## Modeling of Single-Phase Flows in Porous Media and Passive Contaminant Transport

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A porous medium is a material characterized by its ability to store fluids and is found in a variety of contexts, such as sedimentary rocks for oil extraction, underground aquifers, biological tissues, and fuel cells. In general, computational simulations are essential tools for understanding chemical, physical, and flow processes, complementing field observations, laboratory experiments, and analytical models. This work aims to introduce concepts of Numerical Methods for Partial Differential Equations and their applications in modeling flow in porous media. Specifically, numerical simulation methods for elliptic and hyperbolic equations are investigated.

## A Kruzhkov-Type Uniqueness Theorem for a System of Conservation Laws Describing Chemical Flooding

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A system of two hyperbolic conservation laws describing a two-phase, three-component displacement (usually water and oil phases, as well as a component of a chemical agent dissolved in water) is considered. This system is neither truly nonlinear nor strictly hyperbolic, which limits the applicability of general results pertaining to strictly hyperbolic, truly nonlinear systems.

Solutions to some initial-boundary value problems (e.g., the Riemann problem or the problem of injecting a chemical agent slug) for this system have been previously investigated using the transition to Lagrangian coordinates, in which the equations are separated. The solutions are constructed using the method of characteristics, but the uniqueness of the resulting solutions has not been investigated. We used the proposed coordinate change to prove a Kruzhkov-type uniqueness theorem for solving the Cauchy problem under certain constraints on the initial data and the class of admissible weak solutions. A local small parameter criterion (vanishing viscosity) was used to determine the admissibility of discontinuities. The uniqueness result is applied to the analysis of the Riemann problem solutions for an S-shaped flow function changing monotonicity exactly once.

This talk is based on joint work with N. Rastegaev, https://arxiv.org/abs/2512.07639

## An Extension of the Subadditive Ergodic Theorem

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In this talk, it will be presented an extension of the Kingman's subadditive ergodic theorem. Here, we will consider subadditive quantities composed with stochastic deformations, in which the stationarity assumption is missed, and we will show that the same conclusions hold as in the classical case. The proof involves a rather simple reduction to the additive case, where Birkhoff's ergodic theorem can be applied.

## Regular variation, maximal functions, and DiPerna-Lions flows

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Using maximal functions with weights of regular variation, we investigate the regularity of flows generated by Sobolev velocity fields. This is an ongoing joint work with Henrique Borrin (FFCLRP-USP), and it is