Braz J Phys (2013) 43:172–181
DOI 10.1007/s13538-013-0129-y

SOCIEDADE BRASILEIRA DE FÍSICA

PARTICLES AND FIELDS

Elementary Excitations of a Higgs–Yukawa System

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Received: 13 October 2011 / Published online: 27 April 2013

Abstract This work investigates the physics of elementary excitations for the so-called relativistic quantum scalar plasma system, also known as the Higgs–Yukawa system. Following the Nemes–Piza–Kerman–Lin many-body procedure, the random-phase approximation (RPA) equations were obtained for this model by linearizing the time-dependent Hartree–Fock–Bogoliubov equations of motion around equilibrium. The resulting equations have a closed solution, from which the spectrum of excitation modes are studied. We show that the RPA oscillatory modes give the one-boson and two-fermion states of the theory. The results indicate the existence of bound states in certain regions in the phase diagram. Applying these results to recent Large Hadron Collider observations concerning the mass of the Higgs boson, we determine limits for the intensity of the coupling constant g of the Higgs–Yukawa model, in the RPA mean-field approximation, for three decay channels of the Higgs boson. Finally, we verify that, within our approximations, only Higgs bosons with masses larger than 190 GeV/c^2 can decay into top quarks.

Keywords Higgs–Yukawa system · Nemes–Piza–Kerman–Lin procedure · Random-phase approximation · Bound states

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1 Introduction

Recent years have witnessed substantial progress towards understanding the nonequilibrium time evolution of quantum fields. Results have been obtained that proved important in several applications. Examples are found in cosmology, such as the description of quantum-field expectation values in the early universe and subsequent hot stage (big bang) [1, 2]; in high-energy particle physics, such as the description of the dynamics in heavy-ion collision experiments, which seeks to establish experimental signatures for the nonequilibrium evolution of the quark–gluon system and chiral phase transition [3–5]; and in complex many-body quantum systems, such as the description of the dynamics of the Bose–Einstein condensates [6, 7], among other applications [8].

In a previous publication, we have developed a framework to investigate the initial value problem in the context of interacting fermion–scalar field theories [9]. This framework had earlier been developed in the context of the nonrelativistic nuclear many-body dynamics by de Toledo Piza [10] and Nemes and de Toledo Piza [11]. The method led to a set of self-consistent time-dependent equations for the expectation values of linear and bilinear forms of field operators. These dynamical equations acquire a kinetic type structure in which the lowest-order approximation corresponds to the usual Gaussian mean-field approximation. As an application, a zero-order calculation was implemented within the simplest context of a relativistic quantum scalar plasma system, nowadays also called the Higgs–Yukawa system. The usual renormalization prescription was shown to be also applicable to this nonperturbative calculation. In particular, a finite expression for the energy density was obtained, with the numerical results suggesting that the system always has a single stable minimum.

Here, we report an application of the renormalized nonlinear dynamical equations obtained in our previous article [9] and follow Kerman and Lin [12, 13] to investigate the near-equilibrium dynamics around the stationary solution of a Higgs–Yukawa system. We will show that one-boson and two-fermion physics can be studied in the linear approximation of the mean-field equations. We will solve those equations in closed form and find the scattering amplitude, as well as the conditions allowing a two-fermion bound state.

The motivation for this work is the recent observation of a possible Higgs boson around 125 Gev/c^2 [14–16]. Since the early 1990s, the Higgs–Yukawa model has been used to better understand the fermion mass generation via the Higgs mechanism. Recently, the Higgs–Yukawa model has been used to impose limits on the Higgs mass and on the intensity of the Higgs–Yukawa coupling. It has also been used to study the consequences of heavy extra-generation fermions (mass > $600/c^2$ GeV). An important consequence of a fourth fermion generation is the possibility of formation of bound states that can replace the role of the Higgs boson. Recently, these issues have been intensively studied on the lattice [17] or perturbatively—the $1/N$-expansion [18], for example. Our work carries out a nonperturbative calculation of the ground state (vacuum) for an interacting Higgs–Yukawa system.

For clarity and notational purposes, a few key equations from [9] are repeated here. The dynamics of the relativistic quantum scalar plasma model, or Higgs–Yukawa model, is governed by the following lagrangian density [19–32]:

$$\mathcal{L} = \bar{\psi}(i\gamma.\partial - m)\psi + g\bar{\psi}\phi\psi + \frac{1}{8\pi}\left[(\partial\phi)^2 - \mu^2\phi^2\right] - \mathcal{L}_c. \quad (1)$$

The model Hamiltonian is given by the equality

$$H = \int_{\mathbf{x}} \mathcal{H},$$
$$\mathcal{H} = -\bar{\psi}(i\vec{\gamma}.\vec{\partial} - m)\psi - g\bar{\psi}\phi\psi + \frac{1}{8\pi}\left[(4\pi)^2\Pi^2 + |\partial\phi|^2 + \mu^2\phi^2\right] + \mathcal{H}_c, \quad (2)$$

with the shorthand $\int_{\mathbf{x}} = \int d^3x$.

In the Heisenberg picture, $\phi(x)$ and $\Pi(x)$ are scalar spin-0 fields expanded as

$$\phi(\mathbf{x}, t) = \sum_{\mathbf{p}} \frac{1}{(2Vp_0)^{1/2}}\left[b_{\mathbf{p}}(t)e^{i\mathbf{p}.\mathbf{x}} + b^\dagger_{\mathbf{p}}(t)e^{-i\mathbf{p}.\mathbf{x}}\right]$$
$$\Pi(\mathbf{x}, t) = i\sum_{\mathbf{p}}\left(\frac{Vp_0}{2}\right)^{1/2}\left[b^\dagger_{\mathbf{p}}(t)e^{-i\mathbf{p}.\mathbf{x}} - b_{\mathbf{p}}(t)e^{i\mathbf{p}.\mathbf{x}}\right], \quad (3)$$

where $b^\dagger_{\mathbf{p}}(t)$ and $b_{\mathbf{p}}(t)$ are boson creation and annihilation operators, $p_0 = \sqrt{\mathbf{p}^2 + \Omega^2}$ where Ω is the mass parameter of the bosonic fields, $px = p_0 t - \mathbf{p}.\mathbf{x}$, and $\psi(x)$ and $\bar{\psi}(x)$ are fermionic spin-1/2 fields,

$$\psi(\mathbf{x}, t) = \sum_{\mathbf{k},s}\left(\frac{M}{k_0}\right)^{1/2}\frac{1}{\sqrt{V}}\left[u_1(\mathbf{k}, s)a^{(1)}_{\mathbf{k},s}(t)e^{i\mathbf{k}.\mathbf{x}} + u_2(\mathbf{k}, s)a^{(2)\dagger}_{\mathbf{k},s}(t)e^{-i\mathbf{k}.\mathbf{x}}\right]$$
$$\bar{\psi}(\mathbf{x}, t) = \sum_{\mathbf{k},s}\left(\frac{M}{k_0}\right)^{1/2}\frac{1}{\sqrt{V}}\left[\bar{u}_1(\mathbf{k}, s)a^{(1)\dagger}_{\mathbf{k},s}(t)e^{-i\mathbf{k}.\mathbf{x}} + \bar{u}_2(\mathbf{k}, s)a^{(2)}_{\mathbf{k},s}(t)e^{i\mathbf{k}.\mathbf{x}}\right], \quad (4)$$

where $a^{(1)\dagger}_{\mathbf{k},s}(t)$ and $a^{(1)}_{\mathbf{k},s}(t)$ $\left[a^{(2)\dagger}_{\mathbf{k},s}(t) \text{ and } a^{(2)}_{\mathbf{k},s}(t)\right]$ are fermion creation and annihilation operators associated with the positive- (negative-) energy solutions $u_1(\mathbf{k}, s)$ $[u_2(\mathbf{k}, s)]$ of Dirac's equation. Likewise, $k_0 = \sqrt{\mathbf{k}^2 + M^2}$, where M is the mass parameter of the fermionic fields, and $kx = k_0 t - \mathbf{k}.\mathbf{x}$.

In (2), the parameters m and μ are the masses of the fermion and of the scalar particles, respectively, and g is the coupling constant. The last term on the right-hand side, which encompasses the counterterms necessary to remove the later-occurring infinities, is given by the expression

$$4\pi\mathcal{H}_c = \frac{A}{1!}\phi + \frac{\delta\mu^2}{2!}\phi^2 + \frac{C}{3!}\phi^3 + \frac{D}{4!}\phi^4 - \frac{Z}{2}(\partial\phi)^2, \quad (5)$$

where the coefficients A, $\delta\mu^2$, C, and D are infinite constants, to be defined later. To study the dynamical random-phase approximation (RPA) regime, we have to introduce the wave function renormalization constant Z [25].

To deal with condensate and pairing dynamics of the scalar and fermionic fields, we first define the unitary Bogoliubov transformation for the bosonic sector as follows [33]:

$$\begin{bmatrix} d_{\mathbf{p}}(t) \\ d^\dagger_{-\mathbf{p}}(t) \end{bmatrix} = \begin{bmatrix} \cosh\kappa_{\mathbf{p}} + i\frac{\eta_{\mathbf{p}}}{2} & -\sinh\kappa_{\mathbf{p}} + i\frac{\eta_{\mathbf{p}}}{2} \\ -\sinh\kappa_{\mathbf{p}} - i\frac{\eta_{\mathbf{p}}}{2} & \cosh\kappa_{\mathbf{p}} - i\frac{\eta_{\mathbf{p}}}{2} \end{bmatrix}\begin{bmatrix} \beta_{\mathbf{p}}(t) \\ \beta^\dagger_{-\mathbf{p}}(t) \end{bmatrix}, \quad (6)$$

where $d_{\mathbf{p}}$ is the shift boson operator defined by the expression

$$d_{\mathbf{p}}(t) \equiv b_{\mathbf{p}}(t) - B_{\mathbf{p}}(t)\delta_{\mathbf{p},0} \quad \text{with}$$
$$B_{\mathbf{p}}(t) \equiv \langle b_{\mathbf{p}}(t)\rangle = Tr_{\text{BF}}\left[b_{\mathbf{p}}(t)\mathcal{F}\right]. \tag{7}$$

Here, $\mathcal{F}$ is the unitary many-body density operator [9] describing the system state, and the symbol Tr_{BF} denotes a trace over both bosonic and fermionic variables. Partial traces over bosonic or fermionic variables will be denoted Tr_{B} and Tr_{F}, respectively. For simplicity, we restrict our treatment of the fermionic sector to the Nambu transformation [34], parameterized in the following form, which incorporates the unitarity constraints:

$$\begin{bmatrix} a^{(1)}_{\mathbf{k},s} \\ a^{(2)}_{\mathbf{k},s} \\ a^{(1)\,\dagger}_{-\mathbf{k},s} \\ a^{(2)\,\dagger}_{-\mathbf{k},s} \end{bmatrix} = \begin{bmatrix} \cos\varphi_{\mathbf{k}} & 0 & 0 & -e^{-i\gamma_{\mathbf{k}}}\sin\varphi_{\mathbf{k}} \\ 0 & \cos\varphi_{\mathbf{k}} & e^{-i\gamma_{\mathbf{k}}}\sin\varphi_{\mathbf{k}} & 0 \\ 0 & -e^{i\gamma_{\mathbf{k}}}\sin\varphi_{\mathbf{k}} & \cos\varphi_{\mathbf{k}} & 0 \\ e^{i\gamma_{\mathbf{k}}}\sin\varphi_{\mathbf{k}} & 0 & 0 & \cos\varphi_{\mathbf{k}} \end{bmatrix} \begin{bmatrix} \alpha^{(1)}_{\mathbf{k},s} \\ \alpha^{(2)}_{\mathbf{k},s} \\ \alpha^{(1)\,\dagger}_{-\mathbf{k},s} \\ \alpha^{(2)\,\dagger}_{-\mathbf{k},s} \end{bmatrix}. \tag{8}$$

The procedure adopted in [9] to obtain the equations of motion for the Nambu–Bogoliubov parameters $\varphi_{\mathbf{k}}(t)$, $\gamma_{\mathbf{k}}(t)$, $\eta_{\mathbf{p}}(t)$, and $\kappa_{\mathbf{p}}(t)$; for the quasiparticle occupation numbers $\nu^{(\lambda)}_{\mathbf{k},s} = \langle\alpha^{(\lambda)\dagger}_{\mathbf{k},s}\alpha^{(\lambda)}_{\mathbf{k},s}\rangle$ and $\nu_{\mathbf{p}} = \langle\beta^{\dagger}_{\mathbf{p}}\beta_{\mathbf{p}}\ \rangle$; and for the condensates $\langle\phi\rangle$ and $\langle\Pi\rangle$ had been developed earlier in the context of the nonrelativistic nuclear many-body dynamics by de Toledo Piza [10] and Nemes and de Toledo Piza [11]. That approach follows the line of thought of a time-dependent projection technique proposed by Willis and Picard [35] in the context of the master equation for coupled systems. The method consists, essentially, of writing the correlation information of the full density of the system $\mathcal{F}$ in terms of a memory kernel acting on the uncorrelated density $\mathcal{F}_0$, with the help of a time-dependent projector. At this point, a systematic mean-field expansion for two-point correlations can be performed [10]. The lowest order corresponds to the results of the usual Gaussian approximation [34, 36]. The higher orders describe the dynamical correlation effects between the subsystems and are expressed by means of suitable memory integrals added to the mean-field dynamical equations. The resulting equations acquire the structure of kinetic equations, with the memory integrals playing the role of collisional dynamics terms. This systematic expansion scheme for memory effects, in which the mean energy is conserved to all orders [37–40], was implemented, for example, for the Jaynes–Cummings system [41].

In this context, to study the near-equilibrium dynamics around the stationary solution of the Higgs–Yukawa system, Takano Natti et al. [9] focused on a selected set of Gaussian observables, which are related to the expectation values of linear, $\phi(x)$, $\Pi(x)$, and bilinear forms of field operators, such as $\phi(x)\phi(x)$, $\bar{\psi}(x)\psi(x)$, $\psi(x)\psi(x)$, etc. The time evolution of such quantities obeys the Heisenberg equation of motion

$$i\langle\dot{\mathcal{O}}\rangle = Tr_{\text{BF}}[\mathcal{O}, H]\mathcal{F}, \tag{9}$$

where $\mathcal{O}$ can be $\phi(x)$, $\phi(x)\phi(x)$, $\bar{\psi}(x)\psi(x)$, etc. and $\mathcal{F}$ is the state of the system, which is assumed to be spatially uniform, in the Heisenberg picture. As an approximation, we replace the full density $\mathcal{F}$ by a truncated ansatz $\mathcal{F}_0(t)$, whose trace is also unitary and which implements the double mean-field approximation [9]. By construction, $\mathcal{F}_0$ is written as the most general Hermitian–Gaussian functional of the field operators consistent with the assumed uniformity of the system. It will thus be written as the exponential of a general quadratic form in the field operators, which can be reduced to diagonal form by a suitable canonical transformation. In this way, $\mathcal{F}_0$ reproduces the corresponding $\mathcal{F}$ averages for linear or bilinear field operators [34]. In particular, we have used a formulation appropriate for the many-body problem, where $\mathcal{F}_0$ is written in the momentum basis as [34, 36, 41]

$$\mathcal{F}_0 = \mathcal{F}_0^{\text{B}}\mathcal{F}_0^{\text{F}}$$
$$\mathcal{F}_0^{\text{B}} = \prod_{\mathbf{p}}\left[\frac{1}{1+\nu_{\mathbf{p}}}\left(\frac{\nu_{\mathbf{p}}}{1+\nu_{\mathbf{p}}}\right)^{\beta^{\dagger}_{\mathbf{p}}\beta_{\mathbf{p}}}\right] \tag{10}$$

$$\mathcal{F}_0^{\text{F}} = \prod_{\mathbf{k},s,\lambda}\left[\nu^{(\lambda)}_{\mathbf{k},s}\alpha^{(\lambda)\dagger}_{\mathbf{k},s}\alpha^{(\lambda)}_{\mathbf{k},s} + (1-\nu^{(\lambda)}_{\mathbf{k},s})\alpha^{(\lambda)}_{\mathbf{k},s}\alpha^{(\lambda)\dagger}_{\mathbf{k},s}\right], \tag{11}$$

where α ($\alpha^\dagger$) and β ($\beta^\dagger$) stand for Nambu–Bogoliubov quasiparticle annihilation (creation) operators for fermions and bosons, respectively; $\nu^{(\lambda)}_{\mathbf{k},s}$ ($\nu_{\mathbf{p}}$) are the quasi-fermion (quasi-boson) occupation numbers; and $\lambda = 1$ ($\lambda = 2$) is associated with the positive- (negative-) energy solutions.

From (9)–(11), we can directly obtain the equation of motion for the occupation numbers; for the Gaussian variables, now represented by the Nambu–Bogoliubov parameters; and for the condensates. The following expressions result:

$$\dot{\nu}_{\mathbf{p}} = \dot{\nu}^{(1)}_{\mathbf{k},s} = \dot{\nu}^{(2)}_{\mathbf{k},s} = 0 \tag{12}$$

$$\dot{\varphi}_{\mathbf{k}} = \frac{|\mathbf{k}|}{k_0}(M - \bar{m}) \sin\gamma_{\mathbf{k}} \tag{13}$$

$$\begin{aligned} \sin 2\varphi_{\mathbf{k}}\dot{\gamma}_{\mathbf{k}} = {} & \frac{2(\mathbf{k}^2 + \bar{m}M)}{k_0} \sin 2\varphi_{\mathbf{k}} \\ & + \frac{2(M - \bar{m})}{k_0}|\mathbf{k}| \cos 2\varphi_{\mathbf{k}} \cos\gamma_{\mathbf{k}} \end{aligned} \tag{14}$$

$$\dot{\eta}_{\mathrm{p}} e^{-\kappa_{\mathrm{p}}} = \left(\mathbf{p}^2+\Omega^2\right)^{1/2} \left[4\pi e^{2\kappa_{\mathrm{p}}} - \frac{1}{4\pi}\frac{\left(\mathbf{p}^2+\mu^2\right)}{\left(\mathbf{p}^2+\Omega^2\right)} e^{-2\kappa_{\mathrm{p}}}\right] \tag{15}$$

$$\dot{\kappa}_{\mathrm{p}} = -4\pi \left(\mathbf{p}^2 + \Omega^2\right)^{1/2} \eta_{\mathrm{p}} e^{\kappa_{\mathrm{p}}} \tag{16}$$

$$\langle\dot{\phi}\rangle = \frac{4\pi}{(1 + Z)}\langle\Pi\rangle \tag{17}$$

$$\begin{aligned} \langle\dot{\Pi}\rangle = {} & -\frac{1}{4\pi}\left[A + \frac{C}{2}G(\Omega)\right] \\ & -\frac{1}{4\pi}\left[\mu^2 + \delta\mu^2 + \frac{D}{2}G(\Omega)\right]\langle\phi\rangle - \frac{C}{8\pi}\langle\phi\rangle^2 \\ & -\frac{D}{24\pi}\langle\phi\rangle^3 - g\sum_s \int_{\mathbf{k}} \frac{1}{k_0} \\ & \times\left[M\cos 2\varphi_{\mathbf{k}} + |\,\mathbf{k}\,| \sin 2\varphi_{\mathbf{k}} \cos\gamma_{\mathbf{k}}\right]\left(1 - \nu^{(1)}_{\mathbf{k},s} - \nu^{(2)}_{\mathbf{k},s}\right), \end{aligned} \tag{18}$$

where we have introduced the notation

$$G(\Omega) = \int_{\mathbf{p}} \frac{1 + 2\nu_{\mathrm{p}}}{2\sqrt{\mathbf{p}^2 + \Omega^2}}. \tag{19}$$

In (12)–(19), the quantities M and Ω are the mass parameters of the fermionic ψ and bosonic ϕ fields of the Hamiltonian (2), while $\bar{m} \equiv m - g\langle\phi\rangle$ stands for the effective mass of a fermion particle [9].

Another physical quantity of interest is the mean-field energy density of the system,

$$\begin{aligned} \frac{\langle H\rangle}{V} = {} & \frac{1}{V} Tr H \mathcal{F}_0 \\ = {} & -\sum_s \int_{\mathbf{k}} \left[\frac{(\mathbf{k}^2 + \bar{m}M)}{k_0} \cos 2\varphi_{\mathbf{k}} \right. \\ & \left. + \frac{(\bar{m} - M)}{k_0}|\mathbf{k}| \sin 2\varphi_{\mathbf{k}} \cos\gamma_{\mathbf{k}}\right] \\ & \times\left(1 - \nu^{(1)}_{\mathbf{k},s} - \nu^{(2)}_{\mathbf{k},s}\right) \\ & + \frac{1}{8\pi}\left[\langle\Pi\rangle^2 + \mu^2\langle\phi\rangle^2\right] + \frac{1}{4\pi}\left[A + \frac{C}{2}G(\Omega)\right]\langle\phi\rangle \\ & + \frac{1}{8\pi}\left[\delta\mu^2 + \frac{D}{2}G(\Omega)\right]\langle\phi\rangle^2 + \frac{C}{24\pi}\langle\phi\rangle^3 + \frac{D}{96\pi}\langle\phi\rangle^4 \\ & + \frac{1}{8\pi}\left[\mu^2 + \delta\mu^2\right]G(\Omega) + \frac{D}{32\pi}G^2(\Omega). \end{aligned} \tag{20}$$

The above equations describe the real-time evolution of the Higgs–Yukawa system in the double-Gaussian mean-field approximation. The results obtained in (12)–(20), as discussed in [9], are consistent with those in the literature, obtained via different approaches, in particular with those obtained in Ref. [24] on the basis of a Vlasov–Hartree approximation.

Reference [9] showed, in detail, that the usual form of renormalization [24] is applicable to the nonperturbative procedure described in that paper. A simple numerical calculation has also shown that the system always has a single stable minimum, although, as it has been suggested [9], additional investigation is necessary concerning oscillatory modes. The standard approach to this question uses the RPA analysis, with the resulting eigenvalues giving an indication of stability [12, 13, 25].

Finally, we note that dynamical correlation corrections can in principle be systematically added to the double-Gaussian mean-field calculations with the help of a projection technique discussed in [10, 39, 41]. The occupation numbers are then no longer constant, a modification that affects the effective dynamics of the Gaussian observables. The framework presented in this paper also serves as groundwork for finite-density and finite-temperature discussions [42]. In particular, a finite matter-density calculation beyond the mean-field approximation allows one to study such collisional observables as the transport coefficients. The extension of this procedure to nonuniform systems is straightforward, albeit long. In this case, the spatial dependence of the field is expanded in natural orbitals of the extended one-body density. A more general Bogoliubov transformation [33] would relate these orbitals to a momentum expansion.

The results in (12)–(16) are nonlinear time-dependent field equations. A closed solution is not easily constructed.

Here, we consider those equations in the small oscillation regime and find a closed solution offering insight into diverse properties of the theory.

The paper is structured as follows: In Section 2, the RPA equations are derived for this model by considering near-equilibrium dynamics around the stationary solutions obtained in Ref. [9]. Section 3 finds analytical solutions for the RPA equations by using a well-known procedure from the scattering theory. Section 4 discusses renormalization within the context of scattering amplitudes and discusses the existence of bound state solutions. Finally, we apply these results to find limits to the intensity of the coupling constant g of the Higgs–Yukawa model, in the RPA mean-field approximation, for three decay channels of the Higgs boson. Section 5 presents our conclusions.

2 Near-Equilibrium Dynamics

The energy density (20) is a function of the Nambu–Bogoliubov parameters $\varphi_{\mathbf{k}}(t)$, $\gamma_{\mathbf{k}}(t)$, $\eta_{\mathbf{p}}(t)$, and $\kappa_{\mathbf{p}}(t)$, of the quasiparticle occupation numbers $\nu^{(\lambda)}_{\mathbf{k},s} = \langle \alpha^{(\lambda)\dagger}_{\mathbf{k},s} \alpha^{(\lambda)}_{\mathbf{k},s} \rangle$ and $\nu_{\mathbf{p}} = \langle \beta^{\dagger}_{\mathbf{p}} \beta_{\mathbf{p}} \rangle$, and of the condensates $\langle\phi\rangle$ and $\langle\Pi\rangle$. The minimum in (20) corresponds to the ground state of the system. The small-amplitude motion around the minimum is obtained by linearization of the Gaussian motion equations (12)–(16), yielding a set of harmonic oscillators [43]. The eigenvalues and the normal modes of these small oscillations are the RPA solutions. Physically, the RPA solutions are seen as the energy and the wave functions of quantum particles. This section derives the RPA equations of the model, whose solutions are discussed in Section 3.

First, we have to consider the stationary problem [9]. We only have to recall (12)–(16) to see that $\dot{\gamma}_{\mathbf{k}} = \dot{\varphi}_{\mathbf{k}} = \dot{\kappa}_{\mathbf{p}} = \dot{\eta}_{\mathbf{p}} = \langle\dot{\phi}\rangle = \langle\dot{\Pi}\rangle = 0$ under stationary conditions. Reference [9] discussed the renormalization conditions and the solutions for this set of stationary equations in detail. In particular, for the renormalization coefficients of $\mathcal{H}_c$, the following self-consistency renormalization prescription was chosen [9, 25]:

$$D = \pm 48\pi g^4 L(m) \tag{21}$$

$$\delta\mu^2 = \mp 24\pi g^4 L(m) G(\mu) \mp 16\pi g^2 G(0) \pm 24\pi m^2 g^2 L(m) \tag{22}$$

$$C = \mp 48\pi m g^3 L(m) \tag{23}$$

$$A = \pm 24\pi m g^3 L(m) G(\mu) \pm 16\pi m g G(m) \tag{24}$$

with

$$L(m) \equiv \int_{\mathbf{k}} \frac{1}{2\mathbf{k}^2(\mathbf{k}^2+m^2)^{1/2}}, \tag{25}$$

where $M = m$, without loss of generality [9].

The substitution of such counterterms in the stationary equations yields the appropriate cancelations, which makes the equations finite, except for the combination of the type $L(m)[G(\mu) - G(\Omega)]$. Since Ω is an arbitrary expansion mass parameter, one can remove this divergence by setting $\Omega = \mu$ [9]. The resulting finite stationary equations for the system can be regrouped as follows:

$$\sin\gamma_{\mathbf{k}}|_{\text{eq}} = 0 \tag{26}$$

$$\cot 2\varphi_{\mathbf{k}}|_{\text{eq}} = -\frac{(\mathbf{k}^2 + \bar{m}m)}{|\mathbf{k}|(m-\bar{m})} \tag{27}$$

$$\eta_{\text{p}}|_{\text{eq}} = 0 \tag{28}$$

$$\kappa_{\mathbf{p}}|_{\text{eq}} = 0 \quad \text{with} \quad \Omega = \mu \tag{29}$$

$$\langle\Pi\rangle|_{\text{eq}} = 0 \tag{30}$$

$$\frac{\pi}{2}\mu^2\langle\phi\rangle|_{\text{eq}} - g\bar{m}^3\left[\ln\left(\frac{\bar{m}}{m}\right) + \frac{1}{2}\right] = 0. \tag{31}$$

Equations (26)–(31) can be numerically solved for any given μ and g, in units of m, as shown by Takano Natti et al. [9].

To obtain the near-equilibrium dynamics (RPA regime), we examine the fluctuations around the stationary solution, namely,

$$\begin{aligned}
\varphi_{\mathbf{k}} &= \varphi_{\mathbf{k}}|_{\text{eq}} + \delta\varphi_{\mathbf{k}} \\
\gamma_{\mathbf{k}} &= \gamma_{\mathbf{k}}|_{\text{eq}} + \delta\gamma_{\mathbf{k}} \\
\eta_{\mathbf{p}} &= \eta_{\mathbf{p}}|_{\text{eq}} + \delta\eta_{\mathbf{p}} \\
\kappa_{\mathbf{p}} &= \kappa_{\mathbf{p}}|_{\text{eq}} + \delta\kappa_{\mathbf{p}} \\
\langle\phi\rangle &= \langle\phi\rangle|_{\text{eq}} + \delta\langle\phi\rangle \\
\langle\Pi\rangle &= \langle\Pi\rangle|_{\text{eq}} + \delta\langle\Pi\rangle,
\end{aligned} \tag{32}$$

where $\varphi_{\mathbf{k}}|_{\text{eq}}$, $\gamma_{\mathbf{k}}|_{\text{eq}}$, $\eta_{\mathbf{p}}|_{\text{eq}}$, $\kappa_{\mathbf{p}}|_{\text{eq}}$, $\langle\phi\rangle|_{\text{eq}}$ and $\langle\Pi\rangle|_{\text{eq}}$ satisfy (26)–(31) and the deviations $\delta\varphi_{\mathbf{k}}$, $\delta\gamma_{\mathbf{k}}$, $\delta\eta_{\mathbf{p}}$, $\delta\kappa_{\mathbf{p}}$, $\delta\langle\phi\rangle$ and $\delta\langle\Pi\rangle$ are assumed to be small.

Next, we expand (12)–(16) to first order in the fluctuations. The following equations result:

$$\delta\dot{\varphi}_{\mathbf{k}} = g\,\langle\phi\rangle|_{\text{eq}}\,\frac{|\mathbf{k}|}{k_0}\,\delta\gamma_{\mathbf{k}} \tag{33}$$

$$g\,\langle\phi\rangle|_{\text{eq}}\,|\mathbf{k}|\,\delta\dot{\gamma}_{\mathbf{k}} = -4\,k_0\,(\mathbf{k}^2+\bar{m}^2)\,\delta\varphi_{\mathbf{k}} - 2\,g\,|\mathbf{k}|\,k_0\,\delta\langle\phi\rangle \tag{34}$$

$$\delta\langle\dot{\phi}\rangle = \frac{4\pi}{(1+Z)}\,\delta\langle\Pi\rangle \tag{35}$$

$$\delta\langle\dot{\Pi}\rangle = -\left(\frac{\mu^2}{4\pi} + \frac{\delta\mu^2}{4\pi} + \frac{D}{2}\,G(\mu)\right)\delta\langle\phi\rangle - \frac{C}{4\pi}\,\langle\phi\rangle|_{\text{eq}}\,\delta\langle\phi\rangle - \frac{D}{8\pi}(\langle\phi\rangle|_{\text{eq}})^2\delta\langle\phi\rangle + \frac{4\,g}{(2\pi)^3}\int_{\mathbf{k}'}\frac{|\mathbf{k}'|}{(\mathbf{k}'^2+\bar{m}^2)^{1/2}}\delta\varphi_{\mathbf{k}'}. \tag{36}$$

In the RPA regime, the bosonic variables show no dynamical evolution.

The quantities $\delta\gamma_{\mathbf{k}}$ and $\delta\langle\Pi\rangle$ can be eliminated by differentiating (33) and (35) with respect to time, so that (33)–(36) are rewritten as the second-order differential equations

$$\delta\ddot{\varphi}_{\mathbf{k}} = -4\bar{k}_0^2\delta\varphi_{\mathbf{k}} - 2g|\mathbf{k}|\delta\langle\phi\rangle \tag{37}$$

$$(1+Z)\delta\langle\ddot{\phi}\rangle = -\left(\mu^2+\Sigma\right)\delta\langle\phi\rangle + 16\pi g\int_{\mathbf{k}'} h(\mathbf{k}')\delta\varphi_{\mathbf{k}'}, \tag{38}$$

with the notation

$$h(\mathbf{k}) = \frac{|\mathbf{k}|}{\bar{k}_0} \tag{39}$$

where

$$\bar{k}_0 = \sqrt{\mathbf{k}^2+\bar{m}^2} \tag{40}$$

$$\Sigma \equiv \delta\mu^2 + \frac{D}{2}4\pi G(\mu) + C\langle\phi\rangle|_{\text{eq}} + \frac{D}{2}(\langle\phi\rangle|_{\text{eq}})^2. \tag{41}$$

The small oscillation dynamics of the Higgs–Yukawa system is therefore described by coupled equations of linear oscillators, as usual in the RPA treatment [43]. In particular, when $g = 0$, these modes are decoupled and yield two equations describing simple oscillators.

To solve the problem (37, 38), we have to determine the normal modes of the small oscillations and their frequencies. Earlier studies have demonstrated that these elementary excitations can be interpreted as quantum particles. In our case, $\delta\varphi_{\mathbf{k}}$ can be seen as two-fermion spinless wave function [44], while $\delta\langle\phi\rangle$ provides the one-boson physics of the system [12]. The relative momentum of the two-fermion states is $|\mathbf{k}|$ [44], while in the scalar sector, the particles have no momentum dependence.

3 RPA Equations as a Scattering Problem

Section 2 obtained the linear approximation for the Gaussian equations of motion (37, 38). We will now show that these coupled linear oscillator equations can be analytically solved, to determine the wave functions and the elementary excitation spectrum of our system.

We first consider the Fourier transform of the wave functions in the energy representation, i.e., the standard relations

$$\delta\varphi_{\mathbf{k}}(t) = \int d\omega\, \delta\varphi_{\mathbf{k}}(\omega)\, e^{i\omega t}$$

$$\delta\langle\phi\rangle(t) = \int d\omega\, \delta\langle\phi\rangle(\omega)\, e^{i\omega t}, \tag{42}$$

where $\delta\varphi_{\mathbf{k}}(\omega)$ and $\delta\langle\phi\rangle(\omega)$ are now energy-dependent amplitudes.

We then substitute (42) into (37, 38), to obtain the following equations:

$$\left(\omega^2 - 4\bar{k}_0^2\right)\delta\varphi_{\mathbf{k}}(\omega) = 2g\,|\mathbf{k}|\,\delta\langle\phi\rangle(\omega) \tag{43}$$

$$\left(\omega^2-\mu^2+Z\omega^2-\Sigma\right)\delta\langle\phi\rangle(\omega) = -16\pi g\int_{\mathbf{k}'} h(\mathbf{k}')\delta\varphi_{\mathbf{k}'}(\omega). \tag{44}$$

Since the oscillation amplitudes in (43, 44) play the roles of wave functions of quantum particles, it is more convenient to treat this system as a coupled-channel scattering problem with appropriate boundary conditions [13]. The following discussion will focus on the scattering process, where the source is a two-fermion wave. In this case, from (44), we have that

$$\delta\langle\phi\rangle(\omega) = \left(\frac{-16\pi g}{\omega^2-\mu^2+Z\omega^2-\Sigma}\right)\int_{\mathbf{k}'} h(\mathbf{k}')\,\delta\varphi_{\mathbf{k}'}(\omega). \tag{45}$$

Substitution of (45) into (43) then yields the result

$$\left(\frac{\omega^2-4\bar{k}_0^2}{\bar{k}_0}\right)\delta\varphi_{\mathbf{k}}(\omega) = \left(\frac{-32\pi g^2}{\omega^2-\mu^2+Z\omega^2-\Sigma}\right)\frac{|\mathbf{k}|}{\bar{k}_0}\int_{\mathbf{k}'}\frac{|\mathbf{k}'|}{\bar{k}_0'}\delta\varphi_{\mathbf{k}'}(\omega), \tag{46}$$

where the Green's function includes the effects of coupling $\delta\varphi_{\mathbf{k}}$ to $\delta\langle\phi\rangle$.

The potential is separable [45], in the sense that

$$\langle\mathbf{k}|V|\mathbf{k}'\rangle = v(\mathbf{k})\,v(\mathbf{k}') = \frac{|\mathbf{k}|}{\bar{k}_0}\,\frac{|\mathbf{k}'|}{\bar{k}_0'}. \tag{47}$$

In the general solution of (46), the two-fermion wave function $\delta\varphi_{\mathbf{k}}(\omega)$ will have two terms. The first one is the free solution ($g = 0$), which represents an incident wave. The second term is the nontrivial part, arising when $g \neq 0$, which couples different momenta and is associated with the scattered wave [44, 45]. Therefore,

$$\frac{|\mathbf{k}|}{\bar{k}_0}\,\delta\varphi(\mathbf{k},\mathbf{q};\omega) = \alpha\,\delta(\mathbf{q}-\mathbf{k}) + \frac{1}{[\omega^2-4\bar{k}_0^2+i\epsilon]}\times\frac{-32\pi g^2}{[\omega^2-\mu^2+Z\omega^2-\Sigma]}\times\frac{|\mathbf{k}|^2}{\bar{k}_0}\int_{\mathbf{k}'}\frac{|\mathbf{k}|}{\bar{k}_0}\,\delta\varphi(\mathbf{k}',\mathbf{q};\omega), \tag{48}$$

where $\mathbf{q}$ is the relative momentum for two incident quasi-fermions and α is an overall phase factor. The outgoing-wave ($+i\epsilon$) boundary condition was used to solve (46), but other conditions, e.g., the incoming-wave condition ($-i\epsilon$) or Van Kampen wave condition [46], could alternatively have been chosen.

The integral equation (48) can be solved as usual [45]. We integrate both sides with respect to $\mathbf{k}$ to obtain the expression

$$\int_{\mathbf{k}} v(\mathbf{k})\delta\varphi(\mathbf{k},\mathbf{q};\omega) = \frac{\alpha}{1+\left(\frac{32\pi g^2}{\omega^2-\mu^2+Z\omega^2-\Sigma}\right) I^+(\omega)}, \tag{49}$$

where

$$I^+(\omega) = \int_{\mathbf{k}} \frac{|\mathbf{k}|^2}{\bar{k}_0\left[\omega^2-4\bar{k}_0^2+i\epsilon\right]} \tag{50}$$

with $\bar{k}_0 = \sqrt{\mathbf{k}^2+\bar{m}^2}$, while $\bar{m} \equiv m - g\langle\phi\rangle|_{\rm eq}$ stands for the effective mass of a fermion particle and $\langle\phi\rangle|_{\rm eq}$ is given by (31). Substitution of this last result in (48) yields the equality

$$\frac{|\mathbf{k}|}{\bar{k}_0}\delta\varphi(\mathbf{k},\mathbf{q};\omega) = \alpha\,\delta(\mathbf{q}-\mathbf{k}) - \left(\frac{\alpha\bar{k}_0}{\omega^2-4\bar{k}_0^2+i\epsilon}\right) \times\frac{|\mathbf{k}|}{\bar{k}_0}\frac{1}{\Delta^+(\omega)}\frac{|\mathbf{k}|}{\bar{k}_0} \tag{51}$$

with

$$\Delta^+(\omega) = \frac{1}{32\pi g^2}\left(\omega^2-\mu^2+Z\omega^2-\Sigma\right)+I^+(\omega). \tag{52}$$

Finally, substitution of (49) in (44) determines the oscillation frequencies

$$\omega = 2\bar{q}_0^2 = 2\sqrt{\mathbf{q}^2+\bar{m}^2}, \tag{53}$$

where $\mathbf{q}$ is the relative momentum for two incident quasi-fermions with mass $\bar{m}$.

We have therefore found an analytical solution for the elastic channel of the two-fermion scattering problem defined by (43) and (44).

Thanks to the special form of the interacting potential, the following closed expression for scattering matrix can also be obtained [45]:

$$T(\mathbf{k},\mathbf{k}';\omega) = v(\mathbf{k})\frac{1}{\Delta^+(\omega)}v(\mathbf{k}') \tag{54}$$

with $\Delta^+(\omega)$ given by (52).

In summary, this section discussed the solutions of the RPA equations. These elementary excitations describe a coupled-channel scattering problem. The particular case of two-fermion elastic process was studied. Given the simple interacting potential, we were able to obtain closed expressions for the two-fermion wave function and the scattering matrix. Several dynamical behaviors can be read off from $\Delta^+(\omega)$. The remaining problem is the divergent integral $I^+(\omega)$ in (52), which will be removed with the help of counterterms.

In Section 4, we will see that, in addition to the counterterms used in the stationary-state calculation [9], a convenient wave function renormalization constant Z will have to be chosen.

4 Renormalization and Bound State Solution

We now use the framework developed in Refs. [12, 13, 44] to investigate the conditions for the existence of bound states of Dirac spin-1/2 particles in a Higgs–Yukawa system. The standard procedure is to analyze the positions of the poles of the scattering matrix (54). Equation (52), however, contains a divergent integral. We will next show that the divergent terms can be kept directly under control with the help of (21)–(24) and a convenient choice for Z, which yields a finite expression for $\Delta^+(\omega)$.

We therefore substitute the counterterms (21)–(24) in (52) and, after some algebra, obtain the expression

$$\Delta^+(\omega) = \frac{1}{32\pi g^2}\Big[(1+Z)\omega^2-\mu^2+16\pi g^2 G(0) -24\pi g^2 M^2 L(m)\Big]+I^+(\omega) \tag{55}$$

with $I^+(\omega)$ given by (50).

In the interval $0 < \omega < 2\bar{m}$, the integral I_ω is well defined, and we can let $\epsilon = 0$. For $\omega \geq 2\bar{m}$, on the other hand, the spectrum defines a continuum. Straightforward calculation yields the result

$$I(\omega) = Q - \frac{1}{8\pi^2}F(\omega) - \theta\left(\omega^2-4\bar{m}^2\right)\frac{i}{8\pi}\left[\omega^2-4\bar{m}^2\right], \tag{56}$$

with

$$Q = \frac{1}{4\pi}\left[\Lambda^2+\left(\frac{\omega^2}{2}-3\bar{m}^2\right)\log\frac{2\Lambda}{m}\right], \tag{57}$$

where a regularizing momentum cutoff Λ was introduced, and the finite term $F(\omega)$ is given by the relation

$$F(\omega) = \left(\omega^2-6\bar{m}^2\right)\log\left(\frac{\bar{m}}{2m}\right)+\frac{2\left(4\bar{m}^2-\omega^2\right)^{3/2}}{\omega} \tan^{-1}\sqrt{\frac{\omega^2}{4\bar{m}^2-\omega^2}} \qquad 0<\omega^2<4\bar{m}^2 \tag{58}$$

$$F(\omega) = \left(\bar{m}^2-6\omega^2\right)\log\left(\frac{\bar{m}}{2m}\right)+\frac{2\left(\omega^2-4\bar{m}^2\right)^{3/2}}{\omega} \log\frac{\omega+\sqrt{\omega^2-4\bar{m}^2}}{\omega-\sqrt{\omega^2-4\bar{m}^2}} \qquad \omega^2\geq 4\bar{m}^2. \tag{59}$$

In (56), θ is the Heaviside function, defined by the relations

$$\theta\left(\omega^2-4\bar{m}^2\right) = 0 \qquad \text{if} \qquad \omega^2<4\bar{m}^2$$
$$\theta\left(\omega^2-4\bar{m}^2\right) = 1 \qquad \text{if} \qquad \omega^2\geq 4\bar{m}^2. \tag{60}$$

From (55)–(57), we can immediately see that there is still a logarithmic divergence. To cancel it, we choose the following wave function renormalization [25]:

$$Z \equiv 4\pi g^2 L(m) \ . \tag{61}$$

The resulting finite expression is

$$\Delta^+(\omega) = -\frac{\pi\mu^2}{g^2m^2} + F(\omega) - \theta\left(\omega - 4\bar{m}^2\right)\frac{i}{8\pi}\left[\omega^2 - 4\bar{m}^2\right] . \tag{62}$$

The derivation of (62) fixed several counterterms, given by (21)–(24), and (61), to eliminate the divergences. These counterterms are not unique, since they are well defined except for a finite additive constant. This makes the results dependent on the renormalization scheme. The arbitrary finite constants are usually determined by high-energy experiments. The dependence on the renormalization scale is therefore often used to estimate the accuracy of the theory. In the case of a scalar plasma system, or Higgs–Yukawa system, Takano Natti et al. [9] and Alonso and Hakim [24, 25] have discussed the determination of these arbitrary finite constants to obtain the finite stationary (26)–(31) in canonical form.

We next face the problem of obtaining the poles of the scattering matrix when

$$\Delta^+(\omega) = 0 \ . \tag{63}$$

Depending on ω, the system has different dynamical behaviors [44]. For $\omega^2 < 0$, the system is unstable, since the exponentials on the right-hand sides of (42) become real. For $\omega^2 > 0$, by contrast, the system is in the scattering regime. The solution of interest lies in the interval $0 < \omega^2 < 4\bar{m}^2$. In this interval, the system may have a stable bound state if there exist ω_B such that $\Delta^+(\omega_B) = 0$. Figure 1 shows $\Delta^+(\omega)$ as a function of ω/m, when $g = 1$, for three combinations of μ, in unit of m. Also for $g = 1$, $\Delta^+(\omega)$ has a single (no) zero when $\mu/m < 1.794$ ($\mu/m > 1.794$). A natural interpretation considers that, at fixed coupling, the boson mass determines the range of the Yukawa potential. When μ is large, it is more difficult for the fermions to interact, and consequently, the probability of forming a bound state decreases. This behavior is, however, compensated by increases in g, as shown in Fig. 2, which plots the condition (63) in the $(\mu/m\ , \ g)$ plane.

Figure 2 shows that the behavior of $\mu/m \times g$ becomes nearly linear for $g > 50$. We can apply these results, obtained from the Higgs–Yukawa model in the RPA mean-field approximation, to recent observations at the Large Hadron Collider (LHC, ATLAS and CMS collaborations), which led to the announcement of a possible observation of a Higgs boson [14–16]. In the experiments, five decay channels of the Higgs boson ϕ were observed, i.e., $\phi \to \gamma\gamma$, $\phi \to b\bar{b}$, $\phi \to \tau^+\tau^-$, $\phi \to WW$, and $\phi \to ZZ$.

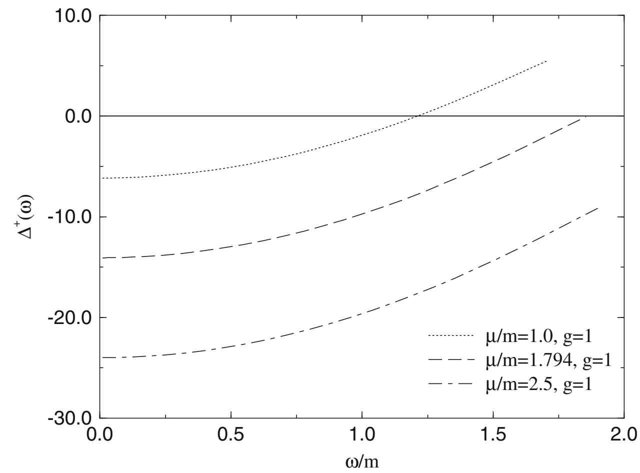

Fig. 1 The behavior of the function $\Delta^+(\omega)$ as a function of energy ω, in unit of m, for several values of the μ/m, when $g = 1$ is fixed

The production of Higgs bosons in proton–proton collisions is known to occur through multiple channels, with branching ratios dependent on the mass of the Higgs boson. Reference [47] presents the branching ratios of the Higgs decay channels as a function of its mass. For Higgs masses below 130 GeV/c^2, the Higgs boson is expected to decay mainly in the following fermions: bottom quarks b, charmed quarks c, and tau leptons τ.

From Fig. 2, we can determine limits to the intensity of the coupling constant g of the Higgs–Yukawa model, in the RPA mean-field approximation, for each decay channel of the Higgs boson. Let us consider a Higgs boson mass of $m_\phi = 125$ GeV/c^2 and masses $m_\mathrm{b} = 4.2$ GeV/c^2, $m_\tau = 1.8$ GeV/c^2, and $m_\mathrm{c} = 1.3$ GeV/c^2 for the botton quarks b, tau leptons τ, and charmed quarks c, respectively [47].

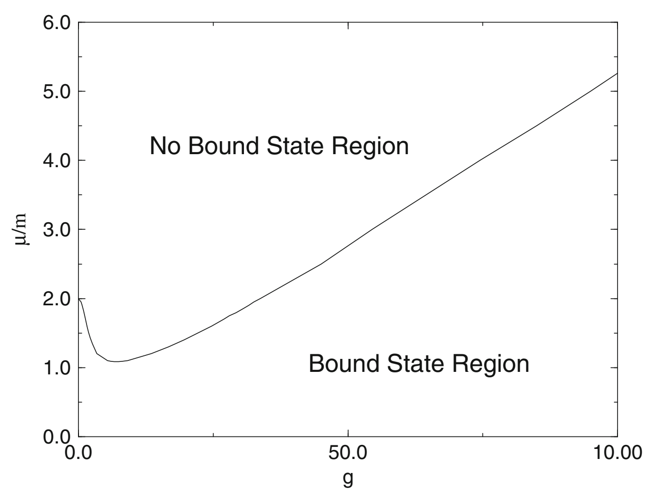

Fig. 2 Existence of bound state of two fermion as a function of parameters μ/m and g

We then have the following ratios between the Higgs boson mass and the masses of the three fermions:

$$\frac{m_\phi}{m_\mathrm{b}} \approx 30 \quad , \quad \frac{m_\phi}{m_\tau} \approx 70 \quad , \quad \frac{m_\phi}{m_\mathrm{c}} \approx 95. \tag{64}$$

Therefore, for these mass ratios and no bound states of fermions (decay channels), the intensity of the Higgs–Yukawa coupling for the decay channel $\phi \to b\bar{b}$ is limited by the condition $g(\phi, b) < 570$. Similarly, for the decay channel $\phi \to \tau^+\tau^-$, one obtains $g(\phi, \tau) < 1{,}300$, and for the decay channel $\phi \to c\bar{c}$, one obtains $g(\phi, c) < 1{,}800$.

Figure 2 shows that, in our approach, only two-fermion bound states can exist for $\mu/m < 1.1$. These results contradict the predictions of the Higgs boson decaying into top quarks, since $m_t = 173$ GeV/c^2 yields $m_\phi/m_t = 125/173 \approx 0.7$. Our calculations indicate that only Higgs bosons with masses larger than 190 GeV/c^2 can decay into top quarks. These results are consistent with those in the literature [47].

Finally, we want to emphasize that the phase diagram in Fig. 2 was obtained in RPA mean-field approximation. As discussed in Refs. [41, 48–50], the contribution of the collisional effects grows with the coupling constant g. For large g, one finds large nonunitary contributions from the collisional effects. For coupling constants in the range $0 < g < 100$, the corrections, i.e., the collisional terms, cannot be neglected. Systematic corrections adding dynamical correlation effects to the RPA mean-field calculations can in principle be readily obtained with the help of a projection technique discussed in Refs. [9, 39, 41]. The resulting occupation numbers are no longer constant and affect the effective dynamics of the Gaussian observables. In particular, a finite matter-density calculation beyond the mean-field approximation would allow the study of such collisional observables as the transport coefficients.

5 Conclusions

Takano Natti et al. [9] treated the initial value problem in a quantum-field theory of interacting fermion–scalar field theories in the Gaussian approximation. Although quite general, the procedure was implemented for the vacuum of an uniform (3 + 1) dimensional relativistic quantum Higgs–Yukawa model. The TDHF renormalized kinetic equations describing the effective dynamics of the Gaussian observables in the mean-field approximation were obtained.

The present work has adapted a nonperturbative framework, the Kerman–Lin procedure [10–12], to investigate the near-equilibrium dynamics close to the stationary solution of arbitrary interacting fermion–scalar field theories. As an application, we have chosen to describe the RPA excitation of the Higgs–Yukawa system at zero temperature.

We have studied the linearized form of the mean-field kinetic equations in Ref. [9] around the stationary (vacuum) solution. In this context, the RPA oscillation amplitudes of excitations were identified with the wave functions of quantum particles and the resulting equations enabled us to study scattering processes, nonperturbatively. These RPA equations were solved analytically by well-known scattering-theory procedures, which yielded a simple form for the scattering amplitude. We have also shown that the usual definitions of counterterms can be applied to the resulting expression, from which relevant physical aspects of the system excitations can be obtained. In particular, the results indicate that bound states exist in a certain region of the phase diagram.

We have applied our results to recent observations at the LHC ATLAS and CMS collaborations. We have obtained limits for the intensity of the coupling constant g of the Higgs–Yukawa model, in the RPA mean-field approximation, for three decay channels of the Higgs boson.

Finally, we comment that, in principle, systematic corrections can readily be added to the RPA mean-field treatment with the help of a projection technique discussed in Refs. [9, 39, 41]. The no longer constant occupation numbers will affect the effective dynamics of the Gaussian observables. The framework in this paper also serves as groundwork to discussions of finite densities and finite temperatures [42]. In particular, a finite matter-density calculation beyond the mean-field approximation allows one to study collisional observables, such as transport coefficients. The extension of this procedure to explore nonuniform systems is straightforward; unfortunately, it is tedious. In this case, the spatial dependence of the field is expanded in natural orbitals of the extended one-body density. These orbitals can be expressed in terms of a momentum expansion by means of a more general Bogoliubov transformation [33, 35].

Acknowledgments The author P. L. Natti thanks the State University of Londrina for the financial support received from the FAEPE programs.

References

1. J. Berges, Introduction to nonequilibrium quantum field theory. AIP Conf. Proc. **739**, 3 (2004)
2. M. Garny, A. Hohenegger, A. Kartavtsev, M. Lindner, Systematic approach to leptogenesis in nonequilibrium QFT: self-energy contribution to the CP-violating parameter. Phys. Rev. D **81**, 085027 (2010)
3. A. Chodos, F. Cooper, W. Mao, A. Singh, Equilibrium and nonequilibrium properties associated with the chiral phase transition at finite density in the Gross-Neveu model. Phys. Rev. D **63**, 096010 (2001)
4. B. Mohanty, J. Serreau, Disoriented chiral condensate: theory and experiment. Phys. Rep. **414**, 263 (2005)

5. R.L.S. Farias, N.C. Cassol-Seewald, G. Krein, R.O. Ramos, Nonequilibrium dynamics of quantum fields. Nucl. Phys. A **782**, 33 (2007)
6. C.Y. Lin, E.J.V. Passos, A.F.R. de Toledo Piza, D.S. Lee, M.S. Hussein, Bogoliubov theory for mutually coherent hybrid atomic molecular condensates: quasiparticles and superchemistry. Phys. Rev. A **73**, 013615 (2006)
7. S. Gopalakrishnan, B.L. Lev, P.M. Goldbart, Atom-light crystallization of BECs in multimode cavities: nonequilibrium classical and quantum phase transitions, emergent lattices, supersolidity, and frustration. Phys. Rev. A **82**, 043612 (2010)
8. M. Eckstein, A. Hackl, S. Kehrein, M. Kollar, M. Moeckel, P. Werner, F.A. Wolf, New theoretical approaches for correlated systems in nonequilibrium. Eur. Phys. J. Special Topics **180**, 217 (2010)
9. E.R. Takano Natti, C.Y. Lin, A.F.R. de Toledo Piza, P.L. Natti, Initial-value problem in quantum field theory: an application to the relativistic scalar plasma. Phys. Rev. D **60**, 125013 (1999)
10. A.F.R. de Toledo Piza, in *Lectures Notes in Physics,* vol 171, ed. by K. Goeke, P.G. Reinhardt. Time-dependent Hartree-Fock and Beyond (Springer, Berlin, 1982)
11. M.C. Nemes, A.F.R. de Toledo Piza, Effective dynamics of quantum systems. Phys. A **137**, 367 (1986)
12. A. Kerman, C.Y. Lin, Time-dependent variational principle for ϕ^4 field theory: 1. RPA approximation and renormalization. Ann. Phys. (N.Y.) **241**, 185 (1995)
13. A. Kerman, C.Y. Lin, Time-dependent variational principle for ϕ^4 field theory: RPA approximation and renormalization (II). Ann. Phys. (N.Y.) **269**, 55 (1998)
14. F. Gianotti, CERN seminar: update on the Standard Model Higgs searches in ATLAS (2012). http://cdsweb.cern.ch/record/1460439/files/ATLAS-CONF-2012-093.pdf. Accessed 1 Apr 2013
15. J. Incandela, CERN seminar: update on the Standard Model Higgs searches in CMS (2012). http://cdsweb.cern.ch/record/1460438/files/HIG-12-020-pas.pdf. Accessed 1 Apr 2013
16. D. Carmia, A. Falkowskib, E. Kuflik, T. Volanskya, J. Zupand, Higgs after the discovery: a status report (2012). http://arxiv.org/pdf/1207.1718.pdf. Accessed 1 Apr 2013
17. P. Gerhold, K. Jansen, J. Kallarackal, The Higgs boson resonance width from a chiral Higgs-Yukawa model on the lattice. Phys. Lett. B **710**, 697–702 (2012)
18. P. Gerhold, K. Jansen, J. Kallarackal, Higgs boson mass bounds in the presence of a very heavy fourth generation quark. J. High Energy Phys. **1101**, article 143 (2011)
19. G. Kalman, Equilibrium and linear response of a classical scalar plasma. Phys. Rev. **161**, 156 (1967)
20. G. Kalman, Relativistic fermion gas interacting through a scalar field. I. Hartree approximation. Phys. Rev. D **9**, 1656 (1974)
21. J.D. Walecka, A theory of highly condensed matter. Ann. Phys. (N.Y.) **83**, 491 (1974)
22. R. Hakim, Statistical-mechanics of relativistic dense matter. Riv. Nuovo Cim. **1**, 1 (1978)
23. M. Wakamatsu, A. Hayashi, Phase-transition to abnormal nuclear-matter at finite temperature and finite barion density. Prog. Theor. Phys. **63**, 1688 (1980)
24. J.D. Alonso, R. Hakim, Quantum fluctuations of the relativistic scalar plasma in the Hartree-Vlasov approximation. Phys. Rev. D **29**, 2690 (1984)
25. J.D. Alonso, R. Hakim, Quasiboson excitation spectrum of the relativistic quantum scalar plasma in the Hartree-Vlasov approximation. Phys. Rev. D **38**, 1780 (1988)
26. J.D. Alonso, A.P. Canyellas, Field theoretical-model for nuclear and neutron matter. 5. Slowly rotating warm cores in neutron-stars. Astrophys. J. **395**, 612 (1992)
27. J. Baacke, K. Heitmann, C. Patzold, Nonequilibrium dynamics of fermions in a spatially homogeneous scalar background field. Phys. Rev. D **58**, 125013 (1998)
28. D. Boyanovsky, H.J. Vega, D.S. Lee, Y.J. Ng, S.Y. Wang, Fermion damping in a fermion-scalar plasma. Phys. Rev. D **59**, 105001 (1999)
29. J. Baacke, D. Boyanovsky, H.J. Vega, Initial time singularities in nonequilibrium evolution of condensates and their resolution in the linearized approximation. Phys. Rev. D **63**, 045023 (2001)
30. O. Scavenius, Á. Mócsy, I.N. Mishustin, D.H. Rischke, Chiral phase transition within effective models with constituent quarks. Phys. Rev. C **64**, 045202 (2001)
31. J. Berges, S. Borsnyi, J. Serreau, Thermalization of fermionic quantum fields. Nucl. Phys. B **660**, 5180 (2003)
32. E.S. Fraga, L.F. Palhares, M.B. Pinto, Nonperturbative Yukawa theory at finite density and temperature. Phys. Rev. D **79**, 065026 (2009)
33. P. Ring, P. Schuck*, The Nuclear Many-Body Problem* (Springer, New York, 1980)
34. P.L. Natti, A.F.R. de Toledo Piza, Initial-condition problem for a chiral Gross-Neveu system. Phys. Rev. D **54**, 7867 (1996)
35. C.R. Willis, R.H. Picard, Time-dependent projection-operator approach to master equations for coupled systems. Phys. Rev. A **9**, 1343 (1974)
36. L.C. Yong, A.F.R. de Toledo Piza, Kinetic approach to the initial-value problem in ϕ^4 field theory. Phys. Rev. D **46**, 742 (1992)
37. J.M. Luttinger, J.C. Ward, Ground-state energy of a many-fermion system. II. Phys. Rev. **118**, 1417 (1960)
38. G. Baym, Self-consistent approximations in many-body systems. Phys. Rev. **127**, 1391 (1962)
39. P. Buck, H. Feldmeier, M.C. Nemes, On energy conservation in the presence of collision terms. Ann. Phys. **185**, 170 (1988)
40. Y.B. Ivanov, J. Knoll, D.N. Voskresensky, Self-consistent approximations to non-equilibrium many-body theory. Nucl. Phys. A **657**, 413 (1999)
41. E.R. Takano Natti, A.F.R. de Toledo Piza, Mean field and collisional dynamics of interacting fermion-boson system: the Jaynes-Cummings model. Phys. A **236**, 321 (1997)
42. P. Tommasini, A.F.R. de Toledo Piza, Non-ideal boson system in the gaussian approximation. Ann. Phys. (N.Y.) **253**, 198 (1997)
43. A. Kerman, S.E. Koonin, Hamiltonian formulation of time-dependent principles for many-body system. Ann. Phys. (N.Y.) **100**, 332 (1976)
44. P.L. Natti, A.F.R. de Toledo Piza, Small oscillations of a chiral Gross-Neveu system. Phys. Rev. D **55**, 3403 (1997)
45. R. Newton*, Scattering Theory of Waves and Particles* (Springer, New York, 1982)
46. M.C. Nemes, A.F.R. de Toledo Piza, J. Providência, Van Kampen waves in extended fermion systems and the random phase approximation. Phys. A **146**, 282 (1987)
47. J. Beringer et al. (Particle Data Group), Review of particle physics. Phys. Rev. D **86**, 010001 (2012) [1528 pages]
48. M.C. Nemes, A.F.R. de Toledo Piza, Nonunitary effects in the time evolution of one-body observables. Phys. Rev. C **27**, 862 (1983)
49. M.C. Nemes, A.F.R. de Toledo Piza, Dynamics of the nuclear one-body density: small amplitude regime. Phys. Rev. C **31**, 613 (1985)
50. B.V. Carlson, M.C. Nemes, A.F.R. de Toledo Piza, Quantum collisional evolution of a one-dimensional fermi gas: numerical solution. Nucl. Phys. A **457**, 261 (1986)