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Abstract

The subject of this paper is the application of multibody system techniques for vehicle chassis modelling aimed at the development of integrated vehicle control. It realises that resulting models can be complex and that simplifications in chassis description is recommendable. For this purpose, it has developed a technique for representing suspension geometry effects which by taking the MBS structure into account, results in small and fast runtime simulation models. Yet, the model is capable of describing the full range of normal operation of the automotive vehicle. Using the previously developed model, comprehensive analysis of all aspects of vehicle motion is carried out. The objectives of such analysis is the determination of a driving envelope in which the use of linearised models of the nonlinear chassis can be justified for control analysis and design. Finally, the numerical and control theoretical properties of the linearised models are addressed. State space representation is used for such purposes.

1 Introduction

The aim of Automotive Engineering is to make the motor vehicle safer, easier to drive, with improved performance to cost ratio and minimised pollutant emissions. Along the years, the techniques used to obtain compromising solutions amongst these conflicting objectives have increased in number and complexity. Important breakthroughs in vehicle development have been reached through its history. The period following the early 1970's has seen a rapid increase in the application of automatic control techniques in automotive vehicles in order to achieve these objectives. These past and current developments in automotive vehicle control have progressed mostly in a piecemeal fashion, whereby individual vehicle subsystems, such as the engine, suspension and braking systems have been studied in isolation

Future applications of control in automotive vehicles will follow a trend towards system integration, leading ultimately to the development of integrated vehicle control systems capable of coordinating the action of the various subsystems. The coordination and integration of automotive vehicle subsystem control requires the interaction amongst the various subsystems to be taken into consideration at the control design stages, i.e., a total system approach to automotive vehicle control is needed.

2 Mathematical Modelling

The use of mathematical modelling in automotive vehicle design is a widespread practice which allows the development of vehicle systems at reduced costs and time and near optimised performance characteristics. This use embraces all aspects of vehicle design and in particular, vehicle motion.

Those aspects of vehicle motion control which are of primary interest span a dynamic range with a frequency cut-off below 50 Hz. Up to this frequency, simulation studies of automotive vehicle dynamics can justifiably be based on lumped parameter models [10]. Current approaches to vehicle dynamics simulation fall into 2 distinct categories. The first approach involves the use of simple mathematical models which are derived from first principles and which are usually assembled manually.

The second approach to vehicle dynamics simulation involves mathematical models which incorporate representations of the vehicle kinematics, tyre characteristics and suspension geometry effects. This approach to vehicle dynamics simulation can result in mathematical models which are highly complex. Consequently, automated model generation facilities based on multibody systems (MBS) modelling techniques are widely employed.

The use of MBS modelling techniques to vehicle dynamics simulation involves two essential considerations, viz. the way in which the MBS model is formulated and the particular MBS program which is to be employed. The MBS model formulation determines how the equations of motion are derived. The MBS program determines the ultimate form of the dynamic equations, as implemented within the computer simulation program. Approaches to MBS modelling based on a combination of Kane's Method [4] and symbolic computation has been shown [9] to generate the most efficient (run time) simulation code.

While the use of those more powerful and complex models makes the idea of an integrated control system very difficult due to the high nonlinearities of the equations of motion as well as the great number of degrees of freedom; it has not been answered yet whether a simplified model, however taking some relevant interactions from a control point of view, into consideration, could result in improved vehicle motion. A positive answer to such a question would possibly mean, for example, a reduced number of transducers and their related apparatus necessary to be implemented in a future vehicle in which these concepts might be applied.

3 Modelling of an Automotive Vehicle for Motion Control Studies

The modelling to be performed in this section is motivated by the recognition of a need for a systematic approach to vehicle simulation which is capable of embracing all aspects of vehicle motion in order to allow the study of automotive vehicle control.

Apart from aerodynamic resistances and gravitational effects, all external forces acting on a vehicle are applied through the wheels. Consequently, total vehicle control will entail the application of control action at the wheels, and will be based on a combination of propulsion, steering and suspension control. Therefore it is necessary to develop vehicle models which are capable of representing all these actions simultaneously.

The modelling of this work is based on an alternative approach [2], in which suspension geometry effects are incorporated in a way which does not involve a detailed representation of the suspension system. The basis of the approach is an empirical black-box representation of the suspension kinematics derived from readily available experimental data describing the suspension trajectory at each wheel hub. From this data, an equivalent swing axle or trailing arm model is derived using techniques from differential geometry.

The simulation model is constructed using the MBS package SD/FAST [11], and the ACSL [1] continuous simulation language. SD/FAST combines Kane's formulation for a MBS model with an equation generator incorporating a transparent symbolic computation facility, while ACSL provides the simulation environment necessary to solve these equations.

3.1 Schematic Description of Vehicle Model

A schematic view of the nonlinear chassis model is presented in figure 1. The model represents the kinematics of the sprung mass and the four wheels and incorporates the effects of geometrical constraints associated with the suspension. At each wheel, the suspension geometry effects are represented in a black box manner as a swing axle [3] connecting the wheel hub H to the sprung mass via a single degree of freedom rotational joint (pin joint) at the point. The pin position and the arm length of the swing axle are derived empirically from data defining the wheelbase/bump and track/bump characteristics at each wheel hub.

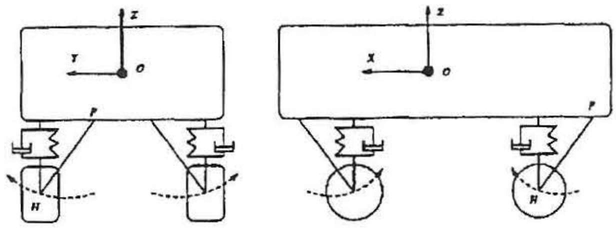

Figure 1: Nonlinear Chassis Model

In all cases, the polynomial of lowest order which adequately fitted the data was chosen. A comparison of these polynomial representations and the corresponding wheelbase and track characteristics data is presented in figures 2 and 3.

The variation in fore-aft and lateral wheel displacements described in these figures appears small. However, the effect of these variatios on the vehicle pitch and roll characteristics is highly significant, particularly during vehicle manoeuvres associated with large acceleration levels.

The model provides a ten degree of freedom representation of the chassis, with six degrees of freedom resulting from the sprung mass and one degree of freedom from each swing axle. This results in a nonlinear twenty state model, in which the states correspond to ten generalised coordinates and ten generalised speeds. This swing axle model provides the simplest possible representation of the suspension system which is consistent with a nonlinear chassis model intended for motion control studies. This chassis model represents the first stage in the development of a vehicle dynamics model for use in motion control studies. The present model is limited, particularly in its representation of the tyre and wheel. However, the model can easily be extended to cater for complete motion control studies through inclusion of a more sophisticated representation of the tyre and wheel, as it has been done in further studies. The chassis model

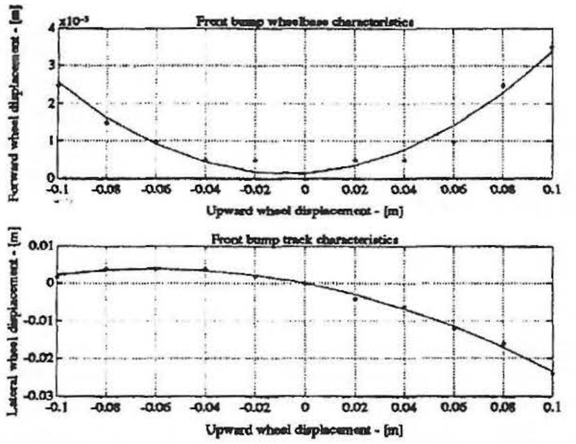

Figure 2: Front Wheelbase and Track Characteristics

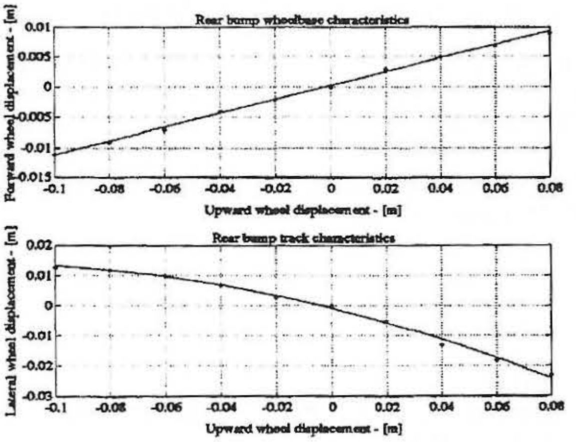

Figure 3: Rear Wheelbase and Track Characteristics

developed in this chapter is based on data corresponding to a luxury European saloon car. This data was selected for illustration purposes only. In particular, it should be noted that the chassis modelling approach presented here is not restricted to luxury saloon cars but is generic and applicable to a wide variety of automotive vehicle types.

3.2 Topological Representation of MBS Model

The chassis model is considered as a multibody system made up from five hinge connected rigid bodies organised in a tree topology, as described in figure 4. The central, or base body corresponds to the sprung mass and the four branch bodies to each of the four wheels and associated suspension links.

Associated with the base body and each branch body is a local right-handed coordinate frame with fixed orientation and location (at the centre of mass) in the respective body.

The geometrical description of the MBS model can be specified by a set of three vectors for each of the branch bodies. The first of the three vectors $\boldsymbol{op}$ describes the position of the pint joint relative to the centre of gravity of the base body. This vector is fixed in the base body. The second vector $\boldsymbol{hp}$ describes the position of the pin joint relative to the centre of gravity of the branch body and is fixed in the branch body. The third vector $\boldsymbol{b}$ describes the orientation of the pin joint axis.

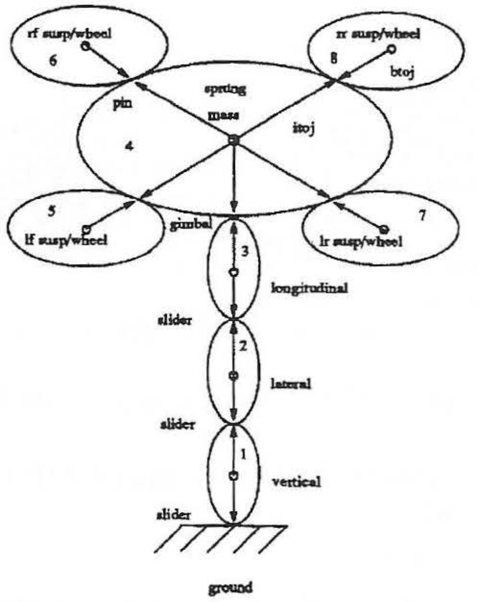

Figure 4: Topological Representation of Chassis

		Left	Right
F	op	(1.441, 0.303, -0.242)	(1.441, -0.303, -0.242)
	hp	(0.121, -0.447, 0.058)	(0.121, 0.447, 0.058)
	b	(0.966, 0.257, -0.029)	(0.966, -0.257, -0.029)
R	op	(-1.516, 0.105, -0.157)	(-1.516, -0.105, -0.157)
	hp	(-0.016, -0.645, 0.143)	(-0.016, -0.645, 0.143)
	b	(0.994, 0.0, 0.113)	(0.994, 0.0, 0.113)

Table 1: Geometrical description of MBS chassis model

For each branch body, specifications for the vectors $\boldsymbol{op}$, $\boldsymbol{hp}$ and $\boldsymbol{b}$ were obtained relative to this reference coordinate frame. The vectors $\boldsymbol{hp}$ and $\boldsymbol{b}$ were derived using differential geometry and the vector $\boldsymbol{op}$ was derived from the vehicle dimensional data presented in table 2 The resultant values of the vectors $\boldsymbol{op}$, $\boldsymbol{hp}$ and $\boldsymbol{b}$ are presented in table 1. To complete the physical description of this MBS model, the mass and inertia matrix for each body is required. This data is also presented in table 2.

Effective weight of wheel	front	50 kg
	rear	40 kg
Radius of wheel		0.2 m
Weight of sprung mass		1600.0 kg
Principle inertias of sprung mass	I_{xx}	500.0 $kg.m^2$
	I_{yy}	3000.0 $kg.m^2$
	I_{zz}	3000.0 $kg.m^2$
Centre of gravity of sprung mass	from front axle	1.32 m
	from rear axle	1.5 m
	above ground	0.5 m
	from right/left side	0.75 m

Table 2: Vehicle inertia and geometric data

3.3 External Forces Acting on MBS Model

3.3.1 Sprung Mass

Apart from gravitational influences, the forces acting on the sprung mass are aerodynamic and suspension forces, which also includes the effect of an antiroll torsion bar at the front axle. Suspension characteristics are given in table 3. Aerodynamic forces and torques have also been modelled, but are not described in this paper.

3.3.2 Unsprung Masses

The forces acting on each unsprung mass are the longitudinal, lateral and vertical tyre forces, the suspension forces and gravitational forces. Tyre characteristics are also described in table 3

3.4 Representation of Suspension and Tyre Forces

The suspension and tyre forces are described by the parameters given in table 3.

		Front	Rear
Susp.	Spring stiffness $kN.m^{-1}$	20.0	27.0
	Damper coefficient $kN.s.m^{-1}$	1.4	2.0
	Antiroll bar stiffness $kN.m^{-1}$	20.0	
Linear tyre	Vertical stiffness $kN.m^{-1}$	250.0	250.0
	Cornering stiffness $kN.rd^{-1}$	66.0	70.0
Steering system compliance $rd.kN^{-1}$		0.0051	

Table 3: Suspension and linear tyre data

3.4.1 Suspension Forces

Each suspension force is modelled as a spring in parallel with a damper. The spring and damper forces are taken as linear functions of the spring displacement, and displacement rate, respectively, relative to the base body. An additional roll torque represents the effect of an antiroll torsion bar acting at the front axle. This torque is modelled, relative to the base body, as a linear function of the left/right difference in spring displacements at the front axle. Static values for the vertical suspension and tyre deflections are computed to balance the weight of the sprung mass in the steady-state.

3.4.2 Tyre Forces

For the linear tyre model, the longitudinal tyre forces represent propulsion and braking action. These forces are modelled as an externally defined forcing term, where it is assumed that wheel spin and wheel lock do not occur. The vertical and lateral tyre forces are represented as linear functions of the vertical wheel displacement and wheel's sideslip angle, respectively.

3.5 Simulation Model of Chassis

The computer simulation model was built using the MBS modelling package SD/FAST and the ACSL continuous simulation language. The SD/FAST program processes an ASCII source code file containing a user-supplied description of a MBS model. An example of part of the program is given below.

```
Body = Right_front_wheel
inb = Sprung_mass
joint = pin
mass = 50
inertia = 0 0 0
inbtojoint = 1.44090214828721 -0.30292586469092
-0.24179219374438
bodytojoint = 0.12090214828721 0.44707413530908
0.05820780625562
pin = 0.965864831945 95 -0.25738705686472
-0.02927506393730
```

The resulting source code, which is generated autonomously by SD/FAST, consists of two main FORTRAN subroutines and several secondary subroutines. Approximately 10 s of CPU time on a SUN 4/330 computer was used to generate the resulting output file. This output file consists of around 1500 lines of FORTRAN code, about 70% of which is contained in the main subroutine defining the state derivatives for the equations of motion of the MBS model.

The approach to simulation model development for an automotive vehicle chassis system, described in this section, has the advantage that the model can be easily extended to include nonlinear tyre and suspension force characteristics without the need to modify the kinematic description of the sprung and unsprung masses. More importantly, the approach facilitates the development of vehicle models which combine a MBS description of the chassis with other vehicle subsystems which are not amenable to MBS modelling techniques, such as the powertrain or a digital control system, but which can be easily represented in a simulation language such as ACSL.

3.6 Model Validation

Unfortunately, the amount of experimental data available was very limited, and therefore extensive validation was not possible to be performed, as it would be desirable. The only experimental results available consisted of the static deflections of the suspension, as it was used to derive the swing axle model. Other results available were an eigenmode analysis resulting from simpler models in use by the vehicle's manufacturer and which had been previously validated by them.

The linear eigenmode analysis was performed with the computer simulation model as a prelude to non-linear simulation experiments. The natural modes of the MBS model and their corresponding frequencies and damping ratios were computed using the linear analysis facilities of ACSL. The linear analysis was carried out with the MBS model in a steady state condition corresponding to a constant forward speed of 20 m/s. The eigenmode analysis resulted in four modes corresponding to the sprung mass and four further modes, two for each of the front and rear wheel pairs. The results were in close agreement with experimental observation and are presented in table 4.

Modes	Frequency		Damping
	Hz	rd/s	Factor
Front-end bounce	1.1	6.88	0.206
Rear-end bounce	1.35	8.46	0.266
Roll on springs	1.81	11.38	0.267
Yaw rate/lateral velocity	1.29	8.13	0.834
In-phase front wheel hop	12.02	75.46	0.200
Out-of-phase front wheel hop	12.50	78.52	0.195
In-phase rear wheel hop	13.00	81.69	0.304
Out-of-phase rear wheel hop	13.05	81.95	0.303

Table 4: Eigenmode analysis

In the next section extensive simulations are carried out in order to determine an operating range and conditions for the use of a linearised model in control system analysis and design. These simulations cover all aspects of vehicle motion under realistic operating conditions considering driver's inputs as well as external disturbances. For that purpose transient and steady-state analysis is performed. However only a few results are present for the reason of space. Full details are given in [2].

4 Simulation and Analysis of Vehicle Motion

In order to confirm the utility of the previously developed model, as well as characterising its behaviour within an operating range which allows the definition of appropriate control strategies, for example, sets of of operating conditions in which linear approximations would still be valid, a large number of tests which exercise the model within the boundaries of its simplifying assumptions was carried out. These tests or experiments, are defined to be either driver inputs or external disturbances and they will be looking at both transient or steady-state behaviour of the simulation model. The analyses to be performed will consist of a detailed study of these simulations and will include eigenvalue analysis, intended at verifying the validity of transient behaviour, and gain analysis, aimed at determining the characteristics of the model's steady-state characteristics.

The intention of these experiments is to allow the definition of a driving envelope in which the interactions present, their nature and magnitude are discussed and a valid range of operating conditions for control studies can be defined.

5 Vehicle Simulation

The simulation runs to be executed comprise two types of manoeuvre; one intended to analyse the vehicle's transient behaviour and the another aimed at obtaining the vehicle steady-state response characteristics. However only a few results are shown in this paper for reasons of space. The interested readermay refer to [2] for full details.

5.1 Transient Manoeuvres

The simulation runs which will be used to analyse the the transient behaviour of the model can be divided in two types; driver related inputs and external disturbances inputs. Driver's inputs will be in the propulsion system, either in acceleration or deceleration manoeuvres, and in the steering system, in cornering manoeuvre. External disturbances will consist of road vertical input.

5.1.1 Steering Input

The simulation runs which will exercise the vehicle's handling or stability and steering control properties will consist of manoeuvres of the so called 'free-control' type, in which steering wheel angle is considered as the input.

Quantities depicted and analysed include yaw rate, lateral velocity and acceleration, roll angle, wheel slip angle, lateral and vertical tyre forces. As it would be expected, the variation of other quantities of motion associated with vehicle's forward and vertical motion are negligible.

5.1.2 Traction/Braking Input

For the acceleration and deceleration simulations, the powertrain and/or braking system dynamics are considered in a very approximate manner. Deterministic functions of time are adopted which account for typical time delays involved in these system's responses and the available level of performance for the type of vehicle being modelled. The input quantities will be either driving or braking forces for the linear tyre which assume no slip in the longitudinal motion of the wheels.

5.1.3 Road Vertical Disturbances

The road vertical inputs consist of crossing obstacles of different types such that all relevant aspects of vehicle motion are exercised. For this purpose, out-of-phase triangular bumps are transversed, together with terminated ramp and sinusoidal road profile inputs.

The quantities of motion which undergo significant variation and are illustrated in the results include sprung mass vertical displacement and acceleration and pitch and roll angles. For the unsprung masses vertical displacement of the wheel's center of gravity, tyre force and suspension travel are included. As an aid to the analysis, the road profile is superposed in some of the simulation results.

5.2 Steady-State Manoeuvres

The primary task of the steady-state manoeuvre simulations is to obtain static and/or steady-state response information in order to assess the linearity of the model for the various quantities of motion involved in response to the already mentioned inputs and disturbances. Another important purpose was to determine directional stability parameters, related to the specific vehicle being modelled, such as static margin, understeer or oversteer behaviour, etc. Simulation runs which are executed include cornering with constant speed and different steering angles, cornering with constant steering angle and various forward velocities and transversing sinusoidal road profiles of different heights and lengths.

5.2.1 Cornering at Different Speeds

In this simulation the vehicle is considered to be driving at various constant speeds when the vehicle is steered by a parabola to step steering input and intended to cover a realistic range of operating conditions of the vehicle.

The variables which are analysed as functions of forward velocity include pitch and roll angles, lateral velocity, yaw rate, sideslip angle and lateral acceleration of the sprung mass and slip angle of the wheels.

5.2.2 Cornering with Different Steering Angles

This simulation is similar to the one previously described, but in this case the forward velocity is kept constant and larger values of lateral acceleration are obtained by performing the cornering manoeuvres at different values of steering angles.

5.2.3 Crossing of Obstacles of Different Heights

In order to investigate the nonlinearities of the quantities of motion in relation to road disturbances, their behaviour are analysed for the vehicle driving over a sinusoidal road profile with various amplitudes.

6 Analysis of Simulation Results

It has been a traditional practice in vehicle dynamics studies [3, 6, 12] to divide the quantities of motion in groups according to which aspects of vehicle motion they are related to. As a result of this, the areas of performance, handling and ride studies have been established. Although the purpose of the present work is to look at the automotive vehicle as a total system, these divisions are well consolidated by their use in practice and also they can be very helpful in understanding overall vehicle behaviour and the relationship amongst the innumerable variables of motion.

The area of performance studies or longitudinal dynamics, is concerned with the behaviour of the vehicle when driving straight ahead or at very small lateral accelerations. Vehicle handling or stability refers to a vehicle's behaviour in response to the application of lateral forces and yawing moments caused by steering inputs and vehicle motion. It is also concerned with vehicle lateral stability when driving straight ahead. Ride studies are concerned with a a vehicle's behaviour as it transverses uneven road surfaces. Under certain operating conditions, these areas can be studied separately, as has often been the case. However, the coupling amongst these variables must be taken into consideration under other conditions, especially limit manoeuvres. The objectives of this simulation analysis is to look at these couplings, their magnitude and nature.

6.1 Definition of a Driving Envelope

The concept of a driving envelope has been originated in aeronautical research and it refers to the operating range of an airplane in terms of its speed, normally expressed in mach number, and the altitude range which it covers. A similar concept called driving range has also been used in automotive engineering [6] and it usually refers to the range of longitudinal and lateral accelerations and forward velocity in which a car can operate, given a certain road condition.

In the present case, the concept of driving envelope are used in a different manner. The idea is to define an operating range, or set of operating ranges, in which a linearised, or set of interrelated linearised conditions, would still be valid, and therefore amenable for control design and analysis purposes. This analysis is based on the previous simulation results.

The quantities which are used to define the envelope are longitudinal, lateral and vertical quantities. Its concept is diagrammatically illustrated in figure 5. The figure represents the limits of linear validity of the models in the X, Y and Z directions.

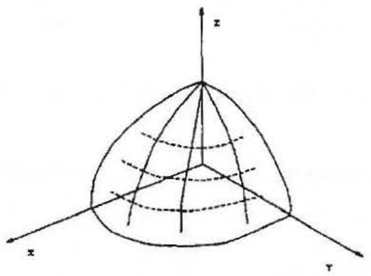

Figure 5: Concept of Driving Envelope

The transient manoeuvre simulations are analysed in comparison to the expected behaviour of a linear system resulting from an eigenvalue-eigenvector analysis around the corresponding operating points. Also, in order to allow a comparison between the various linear system representation, that is, eigenvalues, eigenvectors and system matrices, around different operating points, a certain driving condition will be taken as a *reference* configuration.

The steady-state manoeuvres will be used to obtain the static and/or gain relationship between the various variables of motion and the inputs or disturbances. In this case it will be possible to analyse the strength of the coupling between these variables, and how linear this coupling is. This analysis, together with the previous one, allow the decision of a range of operation in which a linear representation of the system is still valid, and further studies, such as model order reduction, controller analysis and design for active systems, etc. is possible.

6.2 Eigenvalue Analysis (transient)

The analysis of the transient motion of the previous simulation runs is performed based on an eigenvalue/eigenvector analysis of the matrices of the linearised system, determined at certain operating conditions. As described earlier, motion quantities are divided according to which modes they are associated with and to which they contribute more significantly. In this sense, some quantities of motion are either taken into consideration or neglected, according to the circumstances of the simulation run.

6.2.1 Steering Input

Steering inputs are related to a vehicle's handling behaviour which is concerned with the ability of the vehicle to change directions at the driver's request and to maintain directional stability at various operating conditions.

The variables of motion which seem to undergo significant variation during cornering manoeuvres include sprung mass yaw rate, lateral velocity or sideslip angle, lateral acceleration and roll angle, and unsprung mass slip angle. Related quantities which are important for the motion, and/or stability purposes, are the tyre forces. Figure 6 depicts some variables of a typical test.

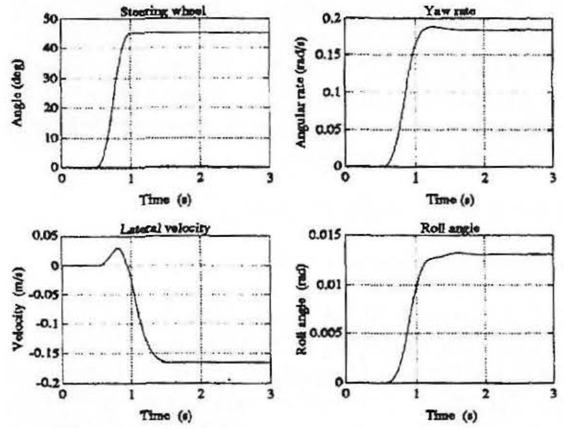

Figure 6: Response of the sprung mass to a steering input

Vehicle directional stability characteristics are related to the eigenvalues of the characteristic equation which is associated with the handling modes. However, a number of parameters which specify a vehicle's handling behaviour, and therefore stability, have been derived from simple models and are well established and understood in the vehicle dynamics community. For handling studies, three *parameters* play a central role in defining vehicle's behaviour:

- front cornering stiffness (gradient);
- rear cornering stiffness (gradient) and
- location of the vehicle's centre of gravity in relation to the wheelbase.

The value of these parameters determine a vehicle's transient and steady-state characteristics, as well as directional stability properties. The handling mode characteristics of a vehicle are shown to be those most susceptible to variations in operating conditions. The *quan-*

	Linear		Nonlinear	
Speed	ω_n	ξ	ω_n	ξ
m/s	rd/s	–	rd/s	–
5.0	31.11	1.0	28.96	1.0
	22.57	1.0	23.62	1.0
10.0	13.99	0.97	13.72	0.96
15.0	9.99	0.90	9.72	0.91
20.0	8.13	0.83	7.85	0.85
40.0	5.80	0.59	5.47	0.61
100.0	5.01	0.29	4.92	0.28

Table 5: Handling mode frequencies and damping factors as function of forward velocity for 2 tyre models

tity of motion which seems to affect the handling characteristics most significantly is the vehicle's *forward velocity*. The eigenvalue analysis of the Jacobian of the nonlinear vehicle model shows that for the vehicle driving in a straight line the handling modes, yaw rate and lateral velocity (or sideslip angle, because forward velocity is assumed to be constant) show the following behaviour with increased speed:

- decrease in natural frequency;
- decrease in damping factor.

This situation is illustrated in table 5.

Because the eigenanalysis at these operating points results in the full set of frequencies, damping factors and modes, The effects of steering angle and forward velocity in the ride and performance modes is also obtained. It could be observed that they have not been affected significantly by forward velocity nor steering angle for the present linear tyre model. The limiting situation for using the linearised model is loss of adhesion which happens do to load transfer effects. The variable of motion which is related to load transfer effect and therefore should be used for a control strategy is *lateral acceleration*. For values of lateral acceleration larger than $5m/s^2$ on a dry road, loss of contact ocurred.

6.2.2 Traction/Braking Input

Traction and braking inputs are associated with a vehicle's performance. In the case of the present model, studies are affected and limited due to the lack of engine, transmission and braking system models, since the dynamics of these subsystems determine vehicle motion. However, it could be observed that that ride modes are execised by performance manoeuvres and that the variation of wheel loads may affect vehicle lateral stability in combined manoeuvres. Figures 7 and 8 represents some variables of motion for such simulation.

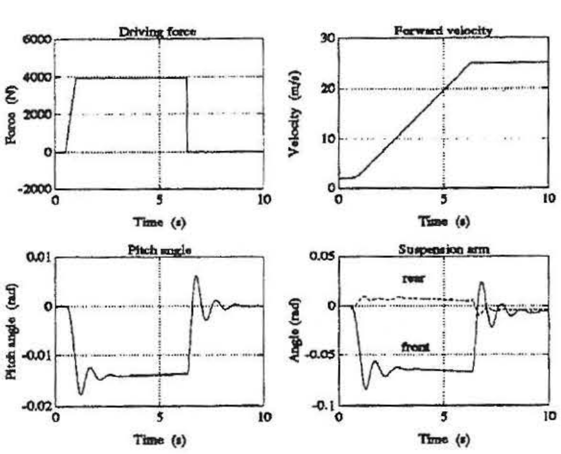

Figure 7: Response of the sprung mass to a acceleration force

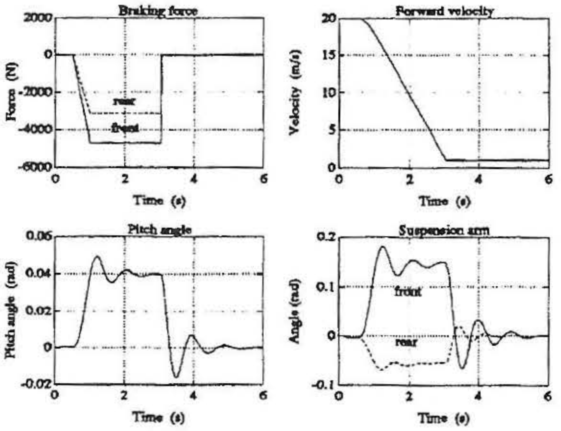

Figure 8: Response of the sprung mass to a deceleration force

6.2.3 Road Vertical Disturbances

Road vertical disturbance simulations are aimed at testing vehicle ride modes. The study of vehicle ride modes assumes a contact point model for the interaction between tyre and road in the vertical direction. This situation is equivalent to a lumped parameter model for the tyre vertical force characteristics. One such test result are illustrated in figures 9 and 10.

Analysing the influence of bump height for the same speed, it can be observed that, for all variables of motion, the changes in transient behaviour is completely negligible. One cannot even distinguish to which height the simulation refers to, if the scale of the y-axis would not be shown!

The behaviour of the sprung mass for the same bump height and for three different speeds shows that, irrespective of the value of the speed, their transient motion *after* the inputs have been applied, in the range in

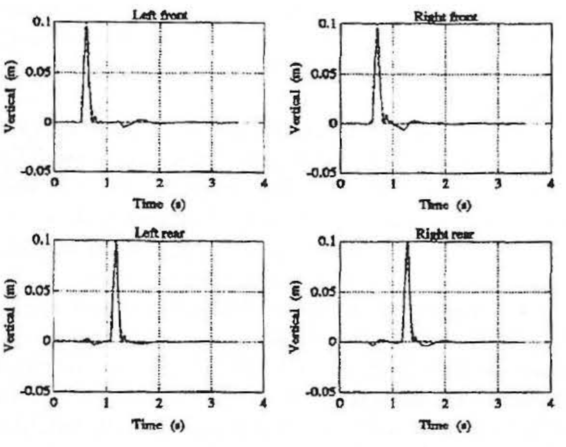

Figure 9: Response of the wheels to a road input

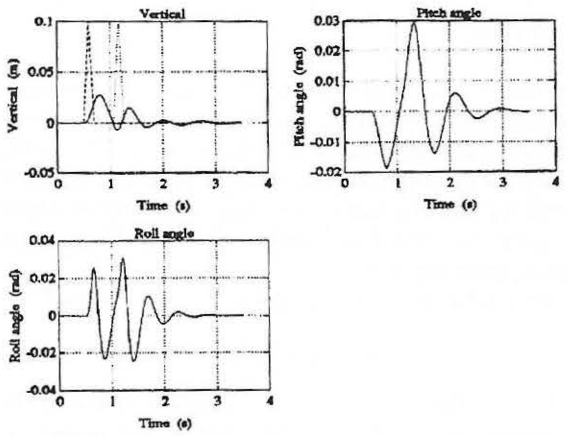

Figure 10: Response of the sprung mass to a road input

which they get excited by the bump input, are nearly the same, in terms of the frequency of oscillation and damping. In this way, vertical displacement and acceleration and pitch and roll angles show similar behaviour for all three speeds for each one of the bump heights.

These results are interesting in the sense that they confirm that the nonlinear vehicle model does not present significant differences in its transient behaviour when under ride mode types of excitation, in terms of *forward speed*. Together with the previous conclusion concerning the effect of bump height and constant speed, these results provide good insight into the use of the linear model approximation of the nonlinear vehicle model for the area of ride performance. It is worth noting that this area's main concern, from a control point of view, is analysis and design of controlled suspension.

6.3 Gain Analysis (steady-state)

The gain, or steady-state analysis, is intended to assess the level of coupling amongst the variables of motion, the amount of nonlinearities in these couplings and also to determine vehicle directional stability parameters.

For this analysis, three types of steady-state simulations have been performed:

- cornering at constant speed with various steering angles;
- cornering with constant steering angle at various speeds;
- crossing of obstacles with sinusoidal profile of different heights, and at different speeds.

The results presented in the table 6 are pitch and roll angles, yaw rate, lateral velocity and acceleration, sideslip angle and slip angle of the wheels. They are calculated as a function of the steering angle for the constant speed manoeuvre. For the constant steering manoeuvres the same quantities as above are calculated in terms of forward velocity.

Constant Speed Input: Steering (rd)			Constant Steering Input: Speed (m/s)		
Output	Gain	Corr.	Out.	Gain	Corr.
a_y (m/s^2)	4.5921	1.0	a_y	0.089	1.0
q_5 (rd)	0.0004	0.98	q_5	0.002e-3	0.79
q_6 (rd)	0.0165	1.0	q_6	0.0003	0.99
u_4 (rd/s)	0.2304	1.0	u_4	0.0013	0.79
V_y (m/s)	-0.2003	-1.0	V_y	-0.0201	-0.96
β (rd/s)	-0.0100	-1.0	β	-0.0007	-0.99
α_f (rd/s)	-0.0447	-1.0	α_f	-0.0009	-0.99
α_r (rd/s)	-0.0277	-1.0	α_r	-0.0005	-0.99

Table 6: Steady-state Gains

As it can be deduced from this table, all variables chosen to be retained showed to be largely affected by the steering input. The only exception being the pitch angle, which showed the smallest influence of all. This result confirms the widespread practice to ignore vehicle pitch from the state variables in handling studies, and when it is considered, its effect is calculated in an approximated manner.

It also confirms real vehicle behaviour, because a vehicle does not suffer large pitch angle variations in any normal operating condition. The other variables chosen for the sprung mass movement, yaw rate (u_4), and lateral velocity in vehicle body (V_y), and slip angle (α_i) for the wheels, are the main quantities in handling studies and their importance for handling studies is obvious.

For the remaining variables of motion related to vehicle handling, it can be seen that the model coupling concerning forward velocity, can be adequately described by linear relations for vehicle steady-state behaviour. This can have interesting design implications from a control point of view, depending which strategy is adopted to implement the control. If forward velocity is adopted as a system *parameter*, the fact that model characteristics change in a linear way for the steady-state behaviour, together with the conclusions already made about the transient behaviour, may allow the implementation of some kind of adaptive control, for example, as a function of vehicle speed.

The sinusoidal profile obstacle crossing manoeuvres are discussed next. The objectives of these simulations are to verify the linearity of the ride modes of the vehicle model.

It can be concluded that the present vehicle model, with linear relations for the spring and damper of the suspension, and for a linear vertical tyre behaviour, as well, can be adequately represented by a linearised model, concerning ride modes behaviour. The limiting factor in this case being the loss of adhesion or compression of the tyre beyond the assumed linear range. These results show that the inertia couplings, in these models, for this mode are weak.

6.4 Determination of a Driving Envelope for Linear Control Analysis and Design

The discussion about a driving envelope for linear control analysis and design is centred at the previous simulation runs.

The discussion will be based on the type of vehicle model adopted and the area of vehicle behaviour intended to be addressed. Issues with regard to strong points as well as limitations of the present model are discussed and conjectures about the effects of possible variations of the model are elaborated.

The areas of vehicle behaviour which are discussed are the same according to the division established for the simulation runs. The effects and consequences of considering them together is also attempted to be discussed.

For the vehicle ride behaviour, with its present tyre and suspension models the range of validity of linear behaviour seems to be limited only by the loss of contact of the wheels with the road. These conclusions are based on the results of the transient and the steady-state analysis of the present simulations. For the transient results it can be observed that there is no significant changes in vehicle transient behaviour as the levels of model inputs are varied.

Concerning vehicle performance, not many conclusions can be drawn due to the limitations of the present model powertrain and braking system dynamics, as well as the longitudinal tyre model and which play an important role in vehicle performance mode and its coupling to other vehicle modes, most notoriously to vehicle handling.

Finally, with respect to the handling modes, it seems that for the linear tyre model, quite adequate linear approximations can be made especially for low value of lateral acceleration, based on the analysis of the present simulation results.

In any case, linear model which are adaptive with vehicle speed seem to be recommendable, to successfully implement such controls. The coupling with the ride modes have been previously described and it does not seem likely that a steering control would affect suspension behaviour, but the contrary would be true.

In the next section the resulting linearised model obtained from the vehicle operating in the range determined in this chapter are analysed from a numerical and control theoretical point of view. The variation in system properties such as conditioning, controllability, observability, principal gains, etc., are analysed for these models at these operating conditions.

7 Linear Model and its Properties

For the analysis of the linear system, time domain representation of the model are discussed. Time domain representation is described by the state space model, in the form of the system matrices, $\boldsymbol{A}$,$\boldsymbol{B}$,$\boldsymbol{C}$,$\boldsymbol{D}$. The discussion concerning the state space representation encompasses structural, numerical and control theoretical properties of the system matrices.

The nonlinear vehicle model can be represented by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) \tag{1}$$

where $\boldsymbol{x}$ the state vector is comprised of the of the generalised coordinates and the generalised speeds. $\boldsymbol{u}$ is the input vector, which includes driver's inputs and external disturbances.

The Jacobian is the partial derivative of the state derivative vector with respect to the state vector and it is a square matrix of dimension equal to the number of state variables.

ACSL through numerical perturbation of the states provides an approximation of the system's Jacobian at an operating point, $\boldsymbol{x}_o$ of the state vector. In this case, it can be written for the autonomous linearised system that

$$\dot{\boldsymbol{x}} = \boldsymbol{J}\boldsymbol{x} \tag{2}$$

where the ij^{th} element of J is given by

$$(J)_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{\boldsymbol{x}_o} \tag{3}$$

ACSL can also determine the linear representation for the nonautonomous system. With the definition of the control variables, $\boldsymbol{u}$, and the output quantities, $\boldsymbol{y}$, a perturbation analysis is performed as before, on the augmented state vector returning the system's matrices in the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{4}$$

where A,B,C,D are the matrices of the state space representation of the nonlinear system described by equation 1. Because ACSL also includes an eigenvalue/eigenvector analysis facility, the Jacobian can be used to determine natural frequencies and modes in a modal analysis study or this same facility can be used for open loop stability analysis in a control system design. According to the way the state vector have been defined in this study, the Jacobian J and the system matrix A coincide.

7.1 Properties of Resulting Matrices

Properties of the system matrices which are analysed include structural, symmetrical and block characteristics. Initially, a discussion of the A matrix and its elements is performed. Afterwards, numerical properties such as conditioning and control related properties are discussed which include open loop stability, controllability, observability and model order reduction.

7.1.1 Elements of the A Matrix

The state vector x comprises the generalised coordinates, q, the generalised speeds u. The kinematical equations describe how the derivatives of the generalised coordinates are related to the generalised speeds. For a vehicle in a given trim condition, either driving straight ahead or in steady-state turning, if the roll and pitch angles are small, then the Jacobian of the kinematical differential equations is the identity matrix.

The A matrix can be further divided into 4 regions (submatrices) according to figure 11.

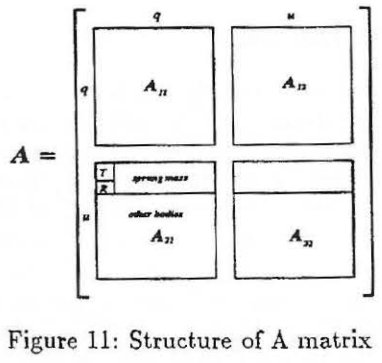

Figure 11: Structure of A matrix

According to the previous reasoning about the kinematical equations, submatrix A_{11} is zero and submatrix A_{12} is the identity matrix, if $m = p$.

The concept of *stability derivatives*, as has been originated and defined in aeronautics is widely used in vehicle dynamics [12]. They are defined as the partial derivatives of the forcing functions in relation to the states. As far as possible, this discussion will refer to the stability derivative concept in order to help clarify the meaning of the elements of the A matrix being discussed.

Submatrices A_{21} and A_{22} refer to the derivatives of the generalised speeds which can be thought of as acceleration terms. They can also be further subdivided. The first six rows of A_{21} and A_{22} correspond to the six degrees of freedom of the sprung mass. The first three of these correspond to the translational acceleration terms and the last three to the angular acceleration ones. The remaining 4 rows correspond to angular accelerations terms of the swing axle.

For the first six rows, which correspond to the six degrees of freedom of rigid body of the sprung mass, these terms possess some analogy to the stability derivative concept previously mentioned. For example, in the equation of $\dot{u}_4$, the proportionality factor of u_4 is related to the damping-in-yaw factor as it is known in vehicle dynamics nomenclature [3, 12] With this model it is possible to assess the influence on the state variables of many degrees of freedom which are ignored in simple models, either comparatively or by themselves. For instance, in this case one has the front suspension displacement yaw torque coefficient, which has not been previously defined in the literature, and such as this one, there are many others in the present model.

7.1.2 Other System Matrices

This section presents and describes the B,C,D matrices. The B matrix is of dimension equal to the number of states by the number of inputs.

The C matrix depends on the observability of the states and which variables are defined as outputs. The concept of observability is discussed in more detail in a future section. In our case, it was assumed observability and the translational velocities of the sprung mass were adopted to be those expressed in the sprung mass frame, instead of those corresponding to the states which are velocities relative to an inertial frame.

The D matrix, because no output was defined to depend on the inputs, is the null matrix. In some situations, depending on the definition of the output variables, it might not be zero, for example if one defines the tyre deflections to be an output variable, the equivalent terms in the D matrix corresponding to the road input would be different from zero.

7.1.3 Conditioning

When working with low order SISO models ($n < 5$) computers are quite forgiving and insensitive to numerical problems. For high order models and MIMO systems the finite precision arithmetic of a computer requires caution to be exercised.

One characteristic of large vehicle models when the linearised equations are obtained is their low condition number, indicating sensitivity of the data to numerical disturbances. In order to evaluate the conditioning of a matrix, the concept of *singular values* is important. The condition number of a matrix is defined as

$$cond(\boldsymbol{A}) = \frac{\overline{\sigma}(\boldsymbol{A})}{\underline{\sigma}(\boldsymbol{A})} \tag{5}$$

where $\overline{\sigma}(\boldsymbol{A})$ is the largest and $\underline{\sigma}(\boldsymbol{A})$ is the smallest singular value of $\boldsymbol{A}$. Note that $\text{cond}(\boldsymbol{A}) \geq 1$. A useful rule of thumb in using the condition number for assessing numerical accuracy of calculations performed with the matrix is that the machine may loose the last $log_{10}[cond(\boldsymbol{A})]$ decimal places of a solution because of round-off errors during Gaussian elimination [7].

The SVD of the $\boldsymbol{A}$ matrix of the linearised model in the reference configuration as described in chapter 5, results for the largest and the smallest singular values

$$\begin{aligned} \overline{\sigma}(\boldsymbol{A}) &= 28416 \\ \underline{\sigma}(\boldsymbol{A}) &= 0.4797 \end{aligned} \tag{6}$$

giving a condition number of $5.9 * 10^4$, which according to the previous rule indicates that for numerical accuracy the computer may loose up to 4 decimal places (in our case out of 16) due to round-off errors.

7.1.4 Controllability/Observability

In order to be able to achieve control systems design which have optimal behaviour in some sense, it is necessary to assess the controllability and observability properties of the open-loop system. Also, these properties establish the conditions for complete equivalence between the state space and the transfer function representations of a system.

A system is said to be completely *state-controllable* if, for any initial time t_o, each initial state $\boldsymbol{x}_o$ can be transferred to any final state $\boldsymbol{x}_f$, in a finite time, $t_f > t_o$, by means of an unconstrained input vector $\boldsymbol{u}(t)$. It means that each mode of the system must be directly affected by the input $\boldsymbol{u}(t)$ and it requires that the controllability matrix has full rank.

A system is said to be completely *state-observable* if every initial state $\boldsymbol{x}_o$ can be exactly determined from the measurements of the output $\boldsymbol{y}(t)$ over the finite time interval $t_o \leq t \leq t_f$. It means that for observability the output must be influenced by each state $\boldsymbol{x}_i$. It can be shown that the system is completely observable if the observability matrix has full rank. If the system is completely controllable and observable, then a state space and the transfer function matrix representations are equivalent and accurately represent the system.

7.1.5 Model Order Reduction

In practice, very good simplifications can be obtained by using a rather simple procedure in which a balanced realisation of the original system is derived and then simply truncated by discarding those parts relating to the state variables most weakly coupled to the inputs and outputs.

For this purpose the joint gramian can be used to reduce the order of the model, if the system is normalised properly. Since it reflects the combined controllability and observability of individual states, it is reasonable to remove those states from the model to which correspond a a small singular value in the joint gramian. Elimination of these states therefore retains the most important input/output characteristics of the original system, as initially intended.

For the problem of this thesis, two alternatives to model order reduction exist. The first one is performed in ACSL and the other in MATLAB using the method previously discussed. The approach to be adopted in this work will make use of both ways, at different stages of the analysis.

The elimination of some states from the state variable vector is performed in ACSL through the use of the `ANALYZ` command with the `'freeze'` option. Later, after this is completed in ACSL, the joint gramians of the balanced realisation of the system matrices are used to determine further model order reduction possibilities. In the resulting model, for the reference configuration, observation of the joint gramian of the balanced realisation for the $\boldsymbol{A}$ matrix shows that there is only one order of magnitude difference between the largest and the smallest singular value, clearly indicating that all states in the present model, according to its present scaling, are important and should be retained.

Also, the combination of ACSL and MATLAB for obtaining the linearised model, designing the controller and simulating it controlling the nonlinear model, interactively, provide a very powerful tool for the analysis and design of control systems.

8 Conclusions

This paper has discussed the importance of the derivation of vehicle chassis models encompassing all aspects of vehicle motion in order to allow the development of integrated vehicle control. It has proposed an approach to modelling vehicle suspension which is based on simple experimental data and uses concepts of differential geometry. The derivation, simulation and analysis of the model was performed using the computer programs SD/FAST, ACSL and MATLAB, respectively. The model has been validated and therefore it could be used for simulation analysis aimed at obtaining operating ranges in which linear approximations woould be valid. For the present model it can be concluded that the performance and ride modes are not affected significantly by variation in operating conditions nor the steady-state response by the magnitude of the inputs. The limiting factors for these aspects of motion are loss of tyre contact. the handling modes are those most affected by changes in operating conditions and the variable of motion which seems to have the largest influence is vehicle speed. However it worth noting that this change do not affect other modes and the coupling occur at the tyre model more than at the vehicle body model.Properties of the resulting linearised models are also discussed and it can be observed that the present model results in a fully controllable, fully observable model. However, even for simple models such as this numerical conditioning of the system matrices are already an area of concern.

References

[1] *ACSL - Advanced Continuous Simulation Language. Reference Manual*, Mitchell and Gautier Associates, 1987.

[2] Costa, A., *Application of Multibody System (MBS) Modelling Techniques to Automotive Vehicle Chassis Simulation for Motion Control Studies* Ph.D. Thesis, University of Warwick, Conventry, England, 1992.

[3] Ellis, J.R., *Vehicle Dynamics*, London Business Books ltd., London, 1969.

[4] Kane, T.R., Levinson, D.A., *Dynamics: Theory and Applications*, McGraw-Hill, New York, 1985.

[5] Kortum, W., Sharp, R., *A Report on the State-of-Affairs on "Application of Computer Codes to Vehicle System Dynamics"*, Vehicle System Dynamics, vol. 20, no. 3-4, pp 177-184, 1991.

[6] Lugner, P., *Horizontal Motion of Automobiles, Theoretical and Praçtical Investigations*, Dynamics of High-Speed Vehicles, CISM Course no. 274, Springer-Verlag, 1982.

[7] Maciejowski, J.M., *Multivariable Feedback Design*, Addison-Wesley Publishing Co., 1989.

[8] *Pro-Matlab: User's Guide - For Sun Workstations*, The MathWorks Inc., 1987.

[9] Rosenthal, D.E., Sherman, M.A., *High Performance Multibody Simulations via Symbolic Equation Manipulation and Kane's Method*, J. Astr. Sci., 34, 223-239, 1986.

[10] Schiehlen, W.O., *Introduction to Vehicle Dynamicsj¡*, Dynamics of High-Speed Vehicles, CISM Course no. 274, Springer Verlag, 1982.

[11] *SD/FAST User's Manual - version b1.1 prej-release edition*, 1990.

[12] Segel, L., *Theoretical Prediction and Experimental Substantiation of The Response of the Automobile to Steering Control*, IMech.E, A.D., London, 1956.

Appendix

Variables of motion and A matrix for a forward velocity of 20 m/s.

- Variables of motion

3 - Vertical Displacement
4 - Yaw angle
5 - Pitch Angle
6 - Roll Angle
7 - Front Left Suspension Angular Displacement
8 - Front Right Suspension Angular Displacement
9 - Rear Left Suspension Angular Displacement
10 - Rear Right Suspension Angular Displacement
11 - Foward Velocity
12 - Lateral Velocity
13 - Vertical Velocity
14 - Yaw Rate
15 - Pitch Rate
16 - Roll Rate
17 - Front Left Suspension Angular Velocity
18 - Front Right Suspension Angular Velocity
19 - Rear Left Suspension Angular Velocity
20 - Rear Right Suspension Angular Velocity

- A Matrix

- Columns 3 through 11

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
-3.0650e+01	-7.4506e-04	-3.3912e+01	0	3.4001e-01	-3.4001e-01	-1.0508e+01	1.0508e+01	-1.0820e-02
1.4552e-07	1.2248e+02	-4.6566e-06	6.0190e+01	6.3453e+00	6.3453e+00	1.8329e+01	1.8329e+01	-1.1642e-07
-3.4511e+01	2.1954e-01	-1.5912e+01	-2.9942e-03	2.8509e+00	-2.8524e+00	3.4643e+00	-3.4643e+00	1.0143e-02
-3.7446e-07	-2.1383e+01	1.6734e-06	-7.6278e+00	4.4467e+00	4.4467e+00	-1.0483e+01	-1.0483e+01	8.6408e-08
-1.8130e+00	1.1660e-02	-3.3433e+01	0	-1.5884e+00	1.5884e+00	4.4020e+00	-4.4020e+00	3.6974e-03
1.6637e-06	3.7719e+01	-9.3090e-05	1.9681e+01	2.9174e+01	2.9174e+01	2.9566e+01	2.9566e+01	-4.6564e-07
-1.1439e+04	1.0659e+03	1.5130e+04	-8.6747e+03	-6.0539e+03	-2.5428e+02	-4.5364e+01	-5.4666e+01	-1.1101e-02
1.1439e+04	1.0667e+03	-1.5130e+04	-8.6747e+03	-2.5428e+02	-6.0539e+03	-5.4666e+01	-4.5363e+01	1.1086e-02
-9.6075e+03	1.2852e+03	-1.4391e+04	-7.2914e+03	-3.5886e+01	-3.4796e+01	-6.9574e+03	-2.8622e+01	-2.4647e-02
9.6075e+03	1.2859e+03	1.4391e+04	-7.2914e+03	-3.4799e+01	-3.5888e+01	-2.8622e+01	-6.9574e+03	2.4676e-02

- Columns 12 through 20

0	1.0000e+00	0	0	0	0	0	0	0
0	0	1.0000e+00	0	0	0	0	0	0
0	0	0	1.0000e+00	0	0	0	0	0
0	0	0	0	1.0000e+00	0	0	0	0
0	0	0	0	0	1.0000e+00	0	0	0
0	0	0	0	0	0	1.0000e+00	0	0
0	0	0	0	0	0	0	1.0000e+00	0
0	0	0	0	0	0	0	0	1.0000e+00
7.4506e-05	0	0	0	0	-9.2387e-03	9.2760e-03	-6.5640e-02	6.5640e-02
-6.1292e+00	0	1.8878e+00	0	0	2.8011e-02	2.8010e-02	1.1801e-01	1.1801e-01
-1.2042e-02	0	0	0	0	3.8176e-01	-3.8179e-01	7.3558e-01	-7.3558e-01
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-5.3368e+01	0	-7.5526e+01	0	0	-3.2987e+01	-1.2517e-01	-3.0953e+00	-3.2809e+00
-5.3417e+01	0	-7.5525e+01	0	0	-1.2517e-01	-3.2988e+01	-3.2812e+00	-3.0950e+00
-6.4294e+01	0	9.9508e+01	0	0	-1.1331e+00	-1.1975e+00	-5.4254e+01	2.4438e-02
-6.4330e+01	0	9.9508e+01	0	0	-1.1984e+00	-1.1331e+00	2.4438e-02	-5.4254e+01