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COMPONENT IMPORTANCE FOR A COHERENT SYSTEM UNDER A HIPERMINIMAL DISTRIBUTION

by

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Component importance for a coherent system under a hyperminimal distribution.

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Abstract In this work we use a hyperminimal distribution for the lifetime of a coherent system to analyses measures of a coherent system components importance.

Keywords: Coherent system, parallel-series decomposition, hyperminimal distribution, component importance.

1.Introduction

As in Barlow and Proschan (1981) the lifetime of an engineering system is defined as a function of its components lifetimes, called structure function. Therefore, if $T_1, T_2, ..., T_n$ represents the component lifetimes, the system lifetime is represented by

$$T = \phi(\mathrm{T}) = \phi((T_1, ..., T_n))$$

where ϕ is the structure function.

The system is coherent if ϕ is nondecreasing in each coordinate and each component is relevant, that is, there exists a configuration of T, at an instant t in which the functioning of the component is fundamental for the system functioning.

Examples of coherent system are the series system

$$T = \min_{1 \leq i \leq n} T_i,$$

the parallel system

$$T = \max_{1 \leq i \leq n} T_i$$

and the k-out-of-n system

$$T = T_{(n-k+1)}$$

where $T_{(1)} \leq T_{(2)}, ..., \leq T_{(n)}$ are the T_i in order.

Generally the coherent system is represented by its parallel-series (series-parallel) decomposition. The parallel-series structures is given by

$$T = \Phi(\mathrm{T}) = \max_{1 \leq j \leq k} \min_{i \in P_j} T_i,$$

where $P_j, 1 \leq j \leq k$ are the minimal path sets, which are minimal sets of elements whose functioning insures the functioning of the system.

The series-parallel structures are given in terms of minimal cut sets which are minimal set of components whose failure causes the system to fail. Then

$$T = \Phi(\mathbf{T}) = \min_{1 \leq j \leq k} \max_{i \in K_j} T_i,$$

where $K_j, 1 \leq j \leq k$ are the minimal cut sets.

Such representations make the distribution function of system lifetime analytically very complicate. An alternative representation for the coherent system distribution function is through the hyperminimal distributions, from Navarro et al. (2007). We apply this concept to analyse several measures of component importance. In Section 2 we define the class of the hyperminimal distributions and verify that the class of the coherent systems is a proper subclass of that. In Section 3 we analyse the importance measures under hyperminimal distributions and give some applications.

2. Hyperminimal distribution.

In this section a multivariate distribution of the random vector $\mathbf{T}$ is denoted by

$$F(t_1, t_2, ..., t_n) = P(T_1 \leq t_1, T_2 \leq t_2, ..., T_n \leq t_n)$$

and the survival function of the random vector $\mathbf{T}$ is given by

$$R(t_1, t_2, ..., t_n) = P(T_1 > t_1, T_2 > t_2, ..., T_n > t_n)$$

An univariate distribution function $F(t)$ is a finite mixture of the distributions functions $F_i(t), i = 1, ..., k$, if

$$F(t) = \sum_{i=1}^{k} \alpha_i F_i(t),$$

where α_i are positive real numbers such that $\sum_{i=1}^{k} \alpha_i = 1$.

If the α_i can be negative we call the representation as a finite generalized mixture.

If $\mathbf{T} = (T_1, ..., T_n)$ is a multivariate random vector with survival function $R(t_1, ..., t_n)$ and distribution function $F(t_1, ..., t_n)$, we introduce:

Definition 2.1 A random variable T has an hyperminimal distribution associated with the random vector $\mathbf{T}$ if its survival function is given by

$$R_T(t) = \sum_{i=1}^{n} \sum_{1 \leq j_1 < ... < j_i \leq n} \alpha_{j_1,...,j_i} R_{j_1,...,j_i}(t, ..., t).$$

where each $R_{j_1,...,j_i}$ is a survival function of the random vector $(T_{j_1},...,T_{j_n})$ and the real parameters $\alpha_{j_1,...,j_i}$ are such that

$$\sum_{i=1}^{n} \sum_{1\leq j_1<...<j_i\leq n} \alpha_{j_1,...,j_i} = 1.$$

In Navarro et. al. (2007), the hypermaximal distribution is defined in a similar way replacing the survival functions by its respective distribution functions. It can be proved that the hyperminimal class of distributions and the hypermaximal class of distributions are the same.

Theorem 2.2 Let $T_1,...,T_n$ be the component lifetimes of a coherent system with lifetime T and with a parallel series representation

$$T = \Phi(\mathrm{T}) = \max_{1\leq j\leq p} \min_{i\in P_j} T_i,$$

where $P_j, 1 \leq j \leq p$ are the minimal path sets.

Then, the system reliability at time t is given by

$$P(T > t) = \sum_{j=1}^{p}(-1)^{j+1} \sum_{1\leq k_1<...<k_j\leq p} P(\bigcap_{k\in N_{k_1,...,k_j}} \{T_k > t\})$$

and $N_{k_1,...,k_j} = \bigcup_{l=1}^{j} P_{k_l}$.

Proof

$$P(T > t) = P(\max_{1\leq j\leq p} \min_{i\in P_j} T_i > t) = P(\bigcup_{j=1}^{p}\{\min_{i\in P_j} T_i > t\}) =$$

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1\leq k_1<...<k_j\leq p} P(\bigcap_{l=1}^{j}\{\min_{i\in P_{k_l}} T_i > t\}) =$$

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1\leq k_1<...<k_j\leq p} P(\bigcap_{l=1}^{j} \bigcap_{i\in P_{k_l}} T_i > t\}) =$$

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1\leq k_1<...<k_j\leq p} P(\bigcap_{k\in N_{k_1,...,k_j}} \{T_k > t\}).$$

Furthermore

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1\leq k_1<...<k_j\leq p} (-1)^{j+1} = \sum_{j=1}^{p}(-1)^{j+1}\binom{p}{j} = 1 + \sum_{j=0}^{p}(-1)^{j+1}\binom{p}{j} = 1.$$

Therefore the system coherent lifetimes has a hyperminimal distribution. Consider now two component lifetimes T_1 and T_2, independents, with exponential distributions with parameters 2 and 1, respectively. The reliability of a series system of those two component is

$$P(T > t) = P(\min(T_1, T_2) > t) = e^{-3t}, t > 0.$$

The reliability of a parallel system of those two components is

$$P(T > t) = P(\max(T_1, T_2) > t) = e^{-2t} + e^{-t} - e^{-3t}, t > 0.$$

Therefore they have hyperminimal distributions. However

$$P(T > t) = 3e^{-2t} - 2e^{-3t}, t \geq 0$$

is a survival function of a lifetime T, has a hyperminimal distribution and it is not a distribution of a coherent system because the unique coherent systems of two components are the series and parallel systems. We conclude that the class of coherent system distributions is a proper subclass of the hyperminimal class of distributions.

3. Importance measures.

The main objective of an importance ranking is to identify those components of a given system whose improvement may result in the greatest improvement for the system and to provide a checklist for failure diagnosis. In the classical theory Birnbaum (1969) and Barlow and Proschan (1975) propose some concepts of component importance.

3.1. Birnbaum importance.

Birbaum (1969) defined the reliability importance of component i, at a fixed instant t, by

$$I^B(i,t) = \frac{\partial \overline{F}(t)}{\partial \overline{F}_i} = E[\Phi(\mathbf{X}(t), 1_i) - \Phi(\mathbf{X}(t), 0_i)], 1 \leq i \leq n$$

where $\mathbf{X}(t) = (X_1(t), ..., X_n(t)), (\mathbf{X}(t), \cdot_i) = (X_1(t), ..., X_{i-1}(t), \cdot, X_{i+1}(t), ..., X_n(t))$ and $X_i(t) = 1_{\{T_i > t\}}$.

Essentially, $I^B(i,t)$ describes the rate of improvement in the systems performance with respect to improvement in the performance of component i.

Theorem 3.1.1. Let $T_1, ..., T_n$ be positive, finite and independent random variables representing the component lifetimes of a coherent system with lifetime T. Then the Birbaum reliability importance of component i for the system at an instant t is

$$I(i,t) = \sum_{j=1}^{p} (-1)^{j+1} \sum_{1 \leq k_1 < ... < k_j \leq p} \{1_{\{i \in \bigcup_{l=1}^{j} P_{k_l}\}} \prod_{k \in \bigcup_{l=1}^{j} P_{k_l} - \{i\}} P(T_k > t).$$

Proof. As above, at a fixed instant t, we let $X_i(t) = 1_{\{T_i > t\}}$. Using the hyperminimal representation of a coherent systems distribution we have

$$I(i,t) = P(\Phi(1_i, \mathbf{X}(t)) = 1) - P(\Phi(0_i, \mathbf{X}(t)) = 1) =$$

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1 \le k_1 < \ldots < k_j \le p} [P(\min_{i \in \bigcup_{l=1}^{j} P_{k_l}} X_i(t) | X_i(t) = 1) - P(\min_{i \in \bigcup_{l=1}^{j} P_{k_l}} X_i(t) | X_i(t) = 0)]$$

$$= \sum_{j=1}^{p}(-1)^{j+1} \sum_{1 \le k_1 < \ldots < k_j \le p} \{1_{\{i \in \bigcup_{l=1}^{j} P_{k_l}\}} P(\min_{i \in \bigcup_{l=1}^{j} P_{k_l} - \{i\}} X_i(t) = 1) +$$

$$1_{\{i \notin \bigcup_{l=1}^{j} P_{k_l}\}} [P(\min_{i \in \bigcup_{l=1}^{j} P_{k_l}} X_i | X_i(t) = 1) - P(\min_{i \in \bigcup_{l=1}^{j} P_{k_l}} X_i(t) = 1 | X_i(t) = 0)]\}.$$

As, by hypothesis, the component lifetimes are independent we have

$$I(i,t) = \sum_{j=1}^{p}(-1)^{j+1} \sum_{1 \le k_1 < \ldots < k_j \le p} \{1_{\{i \in \bigcup_{l=1}^{j} P_{k_l}\}} P(\min_{i \in \bigcup_{l=1}^{j} P_{k_l} - \{i\}} X_i(t) = 1).$$

Using the correspondence $X_i(t) = 1_{\{T_i > t\}}$, where T_i is the lifetime of the component i and t is a fixed instant, we have

$$I(i,t) = \sum_{j=1}^{p}(-1)^{j+1} \sum_{1 \le k_1 < \ldots < k_j \le p} \{1_{\{i \in \bigcup_{l=1}^{j} P_{k_l}\}} \prod_{k \in \bigcup_{l=1}^{j} P_{k_l} - \{i\}} P(T_k > t).$$

Corollary 3.1.2. For a parallel system of n independent component we have

$$I(i,t) = \sum_{j=1}^{p}(-1)^{j+1} \sum_{1 \le k_1 < \ldots < k_j \le p} \{1_{\{i \in \bigcup_{l=1}^{j} \{k_l\}\}} \prod_{i \in \bigcup_{l=1}^{j} \{k_l\} - \{i\}} P(T_i > t).$$

Proof. Without loss of generality we consider $i = n$. For a parallel system of n independent component, the minimal path sets are the unit sets and

$$I(n,t) = \sum_{j=1}^{n}(-1)^{j+1} \sum_{1 \le k_1 < \ldots < k_j \le n} \{1_{\{i \in \bigcup_{l=1}^{j} \{k_l\}\}} \prod_{i \in \bigcup_{l=1}^{j} \{k_l\} - \{i\}} P(T_i > t) =$$

$$1 + \sum_{j=2}^{n}(-1)^{j+1} \sum_{1 \le k_1 < \ldots < k_j \le n} 1_{\{i \in \bigcup_{l=1}^{j} \{k_l\}} \prod_{l=1}^{j} P(T_{k_l} > t) =$$

$$1 - \sum_{j=1}^{n-1}(-1)^{j+1} \sum_{1 \le k_1 < \ldots < k_j \le n-1} \prod_{l=1}^{j} P(T_{k_l} > t) =$$

$$1-P(\bigcup_{k=1}^{n-1}\{T_k>t\})=P(\bigcap_{k=1}^{n-1}\{T_k\le t\})=\frac{P(\max_{1\le k\le n}T_k\le t)}{P(T_n\le t)}=$$

$$P(\max_{1\le k\le n}T_k\le t|T_n\le t)=1-P(\max_{1\le k\le n}T_k>t|T_n\le t)$$

$$=P(\max_{1\le k\le n}T_k>t|T_n>t)-P(\max_{1\le k\le n}T_k>t|T_n\le t)=$$

$$=P(\Phi(1_n,\mathrm{X}(t))=1)-P(\Phi(0_n,\mathrm{X}(t))=1).$$

In the early stages of system development in which we does not have any information about the component reliability it is reasonable to assume that, at a fixed instant t, the components reliability are equals to $\frac{1}{2}$, in the way that we can define the **Birnbaum Structural Importance:**

If the components are independent and identically distributed with survival function at a fixed instant t equal to $P(T_i>t)=\frac{1}{2}$ we define the **Birnbaum Structural Importance** as

$$I(i)=\sum_{j=1}^{p}(-1)^{j+1}\sum_{1\le k_1<\ldots<k_j\le p}\{1_{\{i\in\bigcup_{l=1}^{j}P_{k_l}\}}(\frac{1}{2})^{\#\{\bigcup_{l=1}^{j}P_{k_l}\}-1}.$$

For a parallel system where the minimal path sets are unit sets the Birnbaum structural importance is

$$I(i)=\sum_{j=1}^{p}(-1)^{j+1}\sum_{1\le k_1<\ldots<k_j\le p}\{1_{\{i\in\bigcup_{l=1}^{j}\{k_l\}\}}(\frac{1}{2})^{j-1}.$$

As i is fixed and equal to some k_l the summation has $\binom{n-1}{j-1}$ terms and

$$I(i)=\sum_{j=1}^{n}\binom{n-1}{j-1}(-1)^{j+1}(0,5)^{j-1}=\sum_{j=0}^{n-1}\binom{n-1}{j}(-\frac{1}{2})^{j}=(\frac{1}{2})^{n-1}.$$

3.2. Barlow and Proschan importance.

This Birbaum concept of importance does not incorporate the time dynamics aspects of the problem; this was considered by Barlow and Proschan (1975) who proposed the measure

$$I^{BP}(i)=\int_0^{\infty}E[\Phi(\mathrm{X}(t),1_i)-\Phi(\mathrm{X}(t),0_i)]dF_i(t),$$

where $F_i(t)$ is the absolutely continuous lifetimes distributions of the components $i, 1\le i\le n$, which are assumed independents.

Barlow and Proschan (1975) proved that $I^{BP}(i)=P(T=T_i)$ which is a reasonable interpretation. Using a hyperminimal representation of a coherent system distributions we have

Theorem 3.2.1. Let $T_1, ..., T_n$ be positive, finite and independent random variables with absolutely continuous distribution functions $F_1, ..., F_n$, representing the component lifetimes of a coherent system with lifetime T. Then the Barlow and Proschan reliability importance of the component i for the system is

$$I^{BP}(i) = \int_0^\infty \sum_{j=1}^{p}(-1)^{j+1} \sum_{1\le k_1<...<k_j\le p} 1_{\{i\in\bigcup_{l=1}^{j} P_{k_l}\}} \prod_{k\in\bigcup_{l=1}^{j} P_{k_l}-\{i\}} P(T_k > s)dF_i(s).$$

Proof.

$$P(T = T_i) = P(\max_{1\le j\le p}\min_{k\in P_j} X_k(t) = X_i(t)) = P(\bigcup_{j=1}^{p}\{\min_{k\in P_j} X_k(t) = X_i(t)\}) =$$

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1\le k_1<...<k_j\le p} P(\bigcap_{l=1}^{j}\{\min_{k\in P_{k_l}} X_k(t) = X_i(t)\}) =$$

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1\le k_1<...<k_j\le p} 1_{\{i\in\bigcup_{l=1}^{j} P_{k_l}\}} P(X_i(t) \le \min_{k\in\bigcup_{l=1}^{j} P_{k_l}-\{i\}} X_k(t)) .$$

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1\le k_1<...<k_j\le p} 1_{\{i\in\bigcup_{l=1}^{j} P_{k_l}\}} \int_0^\infty P(\min_{k\in\bigcup_{l=1}^{j} P_{k_l}-\{i\}} T_k > s)dF_i(s) =$$

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1\le k_1<...<k_j\le p} 1_{\{i\in\bigcup_{l=1}^{j} P_{k_l}\}} \int_0^\infty \prod_{k\in\bigcup_{l=1}^{j} P_{k_l}-\{i\}} P(T_k > s)dF_i(s) =$$

$$\int_0^\infty \sum_{j=1}^{p}(-1)^{j+1} \sum_{1\le k_1<...<k_j\le p} 1_{\{i\in\bigcup_{l=1}^{j} P_{k_l}\}} \prod_{k\in\bigcup_{l=1}^{j} P_{k_l}-\{i\}} P(T_k > s)dF_i(s).$$

If we do not know the distribution functions of the components lifetimes we assume that they are identically distributed and define the **Structural Importance of Barlow and Proschan:**

If the component lifetimes are independent and identically distributed we have

$$I(i) = \int_0^\infty \sum_{j=1}^{p}(-1)^{j+1} \sum_{1\le k_1<...<k_j\le p} \{1_{\{i\in\bigcup_{l=1}^{j} P_{k_l}\}} (\overline{F}(s))^{\#\{\bigcup_{l=1}^{j} P_{k_l}\}-1} dF(s) =$$

$$\sum_{j=1}^{p}(-1)^{j+1} \sum_{1\le k_1<...<k_j\le p} \{1_{\{i\in\bigcup_{l=1}^{j} P_{k_l}\}} \int_0^1 (q)^{\#\{\bigcup_{l=1}^{j} P_{k_l}\}-1} dq) =$$

$$\sum_{j=1}^{p}(-1)^{j+1}\sum_{1\leq k_1<...<k_j\leq p}\{1_{\{i\in\bigcup_{l=1}^{j}P_{k_l}\}}\frac{1}{\#\{\bigcup_{l=1}^{j}P_{k_l}\}}.$$

For a parallel system we have

$$\sum_{j=1}^{p}(-1)^{j+1}\sum_{1\leq k_1<...<k_j\leq p}\{1_{\{i\in\bigcup_{l=1}^{j}\{k_l\}\}}\frac{1}{j}=\sum_{j=1}^{p}(-1)^{j+1}\binom{n-1}{j-1}\frac{1}{j}=\frac{1}{n}.$$

3.3. Bergman concept of importance

Bergman (1985) pointed out that many importance measures in reliability theory can be obtained through the study of the change of the system expected lifetime due to different variations of the component lifetime distribution. Assume that the components are independent, and let F_i and G_i be the original and the modified distribution of component i, respectively. Then the importance of component i with respect to this component improvement i given as

$$\int_0^\infty(\overline{G}_i(t)-\overline{F}_i(t))I^B(i,t)dt.$$

$$E[T^*]-E[T]=\int_0^\infty[P(T_i^*>t)-P(T_i>t)]I^B(i,t)dt=$$

$$\int_0^\infty[P(T_i^*>t)-P(T_i>t)]\sum_{j=1}^{p}(-1)^{j+1}\sum_{1\leq k_1<...<k_j\leq p}1_{\{i\in\bigcup_{l=1}^{j}P_{k_l}\}}\prod_{k\in\bigcup_{l=1}^{j}P_{k_l}-\{i\}}P(T_k>t)dt=$$

$$\sum_{j=1}^{p}(-1)^{j+1}\sum_{1\leq k_1<...<k_j\leq p}\{1_{\{i\in\bigcup_{l=1}^{j}P_{k_l}\}}\int_0^\infty[P(T_i^*>t)-P(T_i>t)]\prod_{k\in\bigcup_{l=1}^{j}P_{k_l}-\{i\}}P(T_k>t)dt.$$

In the case of **Natvigs Importance Measure**, Natvig(1979) and Noros (1986), the improvement is obtained through a minimal repair of the component in question.

In this case the transformation results in , $P(T_i^*>t)=\overline{F}_i(t)-\overline{F}_i(t)\ln\overline{F}_i(t)$ and

$$I(i)=\sum_{j=1}^{p}(-1)^{j}\sum_{1\leq k_1<...<k_j\leq p}\{1_{\{i\in\bigcup_{l=1}^{j}P_{k_l}\}}\int_0^\infty\overline{F}_i(t)\ln\overline{F}_i(t)\prod_{k\in\bigcup_{l=1}^{j}P_{k_l}-\{i\}}P(T_k>t)dt.$$

Bueno (2004) defines an **Importance Measure through a Parallel Improvement.** In this case the transformation results in, $P(T_i^*>t)=\overline{F}_i(t)-\overline{F}_i(t)F_i(t)$ and

$$I(i)=\sum_{j=1}^{p}(-1)^{j+1}\sum_{1\leq k_1<...<k_j\leq p}\{1_{\{i\in\bigcup_{l=1}^{j}P_{k_l}\}}\int_0^\infty\overline{F}_i(t)F_i(t)\prod_{k\in\bigcup_{l=1}^{j}P_{k_l}-\{i\}}P(T_k>t)dt.$$

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