_i^2 & N_iN_j \\ N_iN_j & N_j^2 \end{bmatrix}\begin{bmatrix} -V_i & V_i \\ -V_j & V_j \end{bmatrix} dx$$

$$= \frac{n}{6}\left(\begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix} V_i + \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} V_j\right) \tag{10}$$

Case 3: Interpolating h and Q — Jayawardena & White [4] and Ross et al. [5] proposed linear interpolation for Q:

$$Q \approx \sum_{i=1}^{2} Q_i(t)N_i(t) = \mathbf{N}^T\mathbf{Q} \tag{11}$$

besides eqn. (4) for h choosing not to substitute eqn. (2) in eqn. (1). Here the system of eqns. (8) obtained by Galerkin method is replaced by

$$\mathbf{A}\dot{\mathbf{H}} + \mathbf{B}\mathbf{Q} + \mathbf{C} = \mathbf{0} \tag{12}$$

with **B** now assembled from elemental matrices **b** as:

$$\mathbf{b} = \int_e \mathbf{N}\dot{\mathbf{N}}_x^T dx = \frac{1}{L}\int_0^L \begin{bmatrix} -N_i & N_i \\ -N_j & N_j \end{bmatrix} dx = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \tag{13}$$

A and **C** are calculated as before and **Q** as in eqn. (2).

3 Results and Comparison

A simple example of one-dimensional flow along a 100m inclined plane of unit width due to uniform q = 50mm/h inflow is used to test the results from the three approximations presented above. Parameter values used are $\alpha = 0{,}6\ \mathrm{mm}^{1/2}/\mathrm{s}$; $m = 1{,}5$. The exact solution for this problem by the method of characteristics due to Henderson [6] will be used as the reference for comparison purposes. The solution for discharge at the outlet (Q_O) as function of time is obtained by various schemes considering the plane as made up of N_E elements (N_E = 1, 4, 10 and 20).

Figures 1, 2 and 3 present these results for the three finite element interpolation schemes solving the system of first order differential equations by the 4th order Runge-Kutta method which was found to be equivalent or better than the alternative methods suggested in literature (for example Ross et al. [5] use simple finite differences; Taylor et. al [3] and Goodrich et. al [1] prefer middifference trapezoidal time stepping, Vieux et al.[2] employ time-weighting coefficient in the implicit finite difference approximation and Jayawardena and White [4] explore Crank-Nicolson and backward differencing schemes).

An analysis of these figures shows that while the time response according to the approximation of velocity as an average over the element, as suggested by Goodrich et al. [1], overshoots the steady state response (Fig. 1), using linear interpolation function for velocity in eqn. (9), the numerical solution approaches it from below (Fig. 2). These schemes may require a finer discretization of the plane with inordinate number of elements to produce steady-state value of discharge and thus satisfy continuity. The method of Jayawardena and White [4] which uses linear interpolation function for Q instead of v fairs well reaching steady-state discharge with modest number of elements offering, thus, greater possibility of closing the water balance.

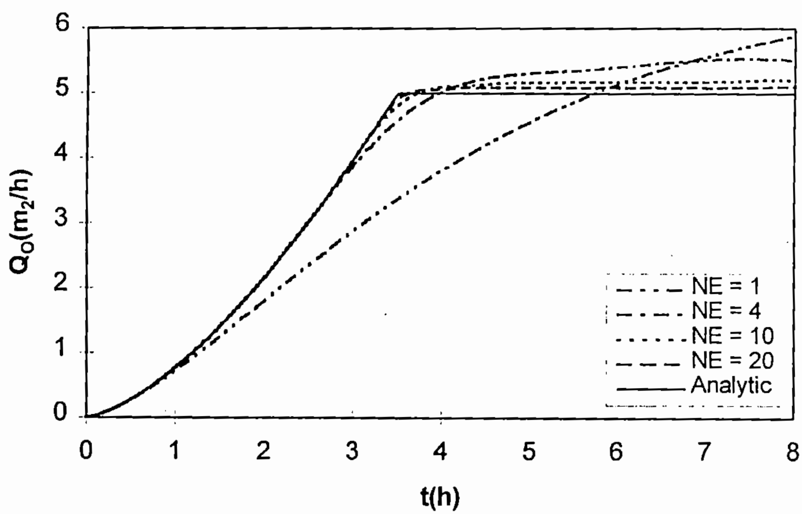

Figure 1. Outlet discharge as function of time by case 1 finite element procedure for various discretizations

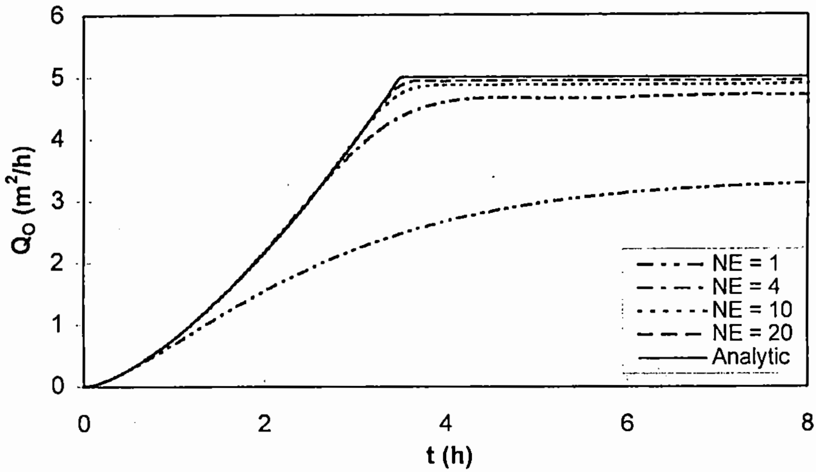

Figure 2. Outlet discharge as function of time by case 2 finite element procedure for various discretizations

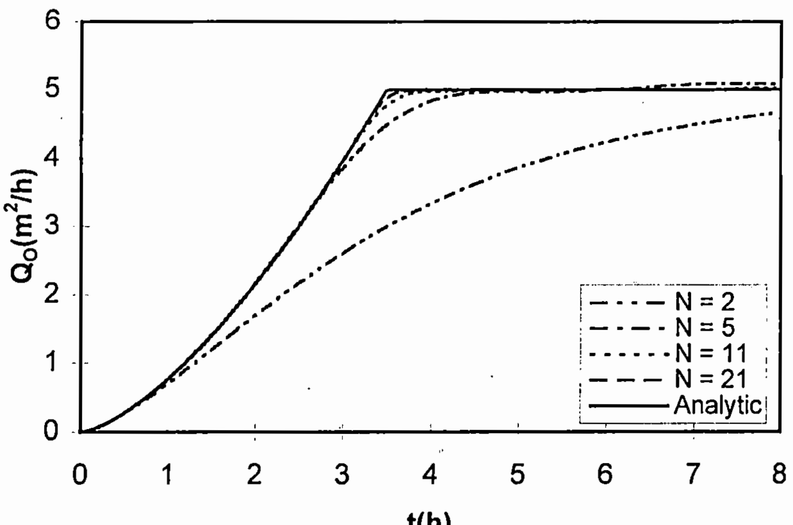

Figure 3. Outlet discharge as function of time by case 3 finite element procedure for various discretizations

Figure 4 shows the approach to steady-state discharge as the discretization becomes finer at the time of concentration $t_C = 3{,}5$ h and at twice t_C. None of the numerical solutions is capable of reproducing the steady-state value at t_C although they come reasonably close to it as the number of elements increases. At $2t_C$, however, the steady state is well approximated by the linear interpolation of discharge with small number of elements. The other two methods may require much larger number of elements.

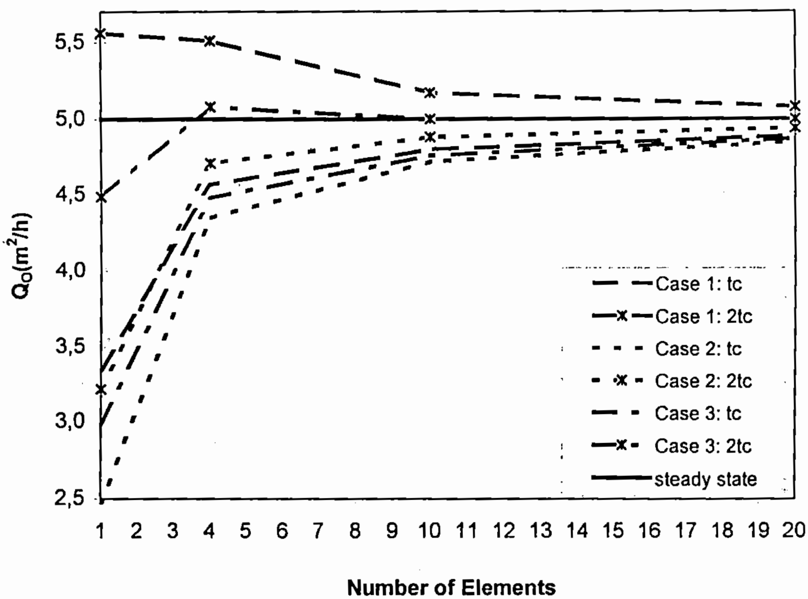

Figure 4: Approach to steady-state outlet discharge for various cases at t_C and 2 t_C as the number of elements increases

4 Conclusions

Numerical schemes employing different combinations of variables and their approximations by linear interpolating functions in the finite element method of solving one-dimensional kinematic overland flow-routing equations were tested against the analytic solution for a simple example. Besides the approximation of flow depth by a linear interpolation function, the other dependent variable approximated was either discharge or velocity. The results of the comparison between various solutions show that the finite element solution of the routing problem expressed in terms of discharge and flow depth matches the analytic solution more closely than when formulated in terms of velocity and depth. Further, the outlet discharge attains steady state in the discharge-depth scheme with much coarser discretization of the physical system than in the alternative schemes. This result should encourage the construction of distributed rainfall-runoff models of watershed response. For example, Franco [7] uses the discharge-depth formulation of kinematic routing as the basis for two-dimensional modeling of overland flow in a hydrographic basin by finite element on a triangular irregular network.

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