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Gröbner Basis in Algebras Extended by Loops

G. Chalom, E. Marcos,
P. Oliveira

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We dedicate this work to the 60th birthday of Otto Kerner.
We also take the opportunity to thank here Prof. Ed. Green for suggesting the subject.

Abstract

Gröbner basis are a very powerful instrument, not only in abstract algebra but also in applications to computer science, as can be seen in [3] (section 4 and 5) and [1]. In this work, we were looking to extend some results obtained in the commutative context to the path algebras context, that is, as was proved in [2], the natural context of Gröbner bases. Also, we discuss, in the last section, some similar results obtained by [2], section 8.

Resumo

As bases de Gröbner tem se mostrado um poderoso instrumento, tanto em álgebra abstrata como em aplicações à computação, como pode ser observado em [3](seções 4 e 5) e[1].

Neste trabalho, procuramos estender resultados obtidos em um contexto comutativo para o contexto de álgebras de caminhos, que é, como foi provado em [2], o contexto natural das bases de Gröbner.

Discutiremos também, na última seção, resultados próximos aos obtidos em [2], seção 8.

1 Preliminaries

In this section, we will define some concepts that will be used in the following sections. All this concepts can be found in [3], with a detailed description of the theory of Gröbner basis.

Let $\mathcal{K}$ be a field and Λ a $\mathcal{K}$-algebra with a $\mathcal{K}$-basis $\mathcal{B} = \{b_i\}_{i \in \mathcal{I}}$ fixed, where $\mathcal{I}$ is a set of index.

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As $\mathcal{B}$ is a $\mathcal{K}$-basis of Λ, for each $a \in \Lambda$, there is a family $(\lambda_i)_{i\in\mathcal{I}}$ such that $a = \sum_{i\in\mathcal{I}} \lambda_i b_i$, where $\lambda_i = 0$, except for a finite number of indices.

If $a = \sum_{i\in\mathcal{I}} \lambda_i b_i$, we will say that b_i *occurs in* a if $\lambda_i \neq 0$.

One of the main facts in the theory of Gröbner basis is the choice of a well ordered basis. We recall that a *well-order* $>$ in $\mathcal{B}$, is a total order in $\mathcal{B}$, where every non empty subset of $\mathcal{B}$ has a minimal element. We recall that one of the consequences of the Choice Axiom, is that any set can be well ordered.

Definition. 1.1 [3] *If $\mathcal{B} = \{b_i\}_{i\in\mathcal{I}}$ is a $\mathcal{K}$-basis of Λ, as a vector space, well ordered by $>$ in $\mathcal{B}$, and if $a = \sum_{i\in\mathcal{I}} \lambda_i b_i$ is non zero, we will call tip of a and denote by $Tip(a)$ the largest basis element in the support of a and its coefficient λ_i is denoted by $CTip(a)$.*

If X is a subset of Λ, we define

(i) $Tip(X) = \{b \in \mathcal{B} : b = Tip(x) \text{ for some } 0 \neq x \in X\}$

(ii) $NonTip(X) = \mathcal{B} \setminus Tip(X)$

So, both $Tip(X)$ and $NonTip(X)$ are subsets of $\mathcal{B}$ depending on the choice of the well order of $\mathcal{B}$.

We assume that, for every $b, b' \in \mathcal{B}$ we have $bb' \in \mathcal{B}$ or $bb' = 0$.

Such $\mathcal{K}$-basis is said a *multiplicative basis* of Λ.

We are not interested in an arbitrary order in $\mathcal{B}$, but we want an order that preserves the multiplicative structure of $\mathcal{B}$.

Definition. 1.2 [3] *We will say that a well order in $\mathcal{B}$, is admissible, if it satisfies the following conditions, for every $p, q, r, s \in \mathcal{B}$:*

(i) If $p < q$ then $pr < qr$, if both are non zero;

(ii) If $p < q$ then $sp < sq$, if both are non zero;

(iii) If $p = sqr$ then $p \geq q$.

We will say that a $\mathcal{K}$-algebra Λ has *Gröbner basis theory* if Λ has a multiplicative basis $\mathcal{B}$ with an admissible order $>$ in this basis. From this point, we assume that the $\mathcal{K}$-algebra Λ has a Gröbner basis theory. Let I be a two-sided ideal in Λ.

Definition. 1.3 [3] *We will say that a set $\mathcal{G} \subset I$ is a Gröbner basis for I with respect to the order $>$, if*

$$\langle Tip(\mathcal{G})\rangle = \langle Tip(I)\rangle$$

as two-sided ideals.

Definition. 1.4 [3] *Let $b_1, b_2 \in X \subset \Lambda$, we will say that b_1 divides b_2 (in X) if there exist $c, d \in X$ such that $b_2 = b_1 d$, $b_2 = cb_1$ or $b_2 = cb_1 d$.*

2 Homogenization

In this section, we present our main results, that extend not only algorithms used in commutative algebra, but also some results obtained in [2].

In [1] we can see the *homogenization process*, for the commutative case, in the polynomial ring on n variables. As the polynomial ring on n commutative variables is a special case of path algebras and that any algebra with 1 that admits Gröbner basis theory is isomorphic to a quotient of a path algebra, see [2], we asked ourselves if the same process (that is, the homogenization process) can be extended, and which results remain true in the case of path algebras. Here, we consider the non commutative version of the homogenization process, for path algebras $\mathcal{K}\mathcal{Q}/I$, where I is a two-sided ideal in $\mathcal{K}\mathcal{Q}$.

In [2], the author used a similar technic of the extension by loops, to construct Gröbner basis to some indecomposable projectives in Mod-$\mathcal{K}\mathcal{Q}$, based on a special admissible order, where the loops where always maximal elements.

In our work, we will consider any admissible order, fixed, and extend it to the extended by loops algebra.

Let $\mathcal{K}$ be a field and Q a finite quiver. Let $\Lambda = \mathcal{K}Q/I$ be the path algebra associated to Q and I a two-sided ideal of $\mathcal{K}Q$. Consider in Λ the multiplicative basis $\mathcal{B}$ and $>$ an admissible order in $\mathcal{B}$.

Then, we define $\tilde{Q}$, where $\tilde{Q}$ has the same vertices of Q and for each vertex i of Q, we add a loop l_i in $\tilde{Q}_1$, and we consider the $\mathcal{K}$-algebra $\Lambda' = \mathcal{K}\tilde{Q}/\tilde{I}$, where $\tilde{I} = \langle I, \alpha Z - Z\alpha\rangle$ as a two-sided ideal of $\mathcal{K}\tilde{Q}$, with $\alpha \in \mathcal{Q}_1$ and $Z = \sum_{i \in |\mathcal{Q}_0|} l_i$. We call Λ' the *Extended by Loops Algebra* of Λ. For Λ' we consider the following basis $\tilde{\mathcal{B}} = \{Z^n b : b \in \mathcal{B} \textit{ and } n \geq 0\}$. Both Λ and Λ' are finitely generated as $\mathcal{K}$-algebras, then $\mathcal{B}$ and $\tilde{\mathcal{B}}$ are finitely generated as semigroups. Then, we fix for $\mathcal{B}$ a minimal generator set U, as a semigroup. For $\tilde{\mathcal{B}}$, consider $\tilde{U} = U \cup \{l_i : i \in |\mathcal{Q}_0|\}$.

Define $\ell(b)$ the *length* of $b \in \mathcal{B}$ (respectively $\tilde{\mathcal{B}}$) as the smaller $n \in \mathbb{N}$ such that $b = b_1 b_2 \cdots b_n$ with $b_i \in U$ (respectively $\tilde{U}$). In $f \in \Lambda$ (Λ'), define the *length* of f by $\ell(f) = \max\{\ell(b) : b \in \mathcal{B}(\tilde{\mathcal{B}})$ *occurs in* $f\}$. We say that an element $f = \sum_{i=1}^{n} \lambda_i b_i$, with $\lambda_i \in \mathcal{K}$ and $b_i \in \mathcal{B}(\tilde{\mathcal{B}})$, is *homogeneous* if $\ell(f) = \ell(b_i)$ for every $1 \leq i \leq n$. An ideal J is *homogeneous* if can be generated by homogeneous elements.

Define in $\tilde{\mathcal{B}}$ the order

$$e_i \prec l_j \prec b, \text{ for every } i, j \in |Q_0| \text{ and } b \in \mathcal{B} \text{ and}$$

$$Z^n b_1 \prec Z^m b_2 \text{ if}: \begin{cases} \text{if } b_1 < b_2 \text{ in } \mathcal{B} \text{ or} \\ \text{if } b_1 = b_2 \text{ in } \mathcal{B} \text{ and } n < m \end{cases}$$

Lets show that this is, in fact, an admissible order. Let $Z^n b_1, Z^m b_2, Z^r b_3, Z^s b_4 \in \tilde{\mathcal{B}}$, then:

(1) if $Z^n b_1 \prec Z^m b_2$ and $Z^n b_1 Z^r b_3$ and $Z^m b_2 Z^r b_3$ are non zero, we have that, if $b_1 < b_2$ then $b_1 b_3 < b_2 b_3$. Now, if $b_1 = b_2$, $n < m$ and so $b_1 b_3 = b_2 b_3$ and $n + r < m + r$. Then, $Z^n b_1 Z^r b_3 \prec Z^m b_2 Z^r b_3$.

(2) in the same way, if $Z^n b_1 \prec Z^m b_2$, then $Z^r b_3 Z^n b_1 \prec Z^r b_3 Z^m b_2$, if the products are non zero.

(3) if $Z^n b_1 = Z^m b_2 Z^r b_3 Z^s b_4 = Z^{m+r+s} b_2 b_3 b_4$, we have that $r \leq n$ and $b_3 \leq b_1$ so $Z_r b_3 \preceq Z^n b_1$.

Therefore, the order $\prec$ given above is an admissible order.

Definition. 2.1 *For $f = \sum_{i=1}^{m} \lambda_i b_i \in \Lambda$, we define the homogenization of f in Λ' by $f^* = \sum_{i=1}^{m} \lambda_i Z^{\ell(f)-\ell(b_i)} b_i$.*

Observe that, for every $f \in \Lambda$, the homogenization of $f$is homogeneous.

Lemma 2.2 *For every $f, g \in \Lambda$, we have $Z^k (fg)^* = f^* g^*$, with $k = \ell(f) + \ell(g) - \ell(fg)$.*

Proof. Let $f = \sum_{i=1}^{n} \lambda_i b_i$ and $g = \sum_{j=1}^{m} \beta_j b_j$, with $\lambda_i, \beta_j \in \Lambda$ and $b_i, b_j \in$

B. As $\ell(f)+\ell(g)\geq \ell(fg)$, we consider $k=\ell(f)+\ell(g)-\ell(fg)$. Then,

$$\begin{aligned}
f^*g^* &= (\sum_{i=1}^{n}\lambda_i b_i)^*(\sum_{j=1}^{m}\beta_j b_j)^* \\
&= (\sum_{i=1}^{n}\lambda_i Z^{\ell(f)-\ell(b_i)}b_i)(\sum_{j=1}^{m}\beta_j Z^{\ell(g)-\ell(b_j)}b_j) \\
&= \sum_{i,j}\lambda_i\beta_j Z^{(\ell(f)+\ell(g))-(\ell(b_i)+\ell(b_j))}b_ib_j \\
&= \sum_{i,j}\lambda_i\beta_j Z^{\ell(fg)-\ell(b_ib_j)}Z^k b_ib_j \\
&= Z^k(\sum_{i,j}\lambda_i\beta_j b_ib_j)^* \\
&= Z^k((\sum_{i=1}^{n}\lambda_i b_i)(\sum_{j=1}^{m}\beta_j b_j))^* \\
&= Z^k(fg)^*
\end{aligned}$$

∎

Now, we define the following application, between the algebras Λ' and Λ:

$$\varphi:\Lambda'\to\Lambda$$

that associates to each element $Z^n b\in\tilde{B}$ the element $b\in B$, for every $n\in\mathbf{N}$. Observe that, for every $b\in B$, there exists $Zb\in\tilde{B}$ such that $\varphi(Zb)=b$. In this way, we have that φ extended by linearity to every element in Λ' is, in fact, an epimorphism of algebras. Also, observe that $ker(\varphi)=\langle Z-1\rangle$.

To simplify the notation, we call $g_*=\varphi(g)$ for every $g\in\Lambda'$.

Lemma 2.3 *For every $f\in\Lambda$ we have $(f^*)_*=f$.*

PROOF. Let $f = \sum_{i=1}^{n} \lambda_i b_i$, with $\lambda_i \in \Lambda$ and $b_i \in B$. Observe that

$$\begin{aligned}
(f^*)_* &= (\sum_{i=1}^{n} \lambda_i Z^{\ell(f)-\ell(b_i)} b_i)_* \\
&= \sum_{i=1}^{n} (\lambda_i Z^{\ell(f)-\ell(b_i)} b_i)_* \\
&= \sum_{i=1}^{n} \lambda_i (Z^{\ell(f)-\ell(b_i)}))_* (b_i)_* \\
&= \sum_{i=1}^{n} \lambda_i b_i \\
&= f
\end{aligned}$$

∎

LEMMA 2.4 *Let $g \in \Lambda'$ homogeneous of length d and let $d' = \ell(g_*)$. Then $d' \leq d$ and $g = Z^{d-d'}(g_*)^*$.*

PROOF. The inequality $d' \leq d$ follows from the definition of g_*. Let $m \in supp_{\bar{B}}(g)$, $m = tZ^i$, with $t \in B$. Then the monomial $m_* \in supp_B(g_*)$ correspondent to m is t. As $\ell(t) = d - i$ the monomial in $supp((g_*)^*)$ correspondent to m_* is $tZ^{d'-(d-i)}$. Then $Z^{d-d'}(g_*)^* = g$. ∎

DEFINITION. 2.5 *Let $F \subset \Lambda$ and $\mathcal{G} \subset \Lambda'$, we define by*

$$F^* = \{f^* : f \in F\}$$

$$\mathcal{G}_* = \{g_* : g \in \mathcal{G}\}$$

LEMMA 2.6 *Let $f \in \Lambda$. Then $\ell(f) = \ell(f^*)$.*

PROOF. Consider $f = \sum_{i=1}^{m} \lambda_i b_i$ with $\lambda_i \in \mathcal{K}$ and $b_i \in \mathcal{B}$, basis of Λ, $1 \leq i \leq m$.

By definition we have that $f^* = \sum_{i=1}^{m} \lambda_i Z^{l(f)-l(b_i)} b_i$. For every summand of f^* we have:

$$\ell(Z^{\ell(f)-\ell(b_i)} b_i) = \ell(Z^{\ell(f)-\ell(b_i)}) + \ell(b_i) = (\ell(f) - \ell(b_i)) + \ell(b_i) = \ell(f)$$

Then, $\ell(f^*) = \max\{\ell(Z^{\ell(f)-\ell(b_i)} b_i) : 1 \leq i \leq m\} = \ell(f)$ ∎

Lemma 2.7 *Let $F = \{f_i\}_{i \in \mathcal{I}}$ be a subset of Λ, not necessarily finite, and $f = \sum_{i=1}^m r_i f_i s_i \in \langle F \rangle$. If $d = \max\{\ell(r_i f_i s_i) : 1 \leq i \leq m\}$ and $d' = \ell(f)$. Then $Z^{d-d'} f^* \in \langle F^* \rangle$.*

Proof. By lemma 2.6, we have that $d = \max\{\ell((r_i f_i s_i)^*) : 1 \leq i \leq m\}$. So, consider $k_i = \ell(r_i) + \ell(f_i) + \ell(s_i) - \ell(r_i f_i s_i)$, $1 \leq i \leq m$.

Let $\overline{f} = \sum_{i=1}^m Z^{d-\ell(r_i f_i s_i)} Z^{k_i} (r_i f_i s_i)^* = \sum_{i=1}^m (Z^{d-\ell(r_i f_i s_i)} r_i)^* f_i^* s_i^*$, by lemma 2.2. So, $\overline{f} \in \langle F^* \rangle$ and is homogeneous (by construction) with $d'' = \ell(\overline{f}) \leq d$. Moreover, using lemma 2.2 and lemma 2.3, we have

$$\begin{aligned}
\overline{f}_* &= \sum_{i=1}^m (Z^{d-\ell(r_i f_i s-i)} Z^{k_i} (r_i f_i s_i)^*)_* \\
&= \sum_{i=1}^m (Z^{d-\ell(r_i f_i s_i)})_* (r_i^*)_* (f_i^*)_* (s_i^*)_* \\
&= \sum_{i=1}^m r_i f_i s_i = f
\end{aligned}$$

Using Lemma 2.4, we can conclude that

$$\overline{f} = Z^{d''-d'} (\overline{f}_*)^* = Z^{d''-d'} f^*$$

As $d'' \leq d$, finally we have:

$$Z^{d-d'} f^* = Z^{d-d''} Z^{d''-d'} f^* = Z^{d-d''} \overline{f} \in \langle F^* \rangle$$

∎

Lemma 2.8 *Let F be a subset of Λ. Then $(\langle F^* \rangle)_* = \langle F \rangle$.*

Proof. Let $f \in \langle F \rangle$. By Lemma 2.7, $Z^k f^* \in \langle F^* \rangle$, for some $k \in \mathbf{N}$, and then

$$f = (f^*)_* = (Z^k f^*)_* \in (\langle F^* \rangle)_*$$

By the other hand, if $g \in \langle F^* \rangle$, say $g = \sum_{i=1}^m r_i (f_i)^* s_i$ with $f_i \in F$ and $r_i, s_i \in \Lambda'$ for $1 \leq i \leq m$, we have

$$\begin{aligned}
g_* &= (\sum_{i=1}^m r_i (f_i)^* s_i)_* \\
&= \sum_{i=1}^m (r_i)_* [(f_i)^*]_* (s_i)_* \\
&= \sum_{i=1}^m (r_i)_* f_i (s_i)_*
\end{aligned}$$

So, $g_* \in \langle F \rangle$. ∎

We reproduce here the Elimination Theorem, found in [2], to discuss and compare the two results. For that, we define some new concepts.

Let $\mathcal{Q}$ be a quiver and $<_{ll}$ a length-lexicographic order defined in the basis of paths $\mathcal{B}$ of $\mathcal{Q}$. Let α be a maximal arrow with respect to $<_{ll}$ in $\mathcal{B}$.

We define the quiver $\mathcal{Q}_\alpha$ in the following way: $(\mathcal{Q}_\alpha)_0 = \mathcal{Q}_0$ and $(\mathcal{Q}_\alpha)_1 = \mathcal{Q}_1 \setminus \{\alpha\}$.

For $\mathcal{T}$ a set of indices, we define the following application $V : \mathcal{T} \longrightarrow \mathcal{Q}_0$. Let $P = \coprod_{i\in\mathcal{T}} V(i)\mathcal{K}\mathcal{Q}$ a (right) projective in $\mathcal{K}\mathcal{Q}$-Mod.

We define $P_\alpha = \coprod_{i\in\mathcal{T}} V(i)\mathcal{K}\mathcal{Q}_\alpha$ a right projective module in $\mathcal{K}\mathcal{Q}_\alpha$-Mod.

Let $\mathcal{B}_P$ be a $\mathcal{K}$-basis of P with order $\prec$ such that:

(1) For every $m_1, m_2 \in \mathcal{B}_P$ and every $b \in \mathcal{B}$, if $m_1 \prec m_2$, then $m_1 b \prec m_2 b$, if $m_1 b$ and $m_2 b$ are non zero.

(2) For every $m \in \mathcal{B}_P$ and every $b_1, b_2 \in \mathcal{B}$, if $b_1 <_{ll} b_2$, then $mb_1 \prec mb_2$, if both are non zero.

(3) For every $m \in \mathcal{B}_P$ and every $b \in \mathcal{B}$, $mb = 0$ or $mb \in \mathcal{B}_P$.

Let $m \in P$, $m = \sum_{i\in\mathcal{T}} \lambda_i m_i b_i$, with $m_i \in \mathcal{B}_P$, $b_i \in \mathcal{B}$ e $\lambda_i \in \mathcal{K}$. We define by $tip(m) = m_i$ if $m_i \preceq m_j$ for every $j \in \mathcal{T}$. For $X \subset P$, we will call by $tip(X) = \{tip(x) : x \neq 0, x \in X\}$.

We say that $\mathcal{G} \subset P$ is right a Gröbner basis for P, with respect to the order $\prec$, if $tip(\mathcal{G})$ generates $tip(P)$ as a right module.

We present, in the following the Elimination Theorem, found in [2].

Theorem 2.9 [2] *Let $\mathcal{Q}$ be a quiver and let $<_{ll}$ be a length-lexicographic order in $\mathcal{B}$, where $\mathcal{B}$ is the set of paths in $\mathcal{Q}$. Let α be a maximal arrow with respect to $<_{ll}$ in $\mathcal{Q}$ and $P = \coprod_{i\in\mathcal{T}} V(i)\mathcal{K}\mathcal{Q}$ a projective in $\mathcal{K}\mathcal{Q}$-Mod. Let $\mathcal{B}_P$ be an ordered basis (as defined above) for P. If $\mathcal{G}$ is a right uniform (reduced) Gröbner basis for P, ento $\mathcal{G}_\alpha = \mathcal{G} \cap P_\alpha$ is a right uniform (reduced) Gröbner basis for P_α.*

As a consequence of the Elimination Theory, Green find a new algebra $\mathcal{K}\mathcal{Q}[T]$, that we will call added by loops.

This algebra $\mathcal{K}\mathcal{Q}[T]$ is an hereditary algebra, obtained adding loops to Q, as above, but without adding any relation. Observe that both are hereditary algebras and the basis of $\mathcal{K}\mathcal{Q}[T]$ is ordered in such a way that the new loops are maximal elements. In this situation, given two ideals and generators sets (Gröbner basis), we can find, as described in [2], a generators set (Gröbner basis), of the intersection of these ideals, constructed by the Elimination Theorem (that can be found, with more details, in [2], section 8).

In our work, we are not doing any additional hypothesis over the given order, unless that the extra loops must be between the vertices and the arrows.

Moreover, we consider the more general case, where $\Lambda = \mathcal{K}\mathcal{Q}/I$, is not necessarily hereditary.

Theorem 2.10 *Let F be a subset of Λ and let $\mathcal{G} \subset \Lambda'$ be homogeneous. If $\mathcal{G}$ is a Gröbner basis for $\langle F^* \rangle$, then $\mathcal{G}_*$ is a Gröbner basis for $\langle F \rangle$.*

Proof. Suppose that $\mathcal{G}$ is a Gröbner basis for $\langle F^* \rangle$. We will prove the theorem, using the definition of Gröbner basis.

As $\mathcal{G}_* \subset \langle F \rangle$, then $\langle Tip(\mathcal{G}_*) \rangle \subseteq \langle Tip(\langle F \rangle) \rangle$, and we only need to verify that $\langle Tip(\langle F \rangle) \rangle \subseteq \langle Tip(\mathcal{G}_*) \rangle$, that is, if given $f \in \langle F \rangle$ there exists $g_* \in \mathcal{G}_*$ such that $Tip(g_*)$ divides $Tip(f)$.

Let $f \in \langle F \rangle$, we can write $f = \sum_{i=1}^{m} \lambda_i b_i$, where $\lambda_i \in \mathcal{K}$ and $b_i \in \mathcal{B}$ for $1 \leq i \leq m$.

Without lost of generality, assume that $Tip(f) = b_1$. Observe that $\ell(b_1) = \ell(f)$, then

$$f^* = \lambda_1 b_1 + \sum_{i=2}^{m} \lambda_i Z^{\ell(f)-\ell(b_i)} b_i$$

it follows by the given order that $Tip(f^*) = b_1 = Tip(f)$.

By lemma 2.7, there exists $k \in \mathbf{N}$ such that $h = Z^k f^* \in \langle F^* \rangle$. By the above observation, $Tip(h) = Z^k Tip(f)$.

As $\mathcal{G}$ is a Gröbner basis for $\langle F^* \rangle$, there exists $g \in \mathcal{G}$ such that $Tip(h) = Z^{k_r} r Tip(g) Z^{k_s} s$, for some $Z^{k_r} r, Z^{k_s} s \in \tilde{\mathcal{B}}$. $Tip(g) \in \tilde{\mathcal{B}}$, so $Tip(g) = Z^{k_g} b$ for some $b \in \mathcal{B}$ and $k_g \in \mathbf{N}$.

Then, $Tip(h) = Z^k b_1 = Z^{k_r} r Tip(g) Z^{k_s} s = Z^{k_r} r Z^{k_g} b Z^{k_s} s = Z^{k_r + k_g + k_s} r b s$.

By hypothesis, g is homogeneous, then $Tip(g) = Z^{k_g} b$ is such that, for every $Z^t b_t \neq Tip(g)$ that occurs in g, we have that $k_g < t$, that implies that $b > b_t$ and so $Tip(g_*) = b$. Then $Tip(f) = r Tip(g_*) s$, and so $\mathcal{G}_*$ is a Gröbner basis for $\langle F \rangle$. ∎

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Cidade Universitária "Armando de Salles Oliveira"
Rua do Matão, 1010 - Cidade Universitária
Caixa Postal 66281 - CEP 05315-970