977014

Boletim Técnico da Escola Politécnica da USP

Departamento de Engenharia Eletrônica

ISSN 1413-2206

BT/PEE/9717

Mixed $H_2/H\infty$ Control of Discrete-Time Markovian Jump Linear Systems

Oswaldo L. V. Costa

Ricardo P. Marques

São Paulo - 1997

O presente trabalho é um resumo da tese de doutorado apresentada por Oswaldo L. V. Costa, sob orientação do Prof. Dr. Ricardo P. Marques: "Algoritmos de Controle para Sistemas Sujeitos a Saltos Markovianos", defendida em 05/05/97, na Escola Politécnica.

A íntegra da tese encontra-se à disposição com o autor e na Biblioteca de Engenharia de Eletricidade da Escola Politécnica/USP.

Costa, Oswaldo Luiz do Valle
Mixed $H_2/H\infty$ control of discrete-time Markovian jump linear systems / O.L.V. Costa, R.P. Marques. -- São Paulo : EPUSP, 1997.
p. -- (Boletim Técnico da Escola Politécnica da USP, Departamento de Engenharia Eletrônica, BT/PEE/9717)

1. Controle automático I. Marques, Ricardo Paulino II. Universidade de São Paulo. Escola Politécnica. Departamento de Engenharia Eletrônica III. Título IV. Série

ISSN 1413-2206 CDD 629.8

MIXED H_2/H_∞ CONTROL OF DISCRETE-TIME MARKOVIAN JUMP LINEAR SYSTEMS*

Oswaldo L. V. Costa** Ricardo P. Marques**

*** Departamento de Engenharia Eletrônica*
Escola Politécnica da Universidade de São Paulo
05508-900 São Paulo SP Brasil
E-mail: oswaldo@lac.usp.br rpm@lac.usp.br

ABSTRACT

In this paper we consider the mixed H_2/H_∞-control problem for the class of discrete-time linear systems with parameters subject to Markovian jumps (MJLS). It is assumed that both the state variable and the jump variable are available to the controller. The transition probability matrix may not be exactly known, but belongs to an appropriated convex set. For this controlled discrete-time Markovian jump linear system, the problem of interest can be stated in the following way. Find a robust (with respect to the uncertainty on the transition Markov probability matrix) mean square stabilizing state and jump feedback controller that minimizes an upper bound for the H_2-norm, under the restriction that the H_∞-norm is less than a pre-specified value δ. The problem of the determination of the smallest H_∞-norm is also addressed. We present an approximated version of these problems via LMI optimization.

1. INTRODUCTION

A great deal of attention has been given nowadays to a class of stochastic linear systems subject to abrupt variations, namely, Markovian jump linear systems (MJLS). This family of system is modeled by a set of linear systems, with the transitions between the models determined by a Markov chain taking values in a finite set. Due to a large number of applications in control engineering, several results on this field can be found in the current literature, regarding applications, stability conditions and optimal control problems (see, for instance, [1]-[11],[13]-[18],[21]).

The mixed H_2/H_∞ and H_∞ control problems for time-invariant discrete-time linear systems has been studied in the current literature usually using a state space approach, leading to non-standard algebraic Riccati equations and Lyapunov-like equations (see, for instance, [12], [19], [20]). The H_2 and H_∞ control problems for MJLS have recently been analyzed in [5], [6], and [11]. For the H_2 control problem a convex programming approach was applied in [5], and numerical algorithms developed. In this paper we study the mixed H_2/H_∞-control and H_∞ control problems of a discrete-time MJLS. We will assume that the transition probability matrix for the Markov chain is not exactly known, but belongs to an appropriated convex set. In this case a robust mean square (state and jump feedback) stabilizing controller is defined as a state-feedback controller, which also depends on the jump Markov variable, that stabilizes in the mean square sense the MJLS for every appropriated Markov transition probability matrix. This kind of concept was first introduced by Rami and El Ghaoui in [21] for continuous-time MJLS. Under these conditions, the mixed H_2/H_∞ control problem of a MJLS can be formulated as follows: we are interested in finding a robust mean square stabilizing controller that minimizes an upper bound for the H_2-norm, under the restriction that the H_∞-norm is less than a pre-specified value δ. The problem of minimizing the H_∞-norm is also addressed. We trace a parallel with the discrete-time linear system theory of H_2/H_∞ and H_∞ control to derive our results. When restricted to the case with no jumps, the equations presented here can be seen as dual to the ones derived in [12]. As in [12], we present an approximated version of the mixed H_2/H_∞ and H_∞ control problems of MJLS based on linear matrix inequalities (LMI) optimization.

The paper is organized in the following way. Section 2 presents the notation that will be used throughout the work. Section 3 deals with previous results derived for stability, H_2 and H_∞-control of MJLS, as well as some other auxiliary results. Section 4 presents a sufficient condition for the existence of a mean square stabilizing controller that makes the H_∞-norm of the MJLS less than a pre-specified value δ. The condition is written in terms of the existence of a solution $P = (P_1,...,P_N)$ and $K = (K_1, \dots, K_N)$ for a set of coupled Lyapunov-like equations. This solution P leads to an upper bound for the H_2-norm of the MJLS, so that an approximation for the mixed H_2/H_∞-control problem for the MJLS can be

* This work was supported in part by CNPq (Brazilian National Research Council) and FAPESP (Research Council of the State of São Paulo).

determined by minimizing this functional over the set of solutions P and K. The H_∞-control problem can also be addressed through this Lyapunov-like equation. In section 5 we consider the case in which the transition probability matrix belongs to an appropriated convex set and, using the results of section 4, derive a LMI optimization problem that leads to an approximation for the mixed H_2/H_∞ and H_∞-control problems. Some numerical examples are presented in section 6 and the paper is concluded in section 7 with some final comments.

2. NOTATION

We shall write $\mathbb{C}^n$ and $\mathbb{R}^n$ to denote the n-dimensional complex and real spaces respectively, and $\mathbb{M}(\mathbb{C}^n,\mathbb{C}^m)$ the normed linear space of all m by n complex matrices. For simplicity we set $\mathbb{M}(\mathbb{C}^n,\mathbb{C}^n) = \mathbb{M}(\mathbb{C}^n)$. We write * to indicate the adjoint operator and, for real matrices, $'$ will indicate transpose. $L \geq 0$ and $L>0$ will be used if a self-adjoint matrix is positive semi-definite or positive definite respectively and we write $\mathbb{M}(\mathbb{C}^n)^+ = \{L \in \mathbb{M}(\mathbb{C}^n);\ L = L^* \geq 0\}$. We denote by $\|\cdot\|$ either the induced norm in $\mathbb{M}(\mathbb{C}^n)$ or the standard norm in $\mathbb{C}^n$.
Let $\mathcal{H}^{m,n}$ be the linear space made up of all N-sequence of matrices $V = (V_1,...,V_N)$, $V_i \in \mathbb{M}(\mathbb{C}^m,\mathbb{C}^n)$. For $V \in \mathcal{H}^{m,n}$ we define the following norm $\|\cdot\|_2$:

$$\| V \|_2 = \left(\sum_{i=1}^{N} \mathrm{tr}\left(V_i^* V_i\right)\right)^{1/2}$$

where tr(.) denotes the trace operator.

It is easy to verify that $\mathcal{H}^{m,n}$ equipped with the norm $\|\cdot\|_2$ is a complex Hilbert space with inner product given by:

$$< V;H > \; = \sum_{i=1}^{N} \mathrm{tr}\left((V_i^* H_i)\right).$$

We set $\mathcal{H}^{n,n} = \mathcal{H}^n$ and $\mathcal{H}^{n+} = \{V = (V_1,...,V_N) \in \mathcal{H}^n;\ V_i \in \mathbb{M}(\mathbb{C}^n)^+,\ i = 1,...,N\}$. For $H = (H_1,...,H_N)$ and $V = (V_1,...,V_N)$ in $\mathcal{H}^{n+}$ the notation $H \leq L$ ($H < L$) indicates that $H_i \leq L_i$ ($H_i < L_i$) for each $i = 1,...,N$.

For an increasing filtration $\{\mathcal{F}_k\}$ defined on a probability space $(\Omega,\mathcal{F},\mathcal{P})$, we set $\ell_2^r(\mathcal{F}_k)$ as the Hilbert space formed by the sequence of second order random variables $z = (z(0),z(1),...)$ with $z(k) \in \mathbb{R}^r$ and $\mathcal{F}_k$-adapted for each k=0,1,.., and such that

$$\| z \|_2^2 := \sum_{k=0}^{\infty} \| z(k) \|_2^2 < \infty$$

where

$$\| z(k) \|_2^2 := E(\| z(k) \|^2) .$$

For any complex Banach space $\mathbb{Z}$ we denote by $\mathbb{B}(\mathbb{Z})$ the Banach space of all bounded linear operators of $\mathbb{Z}$ into $\mathbb{Z}$ with the uniform induced norm represented by $\|\cdot\|$ and for $L \in \mathbb{B}(\mathbb{Z})$ we denote by $r_\sigma(L)$ the spectral radius of L.

Finally, the following well known result will be useful.

Remark 1: If $R > 0$ then $W = \begin{bmatrix} Q & S \\ S' & R \end{bmatrix} \geq 0$ if and only if $Q \geq SR^{-1}S'$

3. AUXILIARY RESULTS

3.1. STABILITY RESULTS

Consider the following stochastic system on an appropriated probability space $(\Omega,\{\mathcal{F}_k\},\mathcal{F},P)$,

$$x(k+1) = \tilde{A}_{\theta(k)}x(k) \qquad (1.a)$$
$$x(0) = x_0\ ,\ \theta(0) = \theta_0 \qquad (1.b)$$

where $\{\theta(k);\ k=0,1,...\}$ is a discrete-time Markov chain with finite state space $\{1,...,N\}$ and defined on the probability space $(\Omega,\{\mathcal{F}_k\},\mathcal{F},P)$, with transition probability matrix $\mathbb{P} = [\ p_{ij}\ ]$. We consider $\tilde{A} = (\tilde{A}_1,...,\tilde{A}_N) \in \mathcal{H}^n$ real, and x_0 a second order random variable in $\mathbb{R}^n$.

We set $Q(k) = (Q_1(k),...,Q_N(k))$, where

$$Q_j(k) := E(\ x(k)\, x(k)'\, 1_{\{\theta(k)=j\}}\ ) \in \mathbb{M}(\mathbb{R}^n)^+ \qquad (2)$$

and $1_{\{.\}}$ stands for the Dirac measure.

For $S = (S_1,...,S_N) \in \mathcal{H}^n$ we define the operator $\mathcal{T} \in \mathbb{B}(\mathcal{H}^n)$ as: $\mathcal{T}(S) = (\mathcal{T}_1(S),...,\mathcal{T}_N(S))$ where

$$\mathcal{T}_j(S) = \sum_{i=1}^{N} p_{ij}\tilde{A}_i\, S_i\tilde{A}_i'. \qquad (3)$$

It is easy to verify that $\mathcal{L} := \mathcal{T}^*$ is given by:

$$\mathcal{L}_i(S) = \tilde{A}_i'\left(\sum_{j=1}^{N} p_{ij}S_j \right) \tilde{A}_i.$$

In particular, $r_\sigma(\mathcal{L}) = r_\sigma(\mathcal{T})$. The following result, shown in Proposition 3 of [7], provides a connection between (2) and (3):

Proposition 1: For every k = 0,1,2,...

$$Q(k+1) = \mathcal{T}(Q(k)).$$

We make the following definitions:

Definition 1: Model (1) is mean square stable (MSS) if $\| Q(k) \|_2^2 \to 0$ as $k \to \infty$ for any initial condition x_0 and initial distribution for θ_0.

The next result is in Theorems 1 and 2 of [7]:

Proposition 2: The following assertions are equivalent:

a) Model (1) is MSS
b) $r_\sigma(\mathcal{T}) < 1$.
c) $r_\sigma(\mathcal{L}) < 1$.
d) There exists $\alpha \in (0,1)$ and $a \in \mathbb{R}$, $a > 0$, such that for each k = 0,1,2,...

$$E(\| x(k) \|^2) \leq a\,\alpha^k.$$

e) (coupled Lyapunov equations) given any $S = (S_1,...,S_N) > 0$ in $\mathcal{H}^{n+}$ there exists $P = (P_1,...,P_N) > 0$ in $\mathcal{H}^{n+}$ satisfying $P - \mathcal{T}(P) = S$ with $P = \sum_{k=0}^{\infty}\mathcal{T}^k(S)$.

f) (adjoint coupled Lyapunov equations) given any $S = (S_1,...,S_N) > 0$ in $\mathcal{H}^{n+}$ there exists $P = (P_1,...,P_N) > 0$ in $\mathcal{H}^{n+}$ satisfying $P - \mathcal{L}(P) = S$ with $P = \sum_{k=0}^{\infty}\mathcal{L}^k(S)$.

Moreover if $r_\sigma(\mathcal{T}) < 1$ then for any $S \in \mathcal{H}^n$ there exists a unique $P \in \mathcal{H}^n$ such that $P - \mathcal{T}(P) = S$. If $S \geq T \geq 0$

(> 0 respectively) and P - $\mathcal{T}$(P) = S, L - $\mathcal{T}$(L) = T then P ≥ L ≥ 0 (> 0). These results also hold replacing $\mathcal{T}$ by $\mathcal{L}$.

We present now the definition of mean square stabilizability and detectability. Consider $A = (A_1,...,A_N) \in \mathcal{H}^n$, $B = (B_1,...,B_N) \in \mathcal{H}^{m,n}$ and $C = (C_1,...,C_N) \in \mathcal{H}^{n,p}$ real.

Definition 2: We say that (A,B) is mean square stabilizable if there exists $K = (K_1,...,K_N) \in \mathcal{H}^{n,m}$ such that model (1) is MSS with $\tilde{A}_i = A_i - B_iK_i$. In this case we say that K stabilizes (A,B) in the mean square sense and set $\mathbb{K} = \{K \in \mathcal{H}^{n,m};$ K stabilizes (A,B) in the mean square sense$\}$. Similarly, we say that (C,A) is mean square detectable if there exists $H = (H_1,...,H_N) \in \mathcal{H}^{p,n}$ such that model (1) is MSS with $\tilde{A}_i = A_i - H_iC_i$, and we say that H stabilizes (C,A).

The next proposition follows from Proposition 6 in [9]. Consider $D = (D_1,...,D_N) \in \mathcal{H}^{m,p}$ such that $D_i'D_i > 0$ and set $\mathcal{E}_i(L) = \sum_{j=1}^{N} p_{ij}L_j$, i = 1,...,N, for $L = (L_1,...,L_N)$.

Proposition 3: Suppose (C,A) is mean square detectable and $P = (P_1,...,P_N) \geq 0$, $K = (K_1,..., K_N) \in \mathcal{H}^{n,m}$ satisfy

$$\begin{aligned} - P_i + (A_i-B_iK_i)'\mathcal{E}_i(P)(A_i-B_iK_i) + \\ + (C_i-D_iK_i)'(C_i-D_iK_i) \leq 0. \end{aligned} \tag{4}$$

Then $K = (K_1,...K_N) \in \mathbb{K}$.

3.2. THE H_2-NORM

Consider again on $(\Omega,\{\mathcal{F}_k\},\mathcal{F},P)$, the following system

$$\mathcal{G} = \begin{cases} x(k+1) = \tilde{A}_{\theta(k)}x(k) + Jw(k) & (5.a) \\ x(0) = 0\ ,\ \theta(0) = \theta_0 & (5.b) \\ z(k) = \tilde{C}_{\theta(k)}x(k) & (5.c) \end{cases}$$

where $\tilde{A} = (\tilde{A}_1,...,\tilde{A}_N) \in \mathcal{H}^n$, $\tilde{C} = (\tilde{C}_1,...,\tilde{C}_N) \in \mathcal{H}^{n,p}$ and $J \in \mathbb{M}(\mathbb{C}^r,\mathbb{C}^n)$, with $\tilde{A}$, $\tilde{C}$, J real and $JJ' > 0$.

Suppose that $r_\sigma(\mathcal{T}) < 1$ (that is, model (1) is MSS) and w = (w(0),...) is an impulse input.
From Proposition 2.d) we have that $z = (z(0),z(1),...) \in \ell_2^p(\mathcal{F}_k)$. The next definition is a generalization of the H_2-norm from discrete-time deterministic systems to the stochastic Markovian jump case:

Definition 3: We define the H_2-norm of system $\mathcal{G}$ as

$$\| \mathcal{G} \|_2^2 = \sum_{s=1}^{n} \sum_{j=1}^{N} \| z_{s,j} \|_2^2$$

where $z_{s,j}$ represents the output sequence (z(0),z(1),...) given by (5.c) when

a) the input sequence is given by w = (w(0),w(1),...), $w(0) = e_s$, w(k) = 0, k > 0, $e_s \in \mathbb{R}^r$ the unitary vector formed by 1 at the s^{th} position and zero elsewhere, and

b) $\theta(0) = \theta(1) = j$.

For the deterministic case (N=1 and p_{11}=1) the above definition reduces to the usual H_2-norm. As in the deterministic case, we have that the H_2-norm as defined above can be calculated as the solution of the discrete-time coupled gramian of observability and controllability. For this, define $\mathcal{C} = (\tilde{C}_1'\tilde{C}_1,...,\tilde{C}_N'\tilde{C}_N) \in \mathcal{H}^{n+}$, $\mathcal{J} = (JJ',...,JJ') \in \mathcal{H}^{n+}$, and $L = (L_1,...,L_N) \in \mathcal{H}^{n+}$, $P = (P_1,...,P_N) \in \mathcal{H}^{n+}$ the unique solution of the equations (recall that $r_\sigma(\mathcal{T})=r_\sigma(\mathcal{L}) < 1$ and see Proposition 2)

$$L = \mathcal{L}(L) + \mathcal{C} \text{ (observability gramian)} \tag{6}$$

$$P = \mathcal{T}(P) + \mathcal{J} \text{ (controllability gramian).} \tag{7}$$

The next result was proved in [5] and represents a characterization of the H_2-norm in terms of the solution of the observability and controllability gramians.

Proposition 4: $\| \mathcal{G} \|_2^2 = \sum_{j=1}^{N} tr(J'L_jJ) = \sum_{j=1}^{N} tr(\tilde{C}_jP_j\tilde{C}_j')$.

3.3. THE H_∞-NORM

Consider again system $\mathcal{G}$ as in (5) above with $w = (w(0),...) \in \ell_2^r(\mathcal{F}_k)$. The following result was proved in Proposition 2 of [6].

Proposition 5: $r_\sigma(\mathcal{T}) < 1$ if and only if $x = (0,x(1),...) \in \ell_2^n(\mathcal{F}_k)$ for every $w = (w(0),w(1),...) \in \ell_2^r(\mathcal{F}_k)$.

Suppose that $r_\sigma(\mathcal{T}) < 1$. From the above Proposition, $x = (0,x(1),...) \in \ell_2^n(\mathcal{F}_k)$ and thus $z = (0,z(1),...) \in \ell_2^p(\mathcal{F}_k)$. The H_∞-norm of system $\mathcal{G}$ is defined as :

Definition 4: $\| \mathcal{G} \|_\infty^2 := \sup_{\theta_0} \sup_{w \in \ell_2^p(\mathcal{F}_k)} \frac{\|z\|_2}{\|w\|_2}$.

Again, for the deterministic case (N = 1 and p_{11} = 1), definition 4 reduces to the usual H_∞-norm.

4. MIXED H_2/H_∞-CONTROL PROBLEM

Consider now a controlled version of system $\mathcal{G}$

$$\mathcal{G} = \begin{cases} x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u(k) + Jw(k) & (8.a) \\ x(0) = 0,\ \theta(0) = \theta_0 & (8.b) \\ z(k) = C_{\theta(k)}x(k) + D_{\theta(k)}u(k) & (8.c) \end{cases}$$

where $A = (A_1,...,A_N) \in \mathcal{H}^n$, $B = (B_1,...,B_N) \in \mathcal{H}^{m,n}$, $C = (C_1,...,C_N) \in \mathcal{H}^{n,p}$, $D = (D_1,...,D_N) \in \mathcal{H}^{m,p}$ and $J \in \mathbb{M}(\mathbb{C}^r,\mathbb{C}^n)$ are real and: a) $D_i'D_i > 0$ for each i = 1,...,N, b) $C_i'D_i = 0$ for each i = 1,...,N.

For $K = (K_1,...,K_N)$ set $\mathcal{G}_K$ as system (8) with $u(k) = - K_{\theta(k)}x(k)$. We have the following result.

Theorem 1: Suppose (C,A) is mean square detectable and $\delta > 0$ fixed a real number. If there exists $P = (P_1,...,P_N) \geq 0$ and $K = (K_1,...,K_N) \in \mathcal{H}^{n,m}$ such that for each $i = 1,\dots,N$,

$$- P_i + (A_i-B_iK_i)'\mathcal{E}_i(P)(A_i-B_iK_i) + + (C_i-D_iK_i)'(C_i-D_iK_i) + \frac{1}{\delta^2} P_iJJ'P_i \leq 0 \tag{9}$$

then $K = (K_1,\dots K_N) \in \mathbb{K}$ and

$$\| \mathcal{G}_K \|_\infty^2 \leq \delta^2(1 - \nu) \leq \delta^2$$

where $\nu \in (0, \frac{1}{\delta^2} \sum_{i=1}^{N} tr(J'P_iJ))$. Moreover,

$$\| \mathcal{G}_K \|_2^2 \leq \sum_{i=1}^{N} tr(J'P_iJ).$$

Proof: Comparing (4) and (9) it is immediate from Proposition (3) that $K \in \mathbb{K}$. Set $\tilde{A}_i = A_i - B_iK_i$ and $\tilde{C}_i = C_i - D_iK_i$. Recalling from Proposition 5 that, for any $w = (w(0),\dots) \in \ell_2^r(\mathcal{F}_k)$ we have $x = (0,x(1),\dots) \in \ell_2^n(\mathcal{F}_k)$, and that $x(k)$, $\theta(k)$ and $w(k)$ are $\mathcal{F}_k$-measurable, we get from (9) that

$$\begin{aligned} &E(x(k+1)'P_{\theta(k+1)}x(k+1)) = \\ &= E(x(k+1)'E(P_{\theta(k+1)} \mid \mathcal{F}_k)x(k+1)) \\ &\leq E\Big(x(k)'\Big(P_{\theta(k)} - \tilde{C}_{\theta(k)}'\tilde{C}_{\theta(k)} - \frac{1}{\delta^2} P_{\theta(k)}JJ'P_{\theta(k)}\Big)x(k) + \\ &+ w(k)'J'\mathcal{E}_{\theta(k)}(P)\tilde{A}_{\theta(k)}x(k) + x(k)'\tilde{A}_{\theta(k)}\mathcal{E}_{\theta(k)}(P)Jw(k) + \\ &+ w(k)'J'\mathcal{E}_{\theta(k)}Jw(k)\Big) \end{aligned}$$

so that,

$$\begin{aligned} &\| P_{\theta(k+1)}^{1/2}x(k+1) \|_2^2 - \| P_{\theta(k)}^{1/2}x(k) \|_2^2 + \| z(k) \|_2^2 \leq \\ &\leq -\frac{1}{\delta^2} \| J'P_{\theta(k)}x(k) \|_2^2 + \frac{1}{\delta^2} \| J'P_{\theta(k+1)}x(k+1) \|_2^2 + \\ &- \frac{1}{\delta^2} \| J'P_{\theta(k+1)}x(k+1) \|_2^2 - E\Big(w(k)'J'\mathcal{E}_{\theta(k)}(P)Jx(k)\Big) + \\ &+ 2\,E(w(k)'J'\mathcal{E}_{\theta(k)}(P)(\tilde{A}_{\theta(k)}x(k) + Jw(k))) \end{aligned}$$

Thus,

$$\begin{aligned} &\| P_{\theta(k+1)}^{1/2}x(k+1) \|_2^2 - \| P_{\theta(k)}^{1/2}x(k) \|_2^2 + \\ &- \frac{1}{\delta^2} \| J'P_{\theta(k+1)}x(k+1) \|_2^2 + \frac{1}{\delta^2} \| J'P_{\theta(k)}x(k) \|_2 + \\ &+ \| z(k) \|_2^2 \\ &\leq \frac{1}{\delta^2} \| J'P_{\theta(k+1)}x(k+1) \|_2^2 + 2E(w(k)'J'P_{\theta(k+1)}x(k+1)) + \\ &- \delta^2 \| w(k) \|_2^2 + E(w(k)'(\delta^2 I - J'\mathcal{E}_{\theta(k)}(P)J)w(k)) \\ &\leq E(w(k)'(\delta^2 I - J'\mathcal{E}_{\theta(k)}(P)J)w(k)). \end{aligned}$$

Taking the sum for $k = 0$ to ∞, and recalling that $x(0) = 0$, $\| x(k) \|_2 \to 0$ as $k \to \infty$, we get that

$$\begin{aligned} \| z \|_2^2 &\leq \delta^2 \sum_{k=0}^{\infty} E(w(k)'(I - \frac{1}{\delta^2} J'P_{\theta(k+1)}J)w(k)) \\ &\leq \delta^2(1-\nu) \| w \|_2^2 \end{aligned}$$

where $\nu \in (0, \frac{1}{\delta^2} \sum_{i=1}^{N} tr(J'P_iJ))$. Thus,

$$\| \mathcal{G} \|_\infty^2 = \sup_{\theta_0} \sup_{w \in \ell_2^r(\mathcal{F}_k)} \frac{\| z \|_2}{\| w \|_2} \leq \delta(1 - \nu)^{1/2} < \delta.$$

Finally notice from Proposition 4,

$$\| G_K \|_2^2 = \sum_{i=1}^{N} tr(J'S_iJ),$$

where $S_i = \tilde{A}_i'\mathcal{E}_i(S)\tilde{A}_i + \tilde{C}_i'\tilde{C}_i$

From (9) and some $V_i \geq 0$, $i = 1,\dots,N$,

$$P_i = \tilde{A}_i'\mathcal{E}_i(P)\tilde{A}_i + \tilde{C}_i'\tilde{C}_i + \frac{1}{\delta^2} P_iJJ'P_i + V_i'V_i$$

so that, from Proposition 2, $P_i \geq S_i$ for all $i = 1,\dots,N$. This implies that

$$\| G_K \|_2^2 = \sum_{i=1}^{N} tr(J'S_iJ) \leq \sum_{i=1}^{N} tr(J'P_iJ)$$

completing the proof of the Theorem. □

This suggests the following approximation for the mixed H_2/H_∞-control problem: for $\delta > 0$ fixed, find $P = (P_1,\dots,P_N) \geq 0$ and $K = (K_1,\dots,K_N)$ such that

$$\min tr(\sum_{i=1}^{N} J'P_iJ)$$

subject to (9). For the case $N=1$, $p_{11} = 1$, equation (9) can be seen as dual to the one obtained in [12], Lemma 3.1.

5. CONVEX APPROACH

We will assume now that the transition probability matrix $\mathbb{P}$ is not exactly known, but belongs to a convex set $\mathbb{D} := \{\mathbb{P};\ \mathbb{P} = \sum_{t=1}^{q} \alpha_t \mathbb{P}^t,\ \alpha_t \geq 0,\ \sum_{t=1}^{q} \alpha_t = 1\}$, where $\mathbb{P}^t$, $t = 1,\dots,q$, are known transition probability matrices. We make the following definition.

Definition 5: We say that $K = (K_1,\dots,K_N) \in \mathcal{H}^{n,m}$ robustly stabilizes (A,B) in the mean square sense if system (1) with $\tilde{A}_i = A_i - B_iK_i$ is MSS for every $\mathbb{P} \in \mathbb{D}$, and we set $\mathbb{K}_r := \{ K \in \mathcal{H}^{n,m};$ K robustly stabilizes (A,B) in the mean square sense$\}$.

We want to solve the following mixed H_2/H_∞ control problem: given $\delta > 0$, find $K \in \mathbb{K}_r$ which minimizes ζ subject to $\| \mathcal{G}_K \|_2 \leq \zeta$, $\| \mathcal{G}_K \|_\infty \leq \delta$, for every $\mathbb{P} \in \mathbb{D}$. Let us show now that an approximation for this problem can be obtained via a LMI optimization problem. Set $\Gamma_i^t = [\sqrt{p_{i1}^t}\, I \ \dots \ \sqrt{p_{iN}^t}\, I] \in \mathbb{M}(\mathbb{C}^{Nn},\mathbb{C}^n)$ for $i = 1,\dots,N$, $t = 1,\dots,q$, and define the following problem:

Problem I : Set $\mu = \delta^2$. Find $P = (P_1,\dots,P_N) > 0$, $Q = (Q_1,\dots,Q_N) > 0$, $L = (L_1,\dots,L_N) > 0$, $Y = (Y_1,\dots,Y_N)$ such that

$$\xi = \min tr(\sum_{i=1}^{N} J'P_iJ)$$

subject to

$$\begin{bmatrix} Q_i & Q_iA_i'+Y_i'B_i' & Q_iC_i' & Y_i'D_i' & J \\ A_iQ_i+B_iY_i & L_i & 0 & 0 & 0 \\ C_iQ_i & 0 & I & 0 & 0 \\ D_iY_i & 0 & 0 & I & 0 \\ J' & 0 & 0 & 0 & \mu I \end{bmatrix} \geq 0,$$

$$i = 1,\ldots,N \tag{10}$$

$$\begin{bmatrix} L_i & L_i\Gamma_i^t \\ \Gamma_i^{t\prime}L_i & \mathrm{diag}\{Q_\kappa\} \end{bmatrix} \geq 0,\ i = 1,\ldots,N,\ t = 1,\ldots,q \tag{11}$$

$$\begin{bmatrix} P_i & I \\ I & Q_i \end{bmatrix} \geq 0,\ i = 1,\ldots,N \tag{12}$$

where $\mathrm{diag}\{Q_\kappa\}$ is the matrix in $\mathbb{M}(\mathbb{C}^{Nn})$ formed by $Q_1,\ldots,Q_N$ in the diagonal, and zero elsewhere.

Theorem 2 : Suppose Problem I has a solution P, Q, L and Y. Set $K = (K_1,\ldots,K_N)$ as $K_i = -Y_iQ_i^{-1}$, $i = 1,\ldots,N$ and $\xi = \sum_{i=1}^{N}\mathrm{tr}(J'P_iJ)$. Then $K \in \mathbb{K}_r$ and $\|\mathcal{G}_K\|_2 \leq \xi$, $\|\mathcal{G}_K\|_\infty \leq \delta$, for every $\mathbb{P} \in \mathbb{D}$.

Proof: First of all notice that (10), (11) and (12) are equivalent to (see Remark 1)

$$\begin{aligned} Q_i \geq\ & Q_i(A_i-B_iK_i)'L_i^{-1}(A_i-B_iK_i)Q_i + \\ &+ Q_i(C_i-D_iK_i)'(C_i-D_iK_i)Q_i + \\ &+ \mu^{-1}Q_i(Q_i^{-1})JJ'(Q_i^{-1})Q_i \end{aligned} \tag{13}$$

$$L_i \geq L_i(\sum_{j=1}^{N}p_{ij}^tQ_j^{-1})L_i,\ t = 1,\ldots,q, \tag{14}$$

$$P_i \geq Q_i^{-1}, \tag{15}$$

Since we are minimizing $\mathrm{tr}(\sum_{i=1}^{N}J'P_iJ)$ and $JJ' > 0$ by hypothesis, we must have from (15) that $P_i = Q_i^{-1}$. Consider any $\mathbb{P} \in \mathbb{D}$. Then by definition we have that $p_{ij} = \sum_{t=1}^{q}\alpha_t p_{ij}^t$ for some $\alpha_t \geq 0$, $\sum_{t=1}^{q}\alpha_t = 1$. Thus from (14) we get that

$$L_i^{-1} \geq (\sum_{j=1}^{N}p_{ij}Q_j^{-1}) = \mathcal{E}_i(P),$$

and from (13),

$$\begin{aligned} P_i = Q_i^{-1} \geq\ & (A_i-B_iK_i)'L_i^{-1}(A_i-B_iK_i) + \\ &+ (C_i-D_iK_i)'(C_i-D_iK_i) + \mu^{-1}P_iJJ'P_i \\ \geq\ & (A_i-B_iK_i)'\mathcal{E}_i(P)(A_i-B_iK_i) + \\ &+ (C_i-D_iK_i)'(C_i-D_iK_i) + \mu^{-1}P_iJJ'P_i. \end{aligned} \tag{16}$$

The result follows from (16) and Theorem 1. □

Remark 2 : If we desire to minimize the H_∞-norm, then we just have to replace the value function $\mathrm{tr}(\sum_{i=1}^{N}J'P_iJ)$ by μ. Inequalities (12) can be eliminated.

6. NUMERICAL EXAMPLES

This example is adapted from [12] for the case in which we have two modes of operation, with transition probability matrix between the models given by $\mathbb{P}$. The matrices are:

$$A_1 = A_2 = \begin{bmatrix} 0.9974 & 0.0539 \\ -0.1078 & 1.1591 \end{bmatrix},\ B_1 = \begin{bmatrix} -0.0013 \\ -0.0539 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.0013 \\ -0.1078 \end{bmatrix},\ J = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ C_1 = C_2 = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix},$$

$$D_1 = D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

We considered the following cases:

a.1) H_2/H_∞-control problem with $\delta = 80$, and transition probability matrix exactly known, given by:

$$\mathbb{P} = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix}.$$

For this case the solution obtained was K_1 = [1.3655 4.4281], K_2 = [1.5999 4.6390], and the optimal value function was $\xi = 726.4880$.

a.2) the same as above but with $\mathbb{P}$ belonging to $\mathbb{D}$, where $\mathbb{D}$ is defined through the transition probability matrices $\mathbb{P}^1$ and $\mathbb{P}^2$ defined below

$$\mathbb{P}^1 = \begin{bmatrix} 0.65 & 0.35 \\ 0.25 & 0.75 \end{bmatrix},\ \mathbb{P}^2 = \begin{bmatrix} 0.75 & 0.25 \\ 0.15 & 0.85 \end{bmatrix}.$$

For this case the solution obtained was K_1 = [2.1348 4.9492], K_2 = [2.4056 6.0059], and the optimal value function was $\xi = 980.7510$.

b.1) H_∞-control problem with the same data as in a.1) above. The minimal value obtained for μ (= δ^2) was 4366.8, with the controllers given by K_1 = [5.4713 7.0138], K_2 = [4.8935 5.9692].

b.2) the same as above but with $\mathbb{P} \in \mathbb{D}$, where $\mathbb{D}$ is defined as in a.2). The minimal value obtained for μ was 5039.2, with the controllers given by K_1 = [5.8694 6.9641], K_2 = [5.9746 7.3220].

7. CONCLUSIONS

In this paper we have considered the problem of mixed H_2/H_∞-control of discrete-time Markovian jump linear systems (MJLS). It was assumed that both the state variable and the jump variable are available to the controller. The transition probability matrix may belong to an appropriated convex set. We are interested in finding a state and jump feedback controller that robustly stabilizes a MJLS in the mean square sense and minimizes an upper bound for the H_2 norm, under the restriction that the H_∞-norm is less than a pre-specified value δ. This kind of problem has been studied in the current literature for discrete-time deterministic linear systems, usually using a state space approach, leading to non-standards algebraic

Riccati and Lyapunov-like equations. We trace a parallel with the discrete-time linear system theory of H_2/H_∞ and H_∞ control to derive our results. An approximation for the problem was proposed by minimizing a linear functional over the positive semi-definite solutions of a set of coupled Lyapunov-like equations. Furthermore it was shown that this problem can be written in a convex programming formulation, leading to numerical algorithms. The H_∞-control problem was also addressed.

8. REFERENCES

[1] H. Abou-Kandil, G. Freiling and G. Jank, On the solution of discrete-time Markovian jump linear quadratic control problems, Automatica 31, (1995), 765-768.

[2] W.P. Jr. Blair and D.D. Sworder, Feedback control of a class of linear discrete systems with jump parameters and quadratic cost criteria, Int. J. Control 21, (1975), 833-844.

[3] H.A.P. Blom and Y.Bar-Shalom, The interacting multiple model algorithm for systems with Markovian switching coefficients, IEEE Trans. Automat. Control 33, (1988), 780-783.

[4] H.J. Chizeck, A.S. Willsky and D. Castano, Discrete-time Markovian jump linear quadratic optimal control, Int. J. Control 43, (1986), 213-231.

[5] O.L.V. Costa, J.B.R. do Val and J.C. Geromel, A convex programming approach to H_2-control of discrete-time Markovian jump linear systems, to appear.

[6] O.L.V. Costa and J.B.R. do Val, Full information H_∞-control for discrete-time infinite Markov jump parameter systems, J. Math. Analysis and Applic., to appear.

[7] O.L.V. Costa and M.D. Fragoso, Stability results for discrete-time linear systems with Markovian jumping parameters, J. Math. Analysis and Applic. 179, (1993), 154-178.

[8] O.L.V. Costa and M.D. Fragoso, Discrete-time LQ-optimal control problems for infinite Markov jump parameter systems, IEEE Transactions on Automatic Control 40, (1995), 2076-2088.

[9] O.L.V. Costa, Discrete-time coupled Riccati equations for systems with Markov switching parameters, J. Math. Analysis and Applic. 194, (1995), 197-216.

[10] X. Feng, K.A. Loparo, Y. Ji, and H.J. Chizeck, Stochastic stability properties of jump linear systems, IEEE Trans. Automat. Control 37, (1992), 38-53.

[11] M.D. Fragoso, J.B. Ribeiro do Val and D. L. Pinto Jr. Jump linear H_∞-control: the discrete-time case, Control Th. and Adv. Tech. 10, (1995), 1459-1474.

[12] J.C. Geromel, P.L.D. Peres and S.R. Souza, A convex approach to the mixed H_2/H_∞-control problem for discrete-time uncertain systems, SIAM J. on Control and Optimization 33, (1995), 1816-1833.

[13] B.E. Griffiths and K.A. Loparo, Optimal control of jump linear quadratic gaussian systems, Int. J. Control 42, (1985), 791-819.

[14] Y. Ji and H.J. Chizeck, Controllability, observability and discrete-time Markovian jump linear quadratic control, Int. J. Control 48, (1988), 481-498.

[15] Y. Ji and H. Chizeck, Optimal quadratic control of jump linear systems with separately controlled transition probabilities, Int. J. Control 49, (1989), 481-491.

[16] Y. Ji and H.J. Chizeck, Jump linear quadratic gaussian control: steady state solution and testable conditions, Control-Theory and Advanced Technology 6, (1990), 289-319.

[17] Y. Ji and H. J. Chizeck, Controllability, stabilizability and continuous-time Markovian jump linear quadratic control, IEEE Trans. Automat. Control 35 (1990), 777-788.

[18] Y. Ji, H. J. Chizeck, X. Feng and K.A. Loparo, Stability and control of discrete-time jump linear systems, Control-Theory and Advanced Technology 7, (1991), 247-270.

[19] P.P. Khargonekar and M.A. Rotea, Mixed H_2/H_∞ control: a convex optimization approach, IEEE Trans. Automatic Control 36, (1991), 824-837.

[20] I. Kaminer, P.P. Khargonekar and M.A. Rotea, Mixed H_2/H_∞ control for discrete-time systems via convex optimization, in Proc. 1992 American Control Conference, Chicago, USA, 392-396.

[21] M.A. Rami and L. El Ghaoui, Robust stabilization of jump linear systems using linear matrix inequalities, IFAC Symposium on Robust Control Design, Rio de Janeiro, RJ, (1994), 148-151.

BOLETINS TÉCNICOS - TEXTOS PUBLICADOS

BT/PEE/9301 - Oscilador a HEMT - 10 GHz - FÁTIMA S. CORRERA, EDMAR CAMARGO
BT/PEE/9302 - Representação Senoidal da Voz através dos Polos do Filtro Preditor - MARCELO B. JOAQUIM, NORMONDS ALENS
BT/PEE/9303 - Blindagens por Grades Condutoras: Cálculo do Campo Próximo - LUIZ CEZAR TRINTINALIA, ANTONIO ROBERTO PANICALI
BT/PEE/9304 - Sistema de Otimização e Controle de Produção em Minas de Pequeno e Médio Porte - TSEN CHUNG KANG, VITOR MARQUES PINTO LEITE
BT/PEE/9401 - Determinação das Frases de Aplicação Forense para o projeto NESPER e Tese de Mestrado IME/94, com Base em Estudos Fonéticos - MARCONI DOS REIS BEZERRA, EUVALDO F. CABRAL JUNIOR
BT/PEE/9402 - Implementação e Teste de uma Rede Neural Artificial do TIpo KSON (Kohonen Self-Organizing Network) com Entradas Bidimensionais - MARCELO YASSUNORI MATUDA, EUVALDO F. CABRAL JR.
BT/PEE/9403 - Transformada de Walsh e Haar Aplicadas no Processamento de Voz - ALEXANDRE AUGUSTO OTTATI NOGUEIRA, THIAGO ANTONIO GRANDI DE TOLOSA, EUVALDO F. CABRAL JÚNIOR
BT/PEE/9404 - Aplicação de Redes Neurais ao Problema de Reconhecimento de Padrões por um Sonar Ativo - ALEXANDRE RIBEIRO MORRONE, CRISTINA COELHO DE ABREU, EDUARDO KOITI KIUKAWA, EUVALDO F. CABRAL JR.
BT/PEE/9405 - Tudo que se Precisa Saber sobre a Prática da FFT - Transformada Rápida de Fourier (Inclui Software) - ROGÉRIO CASAGRANDE, EUVALDO F. CABRAL JR.
BT/PEE/9406 - A Survey on Speech Enhancement Techniques of Interest to Speaker Recognition - CELSO S. KURASHIMA, EUVALDO F. CABRAL JR.
BT/PEE/9407 - Identificação de Pulsos Decádicos em Linhas Telefônicas - ANTONIO P. TIMOSZCZUK, MÁRCIO A. MATHIAS, EUVALDO F. CABRAL JR.
BT/PEE/9408 - Implementação e Teste de Filtros do Tipo Adaptativo e "Notch" para a Remoção de Interferência de 60 Hz em Sinais de Eletrocardiograma - FLÁVIO ANTÔNIO MENEGOLA, JOSÉ AUGUSTO DE MATTOS, JOSÉ GOMES G. FILHO, SIDNEY SILVA VIANA, EUVALDO F. CABRAL JR.
BT/PEE/9409 - Compressão de Sinais de Voz utilizando Transformadas de Karhunen-Loève, Fourier e Hadamard - IVAN LUIS VIEIRA, LUIZ FERNANDO STEIN WETZEL, EUVALDO F. CABRAL JR.
BT/PEE/9410 - "Ray Tracing" Paralelo - EDUARDO TOLEDO SANTOS, JOÃO ANTONIO ZUFFO
BT/PEE/9411 - Implementação de uma Ferramenta Posicionador para "Gate-Arrays" Tipo Mar de Portas - JORGE W. PERLAZA PRADO, WILHELMUS A. M. VAN NOIJE
BT/PEE/9412 - Tudo que se Precisa Saber Sobre a Teoria da FFT - Transformada Rápida de Fourier - FÁBIO LUÍS ROMÃO, REINALDO SILVEIRA, ROGÉRIO CASAGRANDE, EUVALDO CABRAL JR.
BT/PEE/9413 - Análise do Ruído Sonoro em uma Sala de Aquisição de Amostras de Som com Microcomputador - FÁBIO LUÍS ROMÃO, REINALDO SILVEIRA, EUVALDO CABRAL JR.
BT/PEE/9414 - Cor: Aspectos Relevantes para Visualização de Dados - SÍLVIA DELGADO OLABARRIAGA
BT/PEE/9415 - Projeto de Filtros Digitais IIR com Fase Aproximadamente Linear Utilizando Redução de Ordem - IVAN F. J. RODRIGUES, MAX GERKEN
BT/PEE/9416 - GERAFILTRO: Sistema para Projeto Automático de Filtros Digitais "IIR" (da especificação em alto nível ao leiaute do "ASIC") - RICARDO PIRES, JOSÉ VIEIRA DO VALE NETO
BT/PEE/9417 - Redes Neurais Artificiais Aplicadas à Identificação de Pulsos Decádicos em Linhas Telefônicas - ANTONIO P. TIMOSZCZUK, EUVALDO F. CABRAL JR.
BT/PEE/9501 - Estudo Comparativo de Métodos de Cálculo da Frequência Fundamental - MARCOS COSTA HUNOLD, EUVALDO F. CABRAL JR.
BT/PEE/9502 - Combinando Técnicas de Redes Neurais Artificiais e Informações de Excitação no Reconhecimento Automático do Locutor - ANDRÉ BORDIN MAGNI, EUVALDO F. CABRAL JR.
BT/PEE/9503 - Utilização de Redes Neurais Artificiais para Detecção e Identificação de Falhas em Circuitos - MÁRCIO YUKIO TERUYA, ROBERTO AMILTON BERNARDES SÓRIA, EUVALDO CABRAL JR.
BT/PEE/9504 - Uso de Redes Neurais Artiificiais no Reconhecimento de Locutores no Domínio Temporal - BENEDITO JOSÉ BARRETO FONSECA JÚNIOR, EUVALDO CABRAL JÚNIOR
BT/PEE/9505 - Projeto de Filtros Passivos e Ativos em Técnicas de Circuitos Integrados de Microondas - DAVID VIVEIROS JÚNIOR, DENISE CONSONNI
BT/PEE/9506 - Uma Análise de *Clustering* para as Frases de Projeto NESPER - RONALDO OLIVEIRA MESSINA, EUVALDO F. CABRAL JR.
BT/PEE/9507 - Controle com Estrutura Variável e Modos Deslizantes - Um Estudo para Aplicação em Controle Carga-frequência da Geração - JOSE PAULO F. GARCIA, JOCELYN FREITAS BENNATON
BT/PEE/9508 - Recuperação das Margens de Ganho e de Fase para Sistemas de Fase Não Mínima por Realimentação da Saída - MARCO H. TERRA, VITOR M. P. LEITE
BT/PEE/9509 - Sistema de Inspeção Óptica de Dispositivos Bi-Dimensionais - CASIMIRO DE ALMEIDA BARRETO, PEDRO LUÍS PRÓSPERO SANCHEZ
T/PEE/9510 - Sistema de Partículas Uma Poderosa Técnica de Animação em Computação Gráfica - RENATO CURTO RODRIGUES, JOÃO ANTÔNIO ZUFFO
BT/PEE/9511- Efeito de Ruídos em Sinais de Voz Visualizados em Trajetórias Neurais de Kohonen - CELSO S. KURASHIMA, EUVALDO F. CABRAL JR.
BT/PEE/9601 - "Um Reconhecedor de Sinais Sonoros Utilizando LVQ" - ALEXANDRE TORNICE, EUVALDO CABRAL JR.
BT/PEE/9602 - "Coleção *Artificial Neural Networks*: Uma Visão Geral dos Sistemas Neurais Artificais de Stephen Grossberg" - CHIU HSIUNG HUANG
BT/PEE/9603 - "Reactively-Sputtered TiN Formation Using a RF Magnetron System"- SÉRGIO PAULO AMARAL OSÓRIO, LUIZ SÉRGIO ZASNICOFF
BT/PEE/9604 - Aspectos em Tradução de Linguagens Naturais Através de Redes Neurais Artificiais - CARLOS EDUARDO DANTAS DE MENEZES, EUVALDO F. CABRAL JR.
BT/PEE/9605 - Implementação de Blocos Passa-Tudo Utilizando Realimentação de Erro - SÉRGIO JOSÉ CARNEIRO LEÃO, MAX GERKEN
BT/PEE/9606 - Coleção SANN group Redes Neurais Artificiais: A Rede Neural de Sakoe - ANDRÉ BORDIN MAGNI, EUVALDO F. CABRAL JR.

BT/PEE/9607 - Coleção SANN group Redes Neurais Artificiais: A Rede Neural de Steinbuch - ROBERTO AMILTON BERNARDES SÓRIA, EUVALDO F. CABRAL JR.
BT/PEE/9608 - Desenvolvimento de uma Estrutura de Duplo Nível de Metal para a Confecção de Interconexões em Circuitos Integrados - JOSÉ AUGUSTO DE ALENCAR PEREIRA, LUIZ CARLOS MOLINA TORRES
BT/PEE/9609 - Determinação de Parâmetros de Processo para Fotomáscara "Balzers" Utilizando Gerador de Padrões - JORGE SEKI, MEGUMI SAITO
BT/PEE/9610 - Um Ambiente para Desenvolvimento de Sistemas Distribuídos - PEDRO F. ROSA, JOÃO A. ZUFFO
BT/PEE/9611 - Interpretações Teóricas do Funcionamento Cerebelar: Uma Revisão - MARCUS FRAGA VIEIRA, ANDRÉ FÁBIO KOHN
BT/PEE/9612 - Marcapasso Cardíaco Temporário Microcontrolado de Demanda e Baixo Consumo - FLAVIO ANTONIO MENEGOLA, JOSÉ CARLOS TEIXEIRA DE BARROS MORAES
BT/PEE/9613 - Um Sistema de Planejamento de Ação Baseado em Casos para uma Célula Flexível de Manufatura - RICARDO LUÍS DE FREITAS, MÁRCIO RILLO
BT/PEE/9614 - Aplicações do Boundary-Scan para o Teste de Módulos Multichip - ROBERTO C. COSSI JR., JOSÉ ROBERTO DE A. AMAZONAS
BT/PEE/9615 - A 2.488 Gb/s GaAs 1:4/1:16 Demultiplexer IC with Skip Circuit for Sonet STS-12/48 Systems - TAUFIK ABRÃO, FATIMA S. CORRERA
BT/PEE/9616 - Uma Contribuição para a Construção de Algoritmos em Projetos de Redes - ALLAN DE SOUZA, JOSÉ ROBERTO CASTILHO PIQUEIRA
BT/PEE/9617 - Análise Crítica dos Métodos de Medição do Intervalo QT do Eletrocardiograma - SÍDNEY DA SILVA VIANA, JOSÉ CARLOS TEIXEIRA DE BARROS MORAES
BT/PEE/9618 - Deposição e Caracterização de Filmes de SiO_2 Crescidos pela Técnica de PECVD a Baixa Temperatura - MARCO ALAYO CHÁVEZ, INÉS PEREYRA
BT/PEE/9619 - PARSTOOL: Uma Ferramenta de Auxílio à Simulação de Sistemas Paralelos - LI KUAN CHING, LIRIA MATSUMOTO SATO
BT/PEE/9620 - Análise de um Método de Otimização por Malha no Treinamento de Robôs - OLÍMPIO MURILO CAPELI, JOSÉ CARLOS T. B. MORAES, SADAO ISOTANI
BT/PEE/9701 - Identification of Unstable Mechanical Systems - ROBERTO MOURA SALES, ANSELMO BITTAR, MICHAEL PORSCH, LAÉRCIO LUCCHESI
BT/PEE/9702 - Analysis of the Subthreshold Slope Transition Region in SOI nMOSFET - VICTOR SONNENBERG, JOÃO ANTONIO MARTINO
BT/PEE/9703 - Introduction of the SOI MOSFET Dimensions in the High-Temperature Leakage Drain Current Model - MARCELO BELLODI, JOÃO ANTONIO MARTINO, DENIS FLANDRE
BT/PEE/9704 - Controle de Largura de Banda Dinâmica para Transmissões *Multicast* para Redes de Alta Velocidade - SANG SOON LEE, SERGIO TAKEO KOFUJI
BT/PEE/9705 - Uma Modificação Proposta para o Controle Preditivo Generalizado com Filtro de Kalman - JAIME QUINTERO R., OSWALDO L. V. COSTA
BT/PEE/9706 - Aplicações de Redes Neurais em Previsões Financeiras - OLÍMPIO MURILO CAPELI, EUVALDO F. CABRAL JR.
BT/PEE/9707 - Sistema Microcontrolado, Multicanal e Portátil para Estimulação Neuromuscular Funcional - ROGÉRIO QUIARIM ZARZA, JOSÉ CARLOS TEIXEIRA DE BARROS MORAES
BT/PEE/9708 - Requisitos para o Mapeamento Tecnológico em Projetos de Microeletrônica - LUCIANO DE OLIVEIRA CORRÊA DE BRITO, JOSÉ ROBERTO DE ALMEIDA AMAZONAS
BT/PEE/9709 - Sistemas PRMA com Dados Acoplados - JOSÉ AUGUSTO DE LIMA, PAUL JEAN ETIENNE JESZENSKY
BT/PEE/9710 - Algoritmos Genéticos (AG's) para a Otimização de Controladores Nebulosos - JULIO CESAR CEBALLOS AYA, OSWALDO L. V. COSTA
BT/PEE/9711 - Um Estudo Sobre a Redução de Ruídos em Sinais Caóticos - ERNANE JOSÉ XAVIER COSTA, EUVALDO F. CABRAL JR.
BT/PEE/9712 - Geradores não Lineares de Sequência para uso em Sistemas Spread Spectrum - ANGEL ANTONIO GONZALEZ MARTINEZ, PAUL JEAN ETIENNE JESZENSZKY
BT/PEE/9713 - Modelamento Físico do Sistema Heteroestrutura - Metal - CECÍLIA WETTERLE RODRIGUES, MEGUMI SAITO
BT/PEE/9714 - Tensões Induzidas em Linhas Aéreas por Descargas Atmosféricas Indiretas - Modelagem e Aplicação ao Cálculo de Interrupções - ALEXANDRE PIANTINI, JORGE M. JANISZEWSKI
BT/PEE/9715 - RECMAP - Uma Ferramenta para Otimização em Síntese de Alto Nível Baseada em Reconhecimento Funcional e Mapeamento de Componentes - ANDRÉ GERHARD, JOSÉ VIEIRA DO VALE NETO
BT/PEE/9716 - Estudo da Sinterização de Contatos Al/Ti por Recozimento Térmico Rápido Visando a Aplicação em Circuitos Integrados - ANGELO EDUARDO BATTISTINI MARQUES, ROGÉRIO FURLAN