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Critical behavior in lattice models with two symmetric absorbing states

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Abstract. We analyze nonequilibrium lattice models with up–down symmetry and two absorbing states by mean-field approximations and numerical simulations in two and three dimensions. The phase diagram displays three phases: paramagnetic, ferromagnetic and absorbing. The transition line between the first two phases belongs to the Ising universality class and between the last two, to the direct percolation universality class. The two lines meet at the point describing the voter model and the size ℓ of the ferromagnetic phase vanishes with the distance ε to the voter point as $\ell \sim \varepsilon$, with possible logarithm corrections in two dimensions.

Contents

1. Introduction

Studies of phase transitions and critical phenomena in equilibrium and nonequilibrium systems have shown that the critical behavior can be organized into universality classes, which are identified by a small number of features, among them the symmetry. The most widely studied universality class, due to its experimental implications, is the Ising universality class which includes equilibrium as well as nonequilibrium systems [1, 2]. The main feature of the Ising universality class is the up–down symmetry. Another class, which includes only nonequilibrium systems, is the direct percolation (DP) universality class [3–5]. The main feature that distinguishes this class from the others is the presence of a single absorbing state. That is, if a system displays a continuous phase transition to a single absorbing state with no extra symmetry or conserving laws then its critical behavior belongs to the DP universality class [6, 7].

Our purpose here is to analyze models that possess features pertaining to the two universality classes. Here, we focus on models belonging to the Ising and DP universality classes, that is, systems with up–down symmetry and an absorbing state [8–12]. Due to the up–down symmetry, these systems, in fact, have two symmetric single absorbing states. Lattice models of this type have been studied numerically [8], revealing the existence of two transition points when a parameter is varied: a symmetry breaking point belonging to the Ising universality class and another belonging to the DP universality class. Another numerical study in 2D by the method of Langevin equation [9], with two parameters, has shown that the two critical points evolve into two transition lines that meet at a certain point, giving rise to a third transition line. This study confirm the Ising and DP character of the two lines and the existence of a point on the third line corresponding to the voter model [2], which is not the meeting point. It has been claimed that the symmetry breaking transition is not of the Ising type [11], but Monte Carlo simulations on a 2D interacting monomers model [12] confirmed the Ising transition. In the present study, we wish to clear up this point and analyze the role played by the voter model. This is achieved by
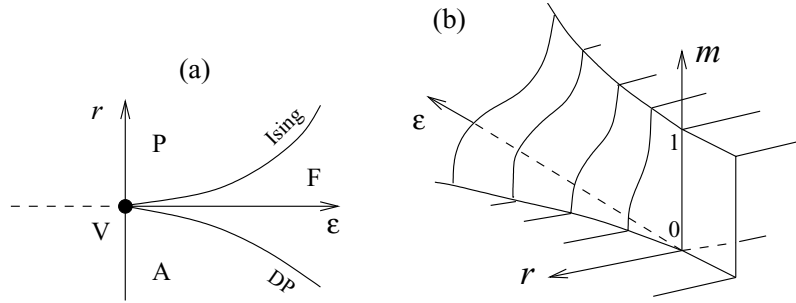
Figure 1. (a) Phase diagram in the field parameters ε and r. The full circle, located at the origin, corresponds to the voter model (V). The phases are: paramagnetic (P), ferromagnetic (F) and absorbing (A). The solid lines are continuous phase transitions belonging to the Ising and DP universality classes and the dashed line is a discontinuous phase transition. (b) Magnetization m as a function of ε and r.

studying lattice models with up–down symmetry and two absorbing states in 2D and 3D by numerical simulations and mean field approach. Our results confirmed the existence of the Ising and DP lines and revealed that two lines meet at the voter point and that this is the only point at the third line which is critical.

The phase diagram in the 2D space of parameters displays three phases: paramagnetic (P), ferromagnetic (F) and absorbing (A), as shown in figure 1(a). The PF line that separates the P and F phases is a critical line belonging to the Ising universality class. The FA line that separates the A phase and the F phase, understood as an active phase, is a critical line belonging to the DP universality class. The two lines, PF and FA, meet at the point corresponding to the voter model. As to the PA line, which separates the P and A phases, it is a discontinuous transition line that ends at the voter point. Along the PA line the jump in the order parameter is a constant and equals the jump observed in the voter model. For an appropriate description of the critical properties around the voter point, it is convenient to introduce two parameters, ε and r, that define a reference frame centered with origin at the voter point. The ε-axis and r-axis are parallel and perpendicular to the direction defined by the discontinuous transition line, as shown in figure 1(a).

The order parameter m of the F phase, shown in figure 1(b), is assumed to obey the following scaling relation, around the voter point,

$$m = \psi_{\pm}(\varepsilon r^{-1/\phi}), \tag{1}$$

where ϕ is the crossover exponent and ψ_+ and ψ_- are valid for $\varepsilon > 0$ and $\varepsilon < 0$, respectively. The universal function $\psi_-(x)$ is a step function. The universal function $\psi_+(x)$ vanishes according to $\psi_+(x) \sim (a - x)^{\beta_{\rm I}}$, where $\beta_{\rm I}$ is the Ising order parameter exponent and a is a positive constant so that, near the Ising critical line, the order Ising parameter m behaves as

$$m = A(\varepsilon_{\rm I} - \varepsilon)^{\beta_{\rm I}}, \tag{2}$$

where $\varepsilon_{\rm I} = ar^{1/\phi}$ and the amplitude A diverges as $A \sim r^{-\beta_{\rm I}/\phi}$. The universal function $\psi_+(x)$ approaches its maximum value 1 according to $1 - \psi_+ \sim (x + b)^{\beta_{\rm DP}}$ where $\beta_{\rm DP}$ is

the DP order parameter exponent and b is a constant so that, near the DP critical line, the DP order parameter $\rho = 1 - m$ behaves as

$$\rho = B(\varepsilon + \varepsilon_{\mathrm{DP}})^{\beta_{\mathrm{DP}}}, \tag{3}$$

where $\varepsilon_{\mathrm{DP}} = br^{\phi}$ and the amplitude B diverges as $B \sim r^{-\beta_{\mathrm{DP}}/\phi}$.

The Ising and DP critical lines occur when $m \to 0$ and $\rho \to 0$, respectively and are given by $\varepsilon = ar^{1/\phi}$ and $\varepsilon = -br^{1/\phi}$ so that the size of the F phase behaves near the voter point as

$$\ell \sim \varepsilon^{\phi}. \tag{4}$$

2. Model

We consider stochastic variables that take two values, $\sigma_i = \pm 1$, called spin variables. They are located on the sites of a regular lattice which is either a 2D triangular lattice or a 3D cubic lattice. Notice that both lattices have the same coordination number, which is six. The system evolves in time according to a continuous time Markov process with a single-site change. The single-site transition rate $w_i(\sigma)$ from $\sigma = (\sigma_1, \ldots, \sigma_i, \ldots, \sigma_N)$ to $\sigma^i = (\sigma_1, \ldots, -\sigma_i, \ldots, \sigma_N)$ is set up in such way as to hold the properties: (a) it has the up–down symmetry, that is, it is invariant under the transformation $\sigma \to -\sigma$; (b) it depends on the neighboring spin variables only through the sum of these spin variables. A transition rate that fulfils these two properties is given by

$$w_i(\sigma) = \frac{1}{2}\{1 - \sigma_i f(s_i)\}, \qquad s_i = \sum_{\delta} \sigma_{i+\delta}, \tag{5}$$

where the sum in δ extends over the six nearest neighbor sites and $f(s)$ is an odd function of s.

The most general form of $f(s)$ has three parameters. Let us denote by σ_j, $j = 1, 2, 3, 4, 5, 6$ the spins of the six nearest neighbor sites of a central site $j = 0$, so that

$$s_0 = \sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 + \sigma_5 + \sigma_6. \tag{6}$$

Taking into account that $\sigma_j = \pm 1$, then s_0 may take only the values $0, \pm 2, \pm 4, \pm 6$, from which it follows that the most general form of an odd function $f(s)$ can be written as

$$f(s) = \frac{a}{2}s + \frac{b}{8}s^3 + \frac{c}{32}s^5, \tag{7}$$

where a, b and c are parameters. Here, it is convenient to define new parameters $p_0 = [1 + f(6)]/2$, $p_1 = [1 + f(4)]/2$ and $p_2 = [1 + f(2)]/2$, that is,

$$p_0 = \frac{1}{2}(1 + 3a + 27b + 243c), \tag{8}$$

$$p_1 = \frac{1}{2}(1 + 2a + 8b + 32c), \tag{9}$$

$$p_2 = \frac{1}{2}(1 + a + b + c). \tag{10}$$

In terms of these parameters, the transition probabilities are as shown in table 1.

Table 1. Probability of transition $\sigma_i \rightarrow -\sigma_i$.

σ_i	-6	-4	-2	0	$+2$	$+4$	$+6$
$+1$	p_0	p_1	p_2	$1/2$	$1-p_2$	$1-p_1$	$1-p_0$
-1	$1-p_0$	$1-p_1$	$1-p_2$	$1/2$	p_2	p_1	p_0

Two well known models are special cases. When $p_0 = 1$, $p_1 = 5/6$ and $p_2 = 2/3$, ($a = 2/3$, $b = c = 0$) we recover the so called voter model [2, 9] whose transition rate is given by

$$w_i^{\text{voter}}(\sigma) = \frac{1}{2}\Big\{1 - \frac{1}{3}\sigma_i s_i\Big\}, \qquad s_i = \sum_\delta \sigma_{i+\delta}. \tag{11}$$

The Glauber–Ising model, whose transition rate is given by

$$w_i^{\text{GI}}(\sigma) = \frac{1}{2}\{1 - \sigma_i \tanh(K s_i)\}, \qquad s_i = \sum_\delta \sigma_{i+\delta}, \tag{12}$$

is recovered when the parameter are connected by

$$p_0 = \frac{1}{2}(1 + \tanh 6K), \tag{13}$$

$$p_1 = \frac{1}{2}(1 + \tanh 4K), \tag{14}$$

$$p_2 = \frac{1}{2}(1 + \tanh 2K). \tag{15}$$

In this case, detailed balance is obeyed and the stationary distribution is the Gibbs distribution associated to the Ising Hamiltonian. In other cases there is no detailed balance as can be seen by the following sequence of transitions. Consider two nearest neighbor sites initially in states $(++)$. All the other nearest neighbors of this pair of spins are positive except one of them which is negative. Next, they change their states according to the cyclic sequence $(++) \rightarrow (+-) \rightarrow (--) \rightarrow (-+) \rightarrow (++)$. Using the rules of table 1 we find that the ratio between the transition probabilities of this path and of its reverse equals $(1 - p_1)^2 p_2 p_0 / (1 - p_0)(1 - p_2)p_1 p_1$, which in general is distinct from unity. Thus, there is no microscopic reversibility and no detailed balance.

To meet the two features of the Ising and DP universality class, that is, up–down symmetry and two equivalent single absorbing states, the transition rate should lead to an absorbing state. This is accomplished by imposing the restriction $p_0 = 1$ because in so doing the transition rate will vanish whenever all spins are up or all spins are down, which are the two states identified as the equivalent absorbing states. In this case, the transition rates do not obey detailed balance, which amounts to saying that the two features mentioned above are only possible in systems that in the stationary states are out of equilibrium. When $p_0 = 1$, the phase diagram is restricted to the plane (p_1, p_2) and the voter model corresponds to the point $(5/6, 2/3)$ of this diagram. As we shall see, the phase diagram displays a paramagnetic phase, a ferromagnetic phase and an absorbing state, as explained before.

3. Mean-field approximation

A qualitative phase diagram is obtained by the use of a mean-field approximation. To this end, we start by writing the function $f(s)$ in the form

$$f(s) = \frac{A}{6}\sum_i \sigma_i + \frac{B}{20}\sum_{i,j,k}\sigma_i\sigma_j\sigma_k + \frac{C}{6}\sum_{i,j,k,\ell,n}\sigma_i\sigma_j\sigma_k\sigma_\ell\sigma_n \tag{16}$$

where the first summation runs over i from 1 to 6 and has 6 terms; the second summation runs over i,j,k with the restriction $1 \leqslant i < j < k \leqslant 6$ and has 20 terms; and the third summation runs over i,j,k,ℓ,n with the restriction $1 \leqslant i < j < k < \ell < n \leqslant 6$ and has 6 terms. The parameters A, B and C are related to p_0, p_1 and p_2 by

$$p_0 = \frac{1}{2}(1 + A + B + C) \tag{17}$$

$$p_1 = \frac{1}{2}(1 + \frac{2}{3}A - \frac{2}{3}C) \tag{18}$$

$$p_2 = \frac{1}{2}(1 + \frac{1}{3}A - \frac{1}{5}B + \frac{1}{3}C) \tag{19}$$

The time evolution of the magnetization $\langle\sigma_0\rangle$ is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\sigma_0\rangle = -2\langle\sigma_0 w_0(\sigma)\rangle = -\langle\sigma_0\rangle + \langle f(s_0)\rangle \tag{20}$$

or

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\sigma_0\rangle = -\langle\sigma_0\rangle + A\langle\sigma_i\rangle + B\langle\sigma_i\sigma_j\sigma_k\rangle + C\langle\sigma_i\sigma_j\sigma_k\sigma_\ell\sigma_n\rangle \tag{21}$$

where translation invariance has been used. Next, we use the simplest mean-field approximation, for which $\langle\sigma_i\sigma_j\sigma_k\rangle = m^3$ and $\langle\sigma_i\sigma_j\sigma_k\sigma_\ell\sigma_n\rangle = m^5$, where $\langle\sigma_i\rangle = m$, to write

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -(1-A)m + Bm^3 + Cm^5. \tag{22}$$

Here, we are interested in the case $p_0 = 1$ which is equivalent to $1 - A = B + C$ so that

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -(B+C)m + Bm^3 + Cm^5, \tag{23}$$

which is identical to the deterministic part of the Langevin equation considered in [9]. In the stationary state, the possible solutions are as follows. (a) $m = 0$, corresponding to the paramagnetic (P) phase; it is stable as long as $B + C > 0$, which is equivalent to $12p_1 + 15p_2 < 20$. (b) $m = \pm 1$, corresponding the absorbing (A) phase; it is stable as long as $B + 2C < 0$, which is equivalent to $p_1 > 5/6$. (c) $m \neq 0$ and $m \neq \pm 1$, corresponding to the ferromagnetic (F) phase. The phase diagram in the plane (p_1, p_2) is shown in figure 2. The P-F line is given by $B + C = 0$, or $12p_1 + 15p_2 = 20$ and the F-A line is given by $B + 2C = 0$, or $p_1 = 5/6$. The point corresponding to the voter model is located at the point where the two lines meet, that is, at $B = C = 0$, or $p_1 = 5/6$ and $p_2 = 2/3$. The P-F and F-A transitions are continuous whereas the P-A transition is discontinuous.

The discontinuous P-A transition line shown in figure 2 was obtained as follows. We start by writing equation (23) in the form $\mathrm{d}m/\mathrm{d}t = -\partial f/\partial m$ where $f = (B+C)\,m^2/2 -$
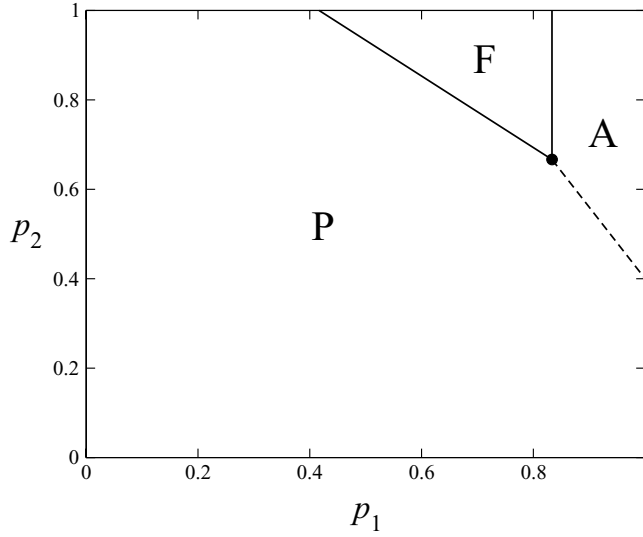
Figure 2. Phase diagram in the plane (p_1, p_2) according to the mean-field approximation. The full circle, located at $(p_1, p_2) = (5/6, 2/3)$, corresponds to the voter model. The phases are: paramagnetic (P), ferromagnetic (F) and absorbing (A). The solid lines are continuous phase transitions and the dashed line is a discontinuous phase transition.

$Bm^4/4 - Cm^6/6 + K$. Interpreting $f(m)$ as a free energy then the discontinuous transition occurs when $f(0) = f(\pm 1)$, which gives $3B + 4C = 0$ or $5p_2 + 8p_1 = 10$.

Inside the F phase, the magnetization m is given by

$$m = \left(\frac{15p_2 + 12p_1 - 20}{15p_2 - 12p_1} \right)^{1/2}. \tag{24}$$

Near the critical line PF, $p_1 = p_{\mathrm{FP}} = (20 - 15p_2)/12$, we may write

$$m = A(p_1 - p_{\mathrm{FP}})^{1/2}, \qquad A = \left[\frac{5}{2} \left(p_2 - \frac{2}{3} \right) \right]^{-1/2} \tag{25}$$

so that the amplitude A diverges with an exponent $1/2$ as one approaches the voter point. Analogously, near the FA transition, $p_1 = p_{\mathrm{FA}} = 5/6$, the order parameter ρ behaves as

$$\rho = B(p_{\mathrm{FA}} - p_1), \qquad B = \left[\frac{5}{4} \left(p_2 - \frac{2}{3} \right) \right]^{-1} \tag{26}$$

so that the amplitude B diverges with an exponent 1 as one approaches the voter point. It is worth noting that the size $\ell = p_{\mathrm{FA}} - p_{\mathrm{PA}}$ of the ferromagnetic phase, given by
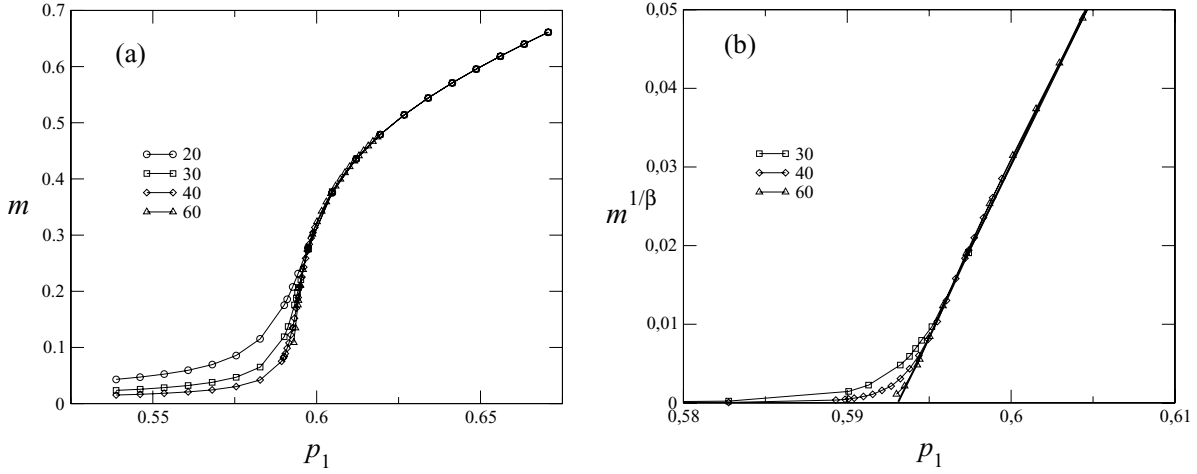
$$\ell = \frac{5}{4} \left(p_2 - \frac{2}{3} \right), \tag{27}$$

vanishes linearly as one approaches the voter point.

4. Simulations

The results obtained from numerical simulations on the cubic and triangular lattice allow us to conclude that the topology of the phase diagram is the same as the one obtained from

Figure 3. (*a*) Magnetization m versus p_1 along $p_2 = 1$. The transition occurs at $p_I = 0.5930$ as can be inferred from the plot (*b*) of $m^{1/\beta}$ versus p_1, where $\beta = 0.326$.

the mean-field approximation. The numerical simulations on these lattices were carried out as follows. At each time step a site of the lattice is chosen at random and updated according to the transition probabilities in table 1, restricted to the case $p_0 = 1$. The averages of several quantities were obtained in the stationary state. Defining $s = (1/N) \sum_i \sigma_i$ where N is the number of sites, then the quantities of interest to analyze the ferromagnetic-paramagnetic line are the magnetization

$$m = \langle |s| \rangle, \tag{28}$$

the susceptibility,

$$\chi = N\{\langle s^2 \rangle - \langle |s| \rangle^2\}, \tag{29}$$

and the Binder cumulant

$$U = 1 - \frac{\langle s^4 \rangle}{3\langle s^2 \rangle^2}. \tag{30}$$

Examples of these quantities, for the cubic lattice with $N = L^3$ sites, calculated along the line $p_2 = 1$, are shown in figures 3 and 4. At the P-F transition point, the Binder cumulant U is a constant, a result that allows the P-F transition line to be located. The critical behavior of m and χ allows one to say that, with respect to the ferromagnetic-paramagnetic transition, the model belongs to the universality class of the Ising model.

To analyze the ferromagnetic-absorbing transition, where the ferromagnetic phase is identified as the active phase, we define the quantity $\rho = 1 - m$, which is the order parameter that characterizes the transition from active to absorbing. In figure 5, we show ρ versus p_1 along the line $p_2 = 1$. The location of the transition is obtained by the extrapolation $\rho \to 0$, which allows us to draw the F-A line. The critical behavior shows that, with respect to this transition, the model belongs to the direct percolation universality class.

Using the data of the P-F and F-A transitions obtained for several values of p_2 we draw the phase diagram of figure 6(*a*). As expected the two lines, P-F and F-A, meet at the voter
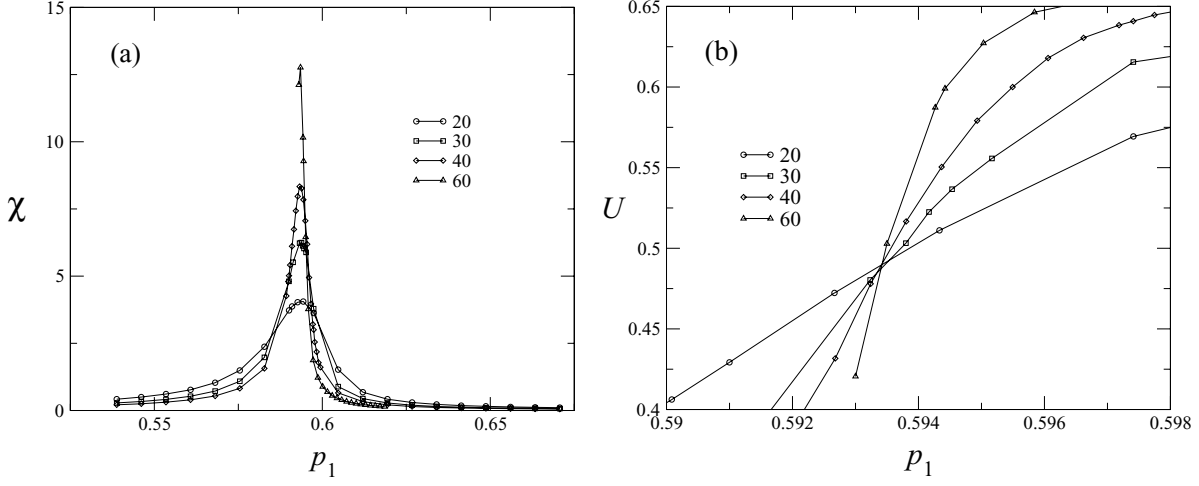
Figure 4. (*a*) Susceptibility χ and (*b*) Binder cumulant U versus p_1 along $p_2 = 1$. The transition occurs at $p_I = 0.5933$.
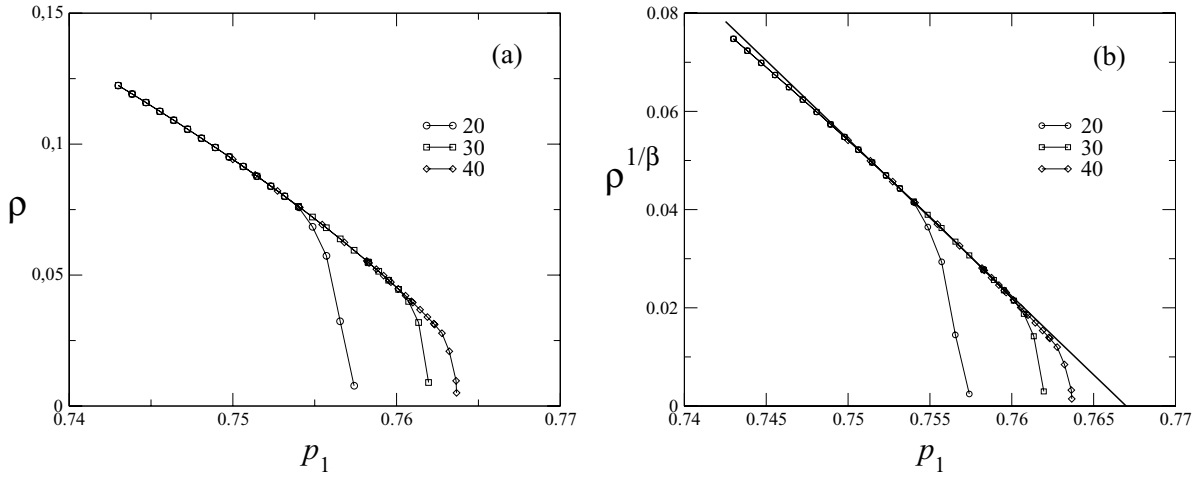

Figure 5. (*a*) Order parameter ρ of the F-A transition as a function of p_1 for $p_2 = 1$. The critical point occurs at $p_{\mathrm{DP}} = 0.7670$ as can inferred from the plot (*b*) where $\beta = 0.81$.

point located at $p_1 = 5/6$ and $p_2 = 2/3$. In figure 6(*b*) we show the size $\Delta p_1 = p_{\mathrm{DP}} - p_I$ of the F phase as a function of the distance from the voter point $\Delta p_2 = p_2 - 2/3$. As can be seen in this figure, the data points have a finite slope at the origin which amounts to saying that the scaling relation

$$\Delta p_1 \sim (\Delta p_2)^\phi, \tag{31}$$

is fulfilled with $\phi = 1.0$.

We have also determined the nature of the phase transition between the P and A phases. As we cross the transition line the order parameter jumps from zero, which is the value of m within the paramagnetic phase, to the maximum value $m = 1$, which is the value of m inside the absorbing phase. The susceptibility χ increases as one approaches the transition line from the paramagnetic phase and reaches a finite value $\chi_0(\Delta p_2)$ which
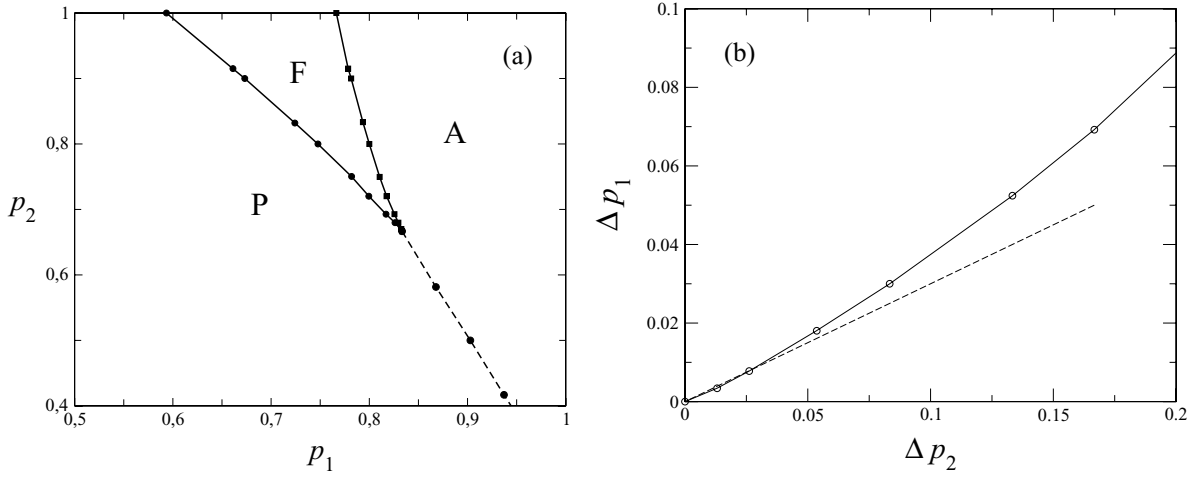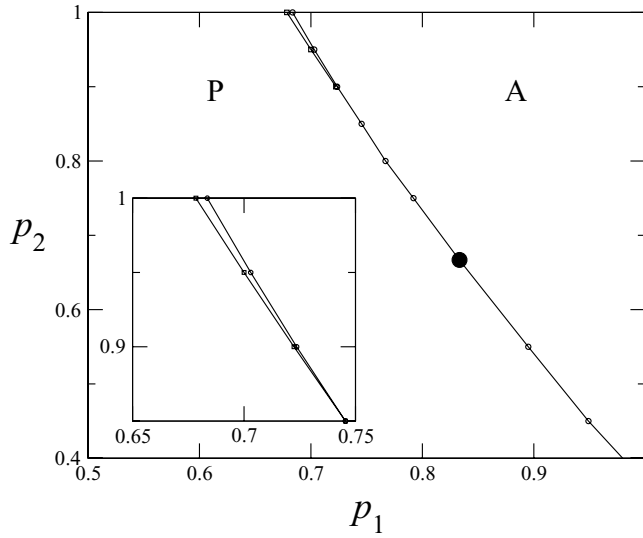
Figure 6. (*a*) Phase diagram in the plane (p_1, p_2) according to simulations on a cubic lattice. The phases are: paramagnetic (P), ferromagnetic (F) and absorbing (A). The solid lines are continuous phase transitions and the dashed line is a discontinuous phase transition. The three lines meet at the point $(p_1, p_2) = (5/6, 2/3)$, corresponding to the voter model. (*b*) Linear size $\Delta p_1 = p_{FA} - p_{PF}$ of the ferromagnetic phase as a function of the distance from the voter point $\Delta p_2 = p_2 - 2/3$. The dashed straight line shows that the data points have a finite slope at the origin.

depends on the distance $\Delta p_2 = p_2 - 2/3$ from the voter point. As one approaches the voter point χ_0 increases without limits and diverges with an exponent $\gamma = 1$. Notice that, according to our numerical results for the susceptibility obtained along $p_2 = 2/3$, the susceptibility χ diverges as one approaches the voter point from the paramagnetic phase with the same exponent $\gamma = 1$.

We have also studied the model defined on a triangular lattice by using the same methods. As can be seen in figure 7, the phase diagram also displays the three phases found in the cubic lattice. However, the ferromagnetic phase is very narrow, becoming very difficult to extract the crossover exponent but the two lines, PF and FA, seem to meet tangentially in consistency with an exponent $\phi = 1$ with logarithm corrections as we will argue below.

5. Discussion

We have studied the phase diagram of a system belonging to the Ising and DP universality classes. The phase diagram in the two parameters has an Ising critical line separating the paramagnetic and the ferromagnetic phase, a DP critical line separating the ferromagnetic phase and the absorbing phase and a discontinuous phase transition line separating the paramagnetic and the absorbing phase. The three lines meet at the voter point in such as way that the size of the ferromagnetic phase vanishes as one approaches the voter point with a crossover exponent ϕ. The crossover exponent should be understood as the ratio between the two exponents related to the two relevant scaling fields ε and r around the

Figure 7. Phase diagram in the plane (p_1, p_2) according to simulations on a triangular lattice. The full circle, located at $(p_1, p_2) = (5/6, 2/3)$, corresponds to the voter model. The phases are: paramagnetic (P), ferromagnetic and absorbing (A). The inset shows an enlargement around showing the ferromagnetic phase between the P-F and F-A transition lines.

voter point. Above the critical dimension d_c of the voter model, these two exponents do not depend on dimension so that the same should happen to the crossover exponent. Taking into account that the upper critical dimension of the voter critical point is $d_c = 2$, we expect the crossover exponent ϕ to have the same value for $d \geqslant 2$ with possible logarithm corrections when $d = 2$. The results coming from mean-field theory give $\phi = 1$ which allows us to conclude that $\phi = 1$ for $d \geqslant 2$, a result confirmed by our calculations in $d = 3$. As to the 2D case, the size of the ferromagnetic phase is too narrow for a precise numerical calculation but is consistent with the result $\phi = 1$ with logarithm corrections.

The results for the susceptibility around the Ising and DP lines are in accordance with these two universality classes. Concerning the discontinuous transition from the paramagnetic to the absorbing transition, we found that the susceptibility is finite at the transition line but diverges as one approaches the voter point.

It would be very interesting if the picture that we have drawn concerning the critical behavior of systems belonging to the Ising and DP universality classes could be confirmed by a renormalization group calculation by means of a Langevin equation formulation.

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