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METASTABILITY FOR THE CONTACT PROCESS

by

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1. Introduction

The problem of explaining theoretically the phenomenon of metastability as shown by undercooled gases, ferromagnets along the histeresis loop and many other systems has been attacked in many different ways during the last years [5], [6]. In the present paper we adopt the theory proposed recently in [1] and we extend the theorems proved there for Harris Contact Model.

In this theory a system is in a metastable situation if:

a) It stays out of its equilibrium situation during a memoryless random time (exponential random time).

b) During this time in which the system is out of equilibrium it stabilizes in the sense that an observer measuring temporal means of some observable quantity will record values which are close to the expectation of this observable in some fixed probability distribution on the configurations of the system. We call this property <u>thermalization</u>.

In section 2 we describe the model. In section 3 we state the results, giving precise meaning to the statements (a) and (b) above, and comparing our results with the results in [1]. In sections 4 and 5 we prove the main theorems. Finally, in section 6 we show the absence of metastability in the subcritical case ($\lambda < \lambda_*$).

2. The Contact Model

The contact model of Harris [2], [3], [4] was motivated by a biological problem: the propagation of an infection. We deal with it because it has some important features: spatial structure(nearest neighbors interaction), and even in one dimension presents critical behaviour. In fact this model is also known in the physics literature [7] with a different name. In this context it is studied numerically in connection with Reggeon Field Theory.

The model is a continuous time Markov process taking its values on the set $P(\mathbb{Z})$ of all the subsets of $\mathbb{Z}$ (we will restrict the definition to the one dimensional case). Particles are distributed in $\mathbb{Z}$ in such a way that each site is empty or occupied by at most one particle. $\xi(t)$ denotes the set of occupied sites at time t. We construct the contact process with the help of a random graph(percolation structure), in the space-time diagram $\mathbb{Z} \times \mathbb{R}_+$. For each $i \in \mathbb{Z}$ consider three independent Poisson processes on $\mathbb{R}_+$: $(\vec{\tau}^{\,i}_n)_{n\in\mathbb{N}}$, $(\overleftarrow{\tau}^{\,i}_n)_{n\in\mathbb{N}}$, $(\tau^{+i}_n)_{n\in\mathbb{N}}$ with parameters λ, λ and 1 respectively. We suppose that for i varying in $\mathbb{Z}$ the processes are all independent. Now, for each $i \in \mathbb{Z}$ we draw arrows in $\mathbb{Z} \times \mathbb{R}_+$, from $(i, \vec{\tau}^{\,i}_k)$ to $(i+1, \vec{\tau}^{\,i}_k)$, $k = 1,2,\ldots$, $i \in \mathbb{Z}$. Secondly we draw arrows from $(i, \overleftarrow{\tau}^{\,i}_k)$ to $(i-1, \overleftarrow{\tau}^{\,i}_k)$, $k=1,2,\ldots$, $i \in \mathbb{Z}$. Finally we put down + signs at each of the points (i, τ^{+i}_k), $k=1,2,\ldots$, $i \in \mathbb{Z}$.

We call a segment linking (x,t) to (x,s) a time segment. We give it the orientation from (x,t) to (x,s) if $s > t$.

Given two points (i,s) and (j,t) in the space time $\mathbb{Z} \times \mathbb{R}_+$, with $s < t$, we say that there is a path from (i,s) to (j,t) if there is a connected chain of oriented time segments and arrows in the random graph, leading from (i,s) to (j,t), following the direction of the time segments and arrows and without passing through a + sign.

Now, given $A \in \mathcal{P}(\mathbb{Z})$, we define the process $(\xi^A(t),\ t \geq 0)$ in the following way: $\xi^A(0) = A$, and for $t > 0$, $\xi^A(t) = \{ j \in \mathbb{Z}$: there is a path from $(i,0)$ to (j,t), form some $i \in A\}$.

Using the same percolation structure we define other related processes. The contact process taking values on $\mathcal{P}(\{-N\text{-}N{+}1,\dots\})$ or on $\mathcal{P}(\{-N,-N+1,\dots,N-1,N\})$ where N is a positive integer. In the first case it is enough to use just the Poisson processes $(\vec{\tau}^{\,i}_n)_{n\in\{-N,-N+1,\dots\}}$, $(\overleftarrow{\tau}^{\,i}_n)_{n\in\{-N+1,\dots\}}$ and $(\tau^{+}_n)_{n\in\{-N,-N+1,\dots\}}$ disregarding the others. In this case we say that there is a path from (i,s) to (j,t), $t > s$ if it can be constructed only with the arrows determined by these processes. In the same way as before, but with these new definition of path, we define for $t > 0$ $\xi^A_{[-N,\infty)}(t) = \{j \in \mathbb{Z}$: there is a path form $(i,0)$ to (j,t) for some $i \in A\}$, $\xi^A_{[-N,\infty)}(0) = A$, where $A \subset \{-N,-N+1,\dots\}$,

Analogously we define the contact process on $\mathcal{P}(\{-N,-N+1,\dots,N-1,N\})$ using to construct the paths only the Poisson processes $(\vec{\tau}^{\,i}_n)_{n\in\{-N,\dots,N-1\}}$, $(\overleftarrow{\tau}^{\,i}_n)_{n\in\{-N+1,\dots,N\}}$ and $(\tau^{+i}_n)_{n\in\{-N,\dots,N\}}$. We will use for it the notation:

$(\xi^A_N(t),\ t \geq 0)$, for any $A \subset \{-N,\dots,N\}$ as initial state.

In this way we have constructed all those processes on the same probability space, and we have some useful relations like

$$\xi_N^A(t) \subset \xi^A(t) \quad \forall A \subset \{-N,\dots,N\} \ , \ t > 0$$

$$\xi_{[-N,\infty)}^A(t) \subset \xi^A(t) \quad \forall\ A \subset \{-N,\dots\},\ t > 0$$

$$\xi^A(t) \subset \xi^B(t) \quad \text{if } A \subset B \subset \mathbb{Z}$$

which hold for all possible trajectories of the processes. This properties are called additivity. We use the convention of omitting the initial condition in the notation when it is the largest possible. So $\xi(t) = \xi^{\mathbb{Z}}(t)$, $\xi_N(t) = \xi_N^{\{-N,\dots,N\}}(t)$, $\xi_{[-N,\infty)}(t) = \xi_{[-N,\infty)}^{\{-N,\dots\}}(t)$.

By elementary Markov process theory, $(\xi_N^A(t),\ t \geq 0)$ is ergodic with invariant measure concentrated at the empty set δ_ϕ. For $(\xi^A(t),\ t \geq 0)$ and $(\xi_{[-N,\infty)}^A(t)\ ,\ t \geq 0)$ the situation is different. There is $\lambda_* \in \mathbb{R}_+$ s.t. if $\lambda < \lambda_*$ both are ergodic, if $\lambda > \lambda_*$ both are not ergodic. In the second case there are for both just two extremal invariant measures: one is δ_ϕ and the other which we denote respectively by μ and $\mu_{[0,\infty)}$ can be obtained as the weak limits $\xi(t) \to \mu$, $\xi_{[-N,\infty)}(t) \longrightarrow \mu_{[-N,\infty)}$ as $t \longrightarrow \infty$. That the critical value of λ for both models is the same λ_* is easy to proove using the technics in [2].

We will use the facts that μ and $\mu_{[0,\infty)}$ have support on the infinite subsets of $\mathbb{Z}$ and that both are ergodic under space translations.

3. Results

We show that for $\lambda > \lambda_*$ the process $(\xi_N(t), t \geq 0)$ behaves metastably for large N, in the sense of conditions (a) and (b) of the introduction. Informally its behaviour is as follows: initially there is a global phenomenon in which $\xi_N(t)$ becomes close to μ restricted to $P(\{-N,\dots,N\})$. As $\lambda > \lambda_*$, the tendency is of expansion, i.e if we where considering the process $(\xi^{\{-N,\dots,N\}}(t), t \geq 0)$, with large N, with great probability $\min\xi^{\{-N,\dots,N\}}(t) \to -\infty$ and $\max\xi^{\{-N,\dots,N\}}(t) \to +\infty$ as $t \to \infty$. Nevertheless the boundary conditions at N and -N prevent this expansion. The system remains in apparent equilibrium with $\xi_N(t)$ close to μ restricted to $P(\{-N,\dots,N\})$ until a great fluctuation carries it to the true equilibrium at δ_ϕ.

Let us introduce some notation:

$$T_N^A = \inf\{t > 0: \xi_N^A(t) = \phi\}$$

$$T_N = \inf\{t > 0: \xi_N(t) = \phi\}$$

As usually we identify $P(\mathbb{Z})$ with $\{0,1\}^{\mathbb{Z}}$, if $\eta \in P(\mathbb{Z})$ we write for any $x \in \mathbb{Z}$, $\eta(x) = 1$ if $x \in \eta$ and $\eta(x) = 0$ if $x \notin \eta$.
We define $\max \eta = \sup\{x \in \mathbb{Z}: \eta(x) = 1\}$, $\min \eta = \inf\{x \in \mathbb{Z}: \eta(x) = 1\}$. Given a cylindrical function $f: P(\mathbb{Z}) \to \mathbb{R}$, the support of f (i.e, the smallest $B \subset \mathbb{Z}$ s.t. $f(A) = f(A \cap B)$, $\forall A \subset \mathbb{Z}$) will be represented by $\Lambda(f)$ or just by Λ if there is no possibility of confusion. We define the operators of translation τ_i, $i \in \mathbb{Z}$ by

$$(\tau_i f)(\eta) = f(\eta^{(i)}) \, , \quad \eta^{(i)}(x) = \eta(x-i)$$

Given f and two numbers $N, L \in \mathbb{N}$, we define

$$I_{f,N}(L) = \{i \in \mathbb{Z}: \Lambda(\tau_i f) \subset [-N+L, N-L] \cap \mathbb{Z}\}$$

$$\tilde{I}_{f,N} = \{i \in \mathbb{Z}: \Lambda(\tau_i f) \subset [-N,N] \cap \mathbb{Z}\}$$

Temporal means of f with respect to the process $(\xi_N(t), t \geq 0)$ will be represented by

$$A_R^N(s,f) = R^{-1} \int_s^{s+R} f(\xi_N(t))dt$$

where s is the instant of the beginning of the measurement and R the time interval of observation. Taking the spatial mean of translations of f we define

$$f = \frac{1}{|\tilde{I}_{f,N}|} \sum_{i \in \tilde{I}_{f,N}} \tau_i(f)$$

$$B_R^N(s,f) = A_R^N(s,f) = \frac{1}{|\tilde{I}_{f,N}|} \sum_{i \in \tilde{I}_{f,N}} A_R^N(s,\tau_i f)$$

Given a probability measure ν on $P(\mathbb{Z})$ we write $\nu(f) = \int f d\nu$ for the expectation of f. We use also E(.) for expectations.

By the definition of $(\xi_N(t), t \geq 0)$, T_N is a continuous random variable, so we can define β_N by $P(T_N > \beta_N) = e^{-1}$.

It is clear that as N increases, T_N also increases and diverges (in probability). In order to see the jump to the stable situation one has to consider a different time scale.

So we introduce the macroscopic time scale, in opposition to the microscopic original time scale, dividing the microscopic time by β_N. Condition (a) in the introduction holds for the macroscopic time scale. At the end of section 4 we show that it is also possible to use the expected value $E(T_N)$ instead of β_N to rescale time. Now, the meaning of condition (b) in the introduction is the existence of a time scale intermediate between de microscopic and macroscopic one and such that if a time interval of length R_N in the microscopic scale has length 1 in this intermediate scale then for any cylindrical f.

$$A^N_{R_N}(s,f) \cong \mu(f)$$

if

$$s + R_N < T_N .$$

More precisely we state

Theorem 1 - If $\lambda > \lambda_*$ then T_N/β_N converges in distribution to a unit mean random variable, as $N \longrightarrow \infty$.

Theorem 2 - If $\lambda > \lambda_*$, there is a sequence of positive real numbers $(R_N,\ N \geq 1)$ such that

i) $R_N/\beta_N \longrightarrow 0$ as $N \longrightarrow \infty$

ii) for all $\varepsilon > 0$ and $f: \mathcal{P}(\mathbb{Z}) \to \mathbb{R}$ cylindrical, there is $L = L(\varepsilon, f)$ $\mathbb{N}$ such that

$$P[\max_{\substack{\ell \in \mathbb{Z} \\ 0 \leq \ell < K_N}} \max_{i \in I_{f,N}(L)} |A^N_{R_N}(\ell R_N, f) - \mu(f)| > \varepsilon] \to 0$$

as $N \longrightarrow \infty$, where $K_N = \max\{\ell \geq 0 : \ell R_N < T_N\}$.

Theorem 3 - If $\lambda > \lambda_*$ and $(R_N, N \geq 1)$ is a sequence in the conditions of theorem 2 above then

$$P[\max_{\substack{\ell \in \mathbb{Z} \\ 0 \leq \ell < K_N}} |B^N_{R_N}(\ell R_N, f) - \mu(f)| > 0] \to 0$$

as $N \longrightarrow \infty$.

In [1] theorem 1 has been proved for $\lambda > \vec{\lambda}_* > \lambda_*$, where $\vec{\lambda}_*$ is the critical value of λ for the process constructed on the same percolation structure using only the Poisson processes $(\vec{\tau}^{\,i}_n)_{n\in\mathbb{N}}$ and $(\tau^+_n)_{n\in\mathbb{N}}$ (arrows in only one sense). A weaker version of theorem 2 has also been proved in [1] fixing a cylindrical function without considering its translations. The theorem was then of a strictly local (microscopic) sense. The theorem we prove deals with a global observation of all translations (except some with support very close to the edges) of a cylindrical function simultaneously. Theorem 3 is a corollary of theorem 2, it concerns macroscopic observables depending on the whole configuration of the system. An important particular case of it is that with $f = I_{\{\eta:\eta(0)=1\}}$, then $\bar{f}$ is the spatial density of particles. The proof of theorem 2 depends on estimating the decay of temporal correlations of $(\xi(t), t \geq 0)$.

4. Proof of Theorem 1

We will show that for all $s > 0$, $t > 0$

$$(4.1) \qquad \lim_{N\to\infty} \left| P[\frac{T_N}{\beta_N} > s + t] - P[\frac{T_N}{\beta_N} > s] \; P[\frac{T_N}{\beta_N} > t] \right| = 0 \; .$$

This implies the result once by induction it implies that for positive rational r

$$P(\frac{T_N}{\beta_N} > r) \longrightarrow e^{-r}$$

and by monotonicity it follows the same for all positive real values of r.

In order to prove (4.1) we define for $b > 0$

$$(4.2) \qquad F_b = \{A \subset \mathbb{Z}: \frac{|A \cap [-b,1]|}{b} > \frac{\rho}{2} \; , \; \frac{|A \cap [1,b]|}{b} > \frac{\rho}{2}$$

where $\rho = \rho_\lambda = \mu\{\eta: 0 \in \eta\}$, $\rho > 0$ if $\lambda > \lambda_*$

Then

$$(4.3) \qquad [\frac{T_N}{N} > s + t] = \sum_{\substack{A\subset\{-N,\dots,N\} \\ A\neq\emptyset}} P[\frac{T_N}{\beta_N} > s+t \; | \xi_N(\beta_N s)=A].P[\xi_N(\beta_N s)=A]=$$

$$= \sum_{\substack{A \subset \{-N,\dots,N\} \\ A \in F_b}} P[\frac{T_N^A}{\beta_N} > t] \; . \; P[\xi_N(\beta_N s) = A] \; +$$

$$+ \sum_{\substack{A \subset \{-N,\dots,N\} \\ A \notin F_b \\ A \neq \emptyset}} P[\frac{T_N^A}{\beta_N} > t] \; . \; P[\xi_N(\beta_N s) = A] \; =$$

By additivity it is clear that

$$P[\frac{T_N^A}{\beta_N} > x] \leq P[\frac{T_N}{\beta_N} > x]$$

for any $A \subset \{-N,\dots,N\}$ and $x \in \mathbb{R}$

So

$$P[\frac{T_N}{\beta_N} > s] \;\; P[\frac{T_N}{\beta_N} > t] \geq P[\frac{T_N}{\beta_N} > s + t]$$

and using (4.3)

$$\left| P[\frac{T_N}{\beta_N} > s] \; P[\frac{T_N}{\beta_N} > t] - P[\frac{T_N}{\beta_N} > s + t] \right| \leq$$

$$\leq P[\frac{T_N}{\beta_N} > s] \; P[\frac{T_N}{\beta_N} > t] - \sum_{\substack{A \subset \{-N,\dots,N\} \\ A \in F_b}} P[\frac{T_N^A}{\beta_N} > t] P[\xi_N(\beta_N s) = A] \leq$$

$$\leq P[\frac{T_N}{\beta_N} > s] \; P[\frac{T_N}{\beta_N} > t] - \min_{\substack{A \subset \{-N,\dots,N\} \\ A \in F_b}} (P[\frac{T_N^A}{\beta_N} > t]) P_n[\xi(\beta_N s) \in F_b] \; =$$

$$= P[\frac{T_N}{\beta_N} > s] \{P[\frac{T_N}{N} > t] - \min_{\substack{A \subset \{-N,\dots,N\} \\ A \in F_b}} P[\frac{T_N^A}{\beta_N} > t]\} \quad +$$

$$+ \min_{\substack{A \subset \{-N,\dots,N\} \\ A \in F_b}} P[\frac{T_N^A}{\beta_N} > t] \quad \{P[\frac{T_N}{\beta_N} > s] - P[\xi(\beta_N s) \in F_b]\} \quad \leq$$

$$\leq \quad \{P[\frac{T_N}{\beta_N} > t] - \min_{\substack{A \subset \{-N,\dots,N\} \\ A \in F_b}} P[\frac{T_N^A}{\beta_N} > t]\} \quad +$$

$$+ P[\frac{T_N}{\beta_N} > s \quad , \quad \xi(\beta_N s) \notin F_b]$$

The relation (4.1) will be proven once we show that for all $\varepsilon > 0$, there exist $b(\varepsilon)$ and $N(\varepsilon) > b(\varepsilon)$ such that:

$$(4.4) \qquad P[\frac{T_N}{\beta_N} > t] - P[\frac{T_N^A}{\beta_N} > t] < \varepsilon \quad \text{if } N \geq N(\varepsilon), \text{ and } A \in F_{b(\varepsilon)}$$

$$(4.5) \qquad P[\frac{T_N}{\beta_N} > s \ , \ \xi(\beta_N s) \notin F_{b(\varepsilon)}] < \varepsilon \quad \text{if } \ N \geq N(\varepsilon).$$

To prove (4.4) we consider ξ_N and ξ_N^A constructed with the same percolation structure. Then

$$(4.6) \quad P[\frac{T_N}{\beta_N} > t] - P[\frac{T_N^A}{\beta_N} > t] = P[\frac{T_N}{\beta_N} > t \ , \ \frac{T_N^A}{\beta_N} \leq t] \leq P[T_N \neq T_N^A].$$

For $\lambda > \lambda_*$, given $\varepsilon > 0$ there is $n(\varepsilon)$ such that if $n \geq n(\varepsilon)$ then

$$\mu_{[0,\infty)}\{B: B \cap [1,n] = \emptyset\} \leq \frac{\varepsilon}{2}$$

If N is large enough we can take $b = b'(\varepsilon)$ such that $n(\varepsilon) \leq b.\rho/2$, $b < N$. It follows for $A \in F_b$ that $|A \cap [-b,-1]| \geq b.\rho/2 \geq n(\varepsilon)$. So

$$\text{(4.7a)} \qquad P[T^{A\cap[-b,-1]}_{[-N,\infty)} = \infty] \geq P[T^{\{-N,\dots,-N+n(\varepsilon)\}}_{[-N,\infty)} = \infty] \geq 1 - \frac{\varepsilon}{2}$$

where the first inequality is proved in the same way as relation (16) in [2] and the second is a consequence of the self duality of the contact processes [3], [4].

Analogously

$$\text{(4.7b)} \qquad P[T^{A\cap[1,b]}_{[-\infty,N]} = \infty] \geq 1 - \frac{\varepsilon}{2}$$

We define the event

$$E = [T^{A\cap[-b,-1]}_{[-N,\infty)} = T^{A\cap[1,b]}_{(-\infty,N]} = \infty]$$

and the stopping times

$$U = \inf\{t > 0: N \in \xi^{A\ [-b,-1]}_{[-N,\infty)}(t)\}$$

$$V = \inf\{t > 0: -N \in \xi^{A\ [1,b]}_{(-\infty,N]}(t)\} \quad .$$

On E, $T_N > T_N^A > \max(U,V)$. By standart arguments for the contact processes, based on the fact that the interaction is between nearest neighbors we have that for $t > \max(U,V)$.

$$\xi_N(t) = \xi_N^A(t)$$

So, on E , $T_N = T_N^A$ and (4.6) and (4.7) imply (4.4).

To prove (4.5) we construct $(\xi_N(t),\ t \geq 0)$ and $(\xi(t), t \geq 0)$ using the same percolation structure. We take b and L such that $b < N-L < N$, then

$$(4.8) \qquad P[\xi_N(\beta_N s) \notin F_b \ ,\ T_N > \beta_N s] \leq$$

$$\leq P[\xi_N(\beta_N s) \notin F_b \ ,\ T_N > \beta_N s \ ,\ \min \xi_N(\beta_N s) < -N+L \ ,\ \max\xi_N(\beta_N s) > N-L] +$$

$$+ P[\min\xi_N(\beta_N s) \geq -N+L \ ,\ T_N > \beta_N s] + P[\max\xi_N(\beta_N s) \leq N-L,\ T_N > \beta_N s]$$

But on $[T_N > \beta_N s]$

$$(4.9) \quad \xi_N(\beta_N s) \cap [\min\xi_N(\beta_N s), \max\xi_N(\beta_N s)] =$$

$$= \xi(\beta_N s) \cap [\min\xi_N(\beta_N s) \ ,\ \max \xi_N(\beta_N s)]$$

Thus the first summand in (4.8) may be bounded above by $P[\xi(\beta_N s) \notin F_b]$. But by the ergodicity of μ , there is $b''(\varepsilon)$ such that if $b > b''(\varepsilon)$

$$(4.10) \qquad P(\xi(\beta_N s) \notin F_b) \leq \mu(F_b^c) \leq \frac{\varepsilon}{3}$$

where we used the stochastic monotonicity of the convergence $\xi(t) \longrightarrow \mu$, as $t \longrightarrow \infty$.

Finally, we will control the other two terms in (4.8) using again the semi infinite contact process. From the nearst neighbours nature of the interaction if follows that on $[T_N > t]$,

$$\min \xi_N(t) = \min \xi_{[-N,\infty)}(t)$$

So

$$(4.11) \qquad P[\min \xi_N(\beta_N s) \geq -N + L \ , \ T_N > \beta_N s] \leq$$

$$P[\min \xi_{[-N,\infty)}(\beta_N s) \geq -N + L] \leq$$

$$\mu_{[-N,\infty)}\{A \subset [-N,\infty) \cap Z: A \cap [-N,-N+L-1] = \emptyset\}$$

where the last inequality followed by the monotonic convergence of $\xi_{[-N,\infty)}(t)$ to $\mu_{[-N,\infty)}$ as $t \to \infty$. As $\lambda > \lambda_*$, we can take $L(\varepsilon)$ such that for $L > L(\varepsilon)$, the right-hand-side of (4.11) is smaller than $\varepsilon/3$.

The other summand in (4.8) is analogous and (4.4) and (4.5) are proved.

Remark - We can use $E(T_N)$ instead of β_N to rescale the time. This follows from

Proposition 1. - $E(T_N)/\beta_N \longrightarrow 1$ as $N \longrightarrow \infty$

Proof: $$\frac{E(T_N)}{\beta_N} = \frac{1}{\beta_N}\int_0^\infty P(T_N > t)dt = \int_0^\infty P(\frac{T_N}{N} > t)dt$$

but $P(\frac{T_N}{N} > t) \leq P(\frac{T_N}{\beta_N} > [t]) \leq e^{-[t]}$ where [t] is the integer part of t. By the dominated convergence theorem it follows

$$\lim_{N\to\infty} \frac{E(T_N)}{N} = \int_0^\infty \lim_{N\to\infty} P(\frac{T_N}{N} > t)dt = \int_0^\infty e^{-t}dt = 1$$

▲

5. Thermalization

First we prove that the time correlations of $(\xi(t), t \geq 0)$ decay exponentially fast.

Theorem 4 - For any cylindrical function $f: P(\mathbb{Z}) \to \mathbb{R}$ there are constants $C > 0$, $\gamma > 0$, such that

$$|cov(f(\xi(r)) , f(\xi(s)))| \leq C e^{-\gamma|s-r|}$$

Proof: - Without loss of generality we consider $s > r$. We use the notation

$$u = |s-r|$$

given $A \subset \mathbb{Z}$, $\bar{A} = \{\eta \in P(A): \eta \cap A \neq \emptyset\}$

$I_{\bar{A}}(.)$ = indicator of $\bar{A}$.

As any cylindrical function is a finite linear combination of these indicators it is enough to prove for any pair $A,B \subset \mathbb{Z}$, $|A| < \infty$, $|B| < \infty$, that

$$|cov (I_{\bar{A}}(\xi(r)) , I_{\bar{B}}(\xi(s)))| \leq C e^{-\gamma u}$$

we will construct some auxiliary processes. Consider the percolation structure where $(\xi(t), t \geq 0)$ is constructed. Take the inverse time scale $= s-t$ and invert the sense of the arrows. Now consider the processes $(X_\ell, 0 \leq \ell \leq s)$ and $(Y_\ell, u \leq \ell \leq s)$ defined by (we are changing the definition of a path).

$X_v = \{x \in \mathbb{Z}$: there is a path from B at time $\ell = 0$ up to x at time $\ell = v$ following the time segments in the sense of increasing ℓ and the inverted arrows and without passing through a +sign$\}$

$Y_v = \{x \in \mathbb{Z}$: there is a path from A at time $\ell = u$ up to x at time $\ell = v$ following the time segments in the sense of increasing ℓ and the inverted arrows and without passing through a +sign$\}$

The processes $(X_\ell, 0 \leq \ell \leq s)$ and $(Y_\ell, u \leq \ell \leq s)$ have respectively the same laws of $(\xi^B(t), 0 \leq t \leq s)$ and $(\xi^A(t), 0 \leq t \leq r)$, the first under the correspondence $\ell \longrightarrow t$ and the second under $\ell \longrightarrow t+u$.

We define the events

$$A' = [I_{\bar{A}}(\xi(r)) = 1] = [Y_s \neq \emptyset]$$

$$B' = [I_{\bar{B}}(\xi(s)) = 1] = [X_s \neq \emptyset]$$

then

$$|\mathrm{cov}\,(I_{\bar{A}}(\xi(r)),\ I_{\bar{B}}(\xi(s))| = |P(A' \cap B') - P(A').P(B')| =$$

$$\leq |P(B'|A') - P(B')| =$$

$$\sum_{\substack{\eta \subset Z \\ 0<|\eta|<\infty}} P(X_u=\eta).\{P(B'|A',\ X_u=\eta) - P(B'|X_u=\eta)\}|$$

Now, by theorem 2.17, in [3], it follows

$$P(B'|X_u=\eta) \leq P(B'|A',\ X_u=\eta) \leq 1$$

So

$$|\mathrm{cov}\,(I_{\bar{A}}(\xi(r)),\ I_{\bar{B}}(\xi(s)))| \leq$$

$$\sum_{\substack{\eta \subset Z \\ 0<|\eta|<\infty}} P(X_u=\eta)\{1 - P(B'|X_u=\eta)\} =$$

$$= \sum_{\substack{\eta \subset Z \\ 0<|\eta|<\infty}} P(X_u=\eta).P(B'^C|X_u=\eta) =$$

$$= \sum_{\substack{\eta \subset Z \\ 0<|\eta|<\infty}} P(B'^C,\ X_u=\eta) = P(B'^C,\ X_u \neq \emptyset) =$$

$$= P(X_u \neq \emptyset,\ X_s = \emptyset) = P(u < T^B < s) \leq$$

$$\leq P(u < T^B < \infty) \leq Ce^{-\gamma u}$$

where the last inequality was proved in [2] (theorem 5). ▲

Remark: - In the same way one can prove analogous statements for the process $(\xi_{[0,\infty)}(t),\ t \geq 0)$:

$$|\mathrm{cov}(f(\xi_{[0,\infty)}(r)) \ , \ f(\xi_{[0,\infty)}(s)))\ | \leq Ce^{-\gamma|s-r|}$$

Proof of theorem 2

Since T_N is almost surely finite, for any positive number R_N , K_N is a well defined and finite random variable with values on $\mathbb{N}$. Moreover, if the R_N verify condition (i) above it follows by theorem 1 that $P(T_N < R_N) \longrightarrow 0$ as $N \longrightarrow \infty$, ie, $P(K_N=0) \longrightarrow 0$. Let us now assume R_N is a sequence satisfying (i). For $\varepsilon > 0$ and f cylindrical given, let

$$B^N_{k,i} = [|A^N_{R_N}(kR_N \ , \ \tau_i f) - \mu(f)| > \varepsilon]$$

Then, for any integers $m \geq 1$, $L \geq 0$

$$(5.1) \quad P[K_N \geq 1 \ , \bigcap_{0\leq k<K_N} \ \bigcap_{i\in I_{f},N(L)} (B^N_{k,i})^C] =$$

$$= \sum_{j=1}^{\infty} (P[K_N = j] - P[\bigcup_{k=0}^{j-1} \ \bigcup_{i\in I_{f},N(L)} B^N_{k,i} \ , \ K_N = j]) \geq$$

$$\geq P[1 \leq K_N \leq m] - \sum_{j=1}^{m} P[\bigcup_{k=0}^{j-1} \ \bigcup_{i\in I_{f},N(L)} B^N_{k,i} \ , \ K_N = j]) \geq$$

$$\geq P[1 \leq K_N \leq m] - \sum_{j=1}^{m} j(2N+1) \max_{\substack{0<k<j \\ i\in I_{f,N}(L)}} P[B^N_{k,i} \ , \ K_N = j] \geq$$

$$\geq P[1 \leq K_N \leq m] - m^2(2N+1) \max_{j\geq 1} \ \max_{\substack{0\leq k<j \\ i\in I_{f,N}(L)}} P[B^N_{k,i} \ , \ K_N = j]$$

We construct $(\xi_N(t), \ t \geq 0)$, $(\xi(t) \ , \ t \geq 0)$, $(\xi_{[-N,\infty)}(t) \ , \ t \geq 0)$ and $(\xi_{(-\infty,N]}(t) \ , \ t \geq 0)$ using the same percolation structure. Then we have the following relation between events if $0 < L < N$.

$$(5.2) \qquad [\min \xi_N(t) < -N+L \ , \ \max \xi_N(t) > N-L] \subset$$

$$\subset \bigcap_{i \in I_{f,N}(L)} [\tau_i f(\xi_N(t)) = \tau_i f(\xi(t))]$$

$$(5.3) \qquad [T_N > t] \subset [h_L(\xi_N(t)) = h_L(\xi_{[-N,\infty)}(t))]$$

where $h_L(\eta) = I_{\{\xi:\ \xi\ [-N,-N+L]=\emptyset\}}(\eta)$

In the case $k < j$, $i \in I_{f,N}(L)$

$$(5.4) \qquad P[B^N_{k,i}\ ,\ K_N = j] = P[|A^N_{R_N}(kR_N,\ \tau_i f) - \mu(f)| > \varepsilon,\ K_N=j] \leq$$

$$\leq P[|A^N_{R_N}(kR_N\ ,\ \tau_i f) - \frac{1}{R_N}\int_{kR_N}^{(k+1)R_N} \tau_i f(\xi(t))dt]| > \frac{\varepsilon}{2} \quad \text{or}$$

$$|\frac{1}{R_N}\int_{kR_N}^{(k+1)R_N} \tau_i f(\xi(t))dt - \mu(f)| > \frac{\varepsilon}{2}\ ,\ K_N = j]$$

But, by (5.2) and (5.3), if $k < j$, $i \in I_{f,N}(L)$

$$(5.5) \quad [K_N = j] \subset [A^N_{R_N}(kR_N\ ,\ \tau_i f) - \frac{1}{R_N}\int_{kR_N}^{(k+1)R_N} \tau_i f(\xi(t))dt| \leq$$

$$\leq 2\|f\| \frac{1}{R_N}\int_{kR_N}^{(k+1)R_N} (h_L(\xi_N(t)) + h_L(S\xi_N(t)))dt =$$

$$= 2\|f\| \frac{1}{R_N}\int_{kR_N}^{(k+1)R_N} (h_L(\xi_{[-N,\infty)}(t) + h(S\xi_{(-\infty,N]}(t)))dt]$$

where $\|f\| = \sup_{\eta \in \mathbb{Z}} f(\eta)$ and S is the operator defined by

$(S\eta)(x) = \eta(-x)$.

We define now the events

$$\Gamma^R_{k,i} = [\,|\frac{1}{R}\int_{kR}^{(k+1)R} \tau_i f(\xi(t))dt - \mu(f)| > \frac{\varepsilon}{2}]$$

$$\tilde{\Gamma}^{R,L}_k = [2\|f\|\frac{1}{R}\int_{kR}^{(k+1)R} h_L(\xi_{[-N,\infty)}(t))dt > \frac{\varepsilon}{4}]$$

$$\Gamma^N_k = \Gamma^N_{k,0}$$

So (5.4) and (5.5) give us

(5.6) if $k < j$ and $i \in I_{f,N}(L)$ then

$$P[B^N_{k,i}\,,\ K_N = j] \leq P(\Gamma^{R_N}_k) + 2P(\tilde{\Gamma}^{R_N,L}_k)$$

Part (ii) of the theorem will be proved via (5.1) and (5.6) once part (i) is satisfied and we can find $L \in N$, and a sequence $(m_N\ ,\ N \geq 1)$ such that as $N \longrightarrow \infty$

a) $P\ (1 \leq K_N \leq m_N) \longrightarrow 1$

b) $m^2(2N+1)\ (\max_{k\geq 0} P(\Gamma^{R_N}_k) + \max_{k\geq 0} P(\tilde{\Gamma}^{R_N,L}_k)) \longrightarrow 0$

Condition (a) may be written as $P(T_N \leq m_N R_N) \longrightarrow 1$ or, using theorem 1, as $m_N R_N/\beta_N \to \infty$.

Using the notation

$$\alpha_L(R) = \max_{k \geq 0} P(\Gamma_k^R) + \max_{k \geq 0} P(\tilde{\Gamma}_k^{R,L})$$

and including part (i) of the theorem, our problem now is to find $L \in \mathbb{N}$ and two sequences $(R_N,\ N \geq 1)$, $(m_N, N \geq 1)$ such that as $N \longrightarrow \infty$

(5.7.a) $\quad m_N R_N / \beta_N \longrightarrow \infty$

(5.7.b) $\quad m_N^2\ N\ \alpha_L\ (R_N) \longrightarrow 0$

(5.7.c) $\quad R_N / \beta_N \longrightarrow 0$

Lemma 1 below shows that it is possible to choose $L = L(\varepsilon, f)$ such that $\alpha_L(R) \leq C/R$ (C depending on ε and f). Then it is enough to have (5.7.a), (5.7.c) and

(5.8) $\quad m_N^2 \dfrac{N}{R_N} \longrightarrow 0$

Lemma 2 shows that $N/\beta_N \longrightarrow 0$, then

(5.9.a) $\quad m_N = (\beta_N/N)^{1/5}$

(5.9.b) $\quad R_N = \beta_N^{9/10}\ N^{1/10}$

is a solutions of our problem. Δ

Lemma 1 - If $\lambda > \lambda_*$ and $L = L(\varepsilon, f)$ is such that

$\mu_{[0,\infty)}\{A: A \cap [0,L] = \emptyset\} \leq \dfrac{\varepsilon}{16\|f\|}$, $\exists R(\varepsilon, f)$ s.t.

$\alpha_L(R) \leq \frac{C}{R}$ if $R > R(\varepsilon, f)$ (where C depends on ε and f).

Remark - we will use the convention that from expression to expression the value of C can change.

Proof - First we prove that

$$\max_{k\geq 1} P(\Gamma_k^N) \leq \frac{C}{R_N}$$

consider the random variables

$$X_k^R = \frac{1}{R}\int_{kR}^{(k+1)R} f(\xi(t))dt - \mu(f)$$

$$Y_k^R = \frac{1}{R}\int_{kR}^{(k+1)R} f(\xi(t))dt - \frac{1}{R}\int_{kR}^{(k+1)R} Ef(\xi(t))dt$$

Then $E(Y_k^R) = 0$ and by theorem 4.

$$\text{Var}\,(Y_k^R) \leq \frac{2}{R^2}\int_{kR}^{(k+1)R} dr \int_r^{(k+1)R} ds\, |\,\text{cov}(f(\xi(r), f(\xi(s)))| \leq$$

$$\leq \frac{2}{R^2}\int_{kR}^{(k+1)R} dr \int_r^{\infty} ds\, Ce^{-\gamma(s-r)} = \frac{2}{R^2}\cdot CR = \frac{C}{R}$$

(C independs of k)

By Chebyshev inequality, $\forall\delta > 0$

$$P[|Y_k^N| > \delta] \leq \frac{C}{\delta^2 R}$$

On the other hand, $\xi(t) \longrightarrow \mu$ weakly as $t \longrightarrow \infty$, so $Ef(\xi(t)) \longrightarrow \mu(f)$. And

$\frac{1}{R} \int_{kR}^{(k+1)R} Ef(\xi(t))\, dt \longrightarrow \mu(f)$ uniformly in k, as $R \longrightarrow \infty$.

Given $\varepsilon > 0$ we can take $R(\varepsilon)$ such that

$$R > R(\varepsilon,f) \Longrightarrow \left| \frac{1}{R} \int_{kR}^{(k+1)R} Ef(\xi(t))dt - \mu(f) \right| \leq \frac{\varepsilon}{4}$$

for all $k \geq 1$.

For $R_N > R(\varepsilon,f)$ it follows

$$P(\Gamma_k^R) = P[|X_k^R| > \frac{\varepsilon}{2}] \leq P[Y_k^R > \frac{\varepsilon}{4}] \leq \frac{C}{R_N}$$

The other term in $\alpha_L(R)$ can be controled in an analogous way. We define

$$Z_k^R = \frac{1}{R} \int_{kR}^{(k+1)R} h_L(\xi_{[0,\infty)}(t))dt$$

$$W_k^R = Z_k^R - \frac{1}{R} \int_{kR}^{(k+1)R} Eh_L(\xi_{[0,\infty)}(t))dt.$$

It follows

$$E(W_k^R) = 0$$

$$\mathrm{Var}\,(W_k^R) \leq \frac{C}{R}$$

$$P[|W_k^R| > \delta] \leq \frac{C}{\delta^2 R} \qquad (\forall\delta > 0)$$

Also

$$E\, h_L(\xi_{[0,\infty)}(t)) \longrightarrow \mu_{[0,\infty)}(h_L) \quad \text{as } t \longrightarrow \infty, \text{ increasing}$$

monotonicaly to the limit.

So, for any k and R

$$\frac{1}{R}\int_{kR}^{(k+1)R} Eh_L(\xi_{[0,\infty)}(t))dt \le \mu_{[0,\infty)}(h_L) \le \frac{\varepsilon}{16\|f\|}$$

Finally

$$P(\tilde{\Gamma}_k^R) = P[2\|f\| \; | Z_k^R | > \frac{\varepsilon}{4}] \le P[|W_k^R| > \frac{\varepsilon}{16\|f\|}] \le \frac{C}{R} \qquad \Delta$$

<u>Lemma 2</u> - $N/\beta_N \longrightarrow 0$ as $N \longrightarrow \infty$, if $\lambda > \lambda_*$.

<u>Proof</u>: - We construct $(\xi_N(t),\ t \ge 0)$ and $(\xi(t),\ t \ge 0)$ on the same percolation structure. Consider the event

$$A_N = [\xi^{[0,\frac{N}{2}] \cap \mathbb{Z}}(t) \ne \emptyset,\ \forall t \ge 0]$$

By self duality

$$P(A_N) = \mu\{B \subset \mathbb{Z}:\ B \cap [0,\tfrac{N}{2}] = \emptyset\} \longrightarrow 0 \quad \text{as } N \longrightarrow \infty.$$

By standard arguments, on A_N there is an almost surely finite stopping time U_N defined by

$$U_N = \inf\{t > 0:\ N \in \xi^{(0,\frac{N}{2}] \cap \mathbb{Z}}(t)\}.$$

On A_N we have also the inequalities

$$Y_N \leq U_N \leq T_N$$

where Y_N is the instant of the $[\frac{N}{2}]^{th}$ occurence of a Poisson process of rate λ.

So

$$(5.10) \qquad P(T_N \leq \frac{\lambda}{2} \cdot \frac{N}{2}) \leq P(T_N \leq \frac{\lambda}{2} \frac{N}{2}, A_N) + P(A_N^C) \leq$$

$$\leq P(Y_N \leq \frac{\lambda}{2} \frac{N}{2}, A_N) + P(A_N^C) \leq$$

$$\leq P(Y_N \leq \frac{\lambda N}{4}) + P(A_N^C) \longrightarrow 0 \quad \text{as } N \longrightarrow \infty .$$

where we used the law of large number for Y_N.

By theorem 1 ,

$$P(T_N/\beta_N \leq x) \longrightarrow (1 - e^{-x}) I_{[0,\infty)}(x)$$

This combined with (5.10) implies the thesis. ▲

Proof of theorem 3

Theorem 2 implies

$$(5.11) \qquad P\Big[\max_{\substack{\ell \in \mathbb{Z} \\ 0 \le \ell < K_N}} \Big| \frac{1}{I_{f,N}(L)} \sum_{i \in I_{f,N}(L)} A^N_{R_N}(\ell R_N, f) - \mu(f) \Big| > \frac{\varepsilon}{2} \Big] \longrightarrow 0$$

as $N \longrightarrow \infty$, if $L = L(\frac{\varepsilon}{2}, f)$

Using the inequality

$$\left|\frac{a+b}{\alpha+\beta} - \frac{a}{\alpha}\right| = \left|\frac{\alpha b - \beta a}{\alpha(\alpha+\beta)}\right| \le \left|\frac{b}{\alpha+\beta}\right| + \left|\frac{a}{\alpha} \frac{\beta}{\alpha+\beta}\right|$$

with $\alpha = |I_{\Lambda,N}(L)|$, $\beta = |\tilde{I}_{\Lambda N}| - |I_{\Lambda,N}(L)|$,

$$a = \sum_{i \in I_{f,N}(L)} A^N_{R_N}(\ell R_N, \tau_i f)$$

$$b = \sum_{i \in (\tilde{I}_{f,N} - I_{f,N}(L))} A^N_{R_N}(\ell R_N, \tau_i f)$$

we obtain

$$(5.12) \qquad \Big| \frac{1}{|\tilde{I}_{f,N}|} \sum_{i \in \tilde{I}_{f,N}} A^N_{R_N}(\ell R_N, \tau_i f) - \frac{1}{|I_{f,N}(L)|} \sum_{i \in I_{f,N}(L)} A^N_{R_N}(\ell R_N, \tau_i f) \Big| \le$$

$$\le \frac{4L\|f\|}{2N+1-|\Lambda|} \longrightarrow 0 \qquad \text{as} \quad N \longrightarrow \infty .$$

The relations (5.11) and (5.12) imply the thesis. ▲

6. Subcritical Case:

We prove now the absence of metastability in the subcritical case ($\lambda < \lambda_*$).

<u>Theorem 6.</u> - If $\lambda < \lambda_*$, T_N/γ_N does not converge to a exponential random variable, for any sequence $(\gamma_N,\ N \geq 1)$.

<u>Proof</u>: - We show that $P(T_N/\ln N \leq x) \longrightarrow 0$ if $x < 1$ and that there exist $K > 1$ such that $P(T_N/\ln N \leq x) \longrightarrow 1$ if $x > K$.

The first part follows from the fact that $T_N \geq S_N = \max_{i=-N,\dots,N} \tau_1^{+i}$. So

$$P(T_N/\ln N \leq x) \leq P(S_N/\ln N \leq x) = (1-e^{-x\ln N})^{2N+1} =$$

$$= \left(1 - \frac{1}{2N+1} \cdot \frac{2N+1}{N^x}\right)^{2N+1} \longrightarrow 0 \quad \text{as } N \longrightarrow \infty, \text{ if } x < 1.$$

To prove the second part we use theorem 8 in [4]. It states that there is $C = C(\lambda)$ such that

$$P(\max \xi_{(-\infty,0]}(t) > -e^{ct}) \longrightarrow 0 \quad , \quad \text{as} \quad N \longrightarrow \infty$$

Now $\xi_N(t) \subset \xi_{(-\infty,N]}(t)$, so

$$P(T_N/\ln N > 2/C) \leq P(\max \xi_N(2\ln N/C) > -N) \leq$$

$$\leq P(\max \xi_{(-\infty,N]}(2\ln N/C) > -N) = P(\max \xi_{(-\infty,0]}(2\ln N/C) > -2N) \longrightarrow 0$$

as $N \longrightarrow \infty$. Δ

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