$B_s B^* K$ and $B_s B K^*$ vertices using QCD sum rules

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Abstract

The form factors and the coupling constant of the $B_s B^* K$ and $B_s B K^*$ vertices are calculated using the QCD sum rules method. Three-point correlation functions are computed considering both the heavy and light mesons off-shell in each vertex, from which, after an extrapolation of the QCDSR results at the pole of the off-shell mesons, we obtain the coupling constant of the vertex. The form factors obtained have different behaviors but their simultaneous extrapolation reaches the same value of the coupling constant, $g_{B_s B^* K} = 8.41 \pm 1.23$ and $g_{B_s B K^*} = 3.3 \pm 0.5$. We compare our result with other theoretical estimates and compute the uncertainties of the method.
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1. Introduction

In the recent years, many new charmonium and bottomonium states have been observed at the B-factories. As an example, in the bottomonium sector, the Belle Collaboration reported the observation of two charged narrow structures in the $\pi^\pm \Upsilon(nS)$ ($n = 1, 2, 3$) and $\pi^\pm h_b(mP)$ ($m = 1, 2$) mass spectra of the $\Upsilon(5S) \to \Upsilon(nS)\pi^\pm$ and $\Upsilon(5S) \to h_b(mP)\pi^\pm$ decay processes [1].

These narrow structures were called $Z_b(10610)$ and $Z_b(10650)$. As pointed out by the Belle Collaboration, the proximity of the $B\bar{B}^*$ and $B^*\bar{B}^*$ thresholds and the $Z_b(10610)$ and $Z_b(10650)$ masses suggests that these states could be interpreted as weakly bound $B\bar{B}^*$ and $B^*\bar{B}^*$ states. In particular, using the one-boson exchange model and considering S-wave and D-wave mixing, the authors of Ref. [2] were able to explain both, $Z_b(10610)$ and $Z_b(10650)$, as $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states. The main ingredients in the one-boson exchange model are the effective Lagrangians that describe the strong interactions between the heavy and light mesons. These Lagrangians are characterized by the strong coupling constants in the considered vertices which, in general, are not known. These heavy–heavy–light mesons coupling constants are fundamental objects since they can provide essential information on the low energy behavior of the QCD. Depending on their numerical values, a particular molecular state may or may not be bound. Therefore, it is really important to have reliable ways to extract these values based on QCD calculations. However, such low-energy hadron interaction lies in a region which is very far away from the perturbative regime. Therefore, we need some non-perturbative approaches, such as the QCD sum rules (QCDSR) [3–5], to calculate the form factors and coupling constants of these vertices. There are already some QCDSR calculations for the heavy–heavy–light vertices like the $B^*B\pi$ [6], $B_{s0}BK$ [7], $B_s^*BK^*$ [8], $B^*B^*\rho$ [9], $B_sBK_0^*$, $B_s^*BK_1$ [10] and B_s^*BK [11]. In the charm sector, various vertices were evaluated with this approach and the results are systematized in [12]. Here we calculate the form factor and the coupling constant at the B_sB^*K and B_sBK^* vertices in the framework of three-point QCDSR. More specifically, we evaluate the $g_{B_sB^*K}(Q^2)$ and $g_{B_sBK^*}(Q^2)$ form factors in three different ways, considering, one by one, each of the mesons in the vertex to be off-shell. From these form factors, we extract the $g_{B_sB^*K}$ and the $g_{B_sBK^*}$ coupling constants.

2. The QCD sum rule for the B_sB^*K and B_sBK^* vertices

To perform the QCDSR calculation and obtain the form factors and coupling constants of the vertices B_sB^*K and B_sBK^*, we follow our previous works, as Ref. [12]. The starting point is the three-point correlation function given by:

$$\Pi_{\mu(\nu)}^{(B_s)}(p, p') = \int d^4x d^4y \langle 0|T\{j_\mu^{K^{(*)}}(x)(j^{B_s})^\dagger(y)(j_{(\nu)}^{B^{(*)}})^\dagger(0)\}|0\rangle e^{ip'\cdot x}e^{-iq\cdot y}, \quad (1)$$

for the B_s meson off-shell, and:

$$\Pi^{(K)}(p, p')_{(\mu)\nu} = \int d^4x d^4y \langle 0|T\{j_{(\mu)}^{B^{(*)}}(x)(j_\nu^{K^{(*)}})^\dagger(y)(j^{B_s})^\dagger(0)\}|0\rangle e^{ip'\cdot x}e^{-iq\cdot y}, \quad (2)$$

for the K or K^* meson off-shell. In Eqs. (1) and (2), $q = p - p'$ is the momentum of the off-shell meson and p and p' are the momentum of other ones. The currents $j^{(M)}$ are the currents associated with each meson in the vertex and contain the quantum information about the state. The correlation functions in Eqs. (1) and (2) allow to obtain two different form factors corresponding to the same vertex. In this way, the vertex is tested by two different mesons, the heavier and the lighter ones in the corresponding vertex. The calculation of these two correlation functions allows to reduce the uncertainties of the evaluation of the coupling constant of the vertex [12].

Eqs. (1) and (2) contain different numbers of Lorentz structures, and for each structure, we can write a different sum rule. In principle all the structures would give the same result. However, due to different approximations each structure can lead to different results. Therefore, one has to choose the structures less sensitive to the different approximations. To obtain the sum rule,
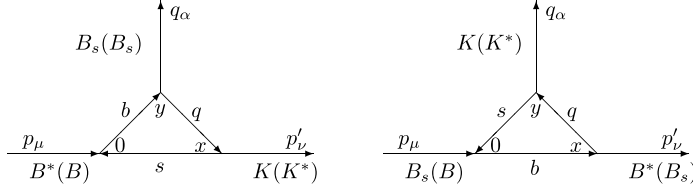
Fig. 1. Perturbative diagrams for the $B_s(B_s)$ off-shell meson (left) and for $K(K^*)$ off-shell meson (right), for $B_s B^* K$ vertex ($B_s B K^*$ vertex).

these functions are calculated in two different ways: using quarks degrees of freedom – the QCD side; and using hadronic degrees of freedom – the phenomenological side. In the QCD side, the correlators are evaluated using Wilson's operator product expansion (OPE). The duality principle allows us to obtain an interval in which both representations are equivalent. Therefore, in this region, we can obtain the QCD sum rule from where the form factors are evaluated. To improve the matching between the two sides, we perform a Borel transformation to both QCD and phenomenological sides.

2.1. The QCD side

The QCD side is obtained using the following meson currents for the $B_s B^* K$ vertex:

$$j_\mu^{B^*}(x) = \bar{b}\gamma_\mu q,$$
$$j_5^{B_s}(0) = i\bar{b}\gamma_5 s,$$
$$j_\nu^{K}(y) = \bar{s}\gamma_\nu \gamma_5 q \tag{3}$$

and the following ones for the $B_s B K^*$ vertex:

$$j_5^{B}(x) = i\bar{q}\gamma_5 b,$$
$$j_5^{B_s}(0) = i\bar{b}\gamma_5 s,$$
$$j_\mu^{K^*}(y) = \bar{q}\gamma_\mu s. \tag{4}$$

Here, q, s and b are the light, strange and bottom quark fields respectively. Each one of these currents has the quantum numbers of the associated meson. In the case of K off-shell meson, we use, as usual in QCDSR, the pseudoscalar current for it, see Refs. [6] ($D^* D\pi$), [13] ($D_s^* DK$) and [11] ($B_s^* BK$). The general expression for the vertices has different structures, which can be written in terms of a double dispersion relation over the virtualities p^2 and p'^2, holding $Q^2 = -q^2$ fixed:

$$\Gamma\left(p^2, p'^2, Q^2\right) = -\frac{1}{4\pi^2} \int\limits_{s_{min}}^{\infty} ds \int\limits_{u_{min}}^{\infty} du \, \frac{\rho(s, u, Q^2)}{(s - p^2)(u - p'^2)}, \tag{5}$$

where the spectral density $\rho(s, u, Q^2)$ can be obtained from the Cutkosky's rules.

The invariant amplitudes receive contributions from all terms in the OPE. In the case of form factors, the main contribution in the OPE is the perturbative term, which is represented in Fig. 1, for the two cases that we are considering, the $B_s(B_s)$ and $K(K^*)$ meson off-shell for the $B_s B^* K$ ($B_s B K^*$) vertex.

In order to obtain the form factor, we have to choose one of the different structures appearing in Eqs. (1) and (2). As commented above, different structures can lead to different results. Therefore, one has to choose the structure less sensitive to higher dimension condensates, that provide a better stability as a function of the Borel mass, and that have a larger pole contribution, when compared with the continuum contribution. This is considered a "good" structure. If there is more than one "good" structure, the others can also be considered to estimate the uncertainties of the method.

For $B_s B^* K$ form factor and in the case B_s off-shell meson, we choose the $p'_\mu p'_\nu$ structure, because it satisfies the criteria above. For K off-shell meson, we can work with both p_ν and p'_ν structures In this case, we are going to work with the p'_ν structure while the other one will be used for the estimate of the uncertainties. The corresponding perturbative spectral densities, which enter in Eq. (5), are:

$$\rho^{(B_s)}(s, u, Q^2) = \frac{3}{2\pi\sqrt{\lambda}}\big[(2m_b - 2m_s)E - 2m_b B\big], \tag{6}$$

for the $p'_\mu p'_\nu$ structure of the B_s off-shell case, and

$$\rho^{(K)}(s, u, Q^2) = -\frac{3}{2\pi\sqrt{\lambda}}\big[A\big(p \cdot p' - 2k \cdot p - m_b m_s + m_b^2\big) + 2\pi\big(m_b^2 - k \cdot p'\big)\big], \tag{7}$$

for the p_ν structure in the K off-shell case. In both cases, the quark condensate contributions are neglected after the Borel transform.

For $B_s B K^*$ vertex, we use the p_μ structure for B_s off-shell meson and for K^* off-shell meson, we have p_μ and p'_μ structures, both giving excellent sum rules. Again we show the results for one structure and the other is used to estimate the uncertainties. The perturbative contribution to the spectral density, when the B_s meson is off-shell is:

$$\rho^{(B_s)}(s, u, Q^2) = \frac{3}{2\pi\sqrt{\lambda}}\big[A\big(p \cdot p' - m_b m_s - 2p \cdot k\big) - p' \cdot k\big] \tag{8}$$

for the p_μ structure. In this case, the quark condensate, $\langle q\bar{q}\rangle$, contribution to the same structure is:

$$\Pi^{\langle q\bar{q}\rangle} = \frac{m_s\langle q\bar{q}\rangle}{(p^2 - m_b^2)(p'^2 - m_s^2)}. \tag{9}$$

For K^* off-shell case, the spectral density, for both structures, is given by:

$$\rho^{(K^*)}(s, u, Q^2) = -\frac{3}{2\pi\sqrt{\lambda}}\Big\{p_\mu\big[A\big(p \cdot p' - m_b m_s - m_b^2\big) + m_b^2 - k \cdot p' - m_b m_s\big]$$
$$+ p'_\mu\big[B\big(p \cdot p' + m_b m_s - m_b^2\big) - k \cdot p + m_b^2\big]\Big\}. \tag{10}$$

In Eqs. (6) to (10) we have defined $\lambda = \lambda(s, u, t) = s^2 + t^2 + u^2 - 2st - 2su - 2tu$, $s = p^2$, $u = p'^2$, $t = -Q^2$ and A, B and E are functions of (s, u, t), given by:

$$A = 2\pi\left[\frac{\overline{k_0}}{\sqrt{s}} - \frac{\overline{|\vec{k}|}}{|\vec{p'}|}\overline{\cos\theta}\frac{\overline{p'_0}}{\sqrt{s}}\right]; \qquad B = 2\pi\frac{\overline{|\vec{k}|}}{|\vec{p'}|}\overline{\cos\theta}; \tag{11}$$

$$E = -\frac{\pi|\vec{k}|^2}{|\vec{p'}|^2}(3\overline{\cos\theta} - 1), \tag{12}$$

where

$$\overline{|\vec{k}|}^2 = \overline{k_0}^2 - m_i^2; \qquad \overline{\cos\theta} = -\frac{2p_0'\overline{k_0} - u - m_i^2 - \eta m_b^2}{2\overline{|\vec{p}'|}\overline{|\vec{k}|}};$$

$$p_0' = \frac{s+u-t}{2\sqrt{s}}; \qquad \overline{|\vec{p}'|}^2 = \frac{\lambda}{4s}; \qquad \overline{k_0} = \frac{s + m_i^2 - \epsilon m_b^2}{2\sqrt{s}}; \tag{13}$$

Finally, the OPE side, is calculated using Eq. (5) with the limits in the integration given by: $s_{min} = (m_b + m_s)^2$ and $u_{min} = t - m_b^2$ for B_s off-shell and $s_{min} = m_b^2 - m_s^2$ and $u_{min} = t + m_b^2 - m_s^2$ for K off-shell, for $B_s B^* K$ vertex. And $s_{min} = (m_b)^2$ and $u_{min} = t + m_b^2$ for B_s and K^* off-shell for the $B_s B K^*$ vertex.

Since we are dealing with heavy quarks, we expect the perturbative contribution to dominate the OPE. Besides, in Ref. [11] it was shown that the gluon condensate contribution to the $D^* D\pi$ and $J/\psi D^* D$ vertices was negligible, as compared with the perturbative contribution. For these reasons, we do not include the gluon condensate and higher dimension condensates in the present work.

2.2. The phenomenological side

The three-point functions from Eqs. (1) and (2), when written in terms of hadron masses, decay constants and form factors, give the phenomenological side of the sum rule.

2.2.1. For the $B_s B^* K$ vertex
The meson decay constants f_K, f_{B_s} and f_{B^*} are defined by the following matrix elements:

$$\langle 0 | j_\nu^K | K(p) \rangle = i f_K p_\nu, \tag{14}$$

$$\langle 0 | j_\mu^{B^*} | B^*(p') \rangle = m_{B^*} f_{B^*} \epsilon_\mu^*(p'), \tag{15}$$

and

$$\langle 0 | j_5^{B_s} | B_s(p) \rangle = \frac{m_{B_s}^2}{m_b + m_s} f_{B_s}, \tag{16}$$

and the vertex function is defined by:

$$\langle K(q) | B^*(-p') B_s(p) \rangle = -g_{B_s B^* K}^{(K)} \epsilon^{\mu*}(p')(2p - p')_\mu, \tag{17}$$

which is extracted from the effective Lagrangian [14]:

$$\mathcal{L}_{B_s B^* K} = i g_{B_s B^* K} \left[B^{*\mu}(\bar{B}_s \partial_\mu K - \partial_\mu \bar{B}_s K) - \bar{B}^{*\mu}(B_s \partial_\mu \bar{K} - \partial_\mu B_s \bar{K}) \right]. \tag{18}$$

Saturating the correlation function with B_s, B^* and K intermediate states we arrive at

$$\Pi_{\mu\nu}^{(B_s)} = \frac{-f_K f_{B_s} f_B m_{B^*} m_{B_s}^2 g_{B_s B^* K}^{(B_s)}(q^2)}{(m_s + m_b)(p'^2 - m_K^2)(q^2 - m_{B_s}^2)(p^2 - m_{B^*}^2)}$$

$$\times \left[p_\mu p'_\nu \left(1 - \frac{(m_K^2 - q^2)}{m_{B^*}^2} \right) - 2p'_\mu p'_\nu \right] + \text{``continuum''}, \tag{19}$$

for an off-shell B_s. Using the matrix element of K meson equal to

$$\langle 0 | j_5^K | K(q) \rangle = \frac{m_K^2}{m_s} f_K, \tag{20}$$

we arrive at an expression for an off-shell K:

$$\Pi_\mu^{(K)} = \frac{-f_{B^*} f_K f_{B_s} m_{B^*} m_{B_s}^2 m_K^2 g_{B_s B^* K}^{(K)}(q^2)}{(m_b + m_s) m_s (p^2 - m_B^2)(p'^2 + m_{B_s^*}^2)(q^2 - m_K^2)}$$
$$\left[-2p_\mu + p'_\mu \left(1 + \frac{(m_{B_s}^2 - q^2)}{m_{B^*}^2} \right) \right] + \text{``continuum''}. \tag{21}$$

2.2.2. For the $B_s B K^*$ vertex

In this case, the effective Lagrangian is [14]:

$$\mathcal{L}_{B_s B K^*} = i g_{B_s B K^*} \left[K^{*\mu} (B \partial_\mu \bar{B}_s - \bar{B}_s \partial_\mu B) + \bar{K}^{*\mu} (B_s \partial_\mu \bar{B} - \bar{B} \partial_\mu B_s) \right], \tag{22}$$

from where we can extract the vertex element, which is given by:

$$\langle K^*(q) | B_s(-p') B(p) \rangle = i g_{B_s B K^*}^{(K^*)} \epsilon^{*\mu}(q) (p + p')_\mu, \tag{23}$$

and the matrix elements which introduce the meson decay constants f_{K^*}, f_{B_s} and f_B are:

$$\langle 0 | j_\mu^{K^*} | K^*(p) \rangle = f_{K^*} \epsilon_\mu^*(q) m_{K^*}, \tag{24}$$

$$\langle B(p) | j_5^B | 0 \rangle = f_B \frac{m_B^2}{m_b^2}, \tag{25}$$

and

$$\langle 0 | j_5^{B_s} | B_s(p') \rangle = f_{B_s} \frac{m_{B_s}^2}{m_b + m_s}. \tag{26}$$

After some algebra we arrive at the following expression:

$$\Pi_\mu^{(B_s)} = -i \frac{f_{K^*} f_B f_{B_s} m_{K^*} m_{B_s}^2 m_B^2}{(m_b^2 + m_s m_b)(p^2 - m_B^2)(p'^2 - m_{K^*}^2)(q^2 - m_{B_s}^2)}$$
$$\times g_{B_s B K^*}^{(B_s)}(q^2) \left[-2p_\mu + p'_\mu \left(1 - \frac{m_B^2 - m_{B_s}^2}{m_{K^*}^2} \right) \right] + \text{``continuum''}, \tag{27}$$

when B_s is off-shell.

For K^* off-shell we arrive at:

$$\Pi_\mu^{(K^*)}(p, p', q) = - \frac{f_B f_{K^*} f_{B_s} m_{K^*} m_{B_s}^2 m_B^2}{(m_s m_b + m_b^2)(p^2 - m_B^2)(p'^2 - m_{B_s}^2)(q^2 - m_{K^*}^2)}$$
$$\times g_{B_s B K^*}^{(K^*)}(q^2) \left[p_\mu \left(1 - \frac{(m_B^2 - m_{B_s}^2)}{m_{K^*}^2} \right) \right.$$
$$\left. + p'_\mu \left(1 - \frac{(m_{B_s}^2 - m_B^2)}{m_{K^*}^2} \right) \right] + \text{``continuum''} \tag{28}$$

Table 1
Hadronic parameters used.

	K	K^*	B_s	B^*	B
m (GeV)	0.49	0.89 ± 5	5.40	5.20	5.28
f (MeV)	$160^{\pm 1.4}_{\pm 44}$	220 ± 5	$208^{\pm 10}_{\pm 39}$	$250^{\pm 10}_{\pm 29}$	191 ± 0.87

3. The sum rule

The sum rule is obtained after performing a double Borel transform ($\mathcal{BB}$), $P^2 = -p^2 \to M^2$ and $P'^2 = -p'^2 \to M'^2$, to both the phenomenological and OPE sides:

$$\mathcal{BB}\left[\Gamma_\mu^{OPE(I)}\right](M, M') = \mathcal{BB}\left[\Gamma_\mu^{phen(I)}\right](M, M'),$$ (29)

where M and M' are the Borel masses and I is the off-shell meson.

In order to eliminate the continuum contribution in the phenomenological side, instead of doing the integrals in Eq. (5) up to ∞, we do the integrals up to the continuum threshold parameters s_0 and u_0. The threshold parameters are defined as $s_0 = (m_i + \Delta_i)^2$ and $u_0 = (m_o + \Delta_o)^2$, where Δ_i and Δ_o are usually taken as 0.5 MeV, and m_i and m_o are the masses of the incoming and outgoing mesons respectively.

In Eqs. (19), (21), (27), (28), $g_{B_s B^* K}^{(I)}(Q^2)$ and $g_{B_s B K^*}^{(I)}(Q^2)$ are the form factors when the I meson is off-shell. As in our previous works, we define the coupling constant as the value of the form factor, $g^{(I)}(Q^2)$, at $Q^2 = -m_I^2$, where m_I is the mass of the off-shell meson.

4. Results and discussion

Table 1 shows the value of the hadronic parameters used in the present calculation. We have used the experimental value for f_K of Ref. [15], for f_{B^*} and f_{B_s} from Ref. [16] and for f_{K^*} of Ref. [17].

We neglect the light quark mass ($m_q = 0.0$ MeV). The strange and bottom quark masses were taken from the Particle Data Group (PDG) and have the values $m_s = 104 + 26 - 34$ MeV and $m_b = 4.20 + 0.17 - 0.07$ GeV respectively. In the next subsections, we show the two form factors used to extract the coupling constants of each vertex.

4.1. $B_s B^* K$ vertex

In the case of a B_s off-shell meson, we work with the $p'_\nu p'_\mu$ structure. In Fig. 2(a) we show the contribution of the pole versus the continuum contribution for the sum rule and in Fig. 2(b) the stability of the form factor as a function o Borel mass, for $Q^2 = 1$ GeV^2 and three different values for the continuum thresholds. We use the usual relation between the Borel masses M' and M [12]: $M'^2 = \frac{m_K^2}{m_{B_s}^2 - m_b^2} M^2$.

From Fig. 2(b) we clearly see a window of stability for $M^2 \geq 10$ GeV^2, and from Fig. 2(a), we can see that the pole contribution is bigger than the continuum contribution for $M^2 \leq 19$ GeV^2. Therefore, there is a Borel window where the sum rule can be used to extract the form factor.

In the case of a K meson off-shell, we have two structures in Eq. (21) that can be used: p'_μ and p_μ. Both structures give good sum rule results, that means a good pole–continuum contribution and good stability. We show in Fig. 3(a) and (b), respectively, the pole–continuum
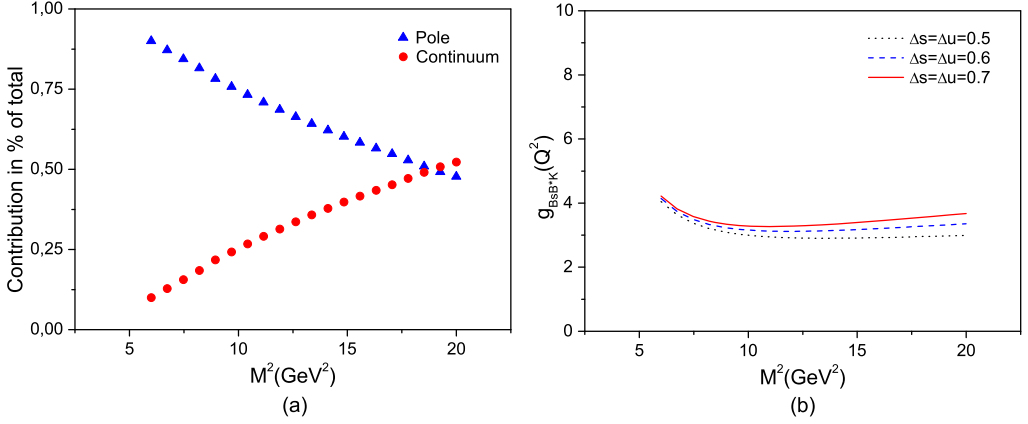
Fig. 2. (a) The pole–continuum contributions for the $B_s B^* K$ sum rule for the B_s off-shell and (b) $g^{(B_s)}_{B_s B^* K}(Q^2 = 1\,\text{GeV}^2)$ stability for different values of the continuum thresholds.
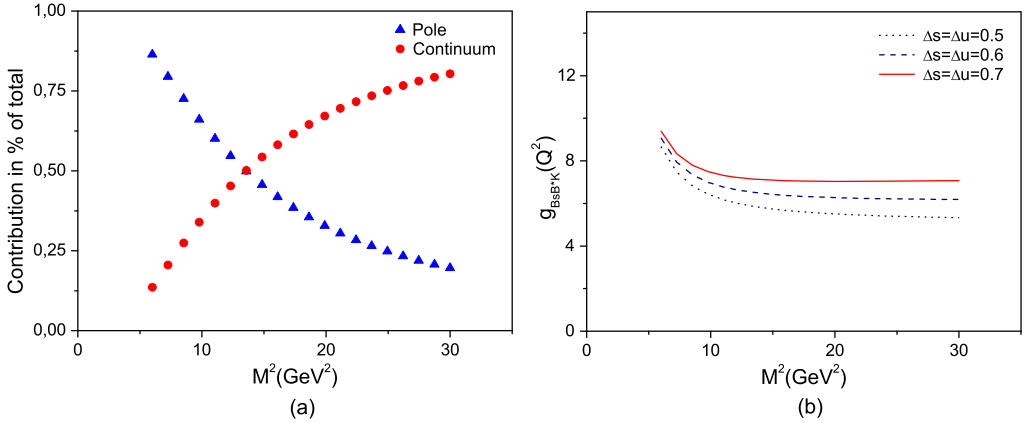

Fig. 3. (a) The pole and continuum contributions for $B_s B^* K$ sum rule with the K off-shell and (b) the $g^{(K)}_{B_s B^* K}(Q^2 = 2\,\text{GeV}^2, M^2)$ stability for different values of the continuum thresholds.

contribution and the stability only for the p_ν structure. For the p'_μ, we obtain a very similar result and we use it to evaluate the uncertainties.

From Fig. 3, we find a Borel window $10\,\text{GeV}^2 \le M^2 \le 14\,\text{GeV}^2$ where the sum rule can be used to extract he form factor.

In Fig. 6 we show the QCDSR results for these two form factors, represented by squares and triangles for the cases of the B_s and K off-shell respectively.

4.2. $B_s B K^*$ vertex

For the case of the B_s meson off-shell, we use the structure p_μ. In Fig. 4(a) and (b) we show the pole–continuum contribution and the Borel mass stability respectively. Once more, we see that we get a Borel window $10\,\text{GeV}^2 \le M^2 \le 12.5\,\text{GeV}^2$ where these both conditions are satisfied and where we can use the sum rule to extract the form factor. Also, for the B_s meson
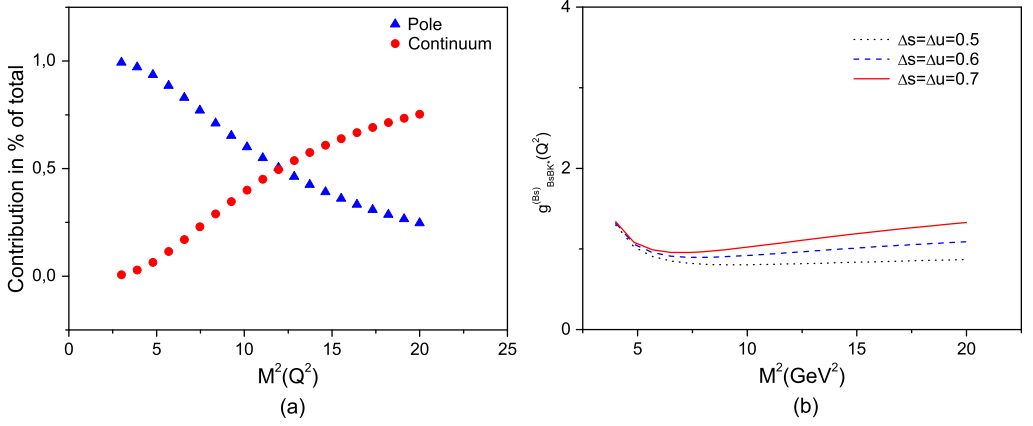
Fig. 4. (a) The pole–continuum contributions for $B_s B K^*$ and in (b) $g_{B_s B K^*}^{(B_s)}(Q^2 = 1 \text{ GeV}^2, M^2)$ stability for different thresholds.
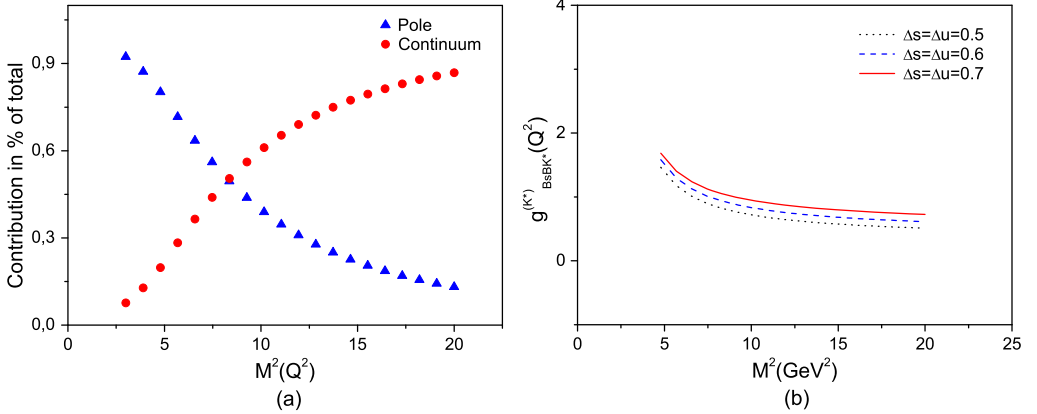

Fig. 5. (a) The pole and continuum contributions for $B_s B K^*$ sum rule for K^* off-shell and (b) the $g_{B_s B K^*}^{(K^*)}(Q^2 = 2 \text{ GeV}^2, M^2)$ stability for different values of the threshold parameters.

off-shell case, we have other structure, p'_ν, to work with. In this case, the result includes the light quarks condensate which do not modified substantially the sum rule. The p'_ν structure will be used to evaluate the uncertainties.

In the case of the K^* off-shell meson, we show the figures of stability and pole–continuum contribution for p'_μ structure, in Fig. 5. In the structure p_μ, we get similar results, that means a good pole–continuum contribution and good stability.

The sum rule result for both form factors are shown in Fig. 7 through triangles and squares for B_s and K^* off-shell respectively.

5. Coupling constant

The coupling constant in the vertex, is obtained when the form factor is extrapolated to $Q^2 = -m_I^2$, where m_I is the mass of the off-shell meson. In order to minimize the uncertainties, we work with two form factors in each vertex, one with the heavier meson in the vertex off-shell and

Table 2
Form factors and coupling constants obtained by extrapolation.

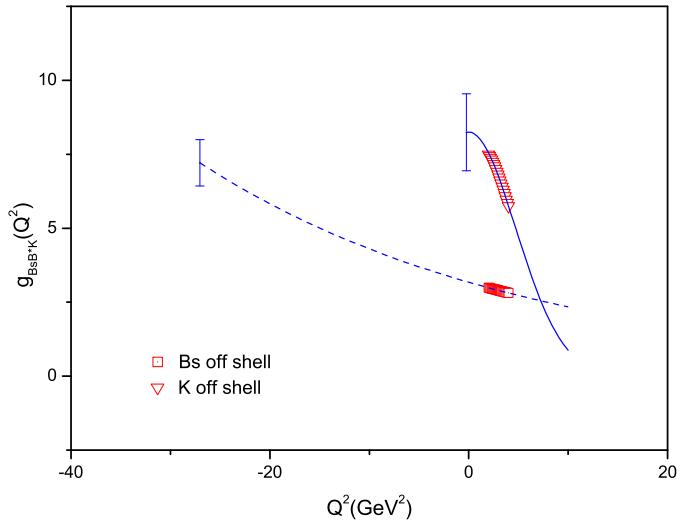| Form factor	Coupling constant
$g_{B_s B^* K}^{(B_s)}(Q^2) = 3.18 \exp^{-Q^2/32.98}$	$g_{B_s B^* K} = 7.55$
$g_{B_s B^* K}^{(K)}(Q^2) = 8.25 \exp^{-(Q^2/6.68)^2}$	$g_{B_s B^* K} = 8.00$
$g_{B_s B K^*}^{(B_s)}(Q^2) = \dfrac{80.88}{58.86+Q^2}$	$g_{B_s B K^*} = 2.72$
$g_{B_s B K^*}^{(K^*)}(Q^2) = 2.49 \exp^{-Q^2/2.63}$	$g_{B_s B K^*} = 3.37$

Fig. 6. The $g_{B_s B^* K}(Q^2)$ form factor, extrapolation function for K and B_s meson off-shell.

the other with the lighter meson off-shell. Both form factors should give the same value for the coupling constant.

In Table 2, we show, for each vertex, the function that fits the QCD sum rule results for the form factors, which is then extrapolated to $Q^2 = -m_f^2$, to determine the coupling constant value. From Table 2 we see that the two form factors in the same vertex, give similar results for the coupling constant.

In Figs. 6 and 7, we show the parametrizations given in Table 2 for the QCDSR results for the form factors $g_{B_s B^* K}(Q^2)$ and $g_{B_s B K^*}(Q^2)$ respectively. The squares and triangles give the QCDSR results for the heavier and lighter off-shell mesons in the vertices respectively. We also show in these figures the values obtained for the coupling constant.

6. Uncertainties of the QCDSR method

Looking at Figs. 6 and 7, we can see a error bar at the endpoint of the curves. To evaluate these error bars, we compute the sum rule taking into account the errors in the masses, decay constants, continuum threshold parameters and also we study a variation on the Borel mass in the window of Q^2.
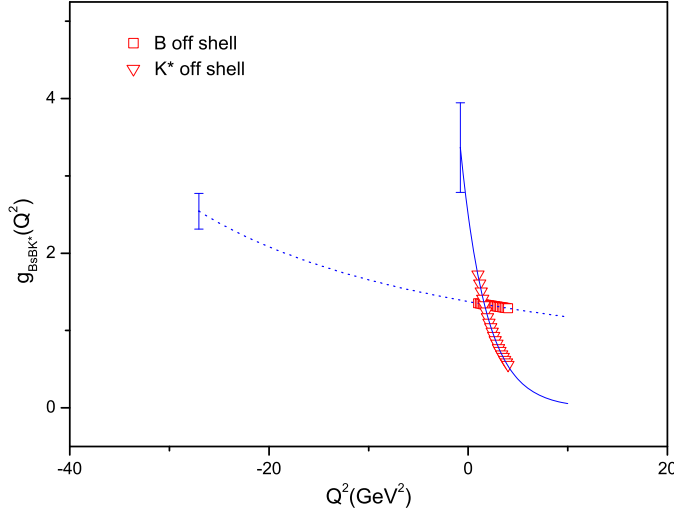
Fig. 7. The $g_{B_s BK^*}(Q^2)$ form factor, extrapolation function for K^* and B_s meson off-shell.

Table 3
Percentage deviation of the coupling constant related to each parameter for both vertices with the meson I off-shell.

Parameters	Deviation %			
	$g_{B_s B^* K}^{(I)}$		$g_{B_s BK^*}^{(I)}$	
	$I = B_s$	$I = K$	$I = B_s$	$I = K^*$
$f_{K^*} = 220 \pm 5$ (MeV)	–	–	1.95	1.94
$f_B = 191 \pm 8.77$ (MeV)	–	–	3.91	3.88
$f_K = 159.8 \pm 1.4 \pm 0.44$ (MeV)	1.04	1.04	–	–
$f_{B^*} = 208 \pm 10 \pm 29$ (MeV)	16.29	16.30	–	–
$f_{B_s} = 250 \pm 10 \pm 35$ (MeV)	12.06	12.07	3.61	3.51
$m_b = 4.20 + 0.17 - 0.07$ (GeV)	15.81	23.39	9.83	28.42
$m_s = 104 + 26 - 34$ (MeV)	1.23	24.71	7.21	19.78
$M^2 \pm 10\%$ (GeV)	4.29	1.85	6.45	18.57
$\Delta s \pm 0.1$ and $\Delta u \pm 0.1$ (GeV)	13.51	4.90	9.32	12.35

Our procedure is that in each computation, all the parameters are kept fixed and only one changes within its intrinsic error. In Table 3, we show the intrinsic uncertainty for each of the sum rule parameters.

We also consider the other good sum rules, for each off-shell meson in each vertex and we also put the third meson off-shell to better estimate the errors.

6.1. Other mesons of shell and other structures

For the $B_s B^* K$ vertex, we calculated the B^* off-shell form factor. Since the B^* meson has a small mass difference when compared with the B_s meson mass, we expect to obtain a similar result to the one obtained for B_s off-shell. Computing the sum rule, we observe that there are two structures: the $p'_\mu p'_\nu$ and $p_\mu p'_\nu$. The $p_\mu p'_\nu$ structure does not give a good stability in the Borel

mass, therefore it is not considered. The other structure, $p'_\mu p'_\nu$, gives a good stability and the pole is larger than the continuum contribution.

To extrapolate the QCDSR results, we use the fit

$$g^{(B^*)}_{B_s B^* K}(Q^2) = 1.21 \exp^{-Q^2/13.65} \tag{30}$$

for the form factor, and the resulting value for the coupling constant is: $g_{B_s B^* K} = 8.64$. This result is very similar to the one showed in Table 2, as expected.

Also for this vertex, in the case of the K meson off-shell, we have other structure, p'_μ, which gives a good sum rule, very similar to the p_μ used before. In this case the form factor is extrapolated by

$$g^{(K)}_{B_s B^* K}(Q^2) = 9.69 \exp^{-(Q^2/7.43)^2}, \tag{31}$$

and the resulting value for the coupling constant is: $g_{B_s B^* K} = 9.45$, in a very good agreement with the results in Table 2.

For $B_s B K^*$, we perform the sum rule for the B off-shell meson. Again, we expect to obtain a similar result to the one obtained for B_s off-shell, since this meson has a small mass difference when compared with the B meson mass.

We get sum rules for the two structures, p_μ and p'_μ. The p_μ structure does not give good stability in the Borel mass. Therefore, it is not considered. The other structure, p'_μ, gives good stability with the pole being bigger than the continuum contribution. The QCDSR results for the form factor can be extrapolated by

$$g^{(B)}_{B_s B K^*}(Q^2) = \frac{104.71}{66.97 + Q^2} \tag{32}$$

and the coupling constant is

$$g_{B_s B K^*} = 2.77.$$

This is a very similar result to the one obtained in Table 2.

7. Conclusion

We have used the three-point QCD sum rules to study the form factors in the vertices $B_s B^* K$ and $B_s B K^*$. In each case, we have considered two different sum rules, for two different mesons off-shell. We have studied the Borel stability and pole dominance in each case and have determined the Borel window where the sum rules can be used. To get the coupling constant in each vertex, we have used the extrapolation method developed in previous works [12]. This extrapolation method has a systematic error which comes from the choice of the analytic form of the extrapolating functions. We consider only monopole, exponential and Gaussian parametrization: $g^{(I)}_{M M_2 M_3}(Q^2) = A \exp^{-Q^2/B}$, $g^{(I)}_{M M_2 M_3}(Q^2) = A \exp^{-(Q^2/B)^2}$ and $g^{(I)}_{M M_2 M_3}(Q^2) = \frac{A}{B+Q^2}$.

There are no physical reason for using these forms. However, they are the usual forms used by experimentalists and also they have only two parameters, A and B, that present some regularity, in all our form factors. For instance, when the heavy meson is off-shell, the form factor is harder as a monopolar function, and the B cut-off parameter is a big number. By the other hand, when the light meson is off-shell, the parametrization is usually exponential or Gaussian and have a smaller B cut-off parameter. These cut-offs are showed in Table 4.

Table 4
Cut-off parameters for both vertices.

| Off-shell meson | $B_s B^* K$ | | $B_s B K^*$ | |
| | Cut-off parameters | | | |
	A	B	A	B
B_s	3.18	32.98	80.88	58.86
B^*	1.21	13.65	–	–
K	8.25	6.68	–	–
B	–	–	104.71	66.97
K^*	–	–	2.49	2.63

We have also performed a very extensive study of the uncertainties to estimate the errors in the coupling constants. After considering all good sum rules and including the uncertainties study, we obtain the coupling constant equal to:

$$g_{B_s B^* K} = 8.41 \pm 1.23 \tag{33}$$

and

$$g_{B_s B K^*} = 3.3 \pm 0.5 \tag{34}$$

We can compare our results with other theoretical predictions using arguments of heavy hadron chiral perturbation theory (HHChPT), where the couplings for the bottom–light vertex $g_{B_s^* B K}$ are related to the charm–light vertex $g_{D_s D^* K}$ through the relation [18,19]:

$$g_{B_s B^* K} = g_{D_s^* D K} \frac{m_B}{m_D}, \tag{35}$$

where $m_{B^*} = 5.2$ and $m_D = 1.8693$ are the experimental masses. For $g_{D_s D^* K}$ we can use our previous QCDSR result [13] of $g_{D^* D K} = 3.02$. Therefore, we obtain $g_{B_s B^* K} = 8.39$, which is in excellent agreement with our result. This result shows that the relation with charm–light vertex can give a good estimate of the couplings in the bottom–light vertex. The uncertainties are about 20%, that is in complete agreement to the technique of QCDSR. The greatest source of uncertainties found in this work is due to the mass of the bottom quark.

Acknowledgements

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