Analysis of unidimensional peridynamic constitutive models

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Objectives

To investigate the one-dimensional equilibrium problem of an infinite linearly elastic bar subjected to a concentrated force in the context of the peridynamic theory and to compare results from this analysis with results from classical linear elasticity theory.

Methods and procedures

We consider the constitutive models proposed by Aguiar & Fosdick [1] and Silling et al. [2] and show that, in the context of one-dimensional peridynamic theory, these models are equivalent to each other under certain constitutive assumptions. In this context, the peridynamic equilibrium problem of an infinite linearly elastic rod is to find the longitudinal displacement field $u:(-\infty,\infty)\to\mathbb{R}$ that satisfies

$$\int_{-\infty}^{\infty} C(\xi)\big(u(x-\xi)-u(x)\big)d\xi + b(x) = 0$$

$\forall\, x \in (-\infty,\infty)$, where $C:(-\infty,\infty)\to\mathbb{R}$ is the micro-modulus of a peridynamic material and $b:(-\infty,\infty)\to\mathbb{R}$ is the body force density. Here, $C(\xi) = 4\,E\exp(-(\xi/l)^2)/(l^3\sqrt{\pi})$, where E is the Young's modulus and l is the peridynamic horizon, and $b(x) = \lim_{a\to\infty}[-\delta(x+2a)+2\delta(x)-\delta(x-2a)]/2$ where $\delta(\cdot)$ is the Dirac delta. The density $b(x)$ corresponds to a unit concentrated force acting at $x=0$. To construct a solution to this problem, we use the solution of a similar problem investigated by Mikata [3] for the case of a dipole of forces separated by a distance $2a$. For comparison reasons, we also consider the equilibrium problem of an infinite linearly elastic rod in the context of the classical theory, which consists of finding $u:(-\infty,\infty)\to\mathbb{R}$ that satisfies $E\,\frac{d^2u}{dx^2} + b(x) = 0$, $\forall\, x \in (-\infty,\infty)$, where $b(x)$ is given above.

Results

We show in Fig. 1 the graph $u(x)/\,(a/E)$ versus $x/\,a$ for different values of l/a and the exact solution of the classical equilibrium problem for an infinite rod. Observe from this figure that the results from the peridynamic theory converge to those of the classical theory as l/a goes to zero.

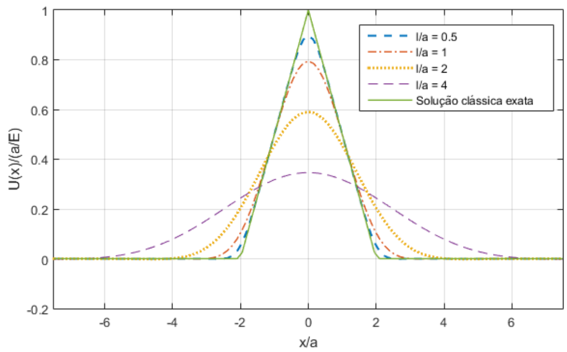

Figure 1: Convergence of peridynamic solution to classical solution as l/a goes to zero.

Conclusions

By considering a unit concentrated force, we have obtained results that can be used to construct analytical solution of unidimensional peridynamic problem for any body force, which allows, for example, to develop a model for predicting failure of the peridynamic material with micromodulus $C(\cdot)$.

References

[1] A. Aguiar and R. Fosdick. Mathematics an Mechanics of Solids, 19(5):502–523, 2014.

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[3] Y. Mikata.. International Journal of Solids and Structures, 49(21):2887 – 2897, 2012.