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Abstract

In the financial markets, there is a well established portfolio optimization model called generalized mean-variance model (or generalized Markowitz model). This model considers that a typical investor, while wanting (expected) returns to be high, also wants returns to be as certain as possible. In this paper we introduce a new media optimization system based on mean-variance model, a novel approach in media planning. After presenting the model in full generality, we discuss possible advantages of the mean-variance paradigm, such as its flexibility in modeling the optimization problem, the ability of dealing with many media performance indices - satisfying most of the media plan needs - and, more important, the property of diversifying the media portfolios in a natural way, without the need to set up *ad hoc* constraints to enforce diversification.

Keywords: media planning, mean-variance optimization, parametric quadratic programming.

1. Introduction

In every media advertising campaign, after the several strategic aspects like market, target, flighting, budget, reach, frequency and other parameters are defined, media planners must build the media portfolio, a schedule that determines where, when and how many spots should be part of the media plan. The planner's challenge is to obtain a media portfolio within budget that reaches the target audience at the right time. Usually previous information (like product, marketing objectives, target, budget and eventually special tactics) help the media planners to decide what media types would be effective in reaching the planned target.

Currently, systems for media optimization are valuable tools for media planners. Their goal is to suggest budgets allocations, based on performance indices of vehicles like ratings, reach, cost per thousand, coverage, composition and others.

Most of the media optimizers known by the authors are based on two major paradigms:

a) Greedy algorithms: these algorithms start from a beginning (small) media portfolio and fill it incrementally by including vehicles (i.e. programs, newspapers, magazines) spots with greater marginal gain until the desired reach-frequency level is achieved;

b) Linear programming: optimizers based on this model return media portfolios with optimum expected return, usually expressed as a linear combination of cost, exposures and other indexes. Linear constraints like maximum investment in a media vehicle, minimum GRP in a particular TV day-part, etc, may be imposed.

The major limitation of these paradigms is that they take into account only the expected returns (first statistical moment). The variance of the returns (second statistical moment) is not considered. Since in general the vehicles with the greatest expected returns are also those with the greatest variances, these algorithms tend to return media portfolios with high risk, i.e. with high probability of not achieving the expected reach-frequency rates at the campaign exhibition time. This forces the planners to diversify their media plans by means of *ad hoc* constraints and bounds.

In the financial markets, there is a well established portfolio optimization model called generalized mean-variance model, or generalized Markowitz model (Markowitz, 1987; Sharpe & Alexander, 1990; Stern & Silva, 1995). This model considers that a typical investor, while wanting (expected) returns to be high, also wants returns to be as certain as possible. This means that the investor, in seeking both to maximize expected return and minimize uncertainty (that is, risk), has two conflicting objectives that must be balanced against each other when making the purchase decision. One interesting consequence of having these two conflicting objectives is that the investor should diversify by purchasing not just one asset but several ones. In Markowitz model, assets expected returns are expressed as their mean returns and assets risks are expressed as their covariance matrix. Optimizers of this family usually return a set of efficient portfolios: an efficient portfolio is one which gives a certain expected (mean) return with the lowest risk (variance).

It is possible to establish an analogy between financial and media portfolios. In a similar way of investors, media planners must purchase assets (vehicles spots) with high cost effectiveness and high probability of achieving the desired gains (reach-frequency results).

In this paper we introduce a new media optimization system based on Markowitz model, a novel approach in media planning (as far as we know). After presenting the model in its full generality, we discuss possible advantages of the mean-variance paradigm, such as its flexibility in modeling the optimization problem, the ability of dealing with many media performance indices -

satisfying most of the media plan needs - and, more important, the property of diversifying the media portfolios in a natural way, without the need of set up *ad hoc* constraints to enforce diversification.

In Section 2 we present the Markowitz financial model, and in Section 3 we discuss its formulation in the media optimization problem. In Section 4, a real world application for the print media in Brazil market is analyzed in details, on real data. We restrict the case to a situation where only magazines were considered for the communication plan. For those readers not used to the media planning terms, we present at the Appendix a brief summary of some terms used in this article (Surmanek, 1993).

2. Mean-variance optimization concepts

In the Markowitz financial model, the objective function describes a balance between expected return and the corresponding risk (or volatility) of each portfolio. One expects to maximize the first component (expected return) and simultaneously to minimize the second component (volatility). The decision variables are interpreted as amounts to be invested in several assets out from a universe of potential candidate assets (i.e. stocks, bonds, commodities, currencies, etc.).

Formally, the investor should choose fractions $x_1, x_2, \ldots, x_M$ to be invested in the assets $1 \ldots M$, subject to the constraints $x_j \geq 0$ $(j = 1 \ldots M)$ and $\sum_{j=1..M} x_j = 1$.

We consider the returns on investment of individual assets $r_1, r_2, \ldots, r_M$ as random variables with a joint probability distribution. The portfolio return is

$$R(x) = \sum_{j=1}^{M} r_j \; x_j \, .$$

The mean expected return of the portfolio is

$$e(x) \equiv E(R(x)) = \sum_{j=1}^{M} \mu_j \; x_j$$

where E indicates the *expectation* of a random variable and $\mu_j = E(r_j)$.

The estimated *variance* of the return on the investment is

$$\sigma^2(x) \equiv Var(R(x)) = \sum_{i=1}^{M} \sum_{j=1}^{M} \sigma_{ij}^2 \; x_i \; x_j$$

where

$$\sigma_{ij}^2 = E((r_i - \mu_i)(r_j - \mu_j))$$

is the estimated *covariance* between r_i e r_j. In particular,

$$\sigma_{jj}^2 = E\left((r_j - \mu_j)^2\right)$$

is the variance of r_j.

In matrix notation we have:

$$x = [x_1, x_2, \ldots, x_M]' \, ;$$

$$\mu = [\mu_1, \mu_2, ..., \mu_M]' \; ; \quad Cov = \begin{bmatrix} \sigma_{11}^2 & \cdots & \sigma_{1M}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{M1}^2 & \cdots & \sigma_{MM}^2 \end{bmatrix};$$

$$e(x) = \mu' x \; ;$$

$$\sigma^2(x) = x' \; Cov \; x$$

where vectors are represented as column matrices and the symbol ' indicates the transpose.

Usually, portfolio must respect some constraints, which are imposed by existing laws and regulation or simply demanded by the investors to their agents. Markowitz model is capable of dealing with equality and inequality linear constraints. Some examples include:

- Pension funds managers are obliged by law to respect investment upper bounds in certain classes of assets, like junk bonds and common stocks.
- Investors could require that a certain minimum percentage of their money should be allocated to more conservative assets or specific sectors of the economy, such as govern bonds.

The linear equality constraints have the form:

$$H \; x = h$$

where H is a $K \times M$ matrix of constraints coefficients, h is a $K \times 1$ vector of bounds and K is the number of equality constraints.

The linear inequality constraints are formulated as:

$$G \; x \leq g$$

where G is a $L \times M$ matrix of constraints coefficients, g is a $L \times 1$ vector of bounds and L is the number of inequality constraints.

2.1 The objective function and efficient portfolios

It is well known that portfolios with higher expected returns have large risk (variances). So, portfolio optimization based only on the expected return brings solutions with high risks and low effectiveness. In the Markowitz model, the objective function to be maximized is a trade-off between the expected return of the portfolio and its variance.

A portfolio $x(\alpha)$ is optimal if it maximizes the utility function

$$\begin{aligned} x(\alpha) \in \arg\max_x U(x) &= \alpha \, e(x) - \sigma^2(x) \\ &= \alpha \; \mu' x \; - \; x' \; Cov \; x \end{aligned}$$

subject to the established linear constrains

$$x \geq 0 \,|\, 1' \, x = 1 \; , \; Te \; x = te \; , \; Tl \; x \leq tl \; .$$

The parameter α can be interpreted as a measure of the investor's risk tolerance. In other words, α is a measure of how much maximizing the expected return is preferred instead of minimizing the investment risk. Higher the α value, higher the importance (for the investor) of the expected return as compared with the portfolio risk.

Each value of the parameter α determines a portfolio $x(\alpha)$ that maximizes $U(x)$. An optimal solution $x(\alpha)$ is called an *efficient solution* or *efficient portfolio.* A portfolio x is efficient if:

- There are not portfolios with expected return equal to the one of x and with smaller variance;
- There are not portfolios with variance equal to the one of x and with a higher expected return.

For simplicity, we denote the expected return and variance of the efficient portfolio $x(\alpha)$ by $e(\alpha) \equiv e(x(\alpha))$ and $\sigma^2(\alpha) \equiv \sigma^2(x(\alpha))$. The curve $e(\alpha)$ versus $\sigma^2(\alpha)$ is called *efficient frontier.*

Figure 1 shows the chart of expected returns (e) vs. risk (σ^2) and the efficient frontier for a specific portfolio optimization problem. Points A, B, C and D on the efficient frontier represent efficient portfolios with expected return and variance ($e(\alpha)$, $\sigma^2(\alpha)$). Point A is determined by a high α value and point D is determined by a low α value.

Any portfolio x whose pair $(e(x), \sigma^2(x))$ is below the efficient frontier is *dominated* by portfolios lying on the frontier. For example, portfolio E is dominated by portfolio B (which has the same risk but a higher expected return) and the portfolio C (which has the same expected return but a lower expected risk).

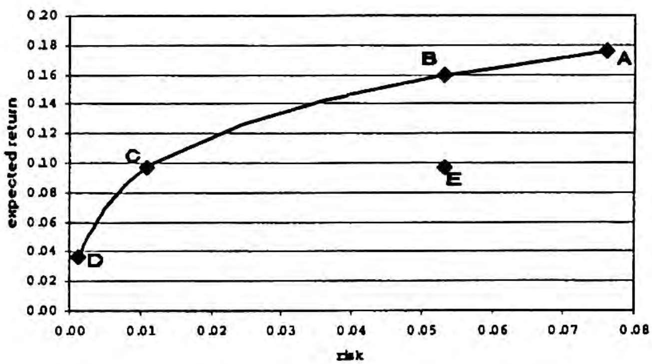

Figure 1 – Example of efficient frontier and some points of interest

By varying the value of α, an optimization solver based on Markowitz model offers to the investor a set of efficient portfolios, letting him choose the most suitable for his purposes (for example, the investor may elect the efficient portfolio which achieves the desired expected return). Furthermore, through analysis of the efficient frontier analysis, the investor may evaluate the return/risk rate of his choice and reveal his demands. Let us take the chart on Figure 1 and consider, for example, an investor who wants an expected return (e) about 0.18. Then the chosen efficient portfolio should be near the point A and achieve a variance (σ^2) about 0.075. However, if the investor accepts a return slightly lower as 0.16, the corresponding efficient portfolio will be near point B and achieve a variance about 0.055. He would conclude that by reducing his expected return demand by 11% he gets a portfolio with a 27% lower variance.

2.2. Risk protection through diversification

In the financial markets, investors look for diversification as a means of reducing risk, and devise portfolios containing assets which tend to have price behaviors opposite to each other. As we briefly discuss below, the main differential aspect of the mean-variance model is that diversification is attained naturally, as a consequence of the trade-off between expected return and risk, without the need of setting up *ad hoc* constraints to enforce diversification.

The utility function $U(x)$ seeks portfolios with high expected return and low variance. Using the definition of $\sigma^2(x)$, it is easy to see that the variance is lower when the correlation coefficients of the selected portfolios

$$Corr_{ij} = \frac{\sigma_{ij}^2}{\sqrt{\sigma_{ii}^2} \cdot \sqrt{\sigma_{jj}^2}}$$

is negative. Two assets i and j have a negative correlation when r_i and r_j behave in opposite directions, i.e., for example an increase in r_i suggests a simultaneous decrease in r_j and conversely.

3. The Markowitz model and the optimization problem in media planning.

The main topic of this section is the application of the general approach described in section 2 to media optimization. Even though the application described in Section 4 is restricted to magazines, the setup below is quite general.

We shall define a "vehicle" as the smallest unit on which one can take a media purchase decision. It could be an issue of a magazine, newspaper or a particular program in a TV channel. The set of vehicles ("decision units") will be denoted by:

$$U = \{1,2,...,M\},$$

where M is the total number of vehicles considered. Two different vehicles will be denoted by u and v.

We assume that the data comes from a sample of interviewees. This set is denoted by I.

$$I = \{1,...,N\},$$

where N is the number of interviewees in the sample. We can assume also that to each interviewee we associate a "weight" w_i. This can be viewed as if each individual in the sample is a representative of a sub-population with weight w_i. We will denote by *Pop* the total size of the target, and then $Pop = \sum_{i \in I} w_i$.

3.1 Some basic statistics

For each person in the sample of interviewees, several social-demographic data are collected, such as sex, age, family income, personal income and questions on media consumption. The media consumption data usually consist of several questions like:

- Consumption (saw/read/listened) of the particular vehicle in a relative longer period of time at least once.
- Consumption of the last issue.

- Time since last consumption.
- Number of saw/read/listened issues among the last ten for example.

Several questions are included in order to get a better understanding of the chance that one particular individual consumes a given vehicle. To know for example that a person saw/read the last edition of a magazine is not enough to ascertain the frequency with which this person actually does it. In this respect questions related to habits are relevant.

From these questions we compute a measure of how much a person consumes of a particular vehicle, or in other words the probability that person i consumes vehicle v. We will denote this measure by $F(i,v) \in [0,1]$. This index will represent a fraction of the total number of opportunities to see/listen/read the vehicle v by the individual i: $F(i,v) = 0$ means that the individual i never reads/sees the vehicle v. $F(i,v) = 1$ means the opposite, i.e., the individual i always see/listen/reads vehicle v. In our work, $F(i,v)$ has the following interpretation: it is an estimate of the probability that the individual i in the population would see/listen/read an advertisement exhibited in vehicle v.

The computation of $F(i,v)$ is a procedure *ad-hoc*, which depends on the available data and the importance assigned by the media planner for each question. In section 4 we present a particular computation of $F(i,v)$ from a magazine consumption data set.

The estimated return of a vehicle v is the mean of the values of $F(i,v)$, weighted by w_i, over all members of the sample. We denote this mean by μ_v.

$$\mu_v = \frac{1}{Pop} \sum_{i=1}^{N} w_i \, F(i,v) .$$

The value $\mu_v \in [0,1]$ is an estimate of the *reach* (also called *rating*, see appendix) of vehicle v, or the probability that an individual from the population will be exposed to an ad placed in vehicle v. If x_v ads are purchased in vehicle v, the average expected number of exposures per individual in that vehicle is

$$\mu_v \, x_v .$$

The estimated covariance between two vehicles u and v is:

$$\sigma_{u,v}^2 = \frac{1}{Pop} \sum_{i=1}^{N} w_i \, (F(i,u) - \mu_u)\,(F(i,v) - \mu_v) .$$

In particular,

$$\sigma_{v,v}^2 = \frac{1}{Pop} \sum_{i=1}^{N} w_i \, (F(i,v) - \mu_v)^2 .$$

The correlation coefficient,

$$Corr_{u,v} = \frac{\sigma_{u,v}^2}{\sqrt{\sigma_{u,u}^2}\,\sqrt{\sigma_{v,v}^2}} ,$$

varies within the interval $[-1,+1]$. When two vehicles u and v have a high positive correlation, that means that both reach approximately the same universe (one who likes vehicle u also likes vehicle v, and *vice versa*). On the other hand when u and v have high negative correlation coefficient, that means that the vehicles have a tendency to reach different or even exclusive targets (individual liking vehicle u doesn't like vehicle v and *vice versa*).

Similarly to what happens in the financial markets, if we base our optimization procedure exclusively on the expected return (which, in our case is represented by μ_v) we will produce schedules with lower reach if the selected vehicles have positive correlation coefficients among them. In this sense, a rational optimization procedure will naturally select vehicles with negative correlation, increasing in this way the probability of obtaining a higher reach.

The *Gross Rating Point* (or simply GRP), another common measure of a schedule's efficiency, represents the cumulative rating points of all vehicles scheduled, i.e., the sum of the expected rating of each spot. Its estimation is quite similar to the average expected number of exposures per individual, except the fact it is presented in a percentage scale:

$$GRP = 100\,\mu'\,x\,.$$

3.2 Objective functions

In our model, the decision variable is a vector $x = [x_1, x_2, \ldots, x_M]'$, where x_v is the number of ads to be inserted in vehicle v.

We denote by c_v the cost of each advertisement insertion in vehicle v and by c the vector $c = [c_1, c_2, \ldots, c_M]'$. The *total cost* of schedule x is:

$$\sum_{v=1}^{M} c_v\, x_v \quad \text{or, in matrix notation,} \quad c'\,x\,.$$

The *expected return* of a schedule x is the average number of exposures per individual in the population, and this is obtained by:

$$e(x) = \mu'\,x\,. \tag{1}$$

Similarly, the expected GRP of a schedule (see appendix) is:

$$GRP(x) = 100\mu'\,x\,. \tag{2}$$

The variance and standard deviation (in hits/individual) of the return are obtained by:

$$\sigma^2(x) = x'\,Cov\;x \quad \text{and} \quad \sigma(x) = \sqrt{\sigma^2(x)}\,,$$

where *Cov* is the covariance matrix among vehicles,

$$Cov = \begin{bmatrix} \sigma_{11}^2 & \cdots & \sigma_{1M}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{M1}^2 & \cdots & \sigma_{MM}^2 \end{bmatrix}.$$

The interpretation of standard deviation in the present context is the following. Suppose a schedule x with $e(x) = m$ and $\sigma(x) = sd$. The average number of impressions per individual is in the interval $[m - sd,\ m + sd]$ with 67% of confidence. So, when two distinct schedules give the same expected return and same cost but different standard deviations, the one with smaller standard deviation (and consequently, smaller variance) is preferable, because its probability of success (achieving the desired average of impressions per individual) is larger.

It is worthwhile to discuss the importance of trade-off between cost-effectiveness and variance. Here we call cost-effectiveness the amount of exposures per monetary unit paid in a vehicle or a schedule. Most of media optimizers are oriented by cost-effectiveness, i.e. they seek schedules composed by vehicles cheaper and with more expected exposures. Since practice shows that these vehicles have in general a large variance, the built schedules variance is also large.

The Markowitz model allows several distinct optimization formulations. Currently, our media optimization tool under development has two optimization modes, described below. Both of them are based on the equilibrium between the cost-effectiveness (which has to be maximized) and the variance (to be minimized).

3.2.1 GRP maximization at a fixed budget:

In this optimization mode, the planner sets up the available budget and the optimizer goal is to return a set of schedules with optimal trade-off between GRP (to be maximized) and variance (to be minimized).

From equations (1) and (2), we see that maximizing expected $GRP(x)$ is equivalent to maximizing $e(x)$. So, a portfolio $x(\alpha)$ is optimal if it maximizes the utility function

$$\begin{aligned} x(\alpha) \in \arg\max_x U(x) &= \alpha\, e(x) - \sigma^2(x) \\ &= \alpha\, \mu' x - x' \, Cov\, x \end{aligned}$$

subject to the linear constraints

$$x \geq 0 \mid c' x = B, \ Hx = h, \ Gx \leq g,$$

where α is a "risk tolerance" parameter, B is the advertisement budget and H, h, G, g are matrices and vectors for equality and inequality constraints presented in section 2.

The parameter α, already discussed in section 2, plays a similar role in the present utility function. Higher the α value, higher the importance of $e(x)$ component as compared with the $\sigma^2(\alpha)$ component. The solver we use automatically varies α, returning not just a unique schedule, but a set of schedules with different expected GRP and variances.

3.2.2 Cost minimization at fixed GRP:

In this optimization mode, the planner sets up the desired GRP and the optimizer goal is to return a schedule set with optimal trade-off between cost and variance (both to be minimized).

A portfolio $x(\alpha)$ is optimal if it maximizes the utility function

$$x(\alpha) \in \arg\max_x U(x) = -\alpha\, c' x - x' \, Cov\, x$$

subject to the linear constrains

$$x \geq 0 \mid 100\, \mu' x = R, \ Hx = h, \ Gx \leq g,$$

where α is a "risk tolerance", R is the desired GRP and H, h, G, g are matrices and vectors for equality and inequality constraints presented in section 2.

Analogously to the previous optimization mode, the solver automatically varies α, returning a set of schedules with different costs and variances.

3.3 Linear constraints in media optimization

The constraints $Hx = h$, $Gx \leq g$ are general enough for covering a large set of restrictions found in problems of media optimization. One kind of restriction has been already shown: the reader may notice that the available budget or the desired GRP (see previous section) are particular cases of linear constraints. At least two other classes of constraints commonly found in media planning also can be formulated as linear constraints through the appropriated setup of Te, te, Tl and tl.

3.3.1 Upper and lower bounds

The media planner may set up the minimum ($x_v \geq D$), maximum ($x_v \leq D$) or exact ($x_v = D$) number of advertisements to be placed in some vehicles. The media planner could define a set of vehicles and specify the bound to be satisfied for each vehicle belonging to this set.

This functionality is desirable in several cases: for example, when there are pre-purchased spaces in some vehicles, or when the advertiser wants concentrate focus in some specific magazines.

3.3.2 Constraints over entire sets

Beyond specifying bounds for *each* vehicle in a set, the media planner frequently may specify bounds to be satisfied for the *group* of vehicles. For sake of clarity, we give two hypothetical examples:

- At least 40% of the available budget must be applied in women magazines;
- No more than 20% of the budget may be applied in tabloids.

3.4 Linear × Integer Programming

The Markowitz model belongs to a so-called Quadratic Programming class - since $\sigma^2(\alpha)$ is quadratic. However, Wolfe (1959) demonstrates how to solve the quadratic programming problem using an adaptation of the Simplex algorithm for linear programming problems, namely the LC-Simplex (Linear-Complementarity Simplex). The average running time of LC-Simplex method is polynomial (Schrijver, 1986), what in practice means that LC-Simplex is fast enough for solving a large family of linear optimization problems. An available (improved) implementation of Wolfe's method is Critical Point (Stern, 1995).

One of the main mathematical differences between portfolio and media optimization problems is that in portfolio optimization the decision variables are fractions of a total amount to be invested in the several available assets, and in practical terms these decision variables can be seen as rational numbers. On the other hand, in media optimization the decision variables are the number of advertisements in each vehicle, i.e., integer numbers. So, media optimization is an integer programming problem (Martin, 1999).

Since integer programming algorithms are much more computer intensive than LC-Simplex, in some cases their use in media optimization could be unfeasible. So we solve the optimization problem using a solver based on LC-Simplex and round the obtained solution (Salkin and Mathur, 1989). There is not a guarantee that the final solution is optimal (and frequently it is not), but practical experience have shown that this approach is a very good approximation.

3.5 Estimation of reach-frequency distribution

In this paper, the schedule's reach-frequency distribution is computed by estimating the expected number of times each individual will be exposed to the ads. Consider a schedule $x=[x_1,x_2,...,x_M]'$, where x_v is the number of advertisements in vehicle v. If $F(i,v)$ is the estimated probability of interviewee i be exposed to one advertisement in vehicle v, then the expected number of exposures of interviewee i in vehicle v is $F(i,v)\ x_v$. Therefore, the expected number of exposures of interviewee i in the schedule is

$$f_i = \sum_{v=1}^{M} F(i,v)\ x_v\ .$$

The reach-frequency distribution is computed by a histogram (weighted by w_i) of f_i, i=1,...,N.

4. An application to media planning in print advertisement

To exemplify the methodology and evaluate its results, in this section we show the application of the proposed model to a set of real data. We simulate some cases where the objective is to find optimal schedules for advertising in magazines. In this study we selected the 87 most known magazines, sold weekly, fortnightly or monthly.

The information source is a large database containing media consumption habits and related information collected through a representative sample in the nine largest metropolitan areas in Brazil: São Paulo, Rio de Janeiro, Belo Horizonte, Brasília, Curitiba, Porto Alegre, Salvador, Recife and Fortaleza. The original sample consists of 18,833 readers of magazines (people who declared that had read at least one issue of magazine in last 6 months) of both sexes and 10 years old or more.

Beyond media consumption habits (to be detailed below), other social-demographics characteristics like sex, age, income and others were collected for each interviewee. Brazilian research institutes employ some default criteria - based on some of these features - for grouping population in five social class: A (the highest), B, C, D and E (the lowest). Figure 2A shows the Brazilian percentages of households in these social classes. Figure 2B shows the percentages of magazine readers in these social classes, estimated from the consumption database. Comparing the graphs 2A and 2B, it is easily seen that in Brazil the habit of reading magazines is more common in higher social classes.

The target for our media plan was composed by women of social classes B, C and D. So, we filtered the sample in order to get only the consumption data for this group. The filtered sample contained 7,159 people. In order to give a snapshot of magazine consumption of the selected group, Figure 2C shows the proportions of interviewees according to the number of distinct magazines read. For example, 29% of the group declared to have read only one or two distinct magazines in last months (without considering the quantity of read issues of each magazine).

For the definition of $F(i,v)$, the answers to four types of questions were considered:

- Magazines read in last months, where the number of months depends on periodicity. For example, we took 6 months for monthly magazines.
- Number of issues read among the last ten.
- Time elapsed since the last issue read.
- Whether the last issue red was in fact the last issue published.

We took $F(i,v)$ as a natural weighted combination of the answer to these questions. Then, we compute μ_v, the coverage estimate for each magazine.

Figure 3 shows the costs (in US$ thousands) and estimated reach for the 20 magazines with largest ratios between reach and cost. Although the ratio reach/cost is not explicitly used in our work, it may be seen as a measure of cost-effectiveness.

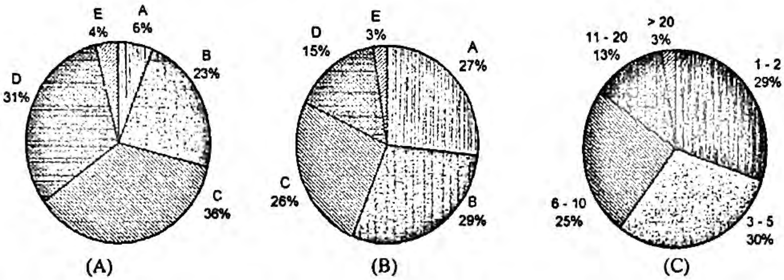

Figure 2 – (A) Distribution of Brazilian households by social class (Source: Associação Brasileira de Empresas de Pesquisa); (B) Distribution of magazine readers by social class; (C) Distribution of intervewees by quantity of different magazines read in last 6 months.

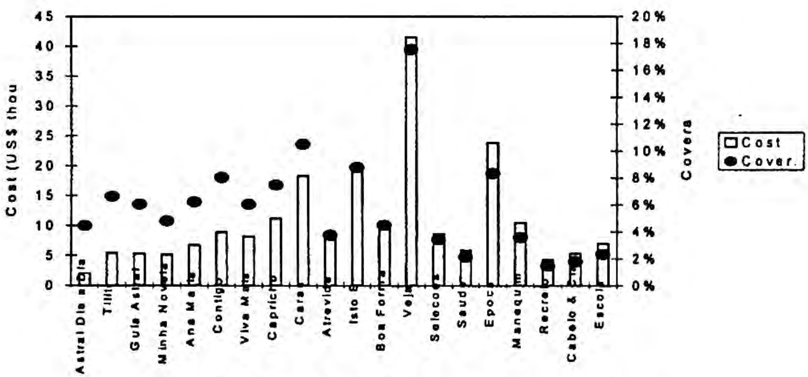

Figure 3 - 20 magazines with largest ratios coverage/cost

4.1 Numerical Results - Case 1

In this case the specified a budget of US$ 370,000 (equivalent to approximately R$ 800,000 in Reais, Brazilian monetary units). The goal was to find schedules with optimal GRP-variance balance. No additional constraints or bounds were established.

As discussed in section 2.2, the mean-variance optimization model gives a set of efficient schedules. The solver automatically varies α and finds the solution which maximizes the utility function $U(x) = \alpha e(x) - v(x)$. So, each efficient schedule corresponds to a distinct trade-off between the expected return $e(x)$ and the variance $\sigma^2(x)$, controlled by the value of α. Analyzing the statistics shown for each schedule, the media planner can select the most appropriate ones.

Table 1 shows the efficient schedules statistics returned by the solver. Each row corresponds to one efficient schedule. The columns are the schedule number (Sched.), the expected GRP, the estimated Reach 1+, the average frequency among the covered people (Freq 1+), the percentages of people exposed one, two, three or above four times to the schedule (%1, %2, %3, %4+), the schedule cost (in US$ 1,000), the standard deviation (StdDev) of return and the α value in the objective function.

Figure 4 shows the reach-frequency distributions for the efficient schedules. Each column corresponds to a schedule and is composed by four areas, each one representing the expected population exposed one (bottom area), two (second area), three (third area) and four or more (top area) times to the advertisements. The schedule's reach 1+ is given by the total column height.

The first schedule (first row in Table 1 and first column in Figure 4) is that with largest α, where the component $e(x)$ is overweight and therefore the efficient schedule is that with maximum GRP (despite the variance). All advertisements are concentrated only in the magazine with maximum reach per dollar, as shown in Table 2. The expected average number of exposures per individual in the whole target is about 5.7 (calculated directly by GRP/100) with a standard deviation of 21.2. This quite large standard deviation may be understood by examining Figure 4: in this schedule, a few people (9.7%) are overexposed to the ads, while the remaining (90.3%) are not exposed at all. This schedule exemplifies the behavior of linear optimizers based just on the cost-effectiveness. When using those optimizers, the media planner must set up several constraints and bounds in order to attain diversified schedules.

As the α value becomes lower, the expected GRP becomes gradually less important as compared with the variance. As consequence, smaller α, smaller the standard deviation and consequently the GRP. This effect can be perceived in Table 1. Notice that the schedules GRP and standard deviations are not strictly descending, due to post-optimization rounding cited in section 3.4. For the same reason, the schedules costs are not exactly the specified. The extreme cases are the last schedules, where α is so small that the variance component $\sigma^2(\alpha)$ dominates the utility function. In these cases, the solver return schedules with minimum variance, even with small GRP. So, the last schedules are too conservative. Therefore, the most suitable schedules are those located in the medium region of the table, as we see now.

Notice that schedules 12, 13 and 14 concentrate more people in the frequency 4+ and less people with just one impression than schedules 15 to 18. The main reason for this phenomenon becomes clear when we analyze these schedules, shown in Table 2. The efficient schedules becomes successively more diversified, as a natural consequence of the relative importance of variance in the utility function and without the need of additional constraints. Figure 4 shows that by purchasing ads in more magazines (and less ads per magazine, since the budget is fixed), the odds of people seeing several ads becomes smaller at the same time that the odds of reaching more people becomes larger.

Table 1 - Efficient schedules for fixed cost of US$ 370,000 – case 1.

Sched.	GRP	Reach 1+	Freq 1+	% 1	% 2	% 3	% 4+	Cost US$ 1,000	StdDev	α
1	565.8	9.7%	58.6	0.0%	0.0%	0.0%	9.7%	372	21.2	171.14
2	426.1	23.7%	18.0	0.0%	0.5%	1.4%	21.7%	369	11.2	62.93
3	411.4	31.5%	13.1	5.2%	2.4%	2.2%	21.7%	372	10.3	55.82
4	386.1	36.5%	10.6	7.6%	3.1%	3.2%	22.6%	367	9.2	47.46
5	318.3	48.7%	6.5	9.8%	8.0%	5.7%	25.3%	370	5.9	23.87
6	310.7	50.0%	6.2	10.3%	8.5%	5.6%	25.6%	373	5.6	19.99
7	287.5	52.1%	5.5	11.3%	9.8%	5.9%	25.1%	366	4.9	15.31
8	270.8	57.8%	4.7	15.4%	11.3%	6.5%	24.7%	378	4.3	12.76
9	244.1	61.7%	4.0	19.7%	10.9%	7.2%	23.9%	364	3.6	9.41
10	253.4	61.9%	4.1	19.2%	11.1%	7.3%	24.3%	374	3.8	9.22
11	206.2	68.4%	3.0	27.3%	13.0%	8.7%	19.4%	368	2.7	7.14
12	185.3	70.1%	2.6	28.2%	16.6%	9.3%	16.1%	371	2.2	4.51
13	179.7	69.9%	2.6	28.1%	17.0%	9.5%	15.3%	376	2.1	3.67
14	175.4	69.7%	2.5	28.3%	17.1%	9.5%	14.8%	373	2.0	3.42
15	159.7	70.1%	2.3	30.9%	17.5%	9.4%	12.3%	371	1.7	2.62
16	148.9	69.4%	2.1	32.2%	17.3%	9.4%	10.5%	365	1.6	2.37
17	150.3	69.7%	2.2	32.1%	17.5%	9.3%	10.7%	372	1.6	1.97
18	151.0	69.9%	2.2	32.2%	17.5%	9.4%	10.8%	376	1.6	1.91
19	140.8	68.3%	2.1	32.8%	17.3%	8.9%	9.3%	374	1.5	1.60
20	133.5	67.6%	2.0	33.8%	17.2%	8.5%	8.1%	375	1.4	1.49
21	129.8	67.1%	1.9	34.5%	16.8%	8.1%	7.7%	377	1.4	1.44
22	125.5	66.2%	1.9	34.6%	16.8%	7.8%	7.1%	375	1.3	1.24
23	121.6	65.4%	1.9	34.8%	16.4%	7.8%	6.4%	375	1.3	1.23
24	119.9	65.0%	1.8	34.9%	16.3%	7.8%	6.1%	375	1.3	1.15
25	118.3	64.6%	1.8	34.9%	16.0%	7.7%	5.9%	375	1.2	1.10
26	122.9	65.7%	1.9	34.8%	16.5%	7.8%	6.6%	374	1.3	1.07
27	116.7	64.3%	1.8	35.1%	15.9%	7.6%	5.7%	376	1.2	1.05
28	101.5	60.5%	1.7	36.2%	14.4%	6.0%	3.9%	378	1.1	0.96
29	98.4	59.6%	1.7	36.3%	14.1%	5.7%	3.6%	378	1.1	0.90
30	93.4	58.3%	1.6	36.5%	13.5%	5.3%	3.1%	375	1.0	0.88

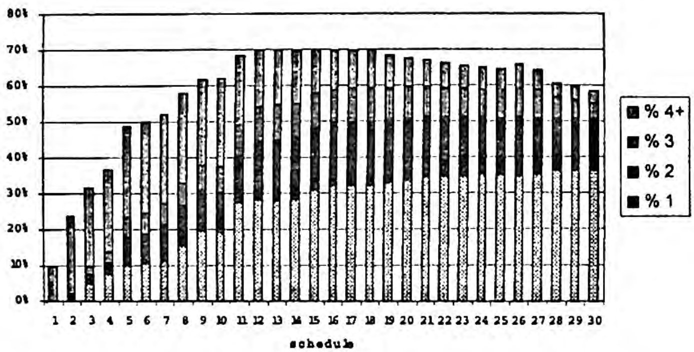

Figure 4 - Reach-frequency distributions for efficient schedules - case 1

Table 2 - Advertisement schedules for case 1.

Magazine	Number of advertisements							
	Sch.1	Sch.12	Sch.13	Sch.14	Sch.15	Sch.16	Sch.17	Sch.18
Ana Maria		3	3	3	2	2	2	2
Caras		2	2	2	2	2	2	2
Contigo		3	2	2	2	1	1	1
Epoca					1	1	1	1
Isto E		1	1	1	1	1	1	1
Minha Novela		1	1	1	1	1	1	1
Recreio							1	1
Tititi		3	2	2	2	1	1	1
Veja		2	2	2	2	2	2	2
Viva Mais		2	2	2	2	2	2	2
Capricho		1	2	2	1	1	1	1
Astral Dia a Dia	128	3	3	2	1	1	1	1
Escola			1	1	1	1	1	1
Familia Crista								1
Guia Astral		3	3	3	2	2	2	2
Manequim						1	1	1
Selecoes		1	1	1	1	1	1	1

4.2 Numerical Results - Case 2

In this case, the goal was to find schedules with optimal cost-variance balance, such that the schedules attained 170 GRP and their costs were below US$ 550,000 (approximately R$ 1,200,000 in Reais, Brazilian monetary units).

Table 3 shows the efficient schedules statistics returned by the solver. The complete table description was given in previous subsection. For a fixed GRP, the solver returns solutions that maximize the utility function $U(x) = -\alpha c(x) - v(x)$, where $c(x)$ is the total cost for schedule x.

The first row corresponds to schedule with largest α, where the component $c(x)$ is overweight and therefore the efficient schedule is that with minimum cost (despite the variance). Analogously to case 1, this schedule concentrates all advertisements only in the magazine with maximum reach per dollar, as shown in Table 4. Although it is the cheapest schedule, its reach is very low: while 9.7% of the target is overexposed to the ads, the remaining people are not exposed ad all. So, this schedule is not suitable for practical objectives. On the other hand, the schedules corresponding to last rows attain the largest reaches and lowest standard deviations, but are too expensive.

Figure 5 shows the reach-frequency distributions for the efficient schedules. We see that the schedules 12 to 15 (bold in Table 3) give high reaches at reasonable costs (from US$ 348,000 to US$ 423,000). Table 4 shows these schedules. The reader can notice that they are similar to those presented in Table 2. This is reasonable, since cost and GRP ranges are similar for both cases, and shows the model coherence.

Table 3 - Efficient schedules with fixed amount of 170 GRP – case 2.

Sched.	GRP	Reach 1+	Freq 1+	% 1	% 2	% 3	% 4+	Cost US$ 1,000	StdDev	α
1	168.0	9.7%	17.4	0.0%	0.1%	0.4%	9.1%	110	6.3	0.58542
2	170.0	23.7%	7.2	2.6%	4.7%	2.9%	13.4%	148	4.5	0.21295
3	170.3	26.3%	6.5	5.0%	4.9%	3.1%	13.3%	154	4.3	0.19028
4	169.9	31.1%	5.5	8.0%	6.3%	3.2%	13.5%	162	4.0	0.16363
5	172.0	45.2%	3.8	16.3%	7.2%	5.6%	16.0%	199	3.2	0.08094
6	172.1	46.8%	3.7	17.3%	7.5%	5.3%	16.7%	205	3.1	0.06424
7	170.8	49.1%	3.5	18.5%	8.1%	5.1%	17.3%	216	2.9	0.04455
8	167.2	52.5%	3.2	21.1%	8.8%	6.0%	16.6%	230	2.7	0.03686
9	167.8	56.3%	3.0	22.5%	11.0%	6.8%	16.0%	251	2.5	0.02578
10	169.9	56.8%	3.0	22.1%	11.4%	7.0%	16.3%	257	2.5	0.02525
11	172.3	63.2%	2.7	27.5%	12.5%	7.6%	15.5%	293	2.4	0.02032
12	**166.3**	**68.8%**	**2.4**	**29.2%**	**17.1%**	**8.8%**	**13.6%**	**348**	**1.9**	**0.01234**
13	**169.4**	**69.4%**	**2.4**	**28.7%**	**17.3%**	**9.5%**	**13.9%**	**366**	**1.9**	**0.00924**
14	**172.2**	**71.9%**	**2.4**	**30.3%**	**17.7%**	**9.7%**	**14.2%**	**402**	**1.9**	**0.00522**
15	**166.5**	**72.2%**	**2.3**	**31.9%**	**16.3%**	**10.5%**	**13.4%**	**423**	**1.8**	**0.00417**
16	168.0	72.5%	2.3	32.0%	16.3%	10.5%	13.7%	429	1.8	0.00138
17	168.7	72.6%	2.3	32.0%	16.4%	10.5%	13.8%	434	1.8	0.00101
18	166.5	72.2%	2.3	31.9%	16.3%	10.5%	13.4%	423	1.8	0.00060
19	164.6	72.3%	2.3	32.4%	16.3%	10.5%	13.1%	457	1.7	0.00047
20	172.8	73.4%	2.4	31.6%	16.8%	10.5%	14.4%	496	1.8	0.00036
21	172.9	73.4%	2.4	31.6%	16.8%	10.6%	14.5%	533	1.8	0.00029
22	172.8	73.4%	2.4	31.7%	16.7%	10.6%	14.5%	531	1.8	0.00027
23	172.9	73.4%	2.4	31.7%	16.7%	10.6%	14.5%	537	1.8	0.00015

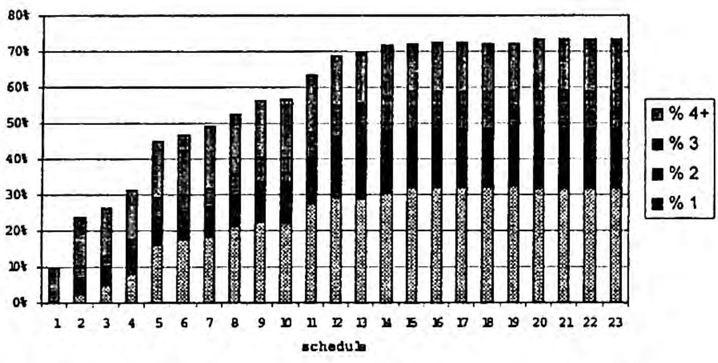

Figure 5 - Reach-frequency distributions for efficient schedules - case 2

Table 4 - Advertisement schedules for case 2

Magazine	Number of advertisements				
	Sch.1	Sch.12	Sch.13	Sch.14	Sch.15
Ana Maria		3	3	2	2
Caras		2	2	2	2
Contigo		2	2	2	1
Epoca				1	1
Isto E		1	1	1	1
Minha Novela		1	1	1	1
Tititi		3	2	2	1
Veja		2	2	2	3
Viva Mais		2	2	2	2
Capricho		1	2	1	1
Astral Dia a Dia	38	2	2	2	1
Boa Forma				1	
Escola			1	1	1
Guia Astral		2	2	2	2
Manequim				1	1
Selecoes		1	1	1	1

5. Principal conclusions and future research

In this paper we establish an analogy between financial and media portfolios. In a similar way of investors, planners must purchase assets (vehicles spots) with high cost effectiveness and high probability of achieving the desired gains (reach-frequency results).

A new media optimization system based on generalized mean-variance model was introduced. We presented several possible advantages of the mean-variance paradigm over other methods like linear programming and greedy algorithms: the flexibility in modeling the optimization problem, the ability of dealing with many media performance indices - satisfying most of the media plan needs - and, most important, the property of diversifying the media portfolios in a natural way. The case study presented here – an application to printed media optimization – shows that the real world behavior of the model corresponds to what is expected.

An area of improvement lies in the definition of $F(i,v)$ and its main statistics μ (mean vector) and *Cov* (covariance matrix). The idea is to estimate μ and *Cov* as mean and covariance of a multivariate normal distribution. This distribution would be the "latent" continuous distribution that determines (by truncation) the values in the observed sample of respondents.

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Appendix – Brief glossary of terms used in media planning

Target: A specific universe of population to which an advertisement campaign is oriented. The target is specified according to affinities between the product and certain personal features like sex, age, social status, etc. For example, the target for a doll campaign is the universe of girls less than 13 years old.

Rating / Coverage: Expected percentage of people in the target who are exposed to a vehicle in a period of time. For example, when 30% of the target is watching a TV program in a certain instant, we say that this program has 30 points of rating. The term *Rating* is commonly used for TV and radio, while *Coverage* is commonly used for magazine and newspapers. In this paper, the reach of a vehicle v is denoted by μ_v (see section 3.1).

Frequency: Given a media schedule containing a total amount of spots, s, its *average frequency*, or simply *frequency*, is the expected number of spots seen by an individual drawn at random from the target.

Reach 1+: Given a media schedule, its *reach* (or *reach* 1+) is the expected percentage of the target exposed one or more times to the scheduled spots. A more general term is *Reach* $s_1 - s_2$, which means the expected percentage of the target who shall see/hear between s_1 and s_2 spots among the total of scheduled spots ($s_1 \leq s_2$). For example, *Reach 3-6* means the percentage of target expected to see between 3 and 6 of the scheduled spots.

Frequency 1+: Is the average number of spots seen by each individual exposed at least once to the scheduled spots.

GRP (*Gross Rating Point*): Cumulative rating points of a schedule, i.e., the sum of products of the vehicles' ratings by their corresponding number of scheduled ads (see section 3.1).

Reach-frequency distribution: Given a schedule, its *reach-frequency distribution* is the set of estimated proportions of the target exposed 0, 1, 2, 3... times to the scheduled ads.

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