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Interpolation by analytic functions on preduals of Lorentz sequence spaces

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INTERPOLATION BY ANALYTIC FUNCTIONS ON PREDUALS OF LORENTZ SEQUENCE SPACES

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ABSTRACT. Let (e_n) be a canonical basis of the predual of the Lorentz sequence space $d_*(w,1)$. We consider the restriction operator R associated to the basis (e_i) from some Banach space of analytic functions into the complex sequence space and we characterize the ranges of R.

1. INTRODUCTION

Let E be a complex Banach space with a Schauder basis (e_i). Let $\mathcal{F}$ be a space of continuous functions on a subset of E which contains the basis (e_i). We will interest in the following formulation of interpolation problems. We consider the *restriction operator* R associated to the basis (e_i) defined by

$$\begin{aligned} R:\mathcal{F} &\to \mathbb{C}^{\mathbb{N}} \\ f &\mapsto (f(x_i))_{i\in\mathbb{N}}, \end{aligned}$$

and then we will ask the range of R for certain spaces of analytic functions. The motivation to study this ranges is based in the papers of Aron-Globevnik [1] and Jaramillo [5]. Indeed, Aron and Globevnik have caracterized the range of R for various nice spaces $\mathcal{F}$ of analytic functions on the space c_0. And Jaramillo has studied the relationship between reflexivity of $\mathcal{F}$, where $\mathcal{F}$ could be certain spaces of real valued infinitely differentiable functions, and the ranges of R.

We are intrerested here in the spaces $\mathcal{F}$ given by $\mathcal{A}^\infty(B_E)$, the space of all the analytic functions on the open unit ball of E and bounded, continuous on the closed unit ball and by the subspace $\mathcal{A}_U(B_E)$ of $\mathcal{A}^\infty(B_E)$ where the functions are uniformly continuous on the closed unit ball, in case $E = d_*(w,1)$ is the preduals of Lorentz sequences space. Also we are interested in the spaces $\mathcal{F}$ giving by n-homogenous polynomials on $d_*(w,1)$. In spite of the canonical basis on predual of Lorentz space has the similar properties of the canonical basis of c_0, the ranges of R from theses spaces above mencioned are totally different in case $E = c_0$.

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Now we fix some notation. In that follow, B_E is the closed unit ball of a complex Banach space E. We consider a decreasing sequence $w = (w_i)_{i \in \mathbb{N}}$ of positive real numbers satisying $w \in c_0 \setminus l_1$ and $w_1 = 1$ and the complex Lorentz sequences space $d(w, 1)$ given by

$$d(w,1) = \Big\{ x = (x_n) : \sup\{ \sum_{n=1}^{\infty} |x_{\pi(n)}| w_n : \pi \text{ is a permutation of } \mathbb{N} \} < +\infty \Big\}.$$

The norm is given by

$$\|x\|_{d(w,1)} := \sup_{\pi \in \Pi} \sum_{i=1}^{\infty} |x_{\pi(i)}| w_i < \infty.$$

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where Π is the set of all permutations of the natural numbers. It is well-known and easy to verify that the above supremum is attained for the decreasing rearrangement of x. The usual vector basis (e_n) is a Schauder basis. A somehow canonical predual $d_*(w, 1)$ of $d(w, 1)$ is given by

$$d_*(w,1) = \Big\{ x = (x_i)_{i \in \mathbb{N}} \in c_0 : \lim_{k \to \infty} \frac{\sum_{n=1}^{k} [x]_i}{\sum_{i=1}^{k} w_i} = 0 \Big\},$$

where $([x]_i)$ is the decreasing rearrangement of $(|x|_i)$ and the norm on $d_*(w, 1)$ is given by

$$\|x\| = \sup_{k \to \infty} \frac{\sum_{n=1}^{k} [x]_i}{\sum_{i=1}^{k} w_i}.$$

The space $d_*(w, 1)$ has a Schauder basis (e_n).

2. POLYNOMIALS

In this section we are interested to caracterize the range of restriction operator R when $\mathcal{F}$ is the space of m-homogeneous polynomials on the predual of Lorentz space $d_*(w, 1)$.

For a complex Banach E with dual E', B_E denotes the closed unit ball of E. $P(^m E)$ denotes the Banach space of all continuous m-homogeneous polynomials on E with the norm $\|P\| = \sup_{x \in B_E} |P(x)|$.

In [1] Aron and Globevnik showed that if $E = c_0$ the range of R for $\mathcal{F} = P(^n c_0))$ is the space $l_1 = c_0'$, for all $n \in \mathbb{N}$. The natural question here is: if the Banach space E has a Schauder basis with similar properties of the canonical basis of c_0 (for example shrinking or unconditional) is it possible the range of $R(P(^nE)) = E'$? We are going to show that in spite of the Schauder basis in $d_*(w,1)$ has the mencined above properties the restrition operator R is totally different in the predual of Lorentz space.

We recall (see [1]) that for every natural number $n \geq 2$, the generalized Rademacher functions (s_j) are defined inductively as follows. Let $\alpha_1 = 1, \alpha_2, \ldots, \alpha_n$ be the complex n-th roots of unit. For $j = 1, \cdots, n$ let $I_j = (\frac{j-1}{n}, \frac{j}{n})$ and let I_{j_1,j_2} denote the j_2-th open subinterval of lenght $\frac{1}{n^2}$ of I_{j_1} ($j_1.j_2 = 1, 2, \cdots, n$). Proceeding like this, it is clear how to define the interval $I_{j_1,\cdots,j_k}$ for any k. Now $s_1 : [0,1] \to \mathbb{C}$ is defined by setting $s_1(t) = \alpha_j$ for $t \in I_j$, where $1 \leq j \leq n$. In general, $s_k(t)$ is defined to be α_j if t belongs to the subinterval $I_{j_1,\cdots,j_k}$ where $j_k = j$. There is no harm in setting $s_k(t) = 1$ for all endpoints t.

The next lemma gives us the basic properties of the sequence (s_k) of generalized Rademacher functions which we will need. The verificatiom of these properties follows exactly the same lines as the corresponding result for the classical Rademacher functions.

Lemma 2.1. *For each $n = 2, 3, \ldots$, the associated Rademacher's functions $\{s_k\}_{k\in\mathbb{N}}$ satisfy the following properties:*

(a) $|s_k(t)| = 1, \forall k \in \mathbb{N}, \forall t \in [0,1]$.

(b) *For any $k_1, \ldots, k_n$,*

$$\int_0^1 s_{k_1}(t) \cdots s_{k_n}(t)dt = \begin{cases} 1, & \text{if } k_1 = k_2 = \cdots = k_n; \\ 0, & \text{otherwise.} \end{cases}$$

Proposition 2.2. *For each $n \in \mathbb{N}$, let $P \in P\left(^n d_*(w,1)\right)$. Then*

$$\|(P(e_i))_i\|_{d(w^n,1)} \leq \|P\|,$$

where w^n denotes the sequence $(w_i^n)_i$.

Proof. Let $P \in P\left(^n d_*(w,1)\right)$. Considering a permutation $\pi : \mathbb{N} \to \mathbb{N}$. We define

$$\lambda_i = \begin{cases} \dfrac{\overline{|P(e_i)|}}{P(e_i)}, & \text{if } P(e_i) \neq 0; \\ 1, & \text{if } P(e_i) = 0. \end{cases}$$

Hence, $\lambda_i P(e_i) = |P(e_i)|$. Let $\beta_i \in \mathbb{C}$ such that $\beta_i^n = \lambda_i$. Let $\check{P}$ denote the symmetric n-linear associated mapping to P. Let (s_j) be the sequence of generalized

Rademacher functions corresponding to n. For each $m \in \mathbb{N}$ we get

$$\begin{aligned}
\sum_{i=1}^{m} w^n_{\pi(i)} |P(e_i)| &= \sum_{i=1}^{m} w^n_{\pi(i)} \lambda_i P(e_i) \\
&= \sum_{i,i_2,\dots,i_n=1}^{m} \Big(\int_0^1 s_i(t) \cdots s_{i_n}(t) dt \Big) w^n_{\pi(i)} \lambda_i \check{P}(e_i, \dots, e_{i_n}) \\
&= \int_0^1 \Big(\sum_{i,i_2,\dots,i_n=1}^{m} \beta_i^n w^n_{\pi(i)} s_i(t) \cdots s_{i_n}(t) \check{P}(e_i, \dots, e_{i_n}) \Big) dt \\
&= \int_0^1 \check{P}\Big(\sum_{i=1}^{m} \beta_i w_{\pi(i)} s_i(t) e_i, \dots, \sum_{i_n=1}^{m} \beta_{i_n} w_{\pi(i_n)} s_{i_n}(t) e_i\Big) dt \\
&= \int_0^1 P\Big(\sum_{i=1}^{m} \beta_i w_{\pi(i)} s_i(t) e_i\Big) dt. \quad (*)
\end{aligned}$$

For each $t \in [0,1]$, we define $z(t) = \sum_{i=1}^{m} \beta_i w_{\pi(i)} s_i(t) e_i$. So, $|z(t)_i| = |\beta_i w_{\pi(i)} s_i(t)| = 1 \cdot w_{\pi(i)} \cdot 1 = w_{\pi(i)}$, if $i \leq m$, and $|z(t)_i| = 0$, if $i > m$.
Hence,

$$\|z(t)\|_{d_*(w,1)} = \sup_l \frac{\sum_{i=1}^{l} [z(t)]_i}{\sum_{i=1}^{l} w_i} \leq \sup_l \frac{\sum_{i=1}^{l} w_i}{\sum_{i=1}^{l} w_i} = 1,$$

In the last inequality we used that the sequence (w_i) is decreasing. Consequently, for each $t \in [0,1]$, $|P(z(t))| \leq \|P\|$. Then, for $(*)$, we get

$$\sum_{i=1}^{m} w^n_{\pi(i)} |P(e_i)| = \int_0^1 P(z(t)) dt \leq \|P\|.$$

Since m is arbitrary, $\sum_{i=1}^{\infty} w^n_{\pi(i)} |P(e_i)| \leq \|P\|$, therefore

$$\|(P(e_i))_i\|_{d(w^n,1)} = \sup_{\pi} \sum_{i=1}^{\infty} w^n_{\pi(i)} |P(e_i)| \leq \|P\|.$$

$\square$

From this proposition we conclude that $R(P({}^n d_*(w,1))) \subset d(w^n, 1)$. Our main is to determine $R(P({}^n d_*(w,1)))$. In order to do that we will establish the following lemma.

Lemma 2.3. *Let $p \geq 1$ and let $k \in \mathbb{N}$. Given $\alpha_1, \ldots, \alpha_k$ positive real numbers, then there exists π^0 in the group of permutation S_k such that for every $x \in B_{d_*(w,1)}$ we have*

$$\sum_{j=1}^{k} \alpha_j |x_j|^p \leq \sum_{j=1}^{k} \alpha_j w^p_{\pi^0(j)}.$$

Before we proof the lemma we need some additional proposition, which the proof is in [2] and let us recall that a point e of a convex subset A of a vector space is called *extremal* if $e = tx + (1-t)y$ for some $t \in (0,1)$ then, it has to be $e = x = y$. We will denote by $ext(A)$ the set of extremal points of A and $B^k_{d_*(w,1)}$ will denote the unit ball of $\mathbb{C}^k$ with the norm given by $d_*(w,1)$.

Proposition 2.4. [2] *The extremal points of $B^k_{d_*(w,1)}$ are the ones that $(x_1, \ldots, x_k, 0, 0, \ldots)$ where there exists $\pi \in S_k$ such that $|x_i| = w_{\pi(i)}$.*

Proof of lemma 2.3: Among all the permutations $\pi \in S_k$, let π^0 be the one that maximize the sum $\sum_{j=1}^{k} \alpha_j w^p_{\pi(j)}$. Let $x \in B_{d_*(w,1)}$ we consider $\tilde{x} = (x_1, \ldots, x_k, 0, 0, \ldots)$. It is easy to see that $\tilde{x} \in B^k_{d_*(w,1)}$. By Krein-Milman's theorem, we have $B^k_{d_*(w,1)} = \overline{\text{co}}(extB^k_{d_*(w,1)})$. So, $\tilde{x} \in \overline{\text{co}}(extB^k_{d_*(w,1)})$. Firstly, we suppose that $\tilde{x} \in \text{co}(extB^k_{d_*(w,1)})$. Hence, $\tilde{x} = \sum_{i=1}^{m} \lambda^i x^i$, where $\lambda^i > 0$, $\sum_{i=1}^{m} \lambda^i = 1$, and for each i, x^i is an extremal point of $B^k_{d_*(w,1)}$. So, for each $1 \leq i \leq m$, there exists $\pi^i \in S_k$ such that $|x^i_j| = w_{\pi^i(j)}, \forall j \leq k$. Then, for each $j \leq k$ we have that

$$\begin{aligned}\sum_{j=1}^{k} \alpha_j |x_j|^p &= \sum_{j=1}^{k} \alpha_j \Big|\sum_{i=1}^{m} \lambda^i x^i_j\Big|^p \leq \sum_{j=1}^{k} \alpha_j \sum_{i=1}^{m} \lambda^i |x^i_j|^p = \sum_{i=1}^{m} \lambda^i \sum_{j=1}^{k} \alpha_j w^p_{\pi^i(j)} \\ &\leq \sum_{i=1}^{m} \lambda^i \sum_{j=1}^{k} \alpha_j w^p_{\pi^0(j)} = \sum_{j=1}^{k} \alpha_j w^p_{\pi^0(j)}.\end{aligned}$$

In case $\tilde{x} \in \overline{\text{co}}(extB^k_{d_*(w,1)}) \setminus \text{co}(extB^k_{d_*(w,1)})$, we can just consider a sequence in $\text{co}(extB^k_{d_*(w,1)})$ which converges to $\tilde{x}$ then it takes the limit. □

In the next theorem we will use the lemma above in order to determine $R(P(^n d_*(w,1)))$.

Theorem 2.5. *For each $n \in \mathbb{N}$, $R(P(^n d_*(w,1))) = d(w^n, 1)$.*

Proof. Using the proposition 2.2 we get, $R(P(^n d_*(w,1))) \subset d(w^n,1)$. On the other hand, let $y = (y_i) \in d(w^n, 1)$ and let consider the polynomial in $d_*(w,1)$ defined by

$$P(x) = \sum_{i=1}^{\infty} y_i x_i^n, \ x = (x_i).$$

By lemma 2.3 P is well defined since, for each $x \in B_{d_*(w,1)}$,

$$\sum_{i=1}^{k} |y_i||x_i|^n \leq \sum_{i=1}^{k} |y_i| w^n_{\pi^0(i)} \leq \|y\|_{d(w^n,1)},$$

for all $k \in \mathbb{N}$. Then $\sum_{i=1}^{\infty} |y_i x_i^n| < \infty$. Obviously $R(P) = y$. ☐

Remark 2.1. The lemma 2.3 could be used to give another proof for the well-known result : if $w \in l_p$, for $p > 1$, then $d_*(w,1) \subset l_p$. It could be done just taking the α_j 's equals to 1 and so we get

$$\sum_{j=1}^{k} |x_j|^p \leq \sum_{j=1}^{k} w^p_{\pi^0(j)} \leq \|w\|_p^p < \infty.$$

3. ANALYTIC FUNCTIONS

In this section, we will discuss the behaivor of the range of the restriction operator R for the following Banach spaces of analytic functions:

$$\mathcal{A}^\infty(B_E) = \{f : B_E \to \mathbb{C} : f \text{ is analytic in } \overset{\circ}{B}_E, \text{ continuous and bounded in } B_E\}$$

and

$$\mathcal{A}_U(B_E) = \{f \in \mathcal{A}^\infty(B_E) : f \text{ is uniformly continuous}\}.$$

We remark that theses space are the natural generalization in infinite dimensional of the disc algebra.

In [1] Aron and Globevnik havei proved that any sequence of 0 and 1 can be interpolated by a function in $\mathcal{A}^\infty(B_{c_0})$ with norm 1. More precisely, if $S \subset \mathbb{N}$ is an arbitrary set, then there exists a function with norm 1 in $\mathcal{A}^\infty(B_{c_0})$ such that $f(e_n) = 1$, if $n \in S$ and $f(e_n) = 0$, if $n \notin S$. Besides, if S is finite, f can be taken in $\mathcal{A}_U(B_{c_0})$. An analogous result in $d_*(w,1)$ holds, since for each $x \in d_*(w,1)$ we have

$$\|x\|_\infty = [x]_1 \leq \sup_k \frac{\sum_{n=1}^{k} [x_n]}{\sum_{n=1}^{k} w_n} = \|x\|_{d_*(w,1)},$$

that means the canonical inclusion $i : d_*(w,1) \to c_0$ is continuous and, consequently, uniformly continous and analytic. More precisely, we have the following lemma.

Lemma 3.1. *Let S and S' be disjointed subsets of $\mathbb{N}$. Then:*

(i) There exists a function $f \in \mathcal{A}^\infty(B_{d_*(w,1)})$ of norm lower or equal than 2 such that

$$f(e_n) = \begin{cases} 1, & \text{if } n \in S; \\ -1, & \text{if } n \in S'; \\ 0, & \text{otherwise.} \end{cases}$$

(ii) If both of the sets S and S' are finite, then the function f above can be taken in $\mathcal{A}_U(B_{d_*(w,1)})$.

Using the previous lemma, we obtain the following properties of $R(\mathcal{F})$ being $\mathcal{F}$ the mentioned spaces.

Proposition 3.2. *(i) Given $x \in l_\infty$, there exists $f \in \mathcal{A}^\infty(B_{d_*(w,1)})$ such that $R(f) = x$ and $\|f\| \leq 4\|x\|_\infty$. Consequently, $R(\mathcal{A}^\infty(B_{d_*(w,1)})) = l_\infty$.*

(ii) Given $x \in c$, there exists $f \in \mathcal{A}_U(B_{d_*(w,1)})$ such that $R(f) = x$ and $\|f\| \leq 10\|x\|_\infty$. Hence $c \subset R(\mathcal{A}_U(B_{d_*(w,1)}))$.

Proof. (i): If $x = 0$, it is enough to take $f \equiv 0$. Let $x \neq 0$. First assume that for each n, $x_n \in \mathbb{R}$. So, for each $n \in \mathbb{N}$, $\frac{x_n}{\|x\|} \in [-1,1]$ and we write $\frac{x_n}{\|x\|}$ in its binary representation, it means, $\frac{x_n}{\|x\|} = \sum_{j=1}^{\infty} 2^{-j}\alpha_{n_j}$, where each α_{n_j} is 0, 1 or -1. For each j, let $S_j = \{n \in \mathbb{N} : \alpha_{n_j} = 1\}$ and $S_j' = \{n \in \mathbb{N} : \alpha_{n_j} = -1\}$, and let consider the function F_j obtained by using the lemma 3.1, item (i). Let $f = \sum_{j=1}^{\infty} 2^{-j}\|x\|F_j$. We can see that

$$f(e_n) = \sum_{j=1}^{\infty} 2^{-j}\|x\|F_j(e_n) = \|x\| \sum_{j=1}^{\infty} 2^{-j}\alpha_{n_j} = \|x\|\frac{x_n}{\|x\|} = x_n,$$

and for this case

$$\|f\| \leq \sum_{j=1}^{\infty} 2^{-j}\|x\|\|F_j\| \leq 2\|x\|_\infty.$$

In the general case, for each $n \in \mathbb{N}$ take $x_n = p_n + iq_n$, where $p_n, q_n \in \mathbb{R}$. Hence using the proof of real case we get that f_p and f_q and we consider $f = f_p + i f_q$ with $\|f\| \leq \|f_p\| + \|f_q\| \leq 4\|x\|_\infty$. So, f is the desired function.

(ii): Let $x \in c$. We assune that for each $n \in \mathbb{N}$, $x_n \in \mathbb{R}$. Let $l = \lim_n x_n$ and let define $\beta_n = x_n - l$ and $\beta = (\beta_n)_n$. Hence, $x_n = l + \beta_n$ and $\|\beta\| \leq 2\|x\|_\infty$. Now using the same argument of item (i) for β, we obtained the functions F_j in $\mathcal{A}_U(B_{d_*(w,1)})$, since $\beta_n \to 0$ we have that the sets $S_j = \{n \in \mathbb{N} : \alpha_{n_j} = 1\}$ and

$S'_j = \{n \in \mathbb{N} : \alpha_{n_j} = -1\}$ are finite Hence, $f = \sum_{j=1}^{\infty} 2^{-j}\|\beta\|F_j + l$ is the function we were looking for, and in this case

$$\|f\| \leq 2\|\beta\|_\infty + |l| \leq 4\|x\| + \|x\| = 5\|x\|.$$

The general case it is similar to the item (i). We write each β_n in the form $p_n + iq_n$, where $p_n, q_n \in \mathbb{R}$ and we get f such that $\|f\| \leq 10\|x\|_\infty$. □

The above proposition gave us $c \subset R(\mathcal{A}_U(B_{d_*(w,1)}))$. But, we characterize, in the next theorem, under some hypothesis on w, the range of the restiction operator R associated to the usual basis of $d_*(w,1)$.

Theorem 3.3. *Let $w \in c_0 \setminus l_1$ be a decreasing sequence of positive real numbers. So, $R(\mathcal{A}_U(B_{d_*(w,1)})) = c$ if, and only if, $w \notin l_p$, $\forall p > 1$. If $w \in l_p$ for some $p > 1$, then $R(\mathcal{A}_U(B_{d_*(w,1)})) = l_\infty$*

Proof. Let us assume that $w \notin l_p$, $\forall p > 1$. In view of proposition 3.2 it will suficient to show $R(\mathcal{A}_U(B_{d_*(w,1)})) \subset c$. Let $f \in \mathcal{A}_U(B_{d_*(w,1)})$. As f is uniform continuous given $\varepsilon > 0$, there exists $\delta > 0$ such that $\|x - y\| < \delta \Rightarrow |f(x) - f(y)| < \varepsilon/2$. Hence, taking $1 - \delta < r < 1$ we have that for all $x \in B_{d_*(w,1)}$, $\|x - rx\| < 1 - r < \delta$ and therefore,

$$|f(rx) - f(x)| < \varepsilon/3, \quad \forall x \in B_{d_*(w,1)}.$$

The function $x \mapsto f(rx)$ is analytic and bounded in $\frac{1}{r}\overset{\circ}{B}_{d_*(w,1)}$. Hence, the power series of $f(r\,\cdot\,)$ at zero converges uniformly in $\frac{1}{r}\overset{\circ}{B}_{d_*(w,1)}$ (see theorem 7.13 in [6]). Then, there exist $m \in \mathbb{N}$ and $P_k \in P({}^k d_*(w,1))$, $k = 0, 1, \ldots, m$, such that

$$|f(rx) - \sum_{k=0}^{m} P_k(x)| < \varepsilon/2, \quad \forall x \in B_{d_*(w,1)}.$$

Therefore, for all $x \in B_{d_*(w,1)}$, we have

$$|f(x) - \sum_{k=0}^{m} P_k(x)| \leq |f(x) - f(rx)| + |f(rx) - \sum_{k=0}^{m} P_k(x)| + < \varepsilon,$$

in particular, $|f(e_n) - \sum_{k=0}^{m} P_k(e_n)| < \varepsilon$.

As $w \notin l_p$, $\forall p \geq 1$, by [7] it follows that, for each $k = 1, \ldots, m$, the polynomials P_k are weakly sequentially continuous, it means P_k applies weakly convergent sequences in convergent sequences. Since (e_n) converges weakly to zero, we have, for each $k = 1, \ldots, m$, $P_k(e_n)$ converges to zero and so

$$\lim_n |f(e_n) - \sum_{k=0}^{m} P_k(e_n)| = |\lim_n f(e_n) - f(0)| \leq \varepsilon.$$

Hence $f(e_n) \to f(0)$ and $R(\mathcal{A}_U(B_{d_*(w,1)})) \subset c$.

In case $w \in l_p$ for some $p > 1$, by remark 2.1 we have that $d_*(w,1) \subset l_N$, for $N > p$. Hence, given any sequence $y = (y_n) \in l_\infty$, we can define a polynomial in $P(^N d_*(w,1))$ by

$$P(x) = \sum_{n=1}^{\infty} y_n x_n^N, \ x = (x_n).$$

Therefore, $R(P) = y$. The proof is now complete. □

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