Distance Transform Network for Shape Analysis

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Abstract

Shape is known as an important source of information in object analyzes and has been studied for many years for this context. In the object classification task, several challenges such as variations in rotation and scale, noise, and degradation make the problem even harder. This paper proposes the Distance Transform Network (DTN), which combines the power of networks and the richness of information provided from Euclidean distance transform for shape analysis. First, a distance map is obtained by the application of the Euclidean distance transform on each contour. Thus, each radius of dilatation is modeled as a network. Then, degree measurements of the dynamic evolution network are used to characterize the contour. Finally, a robust feature vector is composed by characteristics of different radiuses of dilatation. The methodology was tested in seven benchmarks available databases, including two otolith and three sets containing shape of leaves species which presents challenging contours with a lot of intra-class variations. The results against literature methods show that the proposed DTN is effective for natural shapes classification according to the higher success rates obtained in all cases. The advantages of our approach include robustness to degradation and noise, and tolerance to variations in the shapes scale and orientation.

Keywords: Shape classification, Shape analysis, Complex network, Euclidean distance transform

1. Introduction

Shape is a classical visual attribute and it is the most important feature for object characterization with first studies dated from the 60's [5, 1]. Furthermore, the interest on this type of feature is inspired by biological systems where shape matching in biochemical reactions is necessary [13].

Over the years, different approaches have been proposed to represent shapes and can be classified into three main classes: skeleton-based, region-based and contour-based techniques [28]. The category of a method is attributed based on how features are extracted from shape [44]. Skeleton-based methods, for instance, use the medial axes of the shape to extract its features. These methods have as an advantage the robustness for shapes with occlusion and articulation. Backes and Bruno [2] proposed a method in this category that uses the dynamical evolution of the graph, generated by the skeleton, to obtain robustness. The authors create a representation of the shape by a composition of the multi-scale fractal dimension, minimum, average and maximum degree of each network produced by a set of thresholds. Furthermore, path similarity of skeleton segments are achieved by Gaussian smoothing over distances in a pair of and the correspondence is performed by Bayesian analyzes in [7]. Other examples of this category are found in Refs. [11, 9]. Region-based techniques, on the other hand, focus on a global analysis of the image to extract features (e.g., Zernike moments [48] and Hu moments-based methods [23, 26]). Although, the ability to apply the method in generic shapes, the category cannot distinguish objects that are very similar such as different species of leaves.

Finally, contour-based methodologies, considered in this paper, use only the contour information of the shape to extract characteristics. Some examples of this category include Fourier descriptors [43], Curvature Scale Space [31] and Multi-scale fractal dimension [33, 42]. Most of these methods consider the contour as an ordered set of connected points, an intuitive way to deal with a sequence of dots. However, usually, contour-based techniques suffers when silhouette is not complete and the lack of points or occlusion of a shape region can affect the results [6]. To avoid this drawback, Backes, Casanova and Bruno [3] propose to use complex networks (CNs). The introduction of networks to model shapes allows this contour descriptor to deal with non-perfect contours that suffered degradation and are often found in nature.

Inspired by the work proposed in [3], which models the contour pixels as a complex network we developed a novel approach for feature extraction. Different from the previous method, in this paper, the Euclidean distance transform (EDT) is applied on the contour image and then model the pixels that belong to a given radius of dilatation r as a network. The characteristics of different radius of dilatation are used to describe the shape. Therefore, the proposed method adds information about wave propagation and collision of the EDT on the contour. The EDT method calculates the minimum distance from an image pixel to a region of interest [35]. It is a well-known method and largely used in computer vision, shape analysis and pattern recognition [18]. In summary, the EDT has a strong link with morphological mathematics and can be understood as a series of consecutive dilatations. Also, the Euclidean distance transform and morphological mathematics have been successfully used for decades in shape analysis [18]. The reason of this study it is to combine both complex networks model and EDT method with the goal of creating a method that can take advantage of both approaches. While the networks proved to be robust and powerful to shape analysis [3], the EDT of a shape contour can contain rich information since it carries all the information of the contour wave propagation and collisions [18].

Therefore, in this paper, we propose a new shape descriptor, called Distance Transform Network (DTN). In summary, a Euclidean distance transform is applied over the binary image of the contour, as presented in Figure 1 (a)-(b). In the scheme, pixels that belong to a given value of radius of dilatation r are modeled as a network. Then, a set of thresholds are applied to transform the regular network into a t-scale complex network. Thus, given the t-scale network the average and maximum degree are obtained as features, this step is described in Figure 1(c). Finally, for shape representation, degree measurements for different values of thresholds and radius of dilatation are used to obtain the respective feature vector (see Figure 1(d)).

Experiments were performed in two well-known benchmarks: generic shapes and ETH-80. Experimental results on these databases showed the effectiveness of our method compared to other shape methods. Furthermore, the proposed method was applied to the analysis of natural shapes which is the focus of our study, due to the diversity found in nature. For instance, natural objects such as plants contain a variety of differences within the same class, i.e., the same species of a plant can contain smaller or bigger leaves. Therefore, five natural databases were used to evaluate the proposed method: three of leaves species and two for fish recognition. Additionally, in one of the leaves databases, shapes were intentionally reshaped to analyze characteristics such as noise tolerance, scale invariance, rotate invariance and robustness. Experimental results show that the proposed method obtains higher accuracy rates compared to other shape methods even in the presence of degradation, noise, and rotation.

The paper is divided as follows. Section 2.1 shows an introduction of complex networks and the Section 2.2 presents the methodology capable to modify the contour, the Euclidean Distance Transform. In Section 3 the proposal to combine complex networks and EDT is presented. Then, experimental setups are described in Section 4. Finally, recognition results are presented in Section 5. The section also compares and discuss results among all shape descriptors analyzed in this paper. Section 6 presents the conclusion of the paper.

2. Background

2.1. Networks

The networks field can be interpreted as the intersection between graph theory and tools of statistical mechanics which gives a natural multidisciplinary to it, combining Computer Science, Mathematics and Physics [12]. In the 50's, researchers in graph theory conducted by [16, 15] provided the basis of the field. Studies in complex network area were motived by the investigations about small-world networks [45], scale-free networks [8], community structure in networks

[20] and sets of models found in most real networks [12]. The main reason for its popularity is the flexibility and capacity to represent any given structure, natural or discrete, such as real complex systems and computer vision problems [14].

The network theory has been successfully adopted to develop methods in computer vision and pattern recognition. These methods, such as [21, 34, 4], include the study of modeling a problem into a complex network, the analysis of their topological structure and feature extraction. For instance, an image can be modeled as a network and its patterns represented by network measures related to its connectivity. In the literature, the problems of computer vision based on CN include: texture analysis (e.g. [10]), refine edge (e.g. [19]), boundary shape analysis (e.g. [3, 47]), etc.

2.2. Euclidean Distance Transform

The next method presented in this paper is the Euclidean Distance Transform, the EDT. The technique has been used in several tasks such as computer vision, graphics, shape analysis, pattern recognition, among others [18]. In shape analysis, for instance, the method has been used to match objects with the advantage of providing better and smoother results when transformed images are compared [41]. Also, EDT is able to compute morphological operations such as dilations and erosion. Thus, as shown in [30], a multi-scale analysis can be created by the application of these operations. According to [30], the appearance of an object is very dependent on the scale analyzed, which makes the addition of different scales important to obtain a complete comprehend of the object observed. For this reason, EDT is applied in this context to enrich the shape characterization.

Basically, Euclidean distance transform of a binary image I finds the minimum distance of the background pixels B to an object C in the foreground. In this context, the output of the EDT is named distance map S. This distance map is a 2-D matrix with values $R \in r$ and each set of $S_r \in S$ contains all pixels with a minimum distance r to the closest point in the object C, the contour, i.e, S_r has all pixels with the same radius of dilatation r of the interest object.

The radiuses of dilatation forms isolines, pixels with the same value that are an important source of analysis in the distance map. These lines contain important information about the propagation of waves of the EDT and the complexity of the contour. The "shock" between the waves are directly associated with specific characteristics of the object such as curves and size. Therefore, each set of pixels with a radius of dilation, $S_r \in S$, can be analyzed separately and roughly understood as a different scale. Consequently, characteristics of different 'scales' can be combined to obtain more robust features. In order to reduce the computational complexity, this paper uses a linear implementation of the exact Euclidean distance transform from Ref. [29] and available on software Matlab 9.2.

3. Distance Transform Network

In this section, we describe the distance transform network method, also referenced here as DTN, for boundary shape analysis. The proposed method combines complex networks and Euclidean distance transform, taking advantage of the robustness of the former and the extra shape information provided by the latter to create a strong and robust shape descriptor. In the following subsections, we describe: (i) the modeling of a radius of dilatation as a network, (ii) the proposed signature, and (iv) the parameters evaluation.

3.1. Modeling radius of dilatation as network

In this section, we describe the proposal to model the radiuses of dilation as a complex network. First, the Euclidean distance transform is applied considering the contour C as the object of interest (Figure 1(a)). The output of the EDT method is a distance map S shown in Figure 1(b). S is composed by several radiuses of dilatation R formed by pixels at the same distance r of the contour. The goal of this study is to compute the information of how an object progresses along different radiuses of dilation, provided by the EDT, and then analyze

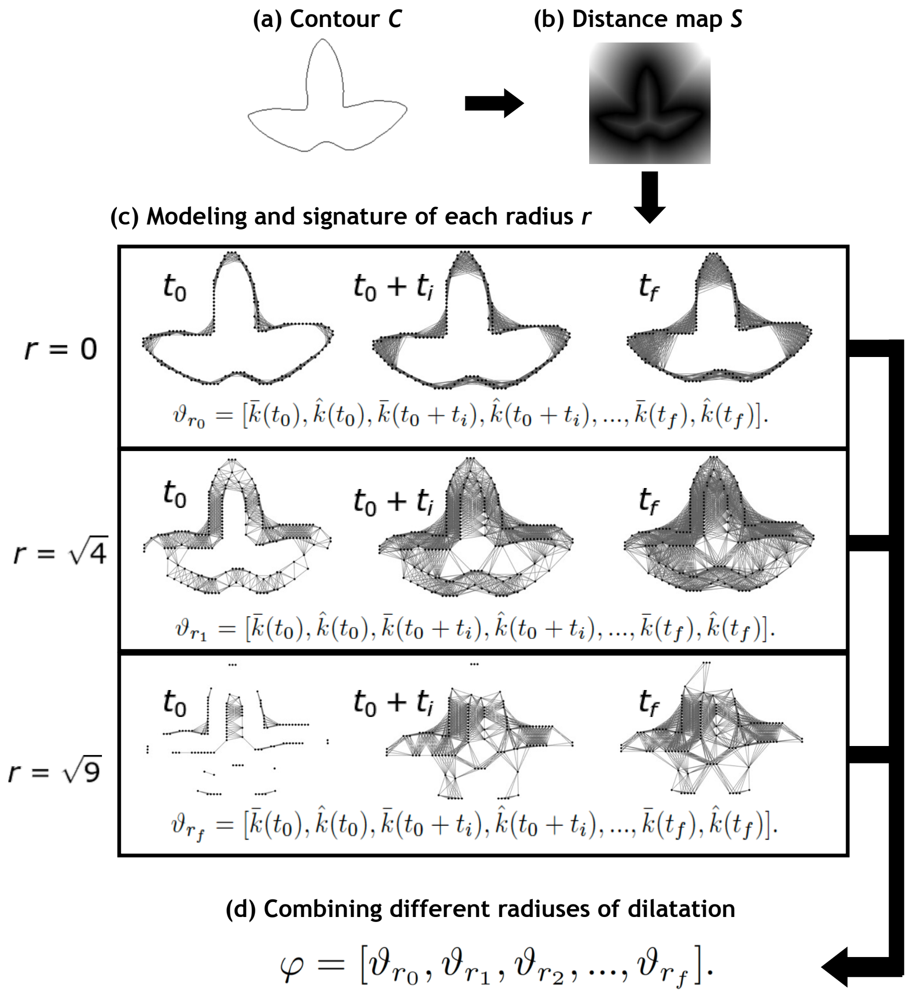


Figure 1: Summarization of the proposed method.

each isoline separately and combine the features of each r to achieve a robust classification. Looking closer, each radius r is composed by a set of points (or pixels) $S_r \in S$ and each point is addressed as $s_i = [x_i, y_i](s_i \in S_r)$, where x_i and y_i are discrete values that correspond the coordinates of the point i in the map.

In order to use the complex networks theory for the problem the subset $S_r \in S$ is modeled as a graph $G_r = (E_r, V_r)$. It is important to highlight that in this study, the points of a radius of dilatation, S_r, are connected to create the graph and not the contour pixels as in the C.N. Degree [3] method. Therefore, a graph is built, where each point s_i of a radius of dilatation r is represented as a vertex $v \in V_r$ of the graph (i.e., $S_r = V_r$). The set of non-directed edges $E_r : V_r \times V_r$ is defined by connecting of all vertices in V_r to each to other. For each edge $e_{i,j} \in E_r$ (where nodes i and j are connected), a non-negative weight w_{ij} is defined:

$$w_{ij} = d(s_i, s_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}. \tag{1}$$

Thus, the network is represented by a $N \times N$ weight matrix W_r,

$$W_r([w_i, w_j]) = d(s_i, s_j). \tag{2}$$

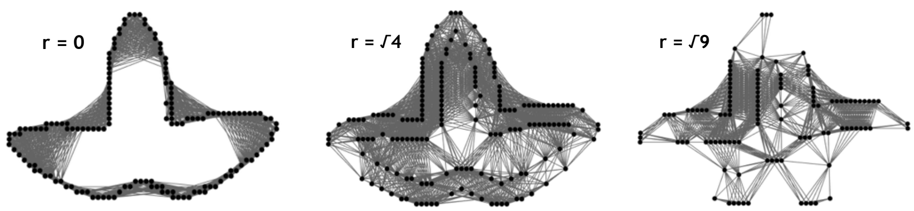
Also, for invariance purposes, the weight matrix is normalized into the interval $[0, 1]$, according to the highest weight:

$$W_r = \frac{W_r}{\max_{w_{ij} \in W_r}}. \tag{3}$$

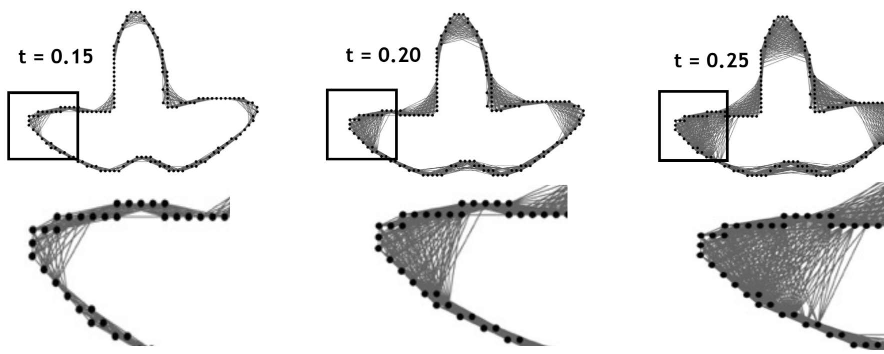
Figure 2(a) presents examples of graph generated by pixels with the same distance in S. Once the network is obtained from a radius of dilatation r, measures of their topological properties can be extracted for pattern recognition. This task is described in the following sections.

3.2. Signature

In a first stage, all vertices of a given radius of dilatation r are connected forming a regular network. However, a regular network has not any topological



(a)



(b)

Figure 2: Examples of network modeling: (a) different radiuses of dilatation r modeled as a complex network; (b) radius of dilatation $r = 0$ modeled as a transformed network by different values of threshold t.

property and it is not considered a complex network. In this way, it is necessary to transform it to highlight important properties that characterize the studied problem.

An approach to transforming the network is to apply a threshold t on its edges, which produces a new set of edges E' [12]. This transformation consists in selecting edges whose weights are smaller than a given threshold t. This strategy can convert an initial regular network that models a contour in a complex network that presents small-world properties [45], a characteristic presented in many real networks [3]. Thus, we propose to apply a transformation $\delta_t(W_r)$ with different values of t over the regular network that models a radius of dilatation r of the EDT. Then, we use degree measures in each output small-world network evolution as feature vector (Figure 1(c)). The adjacency matrix A_t^r of the network G_t^r produced by pixels with distance r from the contour and with the application of a threshold t, is obtained by:

$$A_t^r = \delta_t(W_r) = \forall w \in W_r \begin{cases} a_{ij} = 0 & \text{if } w_{ij} \leq t \\ a_{ij} = 1 & \text{if } w_{ij} > t. \end{cases} \tag{4}$$

As mentioned before, we use a set of thresholds $t \in T$ to transform regular networks with no significant properties to graphs that may contain intrinsic features of the system. This set is defined by an initial threshold t_0 and incremented at a regular interval t_i until a final threshold t_f, creating different networks for each radius r. Figure 2(b) shows the dynamic evolution of the network that models a radius of dilatation ($r = 0$) for different threshold values. As noticed in the figure, as the threshold increases, the number of edges in the curves also increases.

To characterize each network G_t^r produced by the proposed method, degree descriptors are computed from the unweighted matrix A_t^r. To compose the feature vectors we use two measurements: the average degree $\bar{k}$ and the maximum degree $\hat{k}$ of each network derived from the dynamic evolution. However, before computing these measures, a normalization of the degree of the vertices by the number of vertices in the network is performed (Equation 5). The purpose of the normalization is to reduce the network size influence in the representation.

$$\forall k_i = \frac{k_i}{N} \tag{5}$$

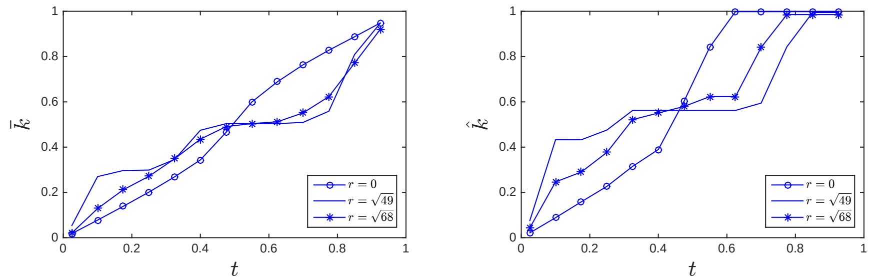
To describe the topology of the network G_r, which models a radius of dilatation r, we proposed a feature vector ϑ_r composed of the average degree $\bar{k}$ and the maximum degree $\hat{k}$. This feature vector consists of the concatenation of $\bar{k}$ and $\hat{k}$ values for each evaluated threshold t:

$$\vartheta_r = [\bar{k}(t_0), \hat{k}(t_0), \bar{k}(t_0 + t_i), \hat{k}(t_0 + t_i), ..., \bar{k}(t_f), \hat{k}(t_f)]. \tag{6}$$

Notice that the previous feature vector ϑ_r, in Equation 6, only computes features for one radius r and several thresholds. However, to obtain a robust feature vector with different information about the shape contour, we concatenate feature vectors ϑ_r for different values of radiuses of dilatation r, as showed in Figure 1(d). To illustrate this approach, Figure 3 presents the final feature φ vector composed by the average degree and maximum degree of different radiuses of dilatation r. As noticed, each radius modeled as network contains different information, which increases the performance in the final shape classification task. To create the feature vector, the radius values ranges between the interval $r_0 \leq r \leq r_f$, which is evaluated in Section 3.4. Thus, the feature vector φ is given by:

$$\varphi = [\vartheta_{r_0}, \vartheta_{r_1}, \vartheta_{r_2}, ..., \vartheta_{r_f}]. \tag{7}$$

The feature vector φ is able of represent a shape considering desirable properties in the shape classification task, such as: rotation and scale invariance, obtained through degree normalization; noise intolerance, reached by the application of the EDT; and degradation invariance, from the fact that the EDT and network modeling does not require the extraction of ordered points from the contour.



(a) Average degree features

(b) Max degree features

Figure 3: Degree features of three radiuses of dilatation of a same contour.

3.3. Computational complexity

In order to evaluate the performance of DTN method in terms of computational cost, we performed a computational complexity analysis. First, the proposed method applies the binary Euclidean distance transform on a shape image, it means that the computational cost of this method must the taken into consideration. Fortunately, The exact EDT of a binary image may be computed in linear time according to the total number of contour pixels (i.e. N_p) [29], which deals with a computational complexity of $O(N_p)$. Then, the next step is to create the network. Considering a radius of dilation with N_d points, each point is connected to all other points of the contour. Therefore $O(N_d + \frac{N_d(N_d-1)}{2})$ operations are required to build the complex network. Next, to create meaningful networks, we apply a set of thresholds T over the network, where $\|T\| = n$ is the number of thresholds applied. The graph cut is performed over a set of radius of dilation of size m. Since n and m are values independent of N_d and $n \ll N_d$ and $m \ll N$ (e.g $n = 13$, $m = 49$ and $N = 5000$), we can ignore n and m in the complexity, leading to a computational complexity of $O(N_p + N_d + \frac{N_d(N_d-1)}{2})$.

3.4. Parameter Evaluation

In this section, we present an evaluation of the parameters of the proposed method and its consequence on shape recognition. DTN method assumes the following parameters: (i) set of thresholds T of the dynamic evolution of the network, and (ii) the initial and final radius of dilatation, r_0 and r_f. To accomplish this task, we use three databases: ETH-80, USP Leaves, and Generic shapes. The classifications of the feature vectors were performed using the LDA classifier [17].

We start by analyzing the behavior of the method for different threshold sets using a constant radius of dilatation $r = 0$ (Table 1 presents the correct classification rate (CCR) for 11 different configurations of threshold set (T_1, T_2, T_3, ..., T_{11})). The set of thresholds consists on an initial threshold t_0, which is incremented by a value t_i until a final value t_f. Note that, regardless of the threshold set used, the proposed method still return good results. As noticed in the table, it is not necessary a large number of network transformations, i.e. the size of T is small, to achieve good results. The output also suggests that a small increment t_i is more appropriate for classification. In the three databases, the best correct classification rate was achieved with the same set of thresholds. Given this evidence, we decided to adopt the set of threshold $T_1, t_0 = 0.025, t_i = 0.075, t_f = 0.950$ for all experiments.

Then, after defining the threshold set, we tested different combination of radiuses of dilatation. Similarly to what occurs for the thresholds, the combination of radiuses of dilatation used to compose the feature vector is defined by an initial radius r_0 and a final radius r_f (in this case, always with increments of 1). Figure 4 presents plots of the correct classification rate for different values of r_0 and r_f for Generic shapes, ETH-80 and USP Leaves databases, respectively. Notice that, in the majority of the values of initial and final radiuses used in the three databases, the proposed method achieves good results. This highlights that the proposed method is not sensitive to these parameters. However, according to the plots, the method achieves the best performance in the three databases for $r_0 = 0$ and $r_f = 113$.

This analysis suggests that the parameters found for the proposed method will achieve good results in any classification configuration besides the databases evaluated in the parameter selection. Therefore, we believe that it is not nec-

Table 1: Results of the proposed method for different sets of threshold T and using radius of dilatation $r = 0$.

T	t_0	t_i	t_f	N. of Features	ETH80	USPLeaves	Generic
T_1	0.025	0.075	0.950	26	**72.43**	**84.00**	**97.00**
T_2	0.025	0.100	0.925	20	71.09	83.50	97.00
T_3	0.025	0.125	0.900	16	70.03	82.16	95.00
T_4	0.025	0.050	0.500	20	72.22	79.16	92.88
T_5	0.025	0.175	0.900	12	67.71	76.16	95.00
T_6	0.050	0.150	0.950	14	68.32	81.00	91.88
T_7	0.050	0.100	0.950	20	70.51	82.83	95.00
T_8	0.050	0.075	0.800	22	70.60	79.33	97.00
T_9	0.050	0.025	0.500	38	72.16	77.66	94.00
T_{10}	0.100	0.050	0.900	34	71.00	82.16	94.88
T_{11}	0.100	0.100	0.900	18	69.35	83.00	95.00

essary a comprehensively search for the parameters of the proposed method for different databases.

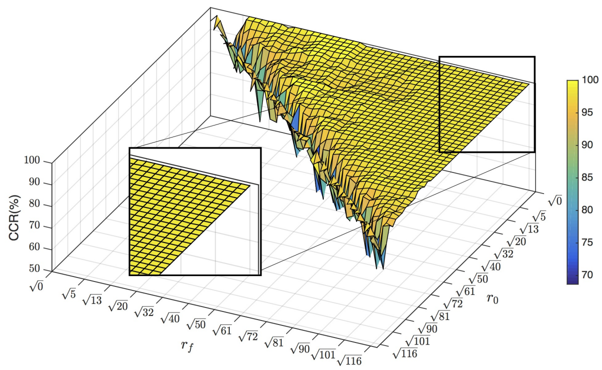
4. Experimental Setup

To evaluate the proposal of this paper, seven different databases were tested: Generic Shapes [38], ETH-80 [25], USP Leaves [3], Swedish Leaves [40], Portuguese Leaves [39], Otolith [36] and Aforo Otolith [27]. USP Leaves database is also evaluated under different conditions: rotation, continuous and random degradation, noise and scale variance. We also compare the results of our method with seven literature shape methods. Finally, we use two machine learning algorithms for classification: Linear Discriminant Analysis (LDA) [17] and Support Vector Machine (SVM) [22].

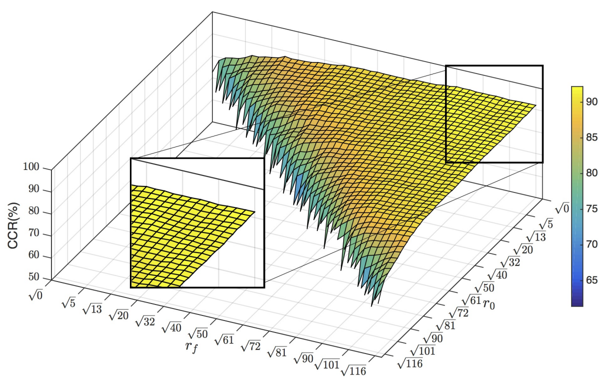
4.1. Shape databases

Generic shapes

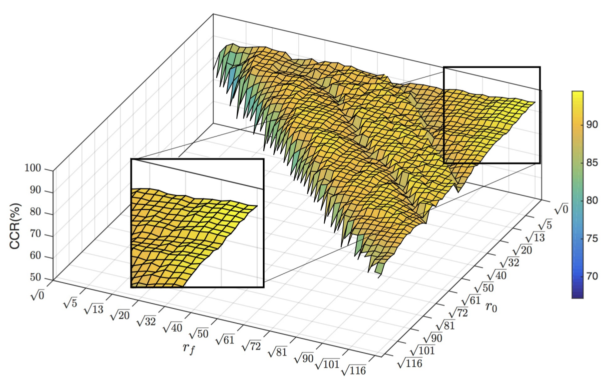
The first database has 99 shape images classified in 9 classes with 11 shapes each [38]. It contains shapes such as rabbits, men, airplanes, tools, and fish. In



(a) Generic Shapes



(b) ETH-80



(c) USP Leaves

Figure 4: Shape recognition performance for three databases: (a) Generic shapes, (b) ETH-80 and (c) USP Leaves as a function of the initial ratio r_0 and the final radius r_f

the set, images were obtained by projections from 3D shapes, which can cause appearance or disappearance of some parts of it. Also, some images suffer from bad segmentation and rotation adding a challenge to object analysis due to deformation.

ETH-80

ETH-80 database contains 3280 images separated in 8 labels [25]. Originally in 3D, the authors also provide segmentation masks (used in the experiments). Categories are represented by human-made and natural objects and for each type of item, different styles are used. For instance, in the cups class, it presents small and big cups while for cars, different brands of toy cars are available. In addition, for each object, 41 images were taken in different viewpoints including upper view. This setup adds variance in rotation and the final contour can be very different for each type of object.

Otolith

Otoliths are often used for fish species classification and therefore two datasets are used here for this purpose. The first, provided by [36], contains 14 species of three families *Engraulidae, Sciaenidae* and *Ariidae*. The database contains images of the right sagittal otoliths, calcium carbonate structures in the ear, captured using a stereomicroscope (Olympus DP25FW, 6.3X magnification) and a digital camera. Images contain differences in position view such as shapes taken in the dorsal edge facing up and posterior end facing the positive direction.

Aforo Otolith

The second natural database for fish recognition was provided by AFORO [27], an open online catalog of otolith images. The website contains images from several contributors, separated by family, species, genus, location, among other specifications. We selected 180 different otolith images containing 20 different species to compose the set.

Swedish Leaves

The biological leaf database containing 15 species with 75 samples per species taken by a 300 dpi color scanner is the fifth set evaluated. According to the authors in [40], the images were not created to make the analysis simple due to the large variation of leaves. A segmentation according to the difference in color of the background and the leaf were necessary to achieve contour information. This preprocessing method was required for all natural databases.

Portuguese Leaves

The database presented in [39] contains 40 different plant species and a total of 340 leaf images. The database provided by the authors has RGB images and binary images (black background and white leaves shapes). The contours were extracted with Matlab to obtain the boundary points.

USP Leaves

This database contains 600 images of 30 leaf species. As different species of plants may contain similar shapes, the classification of this database is a challenging task. Also, in the digitization of the images, overlaps can occur in the adjacency of the objects. As other natural datasets in this work, the USP Leaves is suitable for practical applications due to the presence of one type of major category object, the leaves, divided into subcategories which turns the analysis very hard. Furthermore, in order to evaluate rotation, scale and noise invariance, this database has also supplementary sets containing features generated as follows:

- Rotation: The first modified database of leaves contains images rotated at 7, 35, 132, 201 and 298 resulting in a new set with 3600 shapes. Figure 5(a) shows a sample object rotated by different angles.

- Scale: scaled by a factor of 125%, 150%, 175% and 200% original images generates a different database with original and scaled images. Figure 5(b) shows a sample object scaled by different factors.

- Noise: different levels of uniform noise were added on the images. The noise was generated according to an interval [-l...l] where l is the level of the noise. In the experiments, four different levels were applied to generate a new set with 80 images for each class. Figure 5(c) shows examples of the same object with different levels of noise.

- Random degradation: Levels of degradation, from 15% to 65%, are used to produce this new set. In this case, degradation is applied randomly in the contour and, therefore, sequential points are harder to be obtained. Each level of degradation produces 20 modified images and samples of them are shown in Figure 5(e).

- Continuous degradation: this last modified database is produced by continuous degradation in USP Leaves database. It creates gaps in the contour making recognition harder. Levels of degradation are applied as the same as previously random one but in a continuous manner. Therefore, robustness to this effect can be evaluated. Figure 5(d) shows a sample of this approach.

4.2. Shape Classification Methods

For any new proposed method, it is essential to compare the new results with performances of well-known shape classification methods. Therefore, seven methods from the literature are described in this section and compared with the proposal latter in Section 5.

C.N. degree

Like the proposed method, this feature extractor also models the image as a network. However, it is important to emphasize that the previous work uses the original contour to create the graph while our approach uses the distance map S, output by the EDT of the contour, to create several graphs each one with pixels with value r in S. The method also applies dynamic evolution to transform regular network to significant property graphs. Also, in C.N. degree

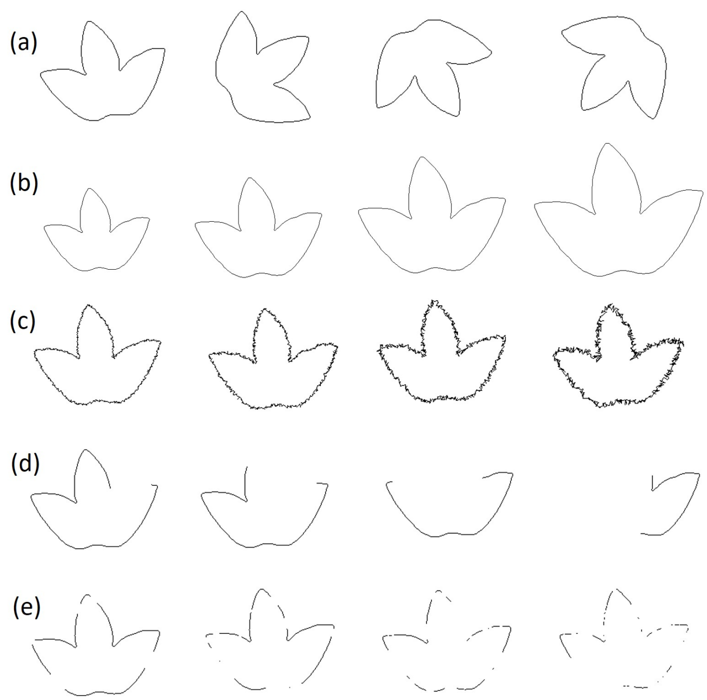


Figure 5: Example of applied artifacts on a contour. (a) rotated contours, (b) scaled contours, (c) noisy contours and contours with (d) continuous degradation and (e) random degradation.

algorithm, features are obtained concatenating measures of the network such as degree and joint degree [3].

Fourier Descriptor

Fourier shape descriptor computes the spectral transform of the contour and uses this information to extract features [32]. In the experiments, the 20 most significant coefficients of the spectrum were used as features of the image.

Curvature Descriptors

The method analyzes the contour as a curve extracting features such as maximum and minimum points which can represent important characteristics of the shape like changes in the directions [46]. The total size of the feature vector obtained for each image in this method is 25.

Zernike Moments

Very simple but useful, this technique computes a set of Zernike moments (order 0 to 7) of the image and uses them as a feature vector. Zernike moments have rotation invariance properties and represent the magnitude of orthogonal complex moments of the shape [48].

Multi-Scale Fractal Dimension

Complexity of the shape can be measured by the fractal dimension. In this method, the contour is understood as a curve and changes in it are related to high complexity [42]. Feature vector contains the 50 most relevant points of the shape to represent the object in a one-dimensional vector.

Segment Analysis

This simple but powerful method describes the contour based on straight lines segments statistics. It considers portions of continuous points and computes the length of the straight line between extreme points of the portion [24]. Average and standard deviation are computed from each line. The different segments are reached by different predefined percentages of the contour size. Final feature vector contains 34 features.

Angular Descriptors of Complex Networks (ADCN)

Another method based on Complex Network, the ADCN analyzes the angle formed by points in the shape contour [37]. A dynamical evolution is also applied in the shape network and angles are computed creating a histogram. According to the results presented in the original study, the approach can achieve robustness against rotation, scale changes, degradation, and noise.

4.3. Classification Setup

The classification of the feature vectors obtained by the methods is performed using two well-known methods LDA [17] and SVM [22] following a stratified 10-fold cross-validation scheme. In the experiments of rotation and scale, it is used a leave-one-out cross-validation scheme. Only in these two setups, the processed samples (i.e. rotated and scaled) from the same shape are used for test and original contours for training (to avoid that information about the test samples were used for training).

5. Results and Discussion

This section compares the performance of the proposed method with the methods described in Section 4.2. In the following, it will be presented and discussed the results obtained for the seven shape databases (Section 4.1) and the analysis of invariance and robustness.

5.1. ETH-80 database

The first results are presented for the ETH-80 database, which contains images of natural and human-made objects such as fruits, vegetables, animals, and vehicles. Table 2 shows CCR, correct classification rate, for ETH-80 recognition according to all methods tested. As noticed, the proposed method obtained one of the highest recognition rates among all shape descriptors analyzed. Results show that the addition of dilation information of the shape improved robustness to viewpoint variance which occurs in the database. The C.N. degree method

Table 2: Comparison results for all databases and literature methods using LDA and SVM classifier.

Methods	**LDA**	**SVM**	**LDA**	**SVM**	**LDA**	SVM
	ETH - 80		Generic Shapes		Aforo Otolith	
Proposed Method	92.02 (± 1.68)	93.47 (± 1.10)	99.00 (± 3.16)	98.00 (± 4.03)	75.00 (± 12.0)	66.23 (± 1.22)
C.N. degree	72.34 (± 2.34)	82.92 (± 2.58)	96.00 (± 6.99)	95.95 (± 5.16)	63.89 (± 7.52)	48.08 (± 1.07)
Fourier	79.26 (± 1.56)	86.12 (± 1.89)	93.88 (± 7.07)	97.97 (± 4.21)	52.77 (± 10.5)	50.35 (± 0.72)
Curvature	62.43 (± 2.03)	72.01 (± 2.61)	78.77 (± 15.2)	84.84 (± 12.5)	55.56 (± 15.0)	49.93 (± 1.31)
Zernike	82.86 (± 2.01)	87.98 (± 1.34)	95.00 (± 5.27)	87.98 (± 1.34)	21.67 (± 4.10)	20.04 (± 1.44)
M. S. fractal	73.96 (± 1.65)	76.73 (± 1.71)	95.00 (± 9.71)	76.73 (± 1.71)	53.33 (± 11.1)	46.89 (± 0.91)
Segment analysis	78.71 (± 2.53)	80.00 (± 2.46)	98.00 (± 4.21)	98.80 (± 3.16)	57.78 (± 8.76)	37.78 (± 0.94)
ADCN	85.39 (± 2.64)	74.35 (± 1.72)	99.00 (± 3.16)	85.78 (± 9.36)	65.55 (± 9.37)	36.32 (± 0.71)
	Otolith		Swedish Leaves		Portuguese Leaves	
Proposed Method	94.90 (± 4.25)	93.55 (± 2.87)	85.41 (± 3.50)	94.61 (± 0.23)	85.00 (± 4.26)	77.79 (± 0.788)
C.N. degree	91.08 (± 4.80)	86.33 (± 5.76)	74.04 (± 3.23)	84.37 (± 0.15)	72.35 (± 7.99)	66.08 (± 0.85)
Fourier	82.63 (± 6.72)	85.43 (± 7.61)	53.77 (± 3.34)	43.14 (± 0.49)	53.53 (± 9.88)	55.33 (± 0.90)
Curvature	90.06 (± 2.40)	89.86 (± 5.51)	56.45 (± 3.68)	74.15 (± 0.56)	64.41 (± 7.52)	65.75 (± 1.07)
Zernike	56.09 (± 8.26)	58.13 (± 8.37)	52.00 (± 4.00)	50.50 (± 0.47)	40.00 (± 10.4)	34.85 (± 0.90)
M. S. fractal	53.33 (± 11.1)	46.89 (± 0.91)	74.84 (± 2.60)	65.05 (± 0.32)	63.82 (± 9.09)	53.52 (± 0.51)
Segment analysis	57.78 (± 8.76)	37.78 (± 0.94)	80.00 (± 2.00)	78.52 (± 0.57)	72.35 (± 5.58)	63.47 (± 0.77)
ADCN	65.55 (± 9.37)	36.32 (± 0.71)	76.18 (± 4.51)	54.77 (± 0.58)	11.52 (± 0.62)	17.64 (± 7.33)
	USP Leaves		Rotated USP Leaves		Scaled USP Leaves	
Proposed Method	94.16 (± 4.24)	93.16 (± 2.41)	89.06	88.67	92.38	93.08
C.N. degree	84.00 (± 5.67)	85.16 (± 3.63)	80.61	84.28	81.46	85.04
Fourier	74.66 (± 6.97)	83.16 (± 4.61)	67.53	77.17	63.25	85.29
Curvature	77.00 (± 6.42)	81.66 (± 3.33)	76.31	83.33	76.92	84.46
Zernike	69.66 (± 5.76)	76.33 (± 5.07)	65.53	76.64	53.38	71.33
M. S. fractal	71.16 (± 4.51)	73.66 (± 5.81)	58.17	68.58	65.25	68.50
Segment analysis	83.33 (± 5.44)	83.50 (± 4.26)	81.47	76.22	82.04	79.75
ADCN	88.50 (± 6.63)	78.83 (± 3.68)	86.36	77.11	86.13	80.75

which was the inspiration for this proposal was increased in almost 20% of CCR using the LDA classifier and 10% using the SVM classifier.

5.2. Generic shapes database

The second database, Generic shapes database, output a high performance for all methods evaluated in this work (see Table 2). Although the simplicity of segment statistics algorithm, it outputs one of the highest CCR among all the methods compared. Using the SVM classifier the proposed method obtained a CCR very close to the segment analysis method. However, again the DTN outperforms all results and obtain the same accuracy that the ADCN using

LDA. Also, the proposed method improves the CCR compared to C.N. degree method. Therefore, it shows the importance of analyzing the complexity of the shapes along different radiuses of dilation.

5.3. Otolith database

The first database containing calcium structures for fish recognition reached 94.90% of success with the DTN approach. The inspiration, C.N. degree, achieved the second place in classification rate with 91.08% (LDA classifier). As notices in Table 2, ADCN works better and output the highest CCR with the linear discriminant classifier associated. However, considering the standard deviation, the differences are minimal. Also, the lowest result is obtained with M.S. Fractal., 50.83 % of CCR (SVM classifier). This may be due to the small changes in shape for most otoliths which cannot be captured by fractal analysis.

5.4. Aforo otolith database

The second database of otoliths, obtained from Aforo library, did not achieve an accuracy as high as the previous database. However, once more, DTN obtained the higher CCR with 75% considering the LDA classifier (see Table 2). Although the database contains fewer images, it has been proved hard to recognize based on all methods results. In this case, in contrast to what occurred in Otolith [36] database, DTN surpass ADCN method considering any classifier. Zernike outputs the lowest CCR reaching around 20% of accuracy for these shapes.

5.5. Swedish Leaves database

We focused on natural shapes for classification and three leaves databases are presented. The first, containing images from Sweden, has a total of 1125 contour of leaves categorized in 15 species. As noticed by Table 2, the proposed method achieves the highest classification rate (94.61%) with a low standard deviation (SVM classifier). The second best method is now the simple Segment with 78.52% of CCR, considering the same classifier and 80% with LDA. With the addition of EDT, which enhances intrinsic features of the shape, the recognition rate is increased in almost 10% comparing the results with C.N. degree. It shows that the application of the preprocessing method is relevant for the analysis.

5.6. Portuguese Leaves database

Named Portuguese Leaves database in this paper, for better database discrimination, the set proposed in [39], outputs 72.35% of CCR in C.N. degree while DTN, the proposed method increased the latter result by almost 13% (SVM classifier). Also, even though ADCN was obtaining a good recognition rate for most of the databases, it was not the case for Portuguese Leaves. The method was not able to capture the main characteristics of the leaves and consequently, its best result was 17.64% (SVM), very low for a classification task. Segment analysis also proved to be a good method for natural shapes, achieved the third place in the podium for this database.

5.7. USP Leaves database

The fifth experiment was conducted in the Leaves database that contains interesting characteristics for shape analysis benchmark. It is composed of real leaves from the Brazilian flora (30 species of plants) [3] and due to the similarities between the leaves from different species and the high varibility within the leaves in the same species, it is a very difficult database and it is capable to analyze very well the power of the shape analysis methods.

Table 2 presents the correct classification rate achieved by DTN and compared methods when applied to the original Leaves database. In this database, the results show a high performance of the proposed method when compared to all other methods, independent of the classifier. For instance, the proposed method improves the recognition performance from 84% to 94.16% over the C.N. degree method and from 83.33% to 94.16% over the Segment analysis method using the LDA classifier.

5.8. *Invariance to scale and rotation*

In order to confirm the properties of rotation and scale invariance discussed in Section 3, we performed experiments in modified databases from original USP Leaves. As can be seen in Table 2, the experimental results using rotated and scaled images confirm the great invariance of the proposed method over these types of transformations. DTN obtained the bests CCR in the two databases independent of the classifier used, LDA or SVM. Notice that the proposed method achieved similar CCR for the two classifiers, while other methods were sensitive to the machine learning algorithms. It suggested a consistent feature vector and consequently a good invariance to scale and rotation.

5.9. *Overall performance*

From Table 2 we can see that the proposed method overcomes the other methods in ETH-80 database and obtain very similar results in Generic Shapes database. On the other hand, the proposed method overcomes all compared methods in natural databases. This demonstrates that DTN is capable to characterize natural and real-world shapes, which are complex and challenging. Indeed, complex networks and EDT are tools widely used to study complexity in real-world problems. In case of shapes, the proposed method works due to the flexibility and capacity of the complex networks to analyze the information of the EDT, overcoming various challenges present in natural shapes such as noise and degradation.

The proposed method significantly improves the correct classification rate compared to the other methods, e.g., from 85.39% (ADCN) to 92.02% on the ETH-80 database, from 91.08% (C.N. Degree) to 94.90% on the Otolith database, from 72.35% (Segment Analysis) to 85.00 on the Portuguese Leaves and from 88.50% (ADCN) to 94.16% on the USP Leaves database (considering the LDA classifier). Furthermore, notice that the other methods do not obtain the same performance on all databases. There are methods that had the second position in a database and a poor performance in other databases. For instance, on the ETH-80 database, the ADCN+LDA achieves 85.39% while on the Portuguese Leaves the result is 11.52%. This suggests that the proposed method works well in general with different databases, while the other methods are sensitive to the specific characteristics of each database.

In terms of classifier, the proposed method obtains similar performances and outperforms all other methods using the LDA and SVM classifiers. In contrast, some methods are very sensitive to the classifier, for example, the results of the ADCD decreases from 99.00% (LDA) to 85.78% (SVM) on the Generic Shapes database, and the results of the Segment Analysis decreases from 57.78% (LDA) to 37.78% (SVM) on the Otolith database. On Rotated and Scaled USP Leaves database, the proposed method also outperformed the ADCN in 2.7% in Rotated Leaves and 6.25% in Scaled Leaves. This indicates that the signature obtained by the proposed method is robust, highlighting the power of combination of complex networks and EDT.

5.10. Robustness against noise

In the experiments performed above in the original USP Leaves databases, the shapes are quite smooth. In order to evaluate the performance of the proposed method under noisy conditions, we apply the methods in modified Leaves database with noise. The results from the noise tolerance experiment using LDA classifier are presented in Table 3. The proposed method demonstrates a great capacity for shape classification even in contours with a high quantity of noise. In the experiments using noise rate Level 1, Level 2 and in the combination of all levels, the proposed method has a good performance compared to other methods. We also emphasize that the Curvature method applies a low-pass filter when it is computed, reducing the noise of the contour. However, the proposed method does not need this kind of preprocessing and it is still able to achieve a good performance in noisy shapes.

5.11. Robustness against degradation

An important property that needed to be evaluated is the robustness of the method to degradation. The degradation in the shape is the lack of information

Table 3: Comparison of the proposed method with literature methods for USP Leaves database corrupted by different levels of noise using the LDA classifier.

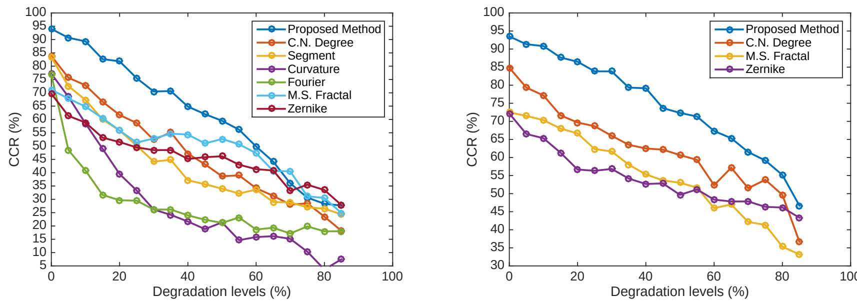
Method	Level 1	Level 2	Level 3	Level 4	All levels
Proposed Method	88.33 (± 3.23)	80.83 (± 5.10)	77.16 (± 3.51)	74.83 (± 5.35)	86.29 (± 2.18)
C.N. degree	79.50 (± 5.55)	78.16 (± 4.87)	78.16 (± 5.29)	76.66 (± 5.38)	77.70 (± 2.76)
Fourier	65.50 (± 4.84)	57.83 (± 6.03)	51.00 (± 6.99)	45.16 (± 5.23)	49.58 (± 1.94)
Curvature	70.33 (± 6.02)	70.16 (± 6.35)	66.33 (± 6.27)	63.16 (± 6.05)	69.25 (± 2.42)
Zernike	67.50 (± 5.89)	66.83 (± 5.35)	66.83 (± 6.77)	63.83 (± 5.61)	62.33 (± 3.86)
M.S fractal dimension	65.00 (± 4.64)	63.50 (± 5.63)	63.33 (± 5.27)	59.66 (± 5.48)	64.20 (± 2.37)
Segment analysis	75.50 (± 6.80)	72.16 (± 5.72)	67.00 (± 5.37)	70.00 (± 5.09)	68.95 (± 1.35)

in its contour. In this way, we evaluate the capacity of a method to recognize a shape when it is not complete. This problem can be easily found due to bad image acquisition. Figure 6(a) presents the correct classification rate in function of continuous degradation level for various methods using the LDA classifier. As the degradation increases the correct classification rate decreases. As noticed in Figure 6(a), the highest correct classification rate is achieved by our proposal, the DTN, for levels 10 to 20.

Another experiment was also performed to evaluate the robustness against random contour degradation. In this experiment, points of different regions of the shape contour were removed. Figure 6(b) shows the results of robustness for several methods for this type of artifact. They also show a high robustness of the proposed method compared to other methods. The CCR's for Fourier, Curvature and Segment statistics methods were not presented in this experiment, due to the requirement of sequential extraction of points in the contour, not possible in this case. This is a problem often found in real applications, therefore, it is important to emphasize that the proposed method does not depend on a contour extraction in a sequential way.

6. Conclusion

In this paper, we have proposed a new method for shape classification based on networks and Euclidean distance transform. Given a contour, or even a subset (dots in sequence or not) of a contour, the DTN method applies the



(a) Robustness in continuous degradation (b) Robustness in random degradation

Figure 6: Comparison of robustness in contour degradation for various methods using LDA classifier.

Euclidean distance transform and models each radius of dilation as a network. Shape is effectively represented by a feature vector composed of the degree measures from the network dynamic evolution, i.e. after the transformation of the original regular network by several thresholds, of different radiuses of dilation.

We have demonstrated the good performance and the robustness of the proposed method in well-know databases of the literature and different properties. First, a study was performed to evaluate the effect of each parameter of the proposed method on the results. The evaluation of the parameters showed that the proposed method is not sensitive to the parameter setup, once it was used the same parameters for all databases. This is a very important characteristic since it suggests that the proposed method could be used with the preconfigured parameters for other databases and the user does not need to perform this setup evaluation anymore.

The experimental results were compared with other literature methods and it was proved that the proposed method has a good performance. In all cases, DTN overcomes the results of its inspiring method, C.N. degree, demonstrating the importance of combine Euclidean distance transform to improve the contour information and complex networks for shape analysis. Segment analysis algorithm also showed good results for some databases and the ADCN method output a good result for most of the databases but failure to recognize Portuguese Leaves images and is dependent on the classifier. It is important to highlight the efficiency of the proposed method to classify natural images by the CCR obtained which are usually challenger to extract consistent features due to illumination, pose and scale changes.

Considering the last experiments, on USP Leaves database, where rotation, scale, noise, and degradation are present and the good performance obtained into the discrimination of the three shape databases considered, we can conclude that the method is robust by its ability to the invariant to different artifacts:

- *Rotation invariance:* the method has a rotation invariance characteristic which was achieved by the normalization of the weight matrix W. In the experiments, it was normalized within the interval $[0, 1]$ maintaining the proportion of the edges weights. Thus, considering images in different rotations, the weight of the largest edge was preserved, ensuring the same properties for the different sets of edges E.

- *Scale invariance:* the property of scale invariance was also noticed in the proposed method, the DTN. It was all, again, due to the normalization of the matrix W. The normalization was responsible to rescale the edges, creating scale tolerance in the classification. For instance, two images in different scales produce different numbers of points in its contour. Once $S = V$, two radiuses of dilatation S_r^a and S_r^b of similar contours in different scales, produce networks with a different number of vertices. Thus, the degree k_i is directly affected by the number of network vertices N. This problem was solved with the normalization of the degree k_i by the number of vertices in the modeled network, as presented previously in Section 3.2 and also used in [3].

- *Noise tolerance:* it is a fact that in the image acquisition process, errors in the contour can be added. This occurs because this task is not perfect, causing a variety of interference and noise. However, the way the contour (i.e., the radiuses of dilatation from contour) was modeled as network allowed us to analyze the shape without decreasing the performance in classification.

- *Degradation robustness:* in the DTN method, the network does not have information about space and sequence of the points of each radius of dilatation. From the distance map, the modeled patterns are equal in the feature space, in theory. This property allowed the method to not require for a sequential extraction of the points of contour. This is important, once the radiuses of dilatation are not continuous. Thus, it is only needed the coordinates of points of the radiuses of dilatation. Specifically for random degradation, the proposed method has shown an as advantageous technique compared to some other shape descriptors such as Fourier, Curvature and Segments descriptor that were not able to classify the shapes due to the sequential requirement. This characteristic is very interesting for real-world problems, that needs to deal with degraded contours or even with contours produced in a sparse and random way.

The Distance Transform Network (DTN) proved its good performance. The capacity to deal with degraded contours and random dots contours, and finally the convenience of no parameter setup needed, make it a very good option for shape analysis.

Acknowledgments

Lucas C. Ribas gratefully acknowledges the financial support grant #2016/23763-8, São Paulo Research Foundation (FAPESP) and grant Nos. 9254772/m, CAPES. Mariane B. Neiva kindly appreciates the support from CAPES which provides financial support No. 132409/2014-3. Odemir M. Bruno thanks the financial support of CNPq (Grant Nos. 307797/2014-7) and FAPESP (Grant No. 14/08026-1).

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