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## KRIGING SURROGATE MODEL APPLIED TO ANALYZE THE DYNAMIC RESPONSE OF COMPOSITE STRUCTURES

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**Abstract.** Computational advance in a structural analysis has been providing increasingly evaluations. In this context, structural optimization problems have also gained more notoriety, and they can be used to assist the quality of the manufacturing process in composite materials. Therefore, this work aims to evaluate a new methodology to analyze the non-conformities in the composite manufacturing process. This methodology relies on Latin Hypercube for the DoE (Design of Experiments) process, FEM (Finite elements method) for the numerical model of the structure, Kriging Metamodel for the development of the substitute numerical model, and EGO (Efficient Global Optimization) for the improvement of the numerical model. The maximum and minimum allowable frequency values are used to check if the frequencies of the plates are within the limits and if there are possible defects in the manufacturing process. These values are obtained through the points used for training the Kriging metamodel. From the results obtained through the failure envelope, it was observed that among the six manufactured composite plates, two of them had some problems in their manufacturing process. Finally, there is a discussion on the manufacturing defect detection by VBM and metamodels.

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### 1. INTRODUCTION

With the advancement of technological development, more and more complex systems appear which need specific materials [1]. The combination of low weight, high strength, high rigidity, and resistance to abrasion and impact is difficult to find in conventional materials [2]. For this reason, advances in composite materials provide new opportunities for high-performance structures. The composite materials are obtained by combining materials with different physicochemical and mechanical characteristics. By combining two or more different materials, it can be obtained a composite material whose properties are different from the properties of each of the components [3]. Thus, it is necessary to carry out strict control, to monitor the loss of structural properties and prevent the structure from collapsing. These losses can occur due to the quality of the manufacturing process, and the degradation suffered during their lifetime by external agents. The development of efficient methodologies and systems for detecting loss of properties fundamental to the interest function is of extreme importance to avoid failure [4].

One of the alternatives is the analysis of the shifts in the vibration characteristics of structures. The vibration-based damage method takes into account the changes in the structure caused by the

damage. These changes could be detected by Structural Health Monitoring (SHM) system, through a set of sensors to acquire and to determine if some significant variation occurred. The method identifies the damage by comparing the mode shapes, natural frequency, and damping data between the healthy and damaged structures. SHM can be defined as an implementation process of damage detection and characterization strategy for engineering structures. Specifically, SHM is a non-destructive structural sensing and evaluation method, which uses a variety of sensors to monitor the structural response, to analyze the structural characteristics to estimate the severity of damage/deterioration, and to evaluate the consequences on the structure in terms of response, capacity, and service-life [5].

On the other hand, computational advancement has been providing increasingly sophisticated analysis of structural systems, mainly by using commercial finite element software for complex structure analysis. Structural optimization problems also gained more notoriety, especially when combined with the study of structural reliability [6]. However, these problems involve a high computational cost. Some researches present the interaction between the commercial software with the optimization technique based on metamodeling. The numerical approach can be used as a substitute model. Besides that, in addition to the Kriging metamodel, these works rely on Efficient Global Optimization (EGO) to solve optimization problems in which, due to the high computational cost of the objective function, it is more efficient to limit the number of evaluations of the objective function [7-10].

This work proposes a methodology to evaluate the state of the composite structure due to the quality of the manufacturing process using an optimized numerical approach. The main objective is achieving a numerical model that represents the natural frequencies of a composite plate reducing the computational effort during this process. The structural dynamic response of composite plates is investigated. The influence of the manufacturing process and the variation of properties presented by the literature review is considered. A design of experiments method is applied to conduct the logic of the test sequence. This sequence of tests is performed by finite elements using the Abaqus™ software as a computational model of the natural frequencies for each case. Therefore, to obtain the numerical model that represents the natural frequencies some techniques, like metamodels, can be used to approximate the expensive computational model. The Kriging and EGO methods are then used to compose the analytical model.

## 2. METHODOLOGY

The workflow of the research is specified in Figure 1(a) and described by:

1. The first phase is called design variables variability analysis. The material properties and geometric dimensions of the composite material samples are collected as input data. The data variability is considered to determine the intact range of each design variable.
2. Latin hypercube method is used to conduct the logic of the test sequence. The hypercube determines the value of each design variable within the range considered intact determined by the previous step [11-12]. With the design of experiments (DoE) built, the next stage is running the tests [13-14].
3. The Finite Element Method is used using the Abaqus™ software to determine the natural frequencies for each test line of the input variables whose value was determined by the Latin Hypercube in the previous step. Thus, for each input data set, there is an output data set obtained by the software.
4. In this step, with the input and output data, the Kriging metamodel is built [15-16].
5. This numerical model aims to provide a model that replaces the FEM in step 3. In other words, this model provides a simplified way to obtain natural frequencies without having to use the Abaqus™ software. In addition to the sample points provided, the model can predict intermediate points. However, this model is subject to errors that can be caused by the lack of sampling points in specific regions.
6. Finally, the Efficient Global Optimization (EGO) algorithm through Expected Improvement (EI) is used to predict regions that need the inclusion of Infill Points (IP) to reconstruct the Kriging metamodel equation [17-19].

The optimization process via metamodel can be divided into the following steps (Figure 1(b)):

1. Define an initial sample space of support points and adjust a metamodel for these points;
2. Calculate the next Infill point using through Expected Improvement (EI);
3. Add this new sample space support point, readjust a model for the new data, and step 2 until reaches a stop critical.

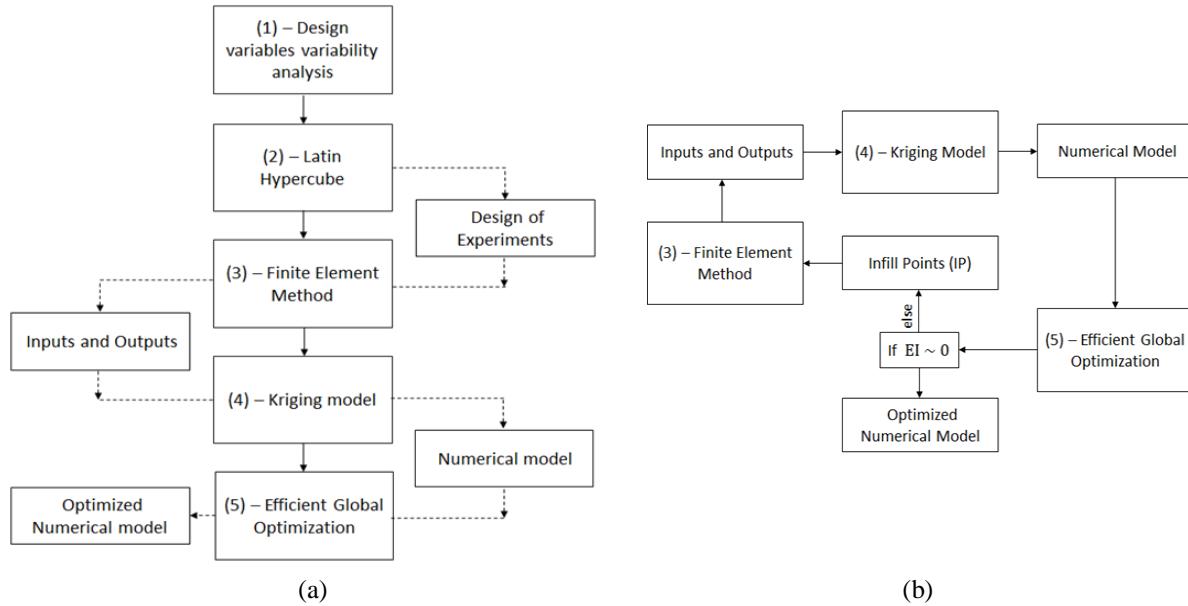


Figure 1 – Methodology: (a) general flow; (b) optimization of metamodel.

## 2.1. Composite Specimens

The methodology is conducted using data from experiments and numerical simulations. The experiments consist of fourteen carbon fiber with epoxy resin (Carbon Fiber Reinforced polymer - CFRP) composite plates made of twelve layers of stacking sequence [0/15/-15/0/15/-15]s at a total thickness (T) of 3.3 mm [20-21]. Geometry data are obtained using a 3D optical scanner. These plates were manufactured by filament winding, with a nominal length of 305mm and a nominal width of 245mm. To conduct the DoE process, the upper and lower limits values of the design variables should be defined. The quality of the DoE representation is improved using reference data to define these boundaries (Tab.1). Regarding the material properties, it was necessary to define the maximum and minimum limits to carry out the simulations. These were obtained through works published by [22-23] as presented in Tab. 2.

Table 1 – Geometric properties range [20-21].

Geometry properties	W (mm)	L (mm)	t (mm)
Mean	305.67	245.21	3.341
St. Deviation	0.93	1.28	0.066
Upper	308.48	249.06	3.538
Lower	302.85	241.36	3.144
Range	5.63	7.7	0.394

The experimental analyses were carried out via vibration tests on composite plates hanged by elastomeric wires to simulate “free-free” boundary conditions. The results of the tests performed can be seen in Tab. 3.

Table 2 – Material properties range [22-23].

Geometry properties	E11 [GPa] [22]	E22 [GPa] [22]	G23 [GPa] [23]	v12 [22]	G12 [GPa] [23]
Mean	130	11.0	3.05	0.31	5.25
St. Deviation	1.77	0.462	0.117	0.003	0.202
Upper	132	11.5	3.2	0.313	5.45
Lower	128	10.6	2.93	0.307	5.05
Range	4	0.9	0.27	0.006	0.4

Table 3 – Experimental Frequency Data [20-21].

Plate Number	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
1	95.379	132.911	250.645	316.229	349.436	405.460
2	104.152	139.884	261.091	320.690	367.079	411.455
3	106.412	144.506	265.334	332.630	377.859	424.862
4	107.616	146.244	268.219	332.774	381.241	426.971
5	102.961	139.202	258.850	320.595	365.101	412.746
6	99.628	136.840	258.484	324.150	361.405	414.755

## 2.2. Numerical model

Finite element models of the laminated composite plates are developed using the commercial Finite Elements software Abaqus<sup>TM</sup>, which incorporates subroutines written in Python. Quadrilateral 8-node shell elements (defined as S8R5), which is an 8-node isoparametric doubly curved thin shell, reduced integration, using five degrees of freedom per node, is used. The computational model is used in two moments. At first, it is used to generate the initial sample data as a basis for the kriging. In a second step, the computational model will be used each time the EGO model requests a new infill point.

The DoE technique used to determine the input variables is the Latin hypercube. The technique is applied through a python routine. This matrix has columns represented by the number of design variables and lines represented by the number of test runs. The result generated is represented by a value between 0 and 1, with zero being the lowest value assumed and 1 being the largest value assumed by a given variable. From Table 4 generated by the Latin Hypercube code, based on the average upper and lower limit value of each variable, it is possible to determine the value of each design variable in each of the 10 test rounds. Tables 1 and 2 presented the design variables' values. These rounds can be used as input to the Abaqus<sup>TM</sup> software, based on finite elements, to determine the first six natural frequencies.

Table 4 – Latin hypercube.

W	L	$\theta(15^\circ)$	$\theta(-15^\circ)$	$\theta(0^\circ)$	t	E <sub>11</sub>	E <sub>22</sub>	G <sub>23</sub>	v <sub>12</sub>	G <sub>12</sub>
0	0.7	0.5	0.7	0.2	0.1	0.3	0.4	0.6	0.2	0.7
0.6	0.9	0.2	0.5	0.8	0.9	0.1	0.9	0.9	0.5	0.9
0.5	0.3	0	0	0	0.6	0.2	0.8	0.1	0	0.1
0.8	0.6	0.7	0.3	0.4	0.2	0.5	0.1	0.2	0.8	0.2
0.9	0.8	0.9	0.4	0.7	0.3	0.8	0.6	0.3	0.9	0.3
0.2	0.1	0.8	0.8	0.6	0.7	0.4	0.5	0.5	0.7	0.8
0.4	0.5	0.3	0.6	0.1	0	0.9	0.3	0.4	0.6	0.4
0.1	0	0.6	0.9	0.3	0.5	0.7	0.7	0	0.4	0.5
0.7	0.4	0.1	0.1	0.9	0.8	0.6	0.2	0.7	0.3	0.6
0.3	0.2	0.4	0.2	0.5	0.4	0	0	0.8	0.1	0

### 3. RESULTS

The results obtained through the analysis between the experimental and computational data, *i.e.*, the Abaqus<sup>TM</sup> software and the program developed in Python language, are presented. Once the computational model is trained with the infill points (IP) suggested by the EI, *i.e.*, evaluated its ability to be used as a tool for assessing the quality of manufacture of composite material plates.

Among the infill points (IP) used for training the metamodel, those that generated the minimum and maximum numerical natural frequencies as a training response are used as the basis for analysis. Each of these two IPs is composed of an input set and an output set. Eleven inputs (the design variables) and six outputs (the first 6 natural frequencies of the composite structure) are evaluated. With the result obtained from the training of the Kriging model and the EGO, it is possible to create a table containing the limits for each frequency (Tab. 5).

Table 5 – Experimental Frequency Data.

Frequency	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
Minimum	102.7747	139.1737	250.4448	303.1655	356.5067	396.8332
Maximum	114.7020	159.3458	281.5885	337.5040	399.5473	451.6348

Figure 2 shows manufacturing limits according to the design parameter *i.e.*, the maximum and minimum numerical limits for the first six experimental natural frequencies. It is possible to observe the plates with a natural frequency out of determined bounds. This indicates a manufacturing problem in the plates that are outside the numerical limits. Analyzing Fig. 2, it is possible to note that plate number 1 is out of boundaries in modes 1, 2, and 5, while plate number 6 is out of boundaries in modes 1 and 2. All the other experimental plates are inside the design limits.

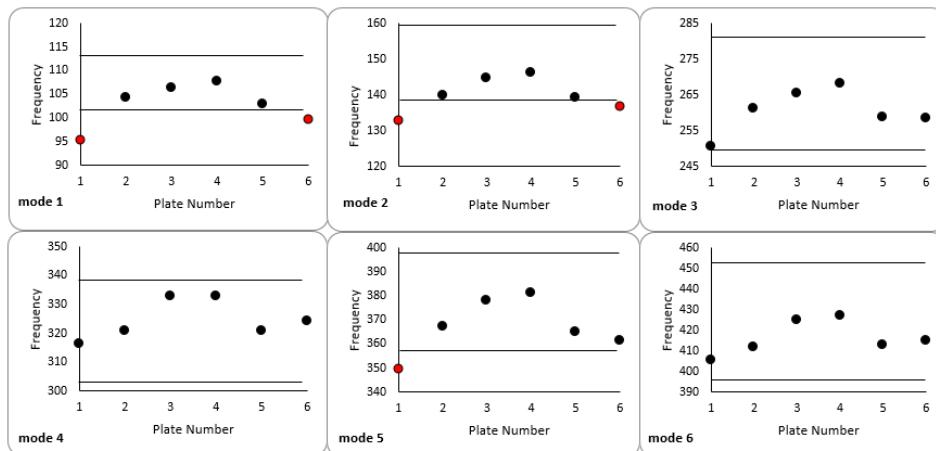


Figure 2 – Maximum and minimum numerical limits of natural frequencies and values of natural frequencies of the experimental composite plates.

A second analysis of the composite material plates is implemented. This analysis is done through the numerical envelope obtained by Abaqus<sup>TM</sup> through information obtained by the kriging model. The FRFs envelopes are obtained considering the maximum and minimum values of numerical frequencies from Kriging metamodel. The FRF final envelope roughly circumvents the experimental ones (Fig. 3). It is observed that the results obtained in the analysis of the envelopes confirmed the results of the previous analysis. This is because the experimental plates FRFs 1 and 6 remain outside of the envelope. This occurs in the first, second, and fifth frequency spikes of FRFs, as in the previous analysis.

Specifically, plate number 1 is out of boundaries in resonances frequencies 1, 2, and 5, while plate number 6 is out of boundaries just in resonance frequencies 1 and 2.

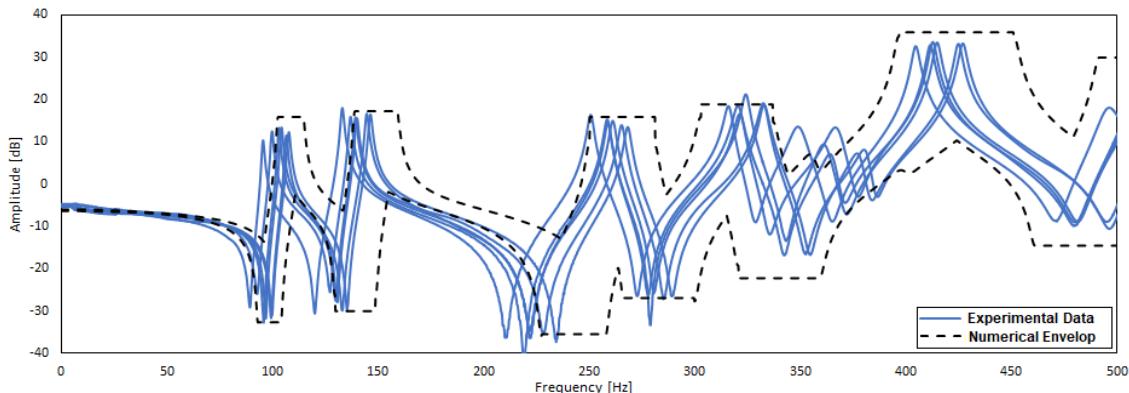


Figure 3 – Numerical and experimental FRFs for the composite plates.

Therefore, in both analyzes, there is an indication that plates 1 and 6 suffered some quality variation in their manufacturing process. The results point to the same trend, which reinforces the method's ability to be used as a quality tool for the manufacturing process of complex structures. From the moment the maximum and minimum limits of natural frequencies are known, it is enough to measure the structure to determine if it is suitable for use, preventing defective parts from being approved in the manufacturing process.

## CONCLUSION

The main objective of this work is to identify problems in the manufacturing process of composite material plates. It is known that changes in the structural properties cause changes in natural frequency. Therefore, as verified in the presented results, it is possible to delimit an interval where the natural frequencies are considered without damage. This interval is constructed using the design variables within the methodology employed. This interval, called an envelope, is presented through natural frequency and FRF. Both are used as a tool for analyzing sample data from composite material plates in the present research. Thus, the applied methodology proved to be efficient in the manufacturing process analysis of composite material plates.

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