DEPARTAMENTO DE CIÊNCIA DA COMPUTAÇÃO

Relatório Técnico

RT-MAC-9505

Categorial Grammar and Harmonic Analysis

Flávio S. Corrêa da Silva
Fábio Kon

junho 95

Categorial Grammar and Harmonic Analysis

Flávio S. Corrêa da Silva   Fábio Kon

Instituto de Matemática e Estatística da Universidade de São Paulo
Cx. Postal 66281 – 05389-970
São Paulo (SP) – Brazil – email: {fcs, kon}@ime.usp.br

Abstract

It is rather commonplace in everyday conversation to refer to the *"Language of Music"*. However, we believe the whole apparatus already built for the analysis of natural language has not been yet as thoroughly used for the analysis of musical phenomena as it could have been. In this article we present some initial ideas towards extending the application of this apparatus for the better understanding of *"Music as Language"*.

In this paper, we apply some techniques from *Categorial Grammar* to represent a simple problem of music theory, which we believe nevertheless to be of widespread interest: functional harmonic analysis. We propose an encoding of the harmonic functions of chords as syntactic categories, and show how the generation of proofs of "harmonic well-formedness" of cadences can be implemented and used as a tool to verify and to display the functional harmonic structuring of cadences.

Keywords: music analysis, harmonic analysis, categorial grammar, syntactic calculus, substructural logics.

1 Introduction

It is commonplace in everyday conversation to refer to the *"Language of Music"*. Indeed, the study of musical phenomena as linguistic objects has been developed by many authors (see e.g. [BCe84, Hol80, LJ83, Sch89]). In this article we present some initial ideas towards extending the application of this apparatus for the better understanding of *"Music as Language"*. More specifically, we employ techniques from *Categorial Grammar* to represent a rather specific and simple problem of music theory, which we believe nevertheless to be of widespread interest: functional harmonic analysis [Bri79].

The aim of Categorial Grammar [Ben87, Ben90, Ben91, Lam58, Lam89] is the analysis of syntactic well-formedness of sentences. The fundamental concept underlying Categorial Grammar is that of *syntactic categories*, which are classes to which words in a sentence must belong. Syntactic categories can be organised as formulae of some substructural logic – e.g. the so-called *Lambek Calculus* [Lam58] – in such way that syntactic well-formedness can be checked via an appropriate *proof theory* related to the logic.

In this paper we propose an encoding of the harmonic functions of chords as syntactic categories and show how the generation of proofs of "harmonic well-foundedness" of cadences can be implemented and used as a tool to verify and to display the functional harmonic structuring of cadences.

In section 2 we briefly review the concepts of Lambek Calculus that we need to use in the rest of the paper. In section 3 we introduce our encoding of harmonic functions of chords as syntactic categories, and show how they can be used to check and to display the functional harmonic structuring of cadences. In section 4 we present a simple PROLOG implementation for checking the harmonic well-foundedness of cadences and displaying functions of chords.

Finally, in section 5 we discuss these results, present some conclusions and propose some future work.

2 The Lambek Calculus

J. Lambek introduced the Syntactic Calculus – most usually called Lambek Calculus nowadays – in [Lam58], as a tool to encode the English grammar, such that well-formedness of sentences could be tested deductively.

Essentially, the Lambek Calculus corresponds to classical propositional logic devoid of any structural rule, in which implication is factorised in two non-commutative connectives. Here, we consider only the implicative fragment of that Calculus, which is sufficient for what we intend to present.

A Gentzen-style presentation of implicative Lambek Calculus can be given by the following rules, in which x, y, z are syntactic categories generated by the members of a set S of basic syntactic categories and $\Gamma, \overline{\Gamma}, \Delta$ are sequences of syntactic categories. The sequences $\overline{\Gamma}$ are assumed to be non-empty.

axiom: $x \vdash x$.

right-inclusion:

$$\frac{\overline{\Gamma}, y \vdash x}{\overline{\Gamma} \vdash x/y}. \qquad \frac{y, \overline{\Gamma} \vdash x}{\overline{\Gamma} \vdash y \backslash x}.$$

left-inclusion:

$$\frac{\overline{\Gamma} \vdash y;\ \Gamma, x, \Delta \vdash z}{\Gamma, x/y, \overline{\Gamma}, \Delta \vdash z}. \qquad \frac{\overline{\Gamma} \vdash y;\ \Gamma, x, \Delta \vdash z}{\Gamma, \overline{\Gamma}, y \backslash x, \Delta \vdash z}.$$

For example, Lambek assumes that $S = \{n, s\}$, in which n stands for "noun" and s stands for "sentence". The words of the English language are attached as labels to formulae, in such way that they can only occur in specific sequences from which the category "s[entence]" can be derived.

Giving a more specific example, if we assume the words `John` and `milk` to be of category "n[oun]", the word `fresh` to be of category "n/n" (a qualifier – must precede the noun it is qualifying to produce a qualified noun) and the word `likes` to be of category $n\backslash s/n$ (a transitive verb – forms a sentence if preceded by a noun – the subject – and followed by another noun – the object of the sentence), we can prove the well-formedness of `John likes fresh milk` with the deduction tree presented in figure 1 (we abbreviate `John`, `likes`, `fresh`, and `milk` by their initials `J`, `l`, `f`, and `m`).

$$\dfrac{\dfrac{\texttt{m}: n \vdash \texttt{m}: n \qquad \dfrac{\texttt{fm}: n \vdash \texttt{fm}: n \quad \texttt{Jlfm}: s \vdash \texttt{Jlfm}: s}{\texttt{Jl}: s/n, \texttt{fm}: n \vdash \texttt{Jlfm}: s}}{\texttt{Jl}: s/n, \texttt{f}: n/n, \texttt{m}: n \vdash \texttt{Jlfm}: s} \qquad \texttt{J}: n \vdash \texttt{J}: n}{\texttt{J}: n, \texttt{l}: n\backslash s/n, \texttt{f}: n/n, \texttt{m}: n \vdash \texttt{Jlfm}: s}$$

Figure 1: Deduction for "John likes fresh milk"

This deduction proves that from the sequence `J`: n, `l`: $n\backslash s/n$, `f`: n/n, `m`: n we can derive the well-formed sentence `Jlfm`: s.

We have employed this Calculus to encode the functional grammar of tonal chords, as detailed in the following sections.

3 Tonal Chords and Syntactic Categories

The set of basic syntactic categories for functional harmony must be large enough to permit the characterisation of all different functions each chord may have in a cadence. We have employed a set of three basic syntactic categories $S = \{a, b, c\}$, a and c loosely corresponding to the concepts of *tonic* and *cadence*, related to Lambek's *noun* and *sentence* functions, and b

corresponding to an intermediate concept leading to the idea of *subdominant*. Intuitively, we have a as tonic, $a \setminus b$ as subdominant (fulfilled when preceded by something of category a) and $b \setminus c/a$ as a full cadence (fulfilled when preceded by some chain of chords of category b and followed by something of category a). In order to present our proposed encoding of chords as representatives of syntactic categories, we must introduce some notation.

We have adopted the (first twelve) MIDI codes for pitch values, and hence the notes C, C♯, D ... are denoted respectively as $0, 1, 2, \ldots$. The syntactic categories of the functions of chords can then be encoded in a dictionary like the one presented in table 1. In this dictionary, $i = 0, 1, \ldots, 11$, and these numbers are operated modulo 12, i.e. $6 + 5 = 11, 6 + 6 = 0, 6 + 7 = 1$ etc. (and, of course, in table 1 we have only a small fragment of one such dictionary).

Major Mode

entry	chord	tonality						
		i	$i+1$	$i+3$	$i+5$	$i+7$	$i+8$	$i+10$
i^1	$i, i+4, i+7$	a			$b\backslash c/a$	$a\backslash b$		
i^2	$i, i+3, i+7, i+10$							$a\backslash b$
$i^{d}3$	$i, i+4, i+7, i+10$				$b\backslash c/a$			
i^4	$i, i+4, i+7, i+11$	a				$a\backslash b$		
i^5	$i, i+4, i+7, i+11, i+2$	a						
i^7	$i, i+3, i+8$		$b\backslash c/a$	$a\backslash b$			a	

Minor Mode

entry	chord	tonality					
		i	$i+1$	$i+4$	$i+5$	$i+7$	$i+9$
i^1	$i, i+4, i+7$				$b\backslash c/a$		
i^3	$i, i+4, i+7, i+10$				$b\backslash c/a$		
i^6	$i, i+3, i+7$	a				$a\backslash b$	
i^7	$i, i+3, i+8$		$b\backslash c/a$				
i^8	$i, i+4, i+9$			$a\backslash b$			a

Table 1: Dictionary of Syntactic Categories of Chords ($i = 1, \ldots, 11$ is the root of the chord)

It should be observed that syntactic categories refer to specific tonalities and modes. We avoid referring explicitly to tonalities and to modes in our deduction trees to preserve our notation as simple as possible. Now, using the notation of table 1, if we attach the perfect major triads $0^1, 5^1, 7^1$ as labels to the categories $a, a\backslash b$ and $b\backslash c/a$, we can derive the well-formedness of the perfect cadence $\{0^1, 5^1, 7^1, 0^1\}$ (figure 2).

$$\dfrac{\dfrac{0^1 : a \vdash 0^1 : a \qquad \dfrac{0^15^1 : b \vdash 0^15^1 : b \quad 0^15^17^10^1 : c \vdash 0^15^17^10^1 : c}{0^15^1 : b, 7^10^1 : b\backslash c \vdash 0^15^17^10^1 : c}}{0^1 : a, 5^1 : a\backslash b, 7^10^1 : b\backslash c \vdash 0^15^17^10^1 : c} \qquad 0^1 : a \vdash 0^1 : a}{0^1 : a, 5^1 : a\backslash b, 7^1 : b\backslash c/a, 0^1 : a \vdash 0^15^17^10^1 : c}$$

Figure 2: Deduction for the perfect cadence

```
| ?- harmon([[c,e,g], [d,f,a], [g,b,d], [c,e,g]], X).

X = [[c,maj]] ?

yes

| ?-
```

Figure 3: Using the Theorem Prover

A theorem prover for this Calculus can be implemented in PROLOG, and in the following section we present a very simple implementation for it.

4 Functional Harmony in PROLOG

The PROLOG code for a simple implementation of a theorem prover for the Calculus presented above is introduced in the appendix following this paper. This program works as follows: given a sequence of chords $[C_1, ..., C_n]$, the procedure **genseq** converts this sequence into a sequence of sets of harmonic functions $\mathcal{F}_i$ that each chord C_i can have. From these, the procedure **cadence** selects the functions $f_i \in \mathcal{F}_i$ such that from $f_i : i = 1, ..., n$ we can derive the function c of any tone and mode. These functions are then presented as solutions, with the corresponding tone and mode of the derived cadence.

For example, if we want to check the well-formedness of the sequence of chords in figure 2, we obtain the following (figure 3). This output indicates that, for the fragment of tonal functional harmony encoded above, the only syntactic category of type "c" that can be derived from the given sequence of chords is that of C major.

5 Conclusions and Future Work

In this paper we presented an encoding of the harmonic functions of chords as syntactic categories, and showed how the generation of proofs of "harmonic well-foundedness" of cadences could be used as a tool to verify and to display the harmonic functional structuring of cadences. We have also presented an implementation of a theorem prover for automating this verification.

Clearly, there is still much to be done on turning Categorial Grammar applied to functional harmonic analysis a more friendly tool for musicians. Nonetheless, our initial experiments suggest that this can be a useful tool, not only for analysis but also for generation of cadences upon certain constraints, e.g. when building accompaniments for given melodies.

Immediate future work shall include the study of applicability of this tool in practical situations of interest for musicians and for students of music, and the extension of our "dictionary" to encompass richer harmonic cadences. It shall also be interesting to further analyse the mathematical properties of tonal harmony under the viewpoint of Lambek Calculus, and to study what the (musical) consequences could be of altering some of these mathematical properties (e.g. by adding some structural rules or different connectives to the Calculus).

The program presented here is also available by ftp at
`ftp.ime.usp.br:/pub/music/lambek`, or directly from the authors.

Acknowledgements: this work has been partially supported by FAPESP grant 93/0603-01, and CNPq grant 300041/93-4.

References

[BCe84] M. Baroni and L. Callegari (eds.). *Musical Grammars and Computer Analysis*. Univ. degli Studi di Bologna e Modena, 1984.

[Ben87] J. Benthem. A Linguistic Turn: New Directions in Logic. In *Logic, Methodology and Philosophy of Science VII*, pages 205–240. Elsevier, 1987.

[Ben90] J. Benthem. Categorial Grammar and Type Theory. *Journal of Philosophical Logic*, 19:115–168, 1990.

[Ben91] J. Benthem. Language in Action. *Journal of Philosophical Logic*, 20:225–263, 1991.

[Bri79] C. M. Brisolla. *Princípios de Harmonia Funcional*. Novas Metas, 1979.

[Hol80] S. R. Holtzman. A Generative Grammar Definition Language for Music. *Interface*, 9:1–48, 1980.

[Lam58] J. Lambek. The Mathematics of Sentence Structure. *American Mathematics Monthly*, 65:154–169, 1958.

[Lam89] J. Lambek. Multicategories Revisited. *Contemporary Mathematics*, 92:217–239, 1989.

[LJ83] F. Lerdahl and R. Jackendoff. *A Generative Theory of Tonal Music*. MIT Press, 1983.

[Sch89] E. F. Schurmann. *A Música como Linguagem*. Brasiliense / CNPq, 1989.

Appendix: A Theorem Prover for Functions of Chords

```
/**************************************************************************/
/* harmon(L, A) - the collection of harmonic justifications for         */
/*                 sequence of chords L is A                             */
/**************************************************************************/

harmon(L, A) :- miditransl(L, Ln), genseq(Ln, F),
    setof([X,Y], cadence(F, X, Y), An), midiback(An, A).

miditransl([H|T], [Hn|Tn]) :- transchord(H, Hn), miditransl(T, Tn).
miditransl([], []).

transchord([H|T], [Hn|Tn]) :- transnote(H, Hn), transchord(T, Tn).
transchord([], []).

transnote(c, 0). transnote(c_sharp, 1). transnote(d_flat, 1).
transnote(d, 2). transnote(d_sharp, 3). transnote(e_flat, 3).
transnote(e, 4).
transnote(f, 5). transnote(f_sharp, 6). transnote(g_flat, 6).
transnote(g, 7). transnote(g_sharp, 8). transnote(a_flat, 8).
transnote(a, 9). transnote(a_sharp, 10).transnote(b_flat, 10).
transnote(b, 11).

midiback([H|T], [Hn|Tn]) :- noteback(H, Hn), midiback(T, Tn).
midiback([], []).

noteback([0, M], [c, M]). noteback([1, M], [c_sharp_d_flat, M]).
noteback([2, M], [d, M]). noteback([3, M], [d_sharp_e_flat, M]).
noteback([4, M], [e, M]).
noteback([5, M], [f, M]). noteback([6, M], [f_sharp_g_flat, M]).
noteback([7, M], [g, M]). noteback([8, M], [g_sharp_a_flat, M]).
noteback([9, M], [a, M]). noteback([10, M], [a_sharp_b_flat, M]).
noteback([11, M], [b, M]).

/**************************************************************************/
/* genseq(S, L) - the collection of candidate sequences of              */
/*                 harmonic functions for S is L                         */
/**************************************************************************/

genseq(S, L) :- genfunct(S, F), remap(F, L).

genfunct([H|T], L) :- genfunct(T, T2), setof(F, function(H, F), S),
    append([S], T2, L).
genfunct([], []).

function([H|T], [Y, Fun, Mod]) :- funct(Lf, [Z, Fun, Mod]),
    match(H, [H|T], Lf), Y is ((Z + H) mod 12).
```

```
/*************************************************************************/
/* funct(H0, F0) - dictionary of harmonic functions                     */
/*************************************************************************/

funct([0,4,7], [0, [a], maj]). funct([0,4,7], [5, [b,e,c,d,a], maj]).
funct([0,4,7], [7, [a,e,b], maj]).
funct([0,3,7,10], [10, [a,e,b], maj]).
funct([0,4,7,10], [5, [b,e,c,d,a], maj]).
funct([0,4,7,11], [0, [a], maj]). funct([0,4,7,11], [7, [a,e,b], maj]).
funct([0,4,7,11,2], [0, [a], maj]).
funct([0,3,8], [1, [b,e,c,d,a], maj]).
funct([0,3,8], [3, [a,e,b], maj]). funct([0,3,8], [8, [a], maj]).
funct([0,4,7], [5, [b,e,c,d,a], min]).
funct([0,4,7,10], [5, [b,e,c,d,a], min]).
funct([0,3,7], [0, [a], min]). funct([0,3,7], [7, [a,e,b], min]).
funct([0,3,8], [1, [b,e,c,d,a], min]).
funct([0,4,9], [4, [a,e,b], min]). funct([0,4,9], [9, [a], min]).

/*************************************************************************/

match(X, [H1|T1], [H2|T2]) :- H1 is ((X + H2) mod 12), match(X, T1, T2).
match(_, [], []).

remap([H|T], L) :- remap(T, T2), combine(H, T2, L).
remap([T], L) :- combine(T, [[]], L).

combine([H|T], L1, L2) :- combine(T, L1, T2), comb(H, L1, H2),
    append(H2, T2, L2).
combine([], _, []).

comb(A, [H1|T1], [H2|T2]) :- comb(A, T1, T2), append([A], H1, H2).
comb(_, [], []).

append([H|T], L1, [H|T2]) :- append(T, L1, T2).
append([], L, L).
```

```
/**********************************************************************/
/* cadence(L, Ton, Mod) - L forms a cadence of Ton - Mod             */
/**********************************************************************/

cadence([H|_], X, Y) :- theor(H, [X, [c], Y]).
cadence([_|T], X, Y) :- cadence(T, X, Y).

theor([H|T], [X, F, Y]) :- theor(T, [X, L, Y]),
    prove(H, [X, L, Y], [X, F, Y]).
theor([[X, F, Y]], [X, F, Y]).

prove([X, L1, Y], [X, [F|T2], Y], [X, L2, Y]) :-
    invert(L1, [F, d|T1]), invert(T1, T1i), append(T1i, T2, L2).
prove([X, L1, Y], [X, [F, e|T2], Y], [X, L2, Y]) :-
    invert(L1, [F|T1]), invert(T1, T1i), append(T1i, T2, L2).

invert([H|T], L) :- invert(T, Ti), append(Ti, [H], L).
invert([], []).

/**********************************************************************/
```

Figure 4: A Theorem Prover for Functions of Chords

Stylistic Musical Choices via Fuzzy Preference Rules

Flávio S. Corrêa da Silva, Fábio Kon

Instituto de Matemática e Estatística da Universidade de São Paulo
Cid. Universitária "ASO" - CP 66281 - 05389-970 - São Paulo (SP) - Brazil
email: {fcs, kon}@ime.usp.br

Abstract. In a previous article we have stretched the analogy of "Music as Language" and employed techniques from Categorial Grammar for functional harmonic analysis. In the present paper we extend those techniques to characterise stylistic preferences in terms of pertinence relations to a fuzzy set of "highly-appraised" harmonic cadences, and develop an experiment assessing the adequacy of the characterisation of aesthetic preferences in terms of fuzzy sets. The structures employed to represent these stylistic preferences are coined Fuzzy-Syntactic Categories. They are triples [fuzzy preference values: chords: syntactic function]. Our experiments have consisted on evaluating the adequacy of this approach by empirically assessing the results of propagating the fuzzy preference values in deduction trees. The results obtained thus far are reported below.

1 Introduction

It is commonplace in everyday conversation to refer to the *"Language of Music"*. Indeed, the study of musical phenomena as linguistic objects has been developed by many authors (see e.g. [1, 6, 11]). In [4] we used some techniques from linguistics – namely, techniques related to *Categorial Grammar* – to represent a rather specific and simple problem of music theory, which we believe nevertheless to be of widespread interest: functional harmonic analysis [3].

The aim of Categorial Grammar [2, 8] is the analysis of syntactic well-foundedness of sentences. The fundamental concept underlying Categorial Grammar is that of *syntactic categories*, which are classes to which words in a sentence must belong. Syntactic categories can be organised as formulae of some substructural logic – e.g. the so-called *Lambek Calculus* [8] – in such way that syntactic well-foundedness can be checked via an appropriate *proof theory* related to the logic.

In [4] we presented an encoding of the harmonic functions of chords as syntactic categories and showed how the generation of proofs of "harmonic well-foundedness" could be implemented and used as a tool to verify and to display the harmonic functional structuring of cadences.

We can apply the same ideas to automatically *generate* accompaniment of melodies, by superimposing chords on selected notes of a melodic line. In this case, the melody acts as a series of *constraints* for the resulting sequence of chords, which can be requested e.g. to contain the notes of the melody upon which they are placed, and to be "well-founded" cadences.

Clearly, for most melodies the sequence of accompanying chords generated this way is not unique. Technically speaking, all different variations are "correct", in the sense that they obey the harmonic rules encoded in the syntactic categories. Stylistic (and personal) judgement, however, shall give preference to some sequences of chords over others. For example, the melodic sequence { C,F,D,C } can have as accompaniments the chords { CEG, FAC, GBD, CEG } or { CEG, DFA, GBD, CEG } (among many others). One sequence of chords may be considered preferable over the other, depending on individual taste or on what musical style is being considered.

It is this type of preference that we intend to capture in our experiment. We represent these preferences as Fuzzy-Syntactic Categories, which are pairs [sequences of chords: harmonic functions], accompanied by *fuzzy membership values* to a set of "highly-appraised" sequences. These values are attached to chords and sequences of chords as labels in terms of a propositional doubly-labelled deductive system [5, 9] (see also [10] for general discussions on labelled deductive systems). The system is doubly-labelled because the fuzzy values are attached to pairs [chords: harmonic function], which are themselves formulae of a propositional labelled deductive system. The values are then propagated to cadences and fragments of cadences according to Zadeh norms and conorms [7, 12, 13]. Our aim is to provide an automatic assessment of generated accompaniments of melodies based on the fuzzy preference rules in use.

In section 2 we briefly review the concepts of Lambek Calculus that we need to use in the rest of this paper. In section 3 we formally introduce the Fuzzy-Syntactic Categories, and show how they can be applied to encode aesthetic preferences among different harmonisations of a single melody. In section 4 we present the empirical results obtained so far. Finally, in section 5 we discuss these results, present some conclusions and propose some future work.

2 The Lambek Calculus

J. Lambek introduced the Syntactic Calculus – most usually called Lambek Calculus nowadays – in [8],

$$\dfrac{m: n \vdash m: n \qquad \dfrac{fm: n \vdash fm: n \qquad Jlfm: s \vdash Jlfm: s}{Jl: s/n,\ fm: n \vdash Jlfm: s}}{\dfrac{Jl: s/n,\ f: n/n,\ m: n \vdash Jlfm: s \qquad\qquad J: n \vdash J: n}{J: n,\ l: n\backslash s/n,\ f: n/n,\ m: n \vdash Jlfm: s}}$$

Figure 1: Deduction for "John likes fresh milk"

as a tool to encode the English grammar, such that grammaticality of sentences could be tested deductively.

Essentially, the Lambek Calculus corresponds to classical propositional logic devoid of any structural rule, in which implication is factorised in two non-commutative connectives. Here, we consider only the implicative fragment of that Calculus, which is sufficient for our application.

A Gentzen-style presentation of implicative Lambek Calculus can be given by the following rules, in which x, y, z are syntactic categories generated by the members of a set $\mathcal{S}$ of basic syntactic categories and $\Gamma, \overline{\Gamma}, \Delta$ are sequences of syntactic categories. The sequences $\overline{\Gamma}$ are assumed to be non-empty.

axiom: $x \vdash x$.

right-inclusion:	left-inclusion:
$\dfrac{\overline{\Gamma}, y \vdash x}{\overline{\Gamma} \vdash x/y}$.	$\dfrac{\overline{\Gamma} \vdash y;\ \Gamma, x, \Delta \vdash z}{\Gamma, x/y, \overline{\Gamma}, \Delta \vdash z}$.
$\dfrac{y, \overline{\Gamma} \vdash x}{\overline{\Gamma} \vdash y \backslash x}$.	$\dfrac{\overline{\Gamma} \vdash y;\ \Gamma, x, \Delta \vdash z}{\Gamma, \overline{\Gamma}, y \backslash x, \Delta \vdash z}$.

For example, Lambek assumes that $\mathcal{S} = \{n, s\}$, in which n stands for "noun" and s stands for "sentence". The words of the English language are attached as labels to formulae, in such way that they can only occur in specific sequences from which the category "[s]entence" can be derived.

Giving a more specific example, if we assume the words **John** and **milk** to be of category "[n]oun", the word **fresh** to be of category "n/n" (a qualifier – must precede the noun it is qualifying to produce a qualified noun) and the word **likes** to be of category $n\backslash s/n$ (a transitive verb – forms a sentence if preceded by a noun – the subject – and followed by another noun – the object of the sentence), we can prove the grammaticality of **John likes fresh milk** with the deduction tree presented in figure 1 (we abbreviate **John**, **likes**, **fresh**, and **milk** with their initials **J**, **l**, **f**, and **m**).

This proves that from the sequence J: n, l: $n\backslash s/n$, f: n/n, m: n we can derive the sentence Jlfm: s.

In [4] we showed how this Calculus could be used for harmonic analysis. We employed a set of three basic syntactic categories $\mathcal{S} = \{a, b, c\}$, a and c loosely corresponding to the functions of *tonic* and *cadence*, related to Lambek's *noun* and *sentence* functions, and b corresponding to an intermediate function of *subdominant*. In order to present an example, we must introduce some notation. We adopt the (first twelve) MIDI codes for pitch values, and hence the notes C,

Table 1: Dictionary of Syntactic Categories of Chords

Major Mode

entry	chord	tonality						
		i	$i+1$	$i+3$	$i+5$	$i+7$	$i+8$	$i+$
i^1	$i, i+4, i+7$	a			$b\backslash c/a$	$a\backslash b$		
i^2	$i, i+3, i+7, i+10$							$a\backslash$
i^3	$i, i+4, i+7, i+10$				$b\backslash c/a$			
i^4	$i, i+4, i+7, i+11$	a				$a\backslash b$		
i^5	$i, i+4, i+7, i+11, i+2$	a						
i^7	$i, i+3, i+8$		$b\backslash c/a$	$a\backslash b$			a	

Minor Mode

entry	chord	tonality					
		i	$i+1$	$i+4$	$i+5$	$i+7$	$i+9$
i^1	$i, i+4, i+7$				$b\backslash c/a$		
i^3	$i, i+4, i+7, i+10$				$b\backslash c/a$		
i^6	$i, i+3, i+7$	a				$a\backslash b$	
i^7	$i, i+3, i+8$		$b\backslash c/a$				
i^8	$i, i+4, i+9$			$a\backslash b$			a

$$\dfrac{\dfrac{0^1 : a \vdash 0^1 : a \qquad \dfrac{0^15^1 : b \vdash 0^15^1 : b \qquad 0^15^17^10^1 : c \vdash 0^15^17^10^1 :}{0^15^1 : b, 7^10^1 : b\backslash c \vdash 0^15^17^10^1 : c}}{0^1 : a, 5^1 : a\backslash b, 7^10^1 : b\backslash c \vdash 0^15^17^10^1 : c} \qquad 0^1 : a \vdash 0^1 : a}{0^1 : a, 5^1 : a\backslash b, 7^1 : b\backslash c/a, 0^1 : a \vdash 0^15^17^10^1 : c}$$

Figure 2: Deduction for the perfect cadence

C♯, D ... are denoted respectively as $0, 1, 2, \ldots$. The syntactic categories of the chords under consideration are as in the dictionary presented in table 1 (in this dictionary, $i = 0, 1, \ldots, 11$, and these numbers are operated modulo 12, i.e. $6+5 = 11, 6+6 = 0, 6+7 = 1$ etc.).

It should be observed that syntactic categories refer to specific tonalities and modes. We avoid referring explicitly to tonalities and to modes in our deduction trees to preserve our notation as simple as possible. Now, using the notation of table 1, if we attach the triads $0^1, 5^1, 7^1$ as labels to the categories $a, a\backslash b$ and $b\backslash c/a$, we can derive the "grammaticality" of the perfect cadence $\{0^1, 5^1, 7^1, 0^1\}$ (figure 2).

Now assume we have a melody for which we want to build an accompaniment. Based on the rhytmic structure of the melody, we select the points in the melody upon which we want to superimpose chords. We require that the harmonic cadences thus obtained have to be "grammatically well-founded", and that each chord must contain the notes in the melody lying at the point upon which it is placed.

For example, assuming that the triad 2^6 is also of category $a\backslash b$, we could similarly deduce that the cadence $\{0^1, 2^6, 7^1, 0^1\}$ is grammatically well-founded, and both $\{0^1, 5^1, 7^1, 0^1\}$ and $\{0^1, 2^6, 7^1, 0^1\}$ (among other alternatives) can be taken as accompaniments for the simple melody $\{0, 5, 2, 0\}$.

The choice upon which accompaniment to choose is based on stylistic and personal preferences. Nonetheless, it seems reasonable to request from a composer/arranger (or attentive listener) that their style and personal preferences be consistent. A rather unrestraining framework for this consistency requirement can be based on the supposition that cadences and segments of cadences can be ordered, so that one would not accept a segment of cadence to entail a second segment they did not like.

One interesting way of envisaging this framework is therefore to admit that segments of cadences have different fuzzy membership degrees to a set of "highly-appraised" cadences such that, if $\alpha \vdash \beta$, then the fuzzy membership degree of β is not smaller that that of α. It should be remarked that what we regard as segments of cadences are pairs [chords: harmonic function], like the ones presented in the example above.

Our framework is detailed below.

3 Fuzzy-Syntactic Categories

We introduce here the concept of *Fuzzy-Syntactic Categories*. Fuzzy-Syntactic Categories are triples of the form $[\mu{:}C{:}c]$, where $\mu = [\mu_1, \mu_2] \subseteq [0, 1]$ is a closed interval containing a ***fuzzy degree of appraisal***, C is a ***sequence of chords*** and c is a ***syntactic category***. Our goal is to obtain the narrowest possible μ.

Assuming that a segment of cadence cannot entail a second segment less-appraised than itself, we formulate the following propagation rule for degrees of appraisal, which is in accordance with Zadeh's triangular norms and conorms for fuzzy sets: if we can have in a proof the sequent $[[\mu_1^1, \mu_2^1] : C_1 : c_1], [[\mu_1^2, \mu_2^2] : C_2 : c_2] \vdash [[\mu_1, \mu_2] : C_1\, C_2 : c]$, then we have that $\mu_1 \geq min\{\mu_1^1, \mu_1^2\}$ and $\mu_2 \geq min\{\mu_2^1, \mu_2^2\}$.

Our computational framework (in course of implementation) is based on constraint-propagation for maintaining consistency among the intervals of degrees of appraisal. At the initial configuration, the set of appraised segments of cadences is empty, and the user builds their own set of degrees of appraisal incrementally, i.e. initially, $\mu = [0, 0]$ for all triples $[\mu{:}Cc]$. Each time the user updates the intervals μ, all intervals that have already occurred in any deduction are updated in order to preserve consistency, and from that point on each newly used segment of cadence is constrained by the existing intervals. The input points for the user to update intervals are individual chords, and compound segments of cadences must obey the propagation rule above.

As an example, let us consider again the perfect cadence $\{0^1, 5^1, 7^1, 0^1\}$. The initial Fuzzy-Syntactic Categories corresponding to our previous deduction

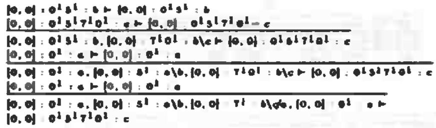

Figure 3: Initial intervals for the perfect cadence

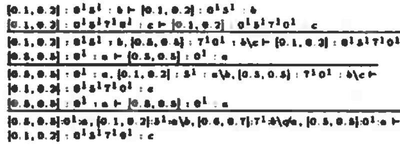

Figure 4: Updated intervals for the perfect cadence

are as in figure 3.

Now, if the user updates these intervals to e.g. $[0.5, 0.5] : 0^1 : a$, $[0.1, 0.2] : 5^1 : a\backslash b$ and $[0.6, 0.7] : 7^1 : b\backslash c/a$, the remaining intervals are changed as in figure 4.

As the process is iterated, the database of intervals for degrees of appraisal is enriched. If now the user wants to evaluate the cadence $\{0^1, 2^6, 7^1, 0^1\}$, by associating e.g. the interval $[0.3, 0.6]$ to $2^6 : a\backslash b$, the resulting intervals become as in figure 5.

We have done an initial experiment on generating a database of intervals for degrees of appraisal. The results are presented in the following section.

4 Empirical Results

Our experiment consisted of encoding a fragment of tonal harmony [3] in terms of Fuzzy-Syntactic Categories, and then of providing all grammatically well-founded harmonisations for a given melody that we could derive from the encoded fragment of tonal harmony. With this in hand, we ordered the set of harmonisations according to personal tastes of three collaborators: an amateur musician, a music theorist, and a professional composer. Each of these collaborators was asked to choose the two "best" and the "worst" harmonisations from the set above.Then we looked for three different labelings with intervals for fuzzy values that reflected these individual tastes, to see whether this methodology was capable of expressing stylistic harmonic preferences, as we have postulated.

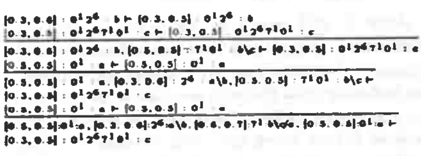

Figure 5: Updated intervals for the altered cadence

Figure 6: Excerpt from "Girl from Ipanema"

The melody we used is a fragment of the well-known "Girl from Ipanema" (Jobim and Mendonça) (figure 6).

Based on table 1, the collection of well-founded cadences to harmonise this melody is the set $\mathcal{H} = \{ 0^5 2^2 7^1 0^1, 0^5 2^2 7^1 0^4, 0^5 2^2 7^1 0^5, 0^5 2^2 7^1 4^7, 0^5 2^2 7^3 0^1, 0^5 2^2 7^3 0^4, 0^5 2^2 7^3 0^5, 0^5 2^2 7^3 4^7, 0^5 2^2 11^7 0^1, 0^5 2^2 11^7 0^4, 0^5 2^2 11^7 0^5, 0^5 2^2 11^7 4^7 \}$. This set was ordered by our collaborators as follows:

amateur musician: $0^5 2^2 7^3 0^5, 0^5 2^2 11^7 0^5, \ldots, 0^5 2^2 11^7 4^7$.

theorist: $0^5 2^2 7^1 0^1, 0^5 2^2 7^3 0^4, \ldots, 0^5 2^2 11^7 0^5$.

composer: $0^5 2^2 7^1 0^4, 0^5 2^2 7^3 0^5, \ldots, 0^5 2^2 11^7 4^7$.

Based on these orderings, we could furnish our Fuzzy-Syntactic Categories with the following intervals:

amateur musician: $[0.5,0.5]:0^5, [0.5,0.5]:2^2, [0.5,0.5]:7^3, [0.4,0.4]:11^7, [0.2,0.2]:7^1, [0.2,0.2]:0^1, [0.2,0.2]:0^4, [0.1,0.1]:4^7$.

theorist: $[0.5,0.5]:0^5, [0.5,0.5]:2^2, [0.5,0.5]:7^1, [0.5,0.5]:0^1, [0.4,0.4]:7^3, [0.4,0.4]:0^4, [0.2,0.2]:4^7, [0.1,0.1]:11^7$.

composer: $[0.5,0.5]:0^5, [0.5,0.5]:2^2, [0.5,0.5]:7^1, [0.5,0.5]:0^4, [0.4,0.4]:7^3, [0.2,0.2]:11^7, [0.2,0.2]:0^1, [0.1,0.1]:4^7$.

By ordering the set $\mathcal{H}$ decreasingly according to these intervals, we obtained the following:

amateur musician: $[0.5,0.5]:0^5 2^2 7^3 0^5, [0.4,0.4]:0^5 2^2 11^7 0^5, [0.2,0.2]:0^5 2^2 7^1 0^1, [0.2,0.2]:0^5 2^2 7^1 0^4, [0.2,0.2]:0^5 2^2 7^1 0^5, [0.2,0.2]:0^5 2^2 7^3 0^1, [0.2,0.2]:0^5 2^2 7^3 0^4, [0.2,0.2]:0^5 2^2 11^7 0^1, [0.2,0.2]:0^5 2^2 11^7 0^4, [0.1,0.1]:0^5 2^2 7^3 4^7, [0.1,0.1]:0^5 2^2 7^1 4^7, [0.1,0.1]:0^5 2^2 11^7 4^7$.

theorist: $[0.5,0.5]:0^5 2^2 7^1 0^1, [0.5,0.5]:0^5 2^2 7^1 0^5, [0.4,0.4]:0^5 2^2 7^3 0^4, [0.4,0.4]:0^5 2^2 7^1 0^4, [0.4,0.4]:0^5 2^2 7^3 0^5, [0.4,0.4]:0^5 2^2 7^3 0^1, [0.2,0.2]:0^5 2^2 7^1 4^7, [0.2,0.2]:0^5 2^2 7^3 4^7, [0.1,0.1]:0^5 2^2 11^7 0^5, [0.1,0.1]:0^5 2^2 11^7 0^1, [0.1,0.1]:0^5 2^2 11^7 0^4, [0.1,0.1]:0^5 2^2 11^7 4^7$.

composer: $[0.5,0.5]:0^5 2^2 7^1 0^4, [0.5,0.5]:0^5 2^2 7^1 0^5, [0.4,0.4]:0^5 2^2 7^3 0^5, [0.4,0.4]:0^5 2^2 7^3 0^4, [0.2,0.2]:0^5 2^2 11^7 0^5, [0.2,0.2]:0^5 2^2 7^1 0^1, [0.2,0.2]:0^5 2^2 7^3 0^1, [0.2,0.2]:0^5 2^2 11^7 0^1, [0.2,0.2]:0^5 2^2 11^7 0^4, [0.1,0.1]:0^5 2^2 7^3 4^7, [0.1,0.1]:0^5 2^2 7^1 4^7, [0.1,0.1]:0^5 2^2 11^7 4^7$.

5 Discussion and Future Work

Clearly, there is still much to be done on turning Fuzzy-Syntactic Categories a more user-friendly tool for musicians. Nonetheless, our initial experimental results indicate that the representation of stylistic preferences in terms of fuzzy sets can be interesting.

Immediate future work on Fuzzy-Syntactic Categories shall include their implementation as a computational tool, and the development of more thorough experiments to assess the applicability of these structures in more complex harmonisations. It shall also be interesting to study the possibilities of automating the generation of the intervals for fuzzy values, perhaps by training an appropriate neural network for this task.

The major restriction that we have imposed is that update values are allowed only for individual chords, to guarantee decidability of our framework. As a consequence, some harmonisations must be deemed indistinguishable, and an evaluation can only be lowered by inserting chords that have not yet occurred in the harmonisation. It shall be interesting to search for more flexible ways to update values guaranteeing decidability.

Acknowledgements: this work was partially supported by FAPESP grant 93/0603-01, and CNPq grant 300041/93-4. Thanks are due to Yára M. C. S. Michelacci (the amateur musician), Maria Helena Câmara (the music theorist) and Hermelino Neder (the composer) for their kind collaboration.

References

[1] M. Baroni and L. Callegari (eds.). *Musical Grammars and Computer Analysis*. Univ. degli Studi di Bologna e Modena, 1984.

[2] J. Benthem. Categorial Grammar and Type Theory. *Journal of Philosophical Logic*, 19:115–168, 1990.

[3] C. M. Brisolla. *Princípios de Harmonia Funcional*. Novas Metas, 1979.

[4] F. S. Correa da Silva and F. Kon. Categorial Grammar and Harmonic Analysis. Technical Report RT-MAC-95xx, Instituto de Matematica e Estatistica, Universidade de Sao Paulo (forthcoming), 1995.

[5] D. M. Gabbay. Abduction in Labelled Deductive Systems – a Conceptual Abstract. In *European Conference on Symbolic and Quantitative Aspects of Uncertainty, LNCS 548*, 1991.

[6] S. R. Holtzman. A Generative Grammar Definition Language for Music. *Interface*, 9:1–48, 1980.

[7] E. P. Klement. Construction of Fuzzy σ-algebras Using Triangular Norms. *Journal of Mathematical Analysis and Applications*, 85:543–565, 1982.

[8] J. Lambek. The Mathematics of Sentence Structure. *American Mathematics Monthly*, 65:154–169, 1958.

[9] A. J. Oliveira and R. J. G. B. Queiroz. Term Rewriting Systems with LDS. In *SBIA 94 – Brazil*, 1994.

[10] R. J. G. B. Queiroz and D. Gabbay. An Introduction to Labelled Natural Deduction. In *Third Advanced Summer School in Artificial Intelligence*, 1992.

[11] E. F. Schurmann. *A Música como Linguagem*. Brasiliense / CNPq, 1989.

[12] L. Zadeh. Fuzzy Sets. *Information and Control*, 8:338–353, 1965.

[13] L. Zadeh. Probability Measures of Fuzzy Events. *Journal of Mathematical Analysis and Applications*, 23:421–427, 1968.

RELATÓRIOS TÉCNICOS

DEPARTAMENTO DE CIÊNCIA DA COMPUTAÇÃO

Instituto de Matemática e Estatística da USP

A listagem contendo os relatórios técnicos anteriores a 1992 poderá ser consultada ou solicitada à Secretaria do Departamento, pessoalmente, por carta ou e-mail(mac@ime.usp.br).

J.Z. Gonçalves, Arnaldo Mandel
COMMUTATIVITY THEOREMS FOR DIVISION RINGS AND DOMAINS
RT-MAC-9201, Janeiro 1992, 12 pp.

J. Sakarovitch
THE "LAST" DECISION PROBLEM FOR RATIONAL TRACE LANGUAGES
RT-MAC 9202, Abril 1992, 20 pp.

Valdemar W. Setzer, Fábio Henrique Carvalheiro
ALGOTRITMOS E SUA ANÁLISE (UMA INTRODUÇÃO DIDÁTICA)
RT-MAC 9203, Agosto 1992, 19 pp.

Claudio Santos Pinhanez
UM SIMULADOR DE SUBSUMPTION ARCHITECTURES
RT-MAC-9204, Outubro 1992, 18 pp.

Julio M. Stern
REGIONALIZAÇÃO DA MATRIZ PARA O ESTADO DE SÃO PAULO
RT-MAC-9205, Julho 1992, 14 pp.

Imre Simon
THE PRODUCT OF RATIONAL LANGUAGES
RT-MAC-9301, Maio 1993, 18 pp.

Flávio Soares C. da Silva
AUTOMATED REASONING WITH UNCERTAINTIES
RT-MAC-9302, Maio 1993, 25 pp.

Flávio Soares C. da Silva
ON PROOF-AND MODEL-BASED TECHNIQUES FOR REASONING WITH UNCERTAINTY
RT-MAC-9303, Maio 1993, 11 pp.

Carlos Humes Jr., Leônidas de O.Brandão, Manuel Pera Garcia
A MIXED DYNAMICS APPROACH FOR LINEAR CORRIDOR POLICIES (A REVISITATION OF DYNAMIC SETUP SCHEDULING AND FLOW CONTROL IN MANUFACTURING SYSTEMS)
RT-MAC-9304, Junho 1993, 25 pp.

Ana Flora P.C.Humes e Carlos Humes Jr.
STABILITY OF CLEARING OPEN LOOP POLICIES IN MANUFACTURING SYSTEMS (Revised Version)
RT-MAC-9305, Julho 1993, 31 pp.

Maria Angela M.C. Gurgel e Yoshiko Wakabayashi
THE COMPLETE PRE-ORDER POLYTOPE: FACETS AND SEPARATION PROBLEM
RT-MAC-9306, Julho 1993, 29 pp.

Tito Homem de Mello e Carlos Humes Jr.
SOME STABILITY CONDITIONS FOR FLEXIBLE MANUFACTURING SYSTEMS WITH NO SET-UP TIMES
RT-MAC-9307, Julho de 1993, 26 pp.

Carlos Humes Jr. e Tito Homem de Mello
A NECESSARY AND SUFFICIENT CONDITION FOR THE EXISTENCE OF ANALYTIC CENTERS IN PATH FOLLOWING METHODS FOR LINEAR PROGRAMMING
RT-MAC-9308, Agosto de 1993

Flavio S. Corrêa da Silva
AN ALGEBRAIC VIEW OF COMBINATION RULES
RT-MAC-9401, Janeiro de 1994, 10 pp.

Flavio S. Corrêa da Silva e Junior Barrera
AUTOMATING THE GENERATION OF PROCEDURES TO ANALYSE BINARY IMAGES
RT-MAC-9402, Janeiro de 1994, 13 pp.

Junior Barrera, Gerald Jean Francis Banon e Roberto de Alencar Lotufo
A MATHEMATICAL MORPHOLOGY TOOLBOX FOR THE KHOROS SYSTEM
RT-MAC-9403, Janeiro de 1994, 28 pp.

Flavio S. Corrêa da Silva
ON THE RELATIONS BETWEEN INCIDENCE CALCULUS AND FAGIN-HALPERN STRUCTURES
RT-MAC-9404, abril de 1994, 11 pp.

Junior Barrera; Flávio Soares Corrêa da Silva e Gerald Jean Francis Banon
AUTOMATIC PROGRAMMING OF BINARY MORPHOLOGICAL MACHINES
RT-MAC-9405, abril de 1994, 15 pp.

Valdemar W. Setzer; Cristina G. Fernandes; Wania Gomes Pedrosa e Flavio Hirata
UM GERADOR DE ANALISADORES SINTÁTICOS PARA GRAFOS SINTÁTICOS SIMPLES
RT-MAC-9406, abril de 1994, 16 pp.

Siang W. Song
TOWARDS A SIMPLE CONSTRUCTION METHOD FOR HAMILTONIAN DECOMPOSITION OF THE HYPERCUBE
RT-MAC-9407, maio de 1994, 13 pp.

Julio M. Stern
MODELOS MATEMATICOS PARA FORMAÇÃO DE PORTFÓLIOS
RT-MAC-9408, maio de 1994, 50 pp.

Imre Simon
STRING MATCHING ALGORITHMS AND AUTOMATA
RT-MAC-9409, maio de 1994, 14 pp.

Valdemar W. Setzer e Andrea Zisman
*CONCURRENCY CONTROL FOR ACCESSING AND COMPACTING B-TREES**
RT-MAC-9410, junho de 1994, 21 pp.

Renata Wassermann e Flávio S. Corrêa da Silva
TOWARDS EFFICIENT MODELLING OF DISTRIBUTED KNOWLEDGE USING EQUATIONAL AND ORDER-SORTED LOGIC
RT-MAC-9411, junho de 1994, 15 pp.

Jair M. Abe, Flávio S. Corrêa da Silva e Marcio Rillo
PARACONSISTENT LOGICS IN ARTIFICIAL INTELLIGENCE AND ROBOTICS.
RT-MAC-9412, junho de 1994, 14 pp.

Flávio S. Corrêa da Silva, Daniela V. Carbogim
A SYSTEM FOR REASONING WITH FUZZY PREDICATES
RT-MAC-9413, junho de 1994, 22 pp.

Flávio S. Corrêa da Silva, Jair M. Abe, Marcio Rillo
MODELING PARACONSISTENT KNOWLEDGE IN DISTRIBUTED SYSTEMS
RT-MAC-9414, julho de 1994, 12 pp.

Nami Kobayashi
THE CLOSURE UNDER DIVISION AND A CHARACTERIZATION OF THE RECOGNIZABLE Z-SUBSETS
RT-MAC-9415, julho de 1994, 29pp.

Flávio K. Miyazawa e Yoshiko Wakabayashi
AN ALGORITHM FOR THE THREE-DIMENSIONAL PACKING PROBLEM WITH ASYMPTOTIC PERFORMANCE ANALYSIS
RT-MAC-9416, novembro de 1994, 30 pp.

Thomaz I. Seidman e Carlos Humes Jr.
SOME KANBAN-CONTROLLED MANUFACTURING SYSTEMS: A FIRST STABILITY ANALYSIS
RT-MAC-9501, janeiro de 1995, 19 pp.

C.Humes Jr. and A.F.P.C. Humes
STABILIZATION IN FMS BY QUASI- PERIODIC POLICIES
RT-MAC-9502, março de 1995, 31 pp.

Fabio Kon e Arnaldo Mandel
SODA: A LEASE-BASED CONSISTENT DISTRIBUTED FILE SYSTEM
RT-MAC-9503, março de 1995, 18 pp.

Junior Barrera, Nina Sumiko Tomita, Flávio Soares C. Silva, Routo Terada
AUTOMATIC PROGRAMMING OF BINARY MORPHOLOGICAL MACHINES BY PAC LEARNING
RT-MAC-9504, abril de 1995,16 pp.

Flávio S. Corrêa da Silva e Fabio Kon
CATEGORIAL GRAMMAR AND HARMONIC ANALYSIS
RT-MAC-9505, junho de 1995,17 pp.