RT-MAE 2013-02

INFLUENCE MEASURES IN QUANTILE REGRESSION MODELS

by

Bruno R. Santos
and
Silvia N. Elian

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Classificação AMS: 62J05

- Fevereiro de 2013 -

Influence measures in quantile regression models

Bruno R. Santos and Silvia N. Elian

Departamento de Estatística
Instituto de Matemática e Estatística,
Universidade de São Paulo
CP 66281 - São Paulo, Brasil
E-mails: bramos@ime.usp.br
selian@ime.usp.br

Abstract

In this paper, we use the asymmetric Laplace distribution to define a new method to determine the influence of a certain observation in the fit of quantile regression models. Our measure is based on the likelihood displacement function and we propose two types of measures in order to determine influential observations in a set of conditional quantiles conjointly or in each conditional quantile of interest. We verify the validity of our average measure in a simulated data set as well in an illustrative example with data about air pollution.

Keywords: Analysis of influence ; Quantile Regression ;Likelihood displacement.

1 Introduction

In regression analysis, it is usual to try to determine whether one observation might influence the fitted model or not. Some important works in this area are Cook (1977), Cook and Weisberg (1982), Atkinson (1981, 1982) and Belsley, Kuh and Welsch (1980), just to name a few. For our work, we consider the measure discussed in Cook, Peña and Weisberg (1988) known as likelihood displacement measure.

However, when we study quantile regression models (Koenker, 2005), we can not find in the literature related studies. Considering L_1 regression, which is a particular case of the quantile regression, Elian et al. (2000) examined a measure based on the likelihood displacement function. With respect to quantile regression models, we have to mention that there is a further difficulty, since one could be interested in studying several conditional quantiles of the response variable. Along with this, it should be noted the concern about the estimation of the model parameters, because these assessments of influence usually compare the likelihood of the models of interest in some way. The second concern might be discarded with the help of recent works, such as Koenker and Machado (1999) and Yu and Zhang (2005), where the quantile regression models are connected with the asymmetric Laplace distribution. With this result, it is possible to calculate a proper likelihood and make comparisons between models.

For the question regarding several conditional quantiles estimates, we propose in Section 2 one possible measure with the intention of identifying influential observations for m fitted models, $m \geq 1$.

The rest of the article is organized as follows. We give a brief introduction to the main methods used in this article, such as the quantile regression, the asymmetric Laplace distribution and the likelihood displacement function, in Section 2. Our proposed measure to determine possible influential observations in quantile regression models is presented with some simulation studies in Section 3. An application to a data set with information about air pollution in US cities is given in Section 4. We finish with some comments in Section 5.

2 Methods

2.1 Quantile regression

Since its introduction in the article of Koenker and Bassett (1978), quantile regression has become not only an alternative to the least squares method, but a major tool to study the correspondence between the response variable Y and explanatory variables not only in a central position, with the conditional mean or median, but also in several other quantiles of the conditional distribution of the response variable. To this end, we give a brief introduction to these models in this subsection.

Consider the following linear regression model

$$y_i = x_i'\beta_\tau + \epsilon_i \tag{1}$$

where y_i is ith observation of the vector $n \times 1$ of the response variable Y, x_i'

is the ith row of the matrix X $n \times p$ with the explanatory variables, ϵ_i is the associated error of this model with the τth quantile equal to zero and β_τ is the vector $p \times 1$ of parameters associated to the τth conditional quantile of the response variable. With these definitions, we conclude that

$$Q_{y_i}(\tau|x_i) = x_i'\beta_\tau,$$

where $Q_{y_i}(\tau|x_i)$ is the τth conditional quantile of y_i given x_i. Koenker and Bassett (1978) defined that the quantile regression estimator, $\hat{\beta}_\tau$, for the parameters of the model in (1) is obtained by finding

$$\min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} \rho_\tau(y_i - x_i'\beta), \tag{2}$$

where $\rho_\tau(u) = u(\tau - I(u < 0))$ and $I(a)$ is the indicator function taking value of one when a is true and zero otherwise.

For applications of these models, we refer to Yu et al. (2003) and Koenker (2005). In the next subsection, we make the connection between the asymmetric Laplace distribution and the minimization problem in (2).

2.2 Asymmetric Laplace distribution

We consider in this article the definition of the asymmetric Laplace distribution (ALD) of Yu and Zhang (2005). In this way, we assume that if $Y \sim \text{ALD}(\mu, \sigma, \tau)$, then its density function is given by

$$f(y; \mu, \sigma, \tau) = \frac{\tau(1-\tau)}{\sigma} \exp\left\{-\frac{\rho_\tau(y-\mu)}{\sigma}\right\},$$

where $0 < \tau < 1$ is the skew parameter, $\sigma > 0$ is the scale parameter and $-\infty < \mu < \infty$ is the location parameter.

If we assume that the errors in (1) are distributed according to an asymmetric Laplace distribution, with $\mu = 0$, and the design matrix X is not stochastic, then we can conclude that y_i is also distributed according with an asymmetric Laplace distribution, with location parameter equal to $x_i'\beta_\tau$. It is reasonable to consider only models with fixed τ, the skew parameter of the distribution.

From this definition, we can calculate the log-likelihood function $L(\beta, \sigma)$ from a sample of n independent observations as

$$L(\beta_\tau, \sigma) = n \log \tau(1-\tau) - n \log \sigma - \frac{1}{\sigma} \sum_{i=1}^{n} \rho_\tau(y_i - x_i'\beta_\tau). \qquad (3)$$

From (3), we have that the maximum likelihood estimator for the scale parameter of the model, σ, is

$$\hat{\sigma} = \frac{1}{n} \sum_{i=1}^{n} \rho_\tau(y_i - x_i'\hat{\beta}_\tau).$$

Beyond that, we can see that the maximum likelihood estimator for β_τ is obtained minimizing the sum in (3). This is the same problem proposed by Koenker (1978) and defined in (2). Linear programming algorithms can be used to obtain $\hat{\beta}_\tau$ (Koenker, 2005).

In the next subsection, we propose an influence measure in quantile regression models using the likelihood displacement function, considering the results presented in this subsection.

2.3 Likelihood displacement

The likelihood displacement function was first suggested by Cook and Weisberg (1982) to help determine influential observations in linear regression models. In Cook et al. (1988), this proposed function is discussed and compared to other measures. We can define it as

$$LD_i(\theta) = 2[L(\hat{\theta}, y) - L(\hat{\theta}_{(i)}, y)], \quad i = 1, \ldots, n, \tag{4}$$

where $\hat{\theta}$ is the maximum likelihood estimator of θ based on all observations and $\hat{\theta}_{(i)}$ is the maximum likelihood estimator based on all observations except the observation i. If the value for this function for the ith observation is large, then this observation is influential because its deletion may cause changes in important conclusions of the model fitted.

In quantile regression models, $\theta = (\beta_\tau, \sigma)$, as we are considering only models with fixed τ. For now on, let $L(\hat{\theta})$ be the log-likelihood obtained from a sample of n observations.

We can rewrite the expression in (4) in our problem, as

$$LD_i(\beta_\tau, \sigma) = 2[L(\hat{\beta}_\tau, \hat{\sigma}) - L(\hat{\beta}_{\tau(i)}, \hat{\sigma}_{(i)})],$$

where $\hat{\beta}_{\tau(i)}$ is the quantile regression estimator of β_τ based on all observations except observation i. It can be shown that $\hat{\sigma}_{(i)}$ is given by

$$\hat{\sigma}_{(i)} = \frac{1}{n-1} \sum_{j=1, j \neq i}^{n} \rho_\tau(y_j - x_j' \hat{\beta}_{\tau(i)}).$$

From (3), we have that

$$L(\hat{\beta}_\tau, \hat{\sigma}) = n\left[\log \tau(1-\tau) - \log \hat{\sigma} - 1\right]$$

and

$$L(\hat{\beta}_{\tau(i)}, \hat{\sigma}_{(i)}) = n\left[\log \tau(1-\tau) - \log \hat{\sigma}_{(i)}\right] - (n-1) - \rho_\tau(y_i - x_i'\beta_{\tau(i)})/\hat{\sigma}_{(i)}.$$

Therefore, the likelihood displacement function for the ith observation is

$$LD_i(\beta_\tau, \sigma) = 2\left[n \log\left(\frac{\hat{\sigma}_{(i)}}{\hat{\sigma}}\right) + \frac{\rho_\tau(y_i - x_i'\beta_{\tau(i)})}{\hat{\sigma}_{(i)}} - 1\right]. \qquad (5)$$

As in Elian at al. (2000), the measure in (5) will be large if the ratio between the estimates of the scale parameter with and without the observation i is too big or if $\rho_\tau(y_i - x_i'\beta_{\tau(i)})/\hat{\sigma}_{(i)}$ is large, or both. Also, we can say that when the ith observation is not influential, then the estimate $\hat{\sigma}_{(i)}$ should be very similar to $\hat{\sigma}$, i.e., $\hat{\sigma}_{(i)}/\hat{\sigma} \cong 1$, and $\rho_\tau(y_i - x_i'\beta_{\tau(i)})$ should be close to the mean of the weighted absolute residuals, $\hat{\sigma}_{(i)}$, resulting in $\rho_\tau(y_i - x_i'\beta_{\tau(i)}) \cong \hat{\sigma}_{(i)}$. With this configuration, the likelihood displacement function for the ith observation should be close to zero, i.e., $LD_i(\beta_\tau, \sigma) \cong 0$. In other words, the likelihood displacement function will be close to zero when the observation i is not influential. On the contrary, large values could denote observations that might be influential.

Another possible approach for this kind of analysis, as suggested by Cook et al. (1988), is to determine the possible influence of the observation i only in the parameter β_τ, using a conditional likelihood function on σ. For this, we must consider the following measure

$$LD_i(\beta_\tau|\sigma) = 2[L(\hat{\beta}_\tau, \hat{\sigma}) - \max_{\sigma} L(\hat{\beta}_{\tau(i)}, \sigma)].$$

We should note that

$$L(\hat{\beta}_{\tau(i)}, \sigma) = n \log \tau(1-\tau) - n \log \sigma - \frac{\sum_{j=1}^{n} \rho_\tau(y_j - x_j'\hat{\beta}_{\tau(i)})}{\sigma}$$

is maximized when $\hat{\sigma}_m = \sum_{j=1}^{n} \rho_\tau(y_j - x_j'\hat{\beta}_{\tau(i)})/n$.

Therefore, we have that the conditional likelihood displacement function on σ is given by

$$LD_i(\beta_\tau|\sigma) = 2n \log \left(\frac{\sum_{j=1}^{n} \rho_\tau(y_j - x_j'\hat{\beta}_{\tau(i)})}{\sum_{j=1}^{n} \rho_\tau(y_j - x_j'\hat{\beta}_\tau)} \right). \tag{6}$$

Again, we conclude that when the ith observation is not influential, then this measure $LD_i(\beta_\tau|\sigma)$ will be close to zero, because we compare the sum of the weighted absolute residuals with and without the observation i and we expect that these values are close when the deletion of this observation does not affect much the fitted model.

It is possible to rewrite (6) as

$$LD_i(\beta_\tau|\sigma) = 2n \log \left(1 + \lambda_{(i)}\right),$$

where

$$\lambda_{(i)} = \frac{\sum_{j=1}^{n} \rho_\tau(y_j - x_j'\hat{\beta}_{\tau(i)}) - \sum_{j=1}^{n} \rho_\tau(y_j - x_j'\hat{\beta}_\tau)}{\sum_{j=1}^{n} \rho_\tau(y_j - x_j'\hat{\beta}_\tau)},$$

which could be interpreted as the relative increase in the sum of weighted absolute residuals when $\hat{\beta}_\tau$ is replaced by $\hat{\beta}_{\tau(i)}$.

Still about influence measures, we could be interested in trying to determine whether an observation is influential in a set of m conditional quantiles. In order to do that, we need to define a new measure regarding the m fitted models. It is reasonable to expect that a certain observation i should be considered influential in this set of models, if this observation is influential in most of the models, if not all. In this way, we suggest the use of an average likelihood displacement measure.

Let $\{\tau\}_m = \{\tau_1, \ldots, \tau_m\}$, $m \geq 1$ be the set of quantiles of interest and $LD_i(\beta_{\tau_j}, \sigma)$, $i = 1, \ldots, n$, $j = 1, \ldots, m$, the likelihood displacement function for the ith observation of the jth model. We should define the average likelihood displacement function, for each i, as follows

$$LD_i(\beta_{\{\tau\}_m}, \sigma) = \frac{\sum_{j=1}^{m} LD_i(\beta_{\tau_j}, \sigma)}{m}.$$

If the ith observation is influential in most of the models, then we should expect that $LD_i(\beta_{\{\tau\}_m}, \sigma)$ would be large compared to other observations. It is important to notice that as m increases, more difficult will be to determine if there is any influential observations in the set of models. Also, if $m = 1$, then we have the likelihood displacement function defined in (5). Using the same arguments, we could define the average conditional likelihood displacement function as

$$LD_i(\beta_{\{\tau\}_m}|\sigma) = \frac{\sum_{j=1}^{m} LD_i(\beta_{\tau_j}|\sigma)}{m}.$$

We have to note that despite the fact that only the parameter β_τ is indexed by the fixed parameter τ, the parameter σ also depends on each τ chosen in the regression analysis.

Moreover, Cook et al. (1988) showed that the values for the likelihood displacement functions could be compared with percentiles of a chi-squared distribution, with the appropriate degrees of freedom. But the required properties of this result are not met for the asymmetric Laplace distribution. Still, we strongly believe from personal experiences with influential analysis in regression models, that some type of comparison between the graphs with the influence measures against the order of observations along with a study of the variation in the estimates of the models with and without a few observations is enough when trying to find influential observations in the data set. In fact, we suggest the following steps in this type of analysis:

1. plot the (conditional) likelihood displacement values against the order of the observations and check if there is any point that is too separated from the others;

2. fit again your models without these points and verify if there is any relevant change on your conclusions.

Again, we do believe that these two steps are sufficient in the search for influential points in quantile regression models. It should be pointed out, though, that we are only considering the possibility of a small set of influential observations in this approach.

In the next section, we use simulated data to show some of the properties of the measures proposed here in this subsection.

3 Simulation studies

In order to to check some properties of the proposed measures in the previous subsection we simulated a simple data set. We consider the following linear model to generate the data

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

where we set $\beta_0 = 0$, $\beta_1 = 1$, x_i is uniform on (0,10) and u_i is standard normal, $i = 1, \cdots, 50$. The variables x and u are mutually independent. For the quantiles, we are interested in $\tau = 0.25, 0.50, 0.75$.

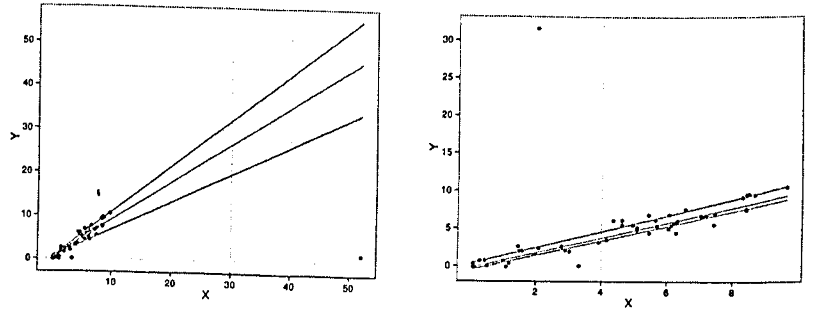
We wanted to check the measures proposed with two situations of outlying observations:

Situation 1: Outlying observations in the explanatory variables, but not in response variable.

Situation 2: Outlying observations in the response variable, but not in the explanatory variable.

In both cases, one observation was moved away from the bulk of points according to each desired situation. The arbitrarily chosen observation to be moved was of number 25, or simply #25.

Situation 1 is pictured in Figure 1 (a), showing the fitted models for each quantile of interest. Clearly, the estimate for the slope parameter is modified by the presence of observation #25, as we expected similar estimates for all conditional quantiles.



(a) (b)

Figure 1: Dispersion graph and the conditional quantiles estimated for $\tau = 0.25, 0.50, 0.75$. (a) Situation 1. (b) Situation 2.

Additionally, Situation 2, where the outlying observations were produced with respect to the response variable, is represented in Figure 1 (b). In this case, the modified observation does not seem to affect the fitted models, for any τ. This could be explained by the fact that quantile regression models are robust to outlying observations in the response variable. Precisely, when we move a certain observation at the response variable value but do not modify the sign of its residual, then the fitted model does not change at all (Koenker and Bassett, 1978).

The conditional likelihood displacement measure proposed for β_τ reach similar conclusions about the influential points in both situations. We show here just the results for Situation 1 in Figure 2 and conclude that the observation #25 should be referenced as highly influential for $\tau = 0.25$ and to a lesser extent for $\tau = 0.50$, but only in Situation 1.

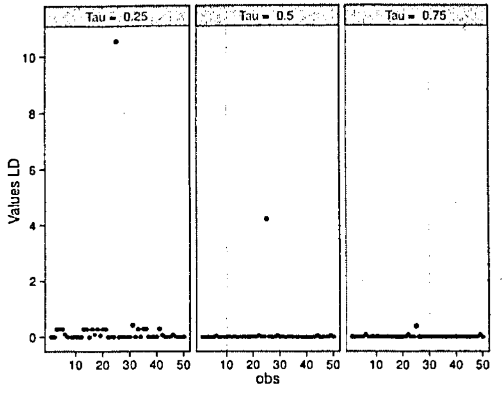


Figure 2: Conditional likelihood displacement measure against the order of observations for Situation 1 and each τ.

In the next section, we will discuss the use of our proposed measures in a real data set.

4 Application

In our application, we examine the data from Sokal and Rohlf (1981) and reproduced in Hand et al. (1994), about the air pollution in 41 US cities, with observations between 1969 and 1971. The response variable is the annual mean concentration of sulphur dioxide, in micrograms per cubic meter. This data can be used to investigate hypotheses about determinants of pollution, as we have other observed variables for each city, such as average annual temperature in degrees Fahrenheit (TEMP), number of manufacturing enterprises employing 20 or more workers (MAN), population size (1970 census) in thousands (POP), average annual wind speed in miles per hour (WIND), average annual precipitation in inches (RAIN).

We consider the following linear model for the conditional quantiles

$$Q_{SO2}(\tau|X) = \beta_0 + \beta_1\text{TEMP} + \beta_2\text{MAN} + \beta_3\text{POP} + \beta_4\text{WIND} + \beta_5\text{RAIN}, \quad (7)$$

where $Q_{SO2}(\tau|X)$ represents the τth conditional quantile of sulphur dioxide given the explanatory variables. With the sample size equal to 41, we construct confidence intervals for the parameters of the models with the rank score method proposed by Gutenbrunner and Jureckova (1992), and suggested by Koenker (2005) for problems with small sample size. We take $\tau = 0.25, 0.50, 0.75$, since we are interested in the effects of the covariates in not just a central position, but also in the lower and the upper tail of the conditional distribution of the concentration of sulphur dioxide in the air.

The estimates for each parameter and correspondent confidence interval inside the brackets are presented in the Table 1.

Table 1: Estimates and confidence interval for the parameters in the model in (7) and $\tau = 0.25, 0.50, 0.75$.

	Conditional quantiles		
Variables	0.25	0.50	0.75
(Intercept)	81.863	96.464	121.028
	[37.036;139.126]	[81.954;144.567]	[16.777;157.411]
TEMP	-0.792	-0.859	-1.118
	[−1.686;−0.168]	[−1.865;−0.723]	[−1.868;−0.343]
MAN	0.042	0.055	0.063
	[−0.025;0.110]	[0.0364;0.082]	[0.033;0.072]
POP	-0.010	-0.028	-0.035
	[−0.081;0.018]	[−0.056;−0.005]	[−0.046;−0.017]
WIND	-4.446	-3.789	-4.688
	[−7.100;−0.192]	[−6.841;−1.883]	[−6.481;2.349]
RAIN	0.277	0.174	0.380
	[0.084;1.379]	[0.084;0.840]	[0.047;0.705]

From a quick overview of the results in Table 1, we can note that the coefficient for the effect of the variable POP is not significant for the conditional quantile 0.25, the same way that the coefficient for the effect of the variable WIND in the quantile 0.75. Also, the variable MAN is not significant in the 0.25th quantile.

Now, to determine the potential influential observations we analyzed the values of the conditional likelihood displacement measure for each τ of interest and also the proposed average measure. The results are displayed in Figures 3 and 4, respectively.

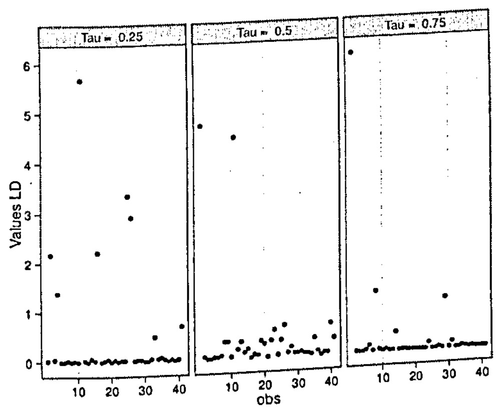


Figure 3: Conditional likelihood displacement values for the air pollution data.

Since distinct points could be considered influential examining Figure 3 for each τ, we took the points dislocated from the bulk of points in Figure 4. Being so, we have the observations #1 and #11 as the ones that stand out from the others. City number one is the city of Phoenix and city number 11 is the city of Chicago.

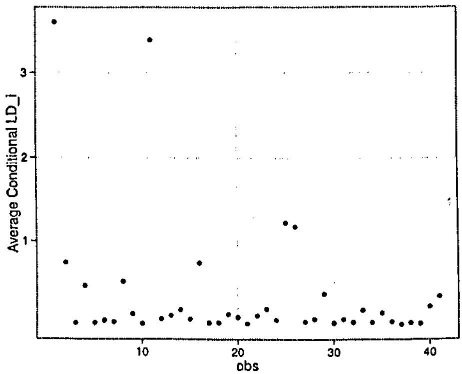


Figure 4: Average conditional likelihood displacement values for the air pollution data.

If we choose not to consider these cities to fit the quantile regression models, we obtain different estimates from the ones in the Table 1, but the conclusions that we reach with the confidence intervals only change with the deletion of Chicago. For this reason, we present only the estimates of the parameters of the models without Chicago in Table 2.

Table 2: Estimates and confidence interval for the parameters in the model in (7) and $\tau = 0.25, 0.50, 0.75$, without the city of Chicago.

	Conditional quantiles		
Variables	0.25	0.50	0.75
(Intercept)	80.508	94.889	118.954
	[30.927;146.903]	[76.748;154.469]	[20.328;162.528]
TEMP	-0.867	-0.799	-1.086
	[−1.962;−0.252]	[−1.839;−0.579]	[−2.402;−0.823]
MAN	0.014	0.075	0.065
	[−0.034;0.105]	[0.004;0.090]	[0.053;0.073]
POP	0.002	-0.040	-0.036
	[−0.085;0.033]	[−0.054;0.006]	[−0.049;−0.019]
WIND	-2.579	-3.726	-4.660
	[−6.987;−0.786]	[−6.712;−2.616]	[−6.352;3.193]
RAIN	0.104	0.113	0.381
	[0.081;1.324]	[0.061;0.876]	[0.082;0.691]

The comparison of the inference for the variable POP gives us a new conclusion about the effect of this variable in the concentration of sulphur dioxide. Instead of having an impact in the conditional median and in the 0.75 quantile, after the deletion of Chicago, the variable POP should not be considered significant in the conditional median, since the range of the confidence interval for this variable changes, adding zero as one of its components. In this way, we should consider that population has a negative effect only in the upper tail of the conditional distribution of the concentration of sulphur dioxide.

One possible explanation for this change is that the city of Chicago is the city with the biggest population among the cities of the sample. Clearly, the presence of this observation is affecting the conclusions about the variable population and its association with the concentration of sulphur dioxide in the quantile regression models of interest.

One important remark that should be made about this example is that the city of Chicago, which had a greater impact on the three models conjointly was identified using the proposed average conditional likelihood displacement measure in this article. This outcome shows the effectiveness of this measure when there is the interest in studying several quantile regression models at once.

5 Concluding Remarks

In this paper, we have proposed influence measures based on the likelihood displacement for quantile regression models. We used the association between quantile regression models and the asymmetric Laplace distribution in order to be able to determine an appropriate quantity that tries to identify influential observations in these models. We proposed the use of an average value of this likelihood displacement measure when dealing with more than one conditional quantile. This suggestion was supported in an illustrative example to study the variables that could affect the pollution in the air in US cities. We gave a few steps to analyze the measures proposed here, since we were not able to compare values of this measure with percentiles of a chi-squared distribution.

Acknowledgements

Special thanks to FAPESP for financial support of the first author.

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