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# A PROBLEM WITH THE BIHARMONIC OPERATOR

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**Abstract**

This work tries an eigenvalue problem for the Shrödinger equation that incorporates the bi-harmonic operator. This problem is associated with a single particle of mass $m = \frac{2}{\hbar^2}$ moving under the influence of an electric force field described by the potential $\phi$. The problem concerns to find the existence of real numbers $\omega$ and real functions $u$, $\phi$ satisfying the system

$$\begin{array}{rcll} -\Delta u + \phi u & = & \omega u & \text{in } \Omega \\ \Delta^2 \phi - \Delta \phi & = & u^2 & \text{in } \Omega \end{array} \tag{1}$$

with the boundary and normalizing conditions

$$u = \Delta\phi = \phi = 0 \quad \text{on} \quad \partial\Omega \quad \text{and} \quad \int_\Omega u^2 = 1. \tag{2}$$

## 1 Introduction

By the classic inspection the function $\phi$ requires necessarily belong to $H := H^2(\Omega) \cap H_0^1(\Omega)$. $H$ is a Hilbert space with the equivalent norm induced by the inner product

$$(u, v)_H = \int_\Omega (\Delta u \Delta v + \nabla u \nabla v)\, dx.$$

Also, it is not difficult to see that the Euler-Lagrange equations of the functional

$$F(u, \phi) = \frac{1}{2}\int_\Omega |\nabla u|^2 dx + \frac{1}{2}\int_\Omega \phi u^2 dx - \frac{1}{4}\int_\Omega |\Delta\phi|^2 dx - \frac{1}{4}\int_\Omega |\nabla\phi|^2 dx, \tag{3}$$

on the manifold

$$M = \left\{(u, \phi) \in H_0^1(\Omega) \times H; \|u\|_{L^2(\Omega)} = 1\right\},$$

give the solutions of (1). Moreover $F$ is a strongly indefinite functional, this means $F$ is neither bounded from above nor from below. Then, the usual methods of the critical points theory can not be directly used. To deal with this difficulty we shall reduce the functional (3) to suitable functional $J$ of the single variable $u$, as that was done by Benci and Fortunato in [1], to which we will apply the genus theory, [2].

## 2 Main Result

**Theorem 2.1.** *Let $\Omega$ be a bounded set in $\mathbb{R}^3$. Then there is a sequence $(\omega_n, u_n, \phi_n)$, with $\{\omega_n\}_{n\in\mathbb{N}} \subset \mathbb{R}$, $\omega_n \to \infty$ and $u_n$, $\omega_n$ are real functions, solving from (1) to (3).*

## References

[1] BENCI AND D. FORTUNATO - An Eigenvalue Problem for the Schrödinger-Maxwell Equations. *Topological Methods in Nonliar Analysis.*,**11**, 283-293, 1998.

[2] RABINOWITZ, P. H. - *Variational methods for nonlinear eigenvalue problems*, Proc. CIME, 1974.