nº 02

FOLIATIONS WHICH ADMIT THE MOST MEAN CURVATURE FUNCTIONS

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Sommaire. Nous considerons ces feuilletages de codimension 1 qui ont la propriété suivant: Toute fonction changant son signe sur la variete feuilletée compact M paraît comme la courbure moyenne des feuilles pour certaine metrique riemannienne sur M.

§ 0. *Introduction.*

In the papers [6] - [9], the family Mean(F), where F is a transversely oriented, codimension-one C^∞-foliation of a compact C^∞-manifold M, was considered. Recall, that $f \in$ Mean(F) if and only if $f \in C^\infty(M)$ and there exists a Riemannian metric g on M for which f is the mean curvature of (the leaves of) F w.r.t.g. Among the others, we proved that if either all the leaves of F are compact or F is transverse to the fibres of an S^1-bundle $M \longrightarrow B$, then

(1) $$\mathrm{Mean}(F) = C^\infty_\pm(M),$$

where $C^\infty_\pm(M)$ consists of zero and all smooth functions which are somewhere positive and elsewhere negative on M. In this note, we prove that the equality (1) holds if and only if there exists a closed transversal intersecting all the levaes of F. This shows that studying the family Mean(F) one can get some information concerning the topology of F. Earlier ([1], [4] and the others),

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it was shown that minimalizability, i.e. the condition "$0 \in \mathrm{Mean}(F)$", forces some topological properties of F.

§ 1. Mean curvature functions.

Let F be a transversely oriented codimension-one C^∞-foliation of a compact oriented manifold M. If g is a Riemannian metric on M, then the mean curvature of F w.r.t. g is the real C^∞-function h(F,g) which assigns to each x of M the mean curvature at x of the leaf L_x passing through x. It is well known that h = h(F,g) satisfies the equation

$$d\omega_g = h\Omega_g ,$$

where ω_g and Ω_g are the volume forms of F and M, respectively. Therefore,

$$\int_M h\,\Omega_g = 0$$

and if U is an open saturated set bounded by closed leaves $L_1,\dots,L_k$, then

$$\int_U h\,\Omega_g = \sum_{i=1}^{k} \mathrm{sgn}(U,L_i).\mathrm{vol}(L_i),$$

where $\mathrm{sgn}(U,L_i) = 1$ when a positively oriented transverse to F vector field turns on L_i inwards U and $\mathrm{sgn}(U,L_i) = -1$ otherwise. Consequently, $h \in C^\infty_\pm(M)$ and if $\mathrm{sgn}(U,L_i) = 1$ (resp., -1) for all boundary components L_i, then h must be positive (resp., negative) somewhere in U.

Let us denote by Adm(F) the set of all smooth functions on M which satisfy the sign conditions described above. Clearly,

(3) $$\mathrm{Mean}(F) \subset \mathrm{Adm}(F) \subset C^{\infty}_{\pm}(M).$$

§ 2. *Novikov components.*

Let us recall that points x and y belong to the same Novikov component of a transversely oriented codimension-one foliation F whenever $L_x = L_y$ or there exists a closed transversal passing through x and y.

Novikov [2] proved that if the foliated manifold M is compact, then there are three possibilities:

(i) All the manifold M is the only Novikov component of F. This holds if and only if there exists a closed transversal intersecting all the leaves of F.

(ii) A component consists of a single closed leaf L. In this case, there are no closed transversals meeting L.

(iii) A component is open, saturated and bounded by a finite number of compact levaes, components of type (ii).

Denote by N(F) the family of all the Novikov components of a foliation F. If A and B ϵ N(F), then we write $A < B$ whenever either $A = B$ or there exists a positively oriented transversal which starts in A and ends in B. The relation $<$ provides a partial ordering of N(F).

Lemma 1. There are maximal elements in (N(F), <).

Proof. Take a finite covering $U = \{U_1,\dots,U_k\}$ of M by closed charts $\phi_i : U_i \longrightarrow D \times \langle 0;1\rangle$ ($D \subset R^n$, n=dimF) distinguished by F. For any i, denote by N_i the Novikov component containing the U_i-plaque corresponding to t = 1 (Figure 1). If $N \in N(F)$ and $N \cap U_i \neq \emptyset$, then $N < N_i$. Therefore, any maximal element of the set

$N_1, \ldots, N_k$ is maximal in $N(F)$.

Lemma 2. If N is a maximal element of $(N(F), <)$, then either $N = M$ or N is of the type (iii) and the positively oriented transverse to F vector field turns inwards N on all the components of the boundary ∂N.

Proof. Obvious.

Lemma 3. If M is the only Novikov component of F, then there exists a nowhere vanishing transverse to F vector field X such that any its trajectory meets all the leaves of F.

Proof. If $\dim F = 1$, then $M = T^2$ and F is equivalent to a suspension of a diffeomorphism of S^1. Therefore, we may assume that $\dim F \geq 2$.

Let γ be a circle embedded into M transversely to F and such that every leaf of F meets γ. Extend γ to an embedding $D \times S^1 \longrightarrow M$, where $D \subset R^n$ ($n = \dim F$) is a closed ball, $D \times \{\theta\}$ ($\theta \in S^1$) lies on a leaf and $\{u\} \times S^1$ ($u \in D$) meets all the leaves of F.

Take an arbitrary non-vanishing vector field Y transverse to F and such that Y is tangent to the circles $\{u\} \times S^1$ for $u \in D$. There are closed plaques $P_1, \ldots, P_k$ such that any trajectory of Y meets $\cup_i P_i$. Since the plaques P_i can be joined by chains of plaques to some points of $D \times S^1$ and since a foliation in a neighbourhood of a relatively compact simply connected domain of a leaf is equivalent to a trivial one, then it follows from [3] that there exists a preserving F diffeomorphism ϕ such that $C_i = \phi(P_i)$ ($i = 1, \ldots, k$) are closed cells lying on $D \times \{\theta_i\}$, $\phi(D \times S^1) \subset D \times S^1$ and $\phi | D' \times S^1 = id$ for some closed ball $D' \subset \operatorname{Int} D$. We may assume that $\theta_i \neq \theta_j$ when $i \neq j$.

Put $Z = \Phi_* Y$. Then Z is transverse to F and every trajectory of Z meets one of the cells C_i.

Now, choose pairwise disjoint open segments $I_1', \ldots, I_k' \subset S^1$ such that $\theta_i \in I_i'$. For any i there exists a diffeomorphism Ψ_i of M such that $\operatorname{supp} \Psi_i \subset D \times I_i'$, $\Psi_i(D \times \{\theta\}) = D \times \{\theta\}$ for any θ, $K_i' = \Psi_i(C_i') = K_i \times \{\alpha_i\}$ and $K_i'' = \Psi_i(C_i'') = K_i \times \{\beta_i\}$, where $I_i = \langle \alpha_i, \beta_i \rangle \subset I_i'$, $C_i' \subset D \times \{\alpha_i\}$ and $C_i'' \subset D \times \{\beta_i\}$ are cells obtained by pushing C_i along trajectories of Z, and $K_i \subset D'$. We may also assume that S^1-saturations of supports of Ψ_i's in $D \times S^1$ are pairwise disjoint.

Gluing together vector fields $Z | M \setminus \cup_i D \times I_i$ and $\Psi_{i*}(\partial/\partial\theta) \mid D \times I_i$ $(i = 1, \ldots, k)$ we obtain a vector field X with the required properties (Figure 2).

§ 3. *Main results.*

Assume that a transversely oriented codimension-one foliation F of a compact manifold M admits several Novikov components. According to Lemmas 1 and 2, there exists a saturated open set U such that any function $f \in \mathrm{Adm}(F)$ is positive somewhere in U. On the other hand, if M is the only component of F, then there are no obstructions related to the equality (2) and the families $\mathrm{Adm}(F)$ and $C^\infty_{\pm}(M)$ coincide. In this manner, we established the following:

Proposition. $\mathrm{Adm}(F) = C^\infty_{\pm}(M)$ if and only if M is the only Novikov component of F.

We have also the following:

Theorem. $\mathrm{Mean}(F) = C^\infty_{\pm}(M)$ if and only if M is the only Novikov component of F.

In the proof, we shall use the following:

Lemma 4 (c.f. [6]). The class Mean(F) is closed under multiplication by positive factors and under composition with diffeomorphisms preserving F. If Riemannian metrics g and g' are conformally equivalent,

$$g' = e^{2\Psi} g$$

then

$$h(F,g') = e^{-\Psi} h(F,g) + n\, g(X, \nabla(e^{-\Psi})),$$

where n = dim F , X is the positively oriented g-unit orthogonal to F vector field on M and $\nabla\alpha$ denotes the gradient (w.r.t.g.) of a smooth function α.

Proof of the Theorem. In view of (3) and the Proposition, it remains to show that if M is the only Novikov component of F and $f \in C^\infty_\pm(M)$, then $f \in$ Mean(F).

If $f \equiv 0$, then $f \in$ Mean(F) according to Rummler's [4,Prop. 1] and Sullivan's [5, Theorem II.20] results.

Suppose that $f(x_1) > 0$ and $f(x_2) < 0$ for some x_1 and x_2 of M. Take a vector field X satisfying the conditions of Lemma 3 and distinguished by both F and X charts ϕ_1 and ϕ_2 such that $x_1 \in U_1$, $x_2 \in U_2$, $f \mid \bar{U}_1 > 0$ and $f \mid \bar{U}_2 < 0$. Next, take the leaf L_1 passing through x_1 and cover it by closed cells $\tilde{V}_1, \ldots, \tilde{V}_m$ such that any trajectory of X meets $U_i \tilde{V}_i$. Choose leaves $L_2, \ldots, L_m$ and $L'_1 \ldots, L'_m$ such that $L_j \cap U_1 \neq \emptyset$, $L'_j \cap U_2 \neq \emptyset$, $L_i \neq L_j$ and $L'_i \neq L'_j$ when $i,j \in \{1,\ldots,m\}$, $i \neq j$. For any i, push the cell $\tilde{V}_i$ along trajectories of X to get cells $V_i \subset L_i$ and $V'_i \subset L'_i$ (Figure 3). Taking cells $\tilde{V}_i$ small enough one may assume that any segment

γ with end points of V_i meets $U_j V_j'$.

Denote by P_i and P_i' the closures of some plaques of $U_1 \cap L_i$ and $U_2 \cap L_i'$, respectively. As in the proof of Lemma 3, one can establish the existence of a diffeomorphism ϕ of M which preserves F and maps V_i onto P_i and V_i' onto P_i' for $i = 1,\ldots,m$.

Take a Riemannian metric g on M such that $|X| = 1$ and X is orthogonal to F, and denote by h the mean curvature function of F with respect to g. Since the function $f_1 = f \circ \phi$ is somewhere positive and elsewhere negative on any segment γ of a trajectory of X with end points on V_i $(i = 1,\ldots,m)$, then there exists a strictly positive C^∞-function $k: M \longrightarrow \mathbb{R}$ such that

(4) $$\int_\gamma f_2 = 0,$$

when $f_2 = k.f_1 - \frac{1}{n} h$, $n = \dim F$ and γ is as above.

From (4), it follows that there exists functions $\lambda_i : V_i \longrightarrow \mathbb{R}$ $(i = 1,\ldots,m)$ such that

$$\lambda_i(x) - \lambda_j(y) = \int_0^s f_2(\phi_t x)dt$$

when (ϕ_t) is the flow of the vector field X, $x \in V_i$ and the trajectory of X passing through x meets V_j at the point $y = \phi_s x$.

The formulae

$$\Psi(\phi_s x) = \lambda_i(x) - \int_0^s f_2(\phi_t x)dt \quad (x \in V_i,\ s \in \mathbb{R},\ i = 1,\ldots,m)$$

define a C^∞-function Ψ satisfying the equation $X_\Psi = -f_2$.

Finally, put $g' = c^{2\Psi} g$. Lemma 4 shows that

$$h(F,g') = ne^{-\Psi}(f \circ \phi).$$

Applying Lemma 4 once again one can see that $f \in \mathrm{Mean}(F)$.

Remark. The realized hitherto study of mean curvature functions for foliations allow us to conjecture that the equality

$$\mathrm{Adm}(F) = \mathrm{Mean}(F)$$

holds for any codimension-one transversely oriented foliation F of any compact manifold.

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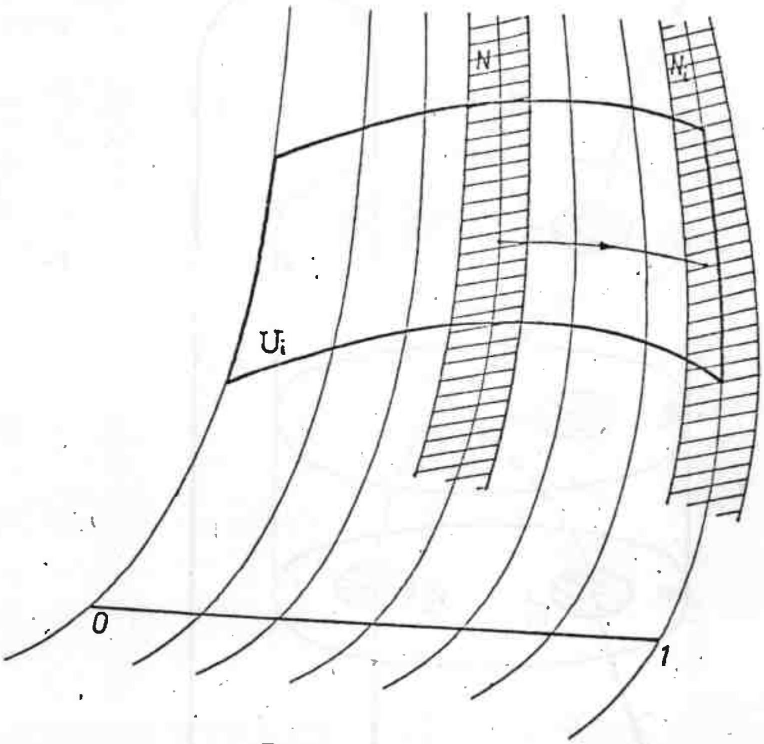

Fig.1

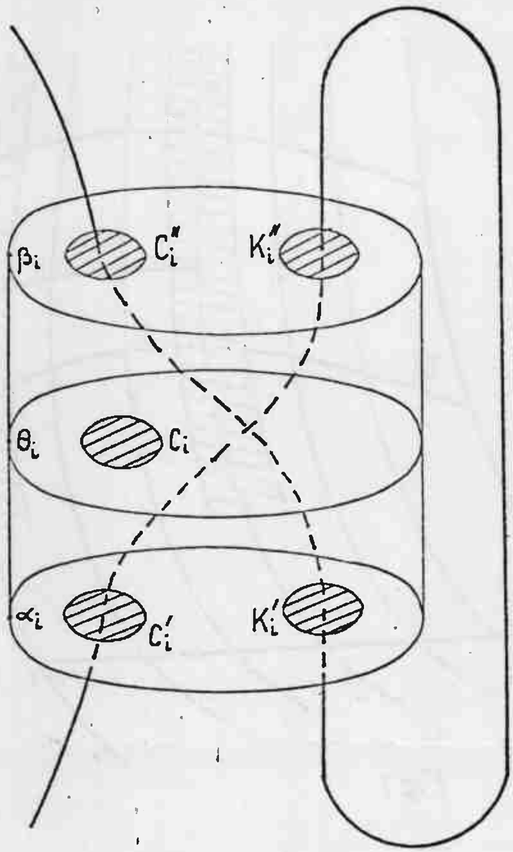

Fig. 2

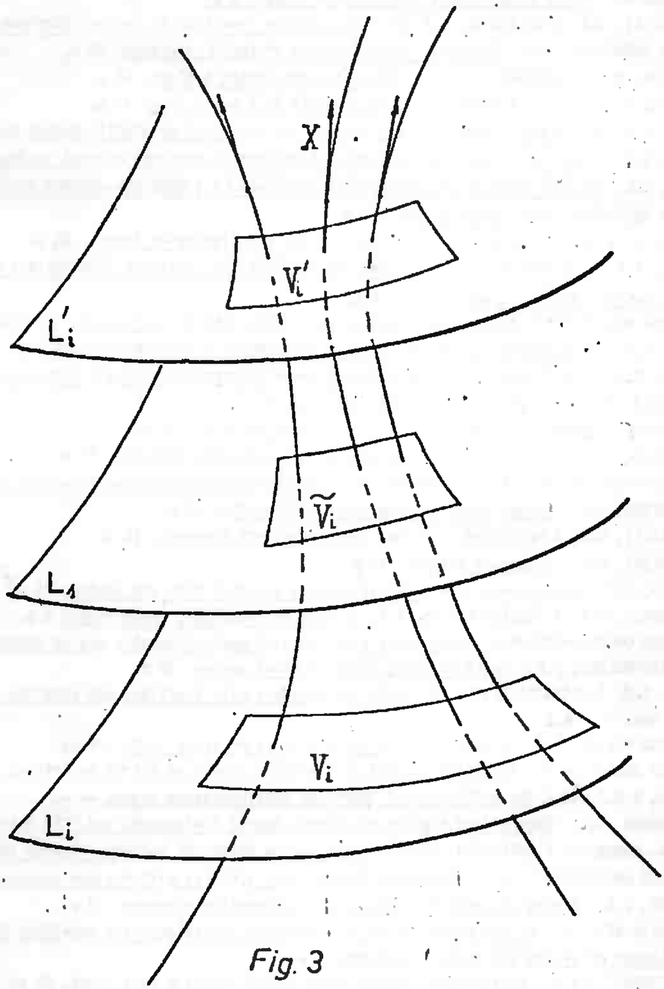

Fig. 3

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