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Edgar G. Goodaire
and
Cēsar Polcino Milies

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CENTRAL IDEMPOTENTS IN ALTERNATIVE LOOP ALGEBRAS

EDGAR G. GOODAIRE AND CÉSAR POLCINO MILIES

Abstract. Let L be a Moufang loop with torsion subloop T and suppose that the loop algebra KL of L over a field K is an alternative algebra. In this paper, we find necessary and sufficient conditions which guarantee that every idempotent of KT is central in KL.

1. Introduction

Let L be a Moufang loop with torsion subloop T and let K be a field. Suppose that the loop algebra KL is an alternative algebra and then let $\mathcal{U}(KL)$ denote the loop of units in KL. In the case where $L = G$ is a group, the study of group-theoretical properties of $\mathcal{U}(KG)$—nilpotence, finite conjugacy classes, closure of torsion units under multiplication—has led naturally to the condition that all idempotents of KT are central in KG. See [3, 6, 11] and [13, Chapter VI]. In the nonassociative case, the authors have recently shown that the idempotents of $\mathbb{Q}T$ are central in $\mathbb{Q}L$ whenever the torsion units in $\mathbb{Z}L$ form a subloop [9]. Thus the condition "idempotents of KT central in KL" seems worthy of independent study. Such is the purpose of this article.

This paper requires some background in the theory of *RA loops* which are, by definition, loops L such that RL is an alternative ring, for any commutative and associative ring R with unity. We record briefly some of the results of the theory which are of particular relevance here. The following theorem, which is implicit in [2, Section 3] and amplified in [10] is fundamental.

Theorem 1.1. *A loop L is RA if and only if*

(i) *$L = G \cup Gu$ is the disjoint union of a nonabelian group G and a single coset Gu;*

(ii) *G has a unique nonidentity commutator, s, which is necessarily central and of order 2;*

(iii) *the map* $g \mapsto g^* = \begin{cases} g & \text{if } g \text{ is central} \\ sg & \text{otherwise,} \end{cases}$
is an involution of G (i.e., an antiautomorphism of order 2);

(iv) *multiplication in L is defined by $g(hu) = (hg)u$, $(gu)h = (gh^*)u$, $(gu)(hu) = g_0h^*g$ for $g, h \in G$, where $g_0 = u^2$ is a central element of G.*

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The loop described in this theorem is denoted $M(G,*,g_0)$. It is a Moufang loop and hence *diassociative*: the subloop generated by any two of its elements is a group. More generally, if three elements associate in some order, they also generate a group.

If g, h and k are elements of a loop L, we denote the *commutator* of g and h by (g,h) and the *associator* of g,h,k by (g,h,k). Thus,

$$gh = (hg)(g,h) \text{ and } gh \cdot k = (g \cdot hk)(g,h,k)$$

for any $g,h,k \in L$. If L is the RA loop $M(G,*,g_0)$, then L has a unique nonidentity commutator which must be, of course, the unique nonidentity commutator of G (and is always denoted s) and this element is also a unique nonidentity associator in L. Thus the subloop $L' = \{1,s\}$ generated by the associators and commutators is a central group of order 2.

The centre of a Moufang loop L is the group

$$\mathcal{Z}(L) = \{a \in L \mid (a,g,h) = (a,g) = 1, \text{ for all } g,h \in L\}. \tag{1.1}$$

(Note that elements of the centre are required to associate with all other pairs of elements.) Using Theorem 1.1, it is easy to show that if $L = M(G,*,g_0)$ is an RA loop, then $\mathcal{Z}(L) = \mathcal{Z}(G)$ and the involution $*$ on G extends to an involution on L with the same definition:

$$\ell^* = \begin{cases} \ell & \text{if } \ell \in \mathcal{Z}(L) \\ s\ell & \text{otherwise.} \end{cases} \tag{1.2}$$

For any commutative and associative ring R with unity, this involution extends linearly to an involution of the loop ring RL. Since $L = G \cup Gu$, every $r \in RL$ can be written in the form $r = x + yu$, where x and y are in the group ring RG. Writing r this way, we have $r^* = x^* + syu$. The centre of RL can be described in two useful ways [10, Proposition 2]:

$$\mathcal{Z}(RL) = \{r \mid r \in RL, r^* = r\} \tag{1.3}$$

$$= \{x + yu \mid x,y \in RG, x^* = x, sy = y\}. \tag{1.4}$$

A Moufang loop is a *torsion* loop if all its elements have finite order. For any prime p, by a *p-element*, we mean an element whose order is a power of p. A *p'-element* is an element of finite order relatively prime to p. Throughout this paper, we shall denote by P and A the sets of p- and p'-elements in a loop, respectively, and make liberal use of the following result [2, Theorem 6].

Lemma 1.2. *Let T be a torsion subloop of an RA loop. Then $T = T_0 \times B$ is the direct product of a 2-loop T_0 and a (central) abelian group B all of whose elements have odd order. Thus, for any prime p, the set P of p-elements and the set A of p'-elements of T are subloops of T. Furthermore, $T = P \times A$.*

We require two more lemmas as well. The first uses the fact that an RA loop has the so-called *LC* property: if two elements of an RA loop commute, then one or the other or the product of both is central [2, Theorem 4].

Lemma 1.3. *Let L be an RA loop and B a commutative subloop of L. Then, for any $x \in L$, the subloop $\langle B, x\rangle$ generated by B and x is a group. In particular, B is an abelian group.*

Proof. Let $b_1, b_2 \in B$. Since $b_1b_2 = b_2b_1$, either b_1 is central or b_2 is central, or b_1b_2 is central. In the first two cases we obtain directly from (1.1) that $(b_1, b_2, x) = 1$ while, if $b_1b_2 = z$ is central, then $(b_1, b_2, x) = (zb_2^{-1}, b_2, x) = 1$ because of diassociativity and the identity

$$(ab, c, d) = (a, c, d)(b, c, d) \tag{1.5}$$

which is valid in any RA loop [2, Theorem 3]. Now a straightforward induction argument making further use of (1.5) gives the result. □

Lemma 1.4. *The set T of torsion elements of an RA loop L is a normal, locally finite subloop of L.*

Proof. That T is a normal subloop is known [9, Lemma 2.1]. To establish local finiteness, let H be a finitely generated subloop of T. If H is commutative, it is a group by Lemma 1.3 and readily seen to be finite. If it is not commutative, it contains the commutator/associator subloop L' of L. Then, as a finitely generated torsion abelian group, H/L' is finite, so H is finite because L' is finite. □

2. The Case of Positive Characteristic

In this section, we establish within the context of alternative algebras an analogue of the following theorem of S. Coelho [4].

Theorem 2.1. *Let K be a field of characteristic $p > 0$ and with prime field $\mathcal{P}$. Let G a group whose torsion elements form a locally finite subgroup T. Let P denote the set of p-elements and A the set of p'-elements in T. Then every idempotent of KT is central in KG if and only if the following four conditions hold.*

(i) *A is an abelian group.*
(ii) *If A is not central, then the algebraic closure $\overline{\mathcal{P}}$ of $\mathcal{P}$ in K is finite and, for all $t \in A$ and $x \in L$, there exists $r \in \mathbb{N}$ such that $xtx^{-1} = t^{p^r}$. Furthermore, each such r is a multiple of the degree $[\overline{\mathcal{P}} : \mathcal{P}]$.*
(iii) *P is a subgroup of G.*
(iv) *$T = P \times A$.*

If G is a group contained in an RA loop L, it is clear from Lemma 1.4 that the torsion elements of G form a locally finite subgroup T of G and, from Lemma 1.2, that the sets P of p-elements and A of p'-elements of G are always subgroups of G with $T = P \times A$. Thus, for such G, we have the following simplification.

Theorem 2.2. *Let K be a field of characteristic $p > 0$ with prime field $\mathcal{P}$ and let G be a group which is contained in an RA loop. Then every idempotent of KT is central in KG if and only if*

(i) *T is an abelian group, and*
(ii) *if T is not central, then the algebraic closure $\overline{\mathcal{P}}$ of $\mathcal{P}$ in K is finite and, for all $t \in A$ and $x \in L$, there exists $r \in \mathbb{N}$ such that $xtx^{-1} = t^{p^r}$. Furthermore, each such r is a multiple of $[\overline{\mathcal{P}} : \mathcal{P}]$.*

In what follows, it is convenient to have a term by which we can refer to the loop elements which actually appear in the representation of a loop ring element. For an element $\alpha = \sum \alpha_\ell \ell$, $\alpha_\ell \in R$, $\ell \in L$, in a loop ring RL, the *support* of α is the set

$$\mathrm{supp}(\alpha) = \{\ell \mid \alpha_\ell \neq 0\}.$$

Also, for $\alpha = \sum \alpha_\ell \ell$, the *augmentation* of α is the element $\epsilon(\alpha) = \sum \alpha_\ell \in R$. The map $\epsilon \colon RL \to R$ is a ring homomorphism. We now have at hand the tools needed to establish the main theorem of this section.

Theorem 2.3. *Let K be a field of characteristic $p > 0$ with prime field $\mathcal{P}$. Let L be an RA loop with torsion subloop T. Then every idempotent of the alternative loop algebra KT is central in KL if and only if $p = 2$ or*

(i) *the set A of p'-elements of L is an abelian group, and*
(ii) *if A is not central, the algebraic closure $\overline{\mathcal{P}}$ of $\mathcal{P}$ in K is finite and, for all $t \in A$ and $x \in L$, there exists $r \in \mathbb{N}$ such that $xtx^{-1} = t^{p^r}$. Furthermore, each such r is a multiple of $[\overline{\mathcal{P}} : \mathcal{P}]$.*

Proof. Suppose every idempotent in KT is central in KL and $p \neq 2$. By Lemma 1.2, we can write $T = T_0 \times B$, where T_0 is a 2-loop and B is a central group consisting of elements of odd order. If T_0 is not commutative, any two noncommuting elements of T_0 will generate a nonabelian group T_1 which is finite because T is locally finite. Since $|T_1|$ is relatively prime to p, the group algebra $\mathcal{P}T_1$ is the direct sum of finite simple associative algebras. If any of these is not commutative, it cannot be a division ring, so it is an $n \times n$ matrix algebra for some $n > 1$. As such, it contains noncentral idempotents, a contradiction. Thus T is commutative and hence, by Lemma 1.3, an abelian group. Since $A \subseteq T$, so also is A an abelian group. If A is not central and $x \in L$, then $G = \langle A, x\rangle$ is a group, by Lemma 1.3. Now (iii) follows directly from Theorem 2.2.

For the converse, suppose first that $p = 2$. Then A is the (central) group B of Lemma 1.2 and so $T = T_0 \times A$ where T_0 is a 2-loop. Let e be an idempotent of KT. Since T is locally finite, replacing T by the loop generated by the support of e, we may assume that T is finite. We have $KA = \oplus K_i$, the direct sum of fields K_i, and $e \in (KA)T_0 = \oplus K_i T_0$. Writing $e = \sum e_i$ with $e_i \in K_i T_0$, we note that each e_i is an idempotent in $K_i T_0$, so its augmentation is 0 or 1. Thus either e_i or $1 + e_i$ (which is also an idempotent) is in the kernel, $\Delta_{K_i}(T_0)$, of the natural map $K_i L \to K_i[L/T_0]$. Since K_i has characteristic 2 and T_0 is a 2-loop, this ideal is nilpotent [13, Lemma I.2.21], so $e_i = 0$ or $e_i = 1$. Thus e is in KA and hence central in KL.

Now suppose that p is odd and that we have (i) and (ii). Write $T = T_0 \times B$ as in Lemma 1.2 and note that T is an abelian group because $T_0 \subseteq A$. Let $e \in KT$ be an idempotent and let $x \in L$. Then $G = \langle T, x\rangle$ is a group whose torsion subgroup is T. By Theorem 2.2, e commutes with x, and it follows that e is central in KL. □

3. The Case of Characteristic 0

Let α and β be nonzero elements of a field F. Recall that a *generalized quaternion algebra* over F is an (associative) algebra (F, α, β) of dimension 4 over F with basis $1, i, j, ij$, where $i^2 = \alpha$, $j^2 = \beta$ and $ij = -ji$. With $\alpha = \beta = -1$, the algebra $(F, -1, -1)$ is known simply as *the quaternion algebra over* F and denoted $H(F)$. With F the field R of real numbers, $H(\mathrm{R})$ is, of course, the well-known quaternion algebra of Sir William Rowan Hamilton.

Let $H = (F, \alpha, \beta)$ be a generalized quaternion algebra. Let γ be another nonzero element of F and let ℓ be an indeterminate. Then the *Cayley-Dickson algebra* $(F, \alpha, \beta, \gamma)$ is the vector space $H \oplus H\ell$ with multiplication defined by

$$(a + b\ell)(c + d\ell) = (ac + \gamma\bar{d}b) + (da + b\bar{c})\ell. \tag{3.1}$$

Here, $\bar{q}$ denotes the conjugate of the quaternion q: for $q = a_1 + a_2 i + a_3 j + a_4 ij$, $\bar{q} = a_1 - a_2 i - a_3 j - a_4 ij$.

We require the following facts about quaternion and Cayley-Dickson algebras.

Lemma 3.1. *The quaternion algebra* $H(F) = (F, -1, -1)$ *is either a division ring or the ring of* 2×2 *matrices of* F. *It is a division ring if and only if the equation* $x^2 + y^2 = -1$ *has no solutions in the field* F. *The Cayley-Dickson algebra* $(F, -1, -1, -1)$ *is either a division ring or* $H \oplus H\ell$ *where* H *is the ring of* 2×2 *matrices over a field and multiplication is given by* (3.1) *with* $\gamma = 1$. *It is a division ring if and only if the equation* $x^2 + y^2 + z^2 + w^2 = -1$ *has no solutions in* F.

Proof. The statement about the quaternion algebra is proven, for instance, in [7, Proposition 8.6.2]. The stated facts concerning the Cayley-Dickson algebra are in [12, Sections III.4 and III.5] and [8, Corollary 3.5]. □

S. Coelho and C. P. Milies have established the following result [5].

Theorem 3.2. *Let* K *be a field of characteristic* 0 *and let* T *denote the set of torsion elements of a group* G. *Then every idempotent of* KT *is central in* KG *if and only if*

(i) *For every* $t \in T$ *and every* $x \in G$, *there exists a positive integer* j *such that* $xtx^{-1} = t^j$. *Furthermore, for every noncentral element* $t \in T$, K *contains no root of unity of order the order of* t.

(ii) *Either* T *is abelian or* $T = A \times Q$ *where* A *is an abelian group and* Q *is the quaternion group of order* 8 *and, for every* $a \in A$ *of order* n *and every* n*-th root of unity* ξ *in an algebraic closure of* K, *the field* $K(\xi)$ *contains no solution of the equation* $x^2 + y^2 = -1$.

The analogue for alternative loop algebras is this:

Theorem 3.3. *Let* K *be a field of characteristic* 0 *and let* L *be an RA loop with torsion subloop* T. *Then every idempotent in* KT *is central in* KL *if and only if*

(i) *for every* $t \in T$ *and every* $x \in L$, *there exists a positive integer* j *such that* $xtx^{-1} = t^j$;

(ii) *for every noncentral element* $t \in T$, K *contains no root of unity of order the order of* t; *and*

(iii) *either*

(a) T *is an abelian group, or*

(b) $T = A \times Q$ *where* A *is an abelian group and* Q *is the quaternion group of order* 8 *and, for every* $a \in A$ *of order* n *and every* n*-root of unity* ξ *in an algebraic closure of* K, *the field* $K(\xi)$ *contains no solutions to* $x^2 + y^2 = -1$, *or*

(c) $T = A \times C$ *where* A *is an abelian group and* C *is the Cayley loop and, for every* $a \in A$ *of order* n *and every* n*-th root of unity* ξ *in an algebraic closure of* K, *the field* $K(\xi)$ *contains no solutions to* $x^2 + y^2 + z^2 + w^2 = -1$.

Proof. Suppose every idempotent of KT is central in KL. Let $t \in T$ have order n. Then the element $e = \frac{1}{n}(1 + t + \cdots + t^{n-1})$ is an idempotent in KT and so central in KL. Thus, for any $x \in L$, we have $xex^{-1} = e$, so that $xtx^{-1} = t^j$ for some integer j which we can assume to be positive. This gives (i) and shows, incidentally, that every subloop of T is commutative or Hamiltonian. If T contains a noncentral element t, of order n, say, write $K\langle t\rangle = \oplus K(\xi_i)$ as the direct sum of cyclotomic fields for various primitive roots of unity, ξ_i, at least one of which, say ξ_1, has order n. Let $x \in L$ be an element which does not commute with t. Since $K(\xi_i) = K\langle t\rangle e_i$ for some idempotent e_i of $K\langle t\rangle$, and since every idempotent of KT is central in KL, conjugation by x defines an automorphism of $K\langle t\rangle$ which induces an automorphism θ of $K(\xi_1)$. Since $xtx^{-1} = t^j \neq t$, $\theta(\xi_1) = \xi_1^j \neq \xi_1$, so $\xi_1 \notin K$. Thus no nth root of unity is in K. This establishes (ii).

If T is commutative, then T is an abelian group by Lemma 1.3 and we have case iii(a). If T is a Hamiltonian group, then it is well-known that $T = A \times Q$ where A is an abelian group and Q is the quaternion group Q of order 8. For any $a \in A$ of order n, T contains the direct product $\langle a\rangle \times Q$, so KT contains $K\langle a\rangle Q$ which is the direct sum $\oplus K(\xi_i)Q$ of group algebras of Q over cyclotomic fields $K(\xi_i)$, where some ξ_i, say ξ_1, is an nth root of unity. Now $K(\xi_1)Q$ is the direct sum of four copies of $K(\xi_1)$ and the quaternion algebra $H(K(\xi_1))$. Since it contains no noncentral idempotents, $H(K(\xi_1))$ must be a division ring so, by Lemma 3.1, we have case iii(b). Finally, if T is not associative, then $T = A \times C$ where A is an abelian group and C is the Cayley loop [1, Section IV.7]. Since the loop algebra of C over a field F is the direct sum of eight copies of F and the Cayley-Dickson algebra $(F, -1, -1, -1)$ (see [8, Proposition 3.9] where C is denoted $M_{16}(Q)$), we see just as before that if $a \in A$ has order n and ξ is an n-th root of unity, then $(K(\xi), -1, -1, -1)$ must be a division algebra. By Lemma 3.1 we have case iii(c).

Conversely, assume that (i), (ii) and (iii) hold. If T is an abelian group and x is any element of L, then $G = \langle T, x\rangle$ is a group by Lemma 1.3, so any idempotent in KT commutes with x by Theorem 3.2. In case iii(b), we proceed as in [5]. Let $e \in KT$ be an idempotent. Replacing T by the subgroup generated by the support of e, we may assume that T is finite. Let A have exponent n and let ξ be a primitive nth root of unity. We have $KA = \oplus K_i$, the direct sum of cyclotomic fields $K_i \subseteq K(\xi)$. By hypothesis, each quaternion algebra $H(K_i)$ is a division ring. Since $KT = (KA)Q = \oplus K_iQ$, and $K_iQ = 4K_i \oplus H(K_i)$, we see that KT is the direct sum of division rings, so each idempotent of KT is central in KT. Since every such idempotent is the sum of primitive idempotents, to show that it is in fact central in KL, it is sufficient to show that each primitive idempotent of K_iQ is central in KL. For this, we first note that any idempotent μ of KA is central in KL, by the following argument. If $x \in L$, then $G = \langle A, x\rangle$ is a group whose torsion subgroup is an abelian group contained in T. By Theorem 3.2, μ is central in KG, so μ commutes with x and it follows that μ is central in KL, as asserted. Next, presenting Q as

$$Q = \langle a, b \mid a^4 = 1, b^2 = a^2, ba = a^{-1}b\rangle,$$

we see that the unique nonidentity commutator of L is $s = (a, b) = a^2$ and so, in view of (1.2), the restriction to Q of the involution on L is defined by

$$g^* = \begin{cases} g & \text{if } g = 1 \text{ or } g = a^2 \\ a^2 g & \text{otherwise.} \end{cases}$$

The primitive idempotents of K_iQ are

$$e_1 = \frac{\mu_i}{8}(1 + a + a^2 + a^3 + b + ab + a^2 + a^3b)$$
$$e_2 = \frac{\mu_i}{8}(1 + a + a^2 + a^3 - b - ab - a^2 - a^3b)$$
$$e_3 = \frac{\mu_i}{8}(1 - a + a^2 - a^3 + b - ab + a^2b - a^3b)$$
$$e_4 = \frac{\mu_i}{8}(1 - a + a^2 - a^3 - b + ab - a^2b + a^3b)$$
$$e_5 = \frac{\mu_i}{8}(1 - a^2)$$

where μ_i is the identity element of K_i which, as an idempotent of KA, is central in KL. For each i, we have $e_i^* = e_i$ so that, by (1.3), e_i is central in KL.

Finally, suppose we have case iii(c). Again letting n be the exponent of A and ξ a primitive nth root of unity, we have $KT = (KA)C = \oplus K_iC$, for certain fields $K_i \subseteq K(\xi)$, and each $K_iC = 8K_i \oplus (K_i, -1, -1, -1)$ is the direct sum of division rings by Lemma 3.1. So every idempotent in KT is central in KT and, to show centrality in KL, it is sufficient to show that any primitive idempotent of K_iC is central in KL. Representing C as $M(Q, *, u)$ for some $u \notin Q$. the primitive idempotents of K_iC are e_5 and, for $j = 1, 2, 3, 4$, the eight elements $e_{j1} = \frac{\mu_i}{2}(1 + u)e_j$ and $e_{j2} = \frac{\mu_i}{2}(1 - u)e_j$. Each μ_i, being an idempotent of KA, is central in KL as before. Since $e_{ij} = x + yu$, with $x, y \in K_iQ$, $x^* - x$ and $sy = y$, each e_{ij} is central because of (1.4) and the proof is complete. □

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Memorial University of Newfoundland, St. John's, Newfoundland, Canada A1C 5S7
E-mail address: edgar@math.mun.ca

Universidade de São Paulo, C. P. 20570, 01452-990 São Paulo, Brasil
E-mail address: polcino@ime.usp.br

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