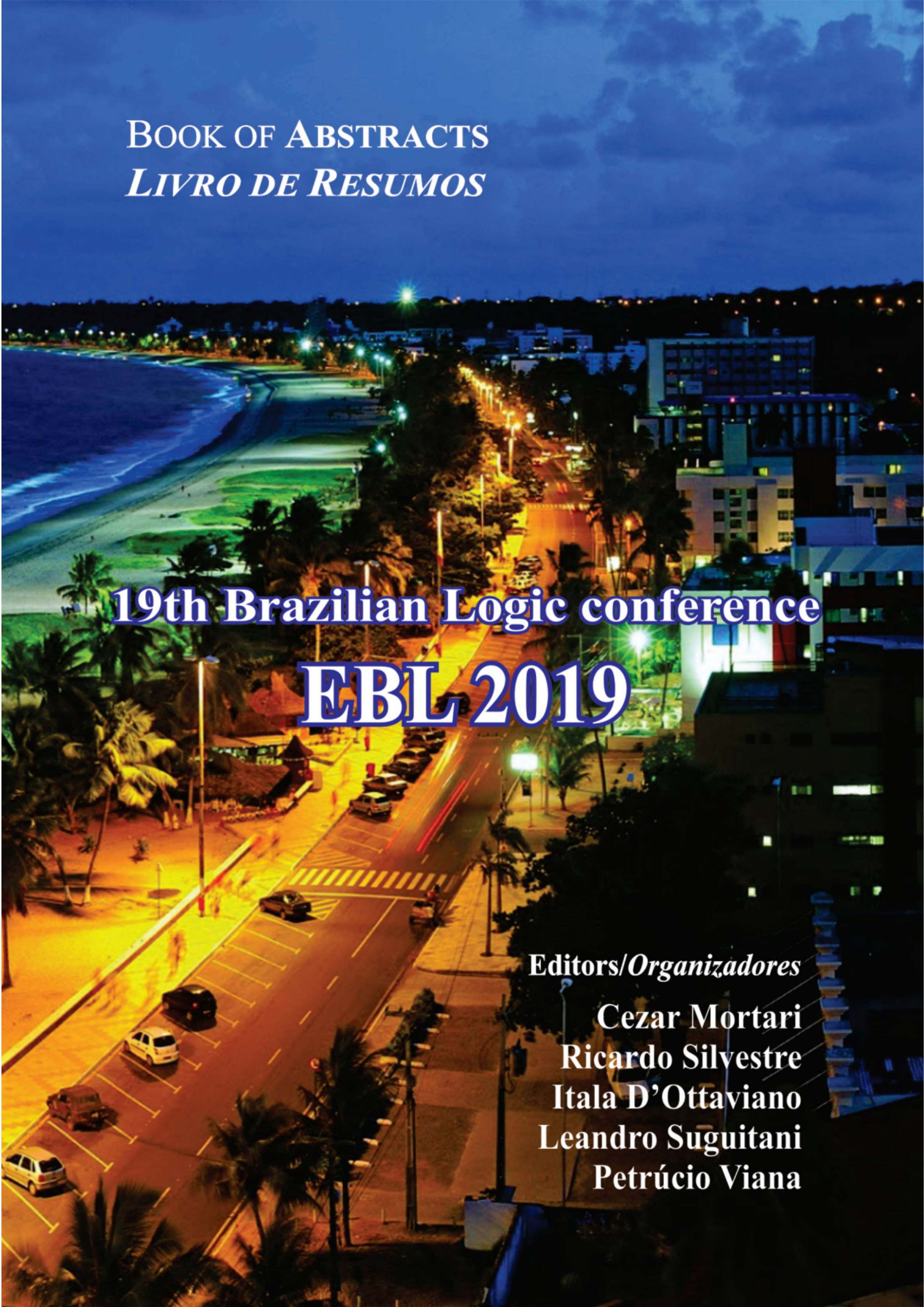

Book of Abstracts
Livro de Resumos

19th Brazilian Logic conference
EBL 2019

Editors/*Organizadores*

Cezar Mortari
Ricardo Silvestre
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Leandro Suguitani
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19TH BRAZILIAN LOGIC CONFERENCE

BOOK OF ABSTRACTS

XIX ENCONTRO BRASILEIRO DE LÓGICA

LIVRO DE RESUMOS

XIX EBL
19th Brazilian Logic Conference

May 6-10th, 2019 — João Pessoa, Brazil

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MÍDIA GRÁFICA E EDITORA LTDA

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B794 19th Brazilian Logic Conference EBL 2019: Book of Abstracts / Cezar Augusto Mortari, Ricardo Silvestre, Itala Maria Loffredo D'Ottaviano, Leandro Suguitani, Petrucio Viana (Organizadores) – João Pessoa : Mídia Gráfica e Editora Ltda, Co-Editora UFCG – EDUFCG, Campina Grande – 2019.
226 p.

ISBN 978-85-2370-166-6

1. Lógica. 2. Matemática. 3. Ciências da Computação. 4. Filosofia.

CDU: 510:6

Von Neumann Regular $\mathscr{C}^\infty$-Rings and Applications to Boolean Algebras

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In this paper we introduce and explore the notion of a "von Neumann regular $\mathscr{C}^\infty$-ring", as well as some of its applications to the theories of Boolean spaces and Boolean algebras.

A $\mathscr{C}^\infty$-ring can be considered, from an universal algebraic viewpoint, as a pair $\mathfrak{A} = (A, \Phi)$, where the carrier A is a (non-empty) set and Φ is a function that assigns to every smooth $n-$ary function, $f : \mathbb{R}^n \to \mathbb{R}$, an $n-$ary function symbol, $\Phi(f) : A^n \to A$, preserving all the equational relationships between real smooth functions. Given two $\mathscr{C}^\infty$-rings $\mathfrak{A} = (A, \Phi)$ and $\mathfrak{B} = (B, \Psi)$, a homomorphisms from $\mathfrak{A}$ to $\mathfrak{B}$ is a function $\varphi : A \to B$ such that for every smooth function $f : \mathbb{R}^n \to \mathbb{R}$, for every $n \in \mathbb{N}$, we have $\Psi(f)\circ\varphi^{(n)} = \varphi\circ\Phi(f)$. These data - together with the ordinary composition of functions and identity morphisms - compose a category that we denote by $\mathscr{C}^\infty$**Rng**.

There is a natural "forgetful functor" from $\mathscr{C}^\infty$**Rng** to **CRing**, obtained by "forgetting" the interpretations of all smooth functions, except for the sum, the product, the opposite and the interpretations of $0, 1 \in \mathscr{C}^\infty(\{*\}, \mathbb{R})$. Such a forgetful functor, that we denote by $\widetilde{U} : \mathscr{C}^\infty\mathbf{Rng} \to \mathbf{CRing}$, provides us with a convenient definition of a *von Neumann regular $\mathscr{C}^\infty$-ring* as a $\mathscr{C}^\infty$-ring $\mathfrak{A} = (A, \Phi)$ whose underlying ring is a von Neumann regular ring in the ordinary sense, that is,

$$\widetilde{U}(\mathfrak{A}) \models (\forall a \in A)(\exists e \in \mathrm{Idemp}(A))(\exists x, y \in A)(a\cdot x = e \ \&\ e\cdot y = a),$$

where $\mathrm{Idemp}(A) = \{b \in A : b^2 = b\}$ is the Boolean algebra consisting of all idempotent members of A.

The category of all von Neumann regular $\mathscr{C}^\infty$-rings, together with their homomorphisms, compose the category we denote by $\mathscr{C}^\infty$**vNRng**. We prove, using different methods, that $\mathscr{C}^\infty$**vNRng** is a reflective subcategory of $\mathscr{C}^\infty$**Rng** and we show, among other things, that the "Moerdijk-Reyes" Zariski spectrum functor (first defined in[13]) restricted to this subcategory, $\mathrm{Spec}^\infty \restriction: \mathscr{C}^\infty\mathbf{vNRng} \to \mathbf{Top}$ has, as its essential image, the category of all Boolean spaces. Also, in this case, the structure sheaf (first defined in [14]) is such that its stalks are $\mathscr{C}^\infty$-fields. In fact, we prove that this property characterizes *all* von Neumann regular $\mathscr{C}^\infty$-rings. Moreover, the subcategory of $\mathscr{C}^\infty$**Rng** consisting of all von Neumann regular $\mathscr{C}^\infty$-rings is characterized as the closure under small limits of the category of $\mathscr{C}^\infty$-fields, *i.e.*, it is the smallest subcategory of $\mathscr{C}^\infty$**Rng** which contain all $\mathscr{C}^\infty$-fields and is closed under small limits.

Finally we show that von Neumann regular $\mathscr{C}^\infty$-rings classify Boolean spaces in the following strong sense: for a fixed $\mathscr{C}^\infty$-field, $\mathbb{K}$, for each pair of Boolean algebras B, B'

and each Boolean algebra homomorphism, $h : B \to B'$, there is a pair of von Neumann-regular $\mathscr{C}^\infty$-rings, V, V', that are also $\mathbb{K}$-algebras, and a $\mathscr{C}^\infty$-homomorphism $f : V \to V'$ such that: $f\restriction_{\mathrm{Idemp}(V)}: \mathrm{Idemp}(V) \to \mathrm{Idemp}(V')$ is (naturally) isomorphic to $h : B \to B'$.

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