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ENERGY FUNCTION FOR POWER SYSTEMS WITH TRANSMISSION LOSSES: EXTENSION OF THE INVARIANCE PRINCIPLE

L.F.C. Alberto , H. M. Rodrigues, N.G. Bretas*

Summary - In many engineering and physical problems, it is very hard to find a Liapunov function satisfying the classical version of the LaSalle's Invariance Principle. This obstacle has been a great problem in the application of energetic methods to the stability analysis of power systems with more realistic models. In this work, an extension of the Invariance Principle, which does not require the Liapunov function to be negative semi-definite, is used to support theoretically the proposal of a new energy function for power systems with transmission losses.

Keywords - invariance principle, transient stability, direct methods, energetic methods, liapunov function.

1. INTRODUCTION

Direct methods have been shown to be suitable for the stability analysis of power systems on real time. Among these methods, the Liapunov's ideas associated to the LaSalle's Invariance Principle have been used to estimate the stability region of power systems. In the last two decades, many authors have adressed the problem of estimating the stability regions and these studies culminated with the development of the BCU method [4], which nowadays is considered the most efficient energetic method to study transient stability. In spite of these advances, the application of these methods to the assessment of stability in real power systems has found many obstacles. The main obstacle is that the energetic methods are still improper to deal with more realistic models. In fact this obstacle is intimately related to the problem of finding a suitable Liapunov function associated to those models.
The most known energy function nowadays was proposed by Athay et al.[3]. That function is an energy type Liapunov function in the Center of Angle(COA) formulation. In this case the loads were modeled as constant impedances and the network was reduced to the electromotive force buses. As consequence, the transfer conductances of the reduced system cannot be neglected and it is not possible to prove that this energy function is a Liapunov function in the usual sense.
In general, in order to find a Liapunov function, many simplifications are made to the power system model. The machines are usually modeled as a constant electromotive force behind the transient reactance, loads are modelled as constant power and the transmission losses are neglected. Also the existence of a infinite bus is required or a hypothesis of uniform damping is made.
Some improvements were made in order to consider more realistic models. Tsolas et al.[10] exhibited a general Liapunov function for a structure preserving power system model with the one-axis-model for the generators but no advance was achieved in the load model.
In other paper Alberto & Bretas [1] tried to take in account more realistic load models. They have provided a general Liapunov function for a structure preserving power system model in which the loads were modeled as a constant active power plus a voltage dependant reactive power and a linear dependency with frequency. However, the transmission losses were still neglected and the load active power was still modeled as a constant power.
Chiang et al.[5] studied the existence of energy functions for power systems with losses and they proved the non existence of general Liapunov functions to power systems in the presence of transfer conductances. In the same article they have proved the existence of a local Liapunov function which can be used for stability studies purpose when the transfer conductances are not

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bigger enough, however their result concerns only with the existence and they did not exhibit such function.

After Chiang et al.[5] studies, the problem of finding a general Liapunov function for power systems considering transfer conductances seemed to be unsolved until an extension of the Invariance Principle was proposed by Rodrigues, Alberto & Bretas [8]. In this paper this extension is used to support the proposal of a new energy function for power systems taking in account the influence of small transfer conductances. It can be shown that this energy function is a Liapunov function in a wider sense, i.e., in the sense of the extension of the Invariance Principle. In this wider sense, the derivative of the Liapunov function is not required to be always negative semidefinite and it can assume positive values in some bounded regions.

This paper is organized as follow. In Section 2, the theoretical results and an example including good estimates for a single Lorenz system are presented. In Section 3, the extended Invariance Principle is used to study the stability of power systems considering transmission losses. Finally, conclusions are presented in Section 4.

2. THE INVARIANCE PRINCIPLE

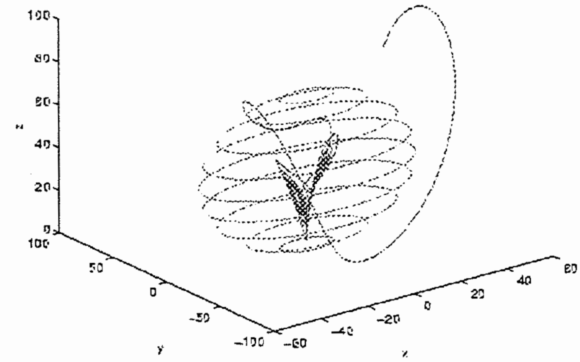
This section starts by reviewing the usual Invariance Principle [6]. Consider the following autonomous differential equation:

$$\dot{x} = f(x) \tag{1}$$

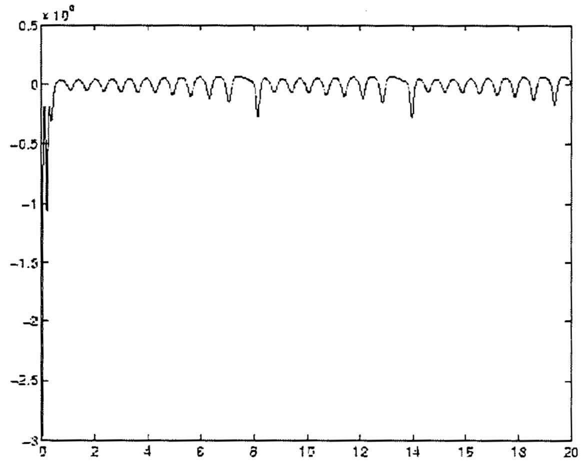
Theorem II.1:Let $V : R^n \rightarrow R_+$ *and* $f : R^n \rightarrow R^n$ *be* C^1 *functions. Let* $L > 0$ *be a constant such that* $\Omega_L = \{x \in R^n : V(x) < L\}$ *is bounded. Suppose that* $\dot{V}(x) \leq 0$ *for every* $x \in \Omega_L$ *and define* $E := \{x \in \Omega_L : \dot{V}(x)=0\}$. *Let B be the largest invariant set contained in E. Then very solution of (1) starting in* Ω_L *converges to B as* $t \rightarrow \infty$.

In this work, more general results than the above one are presented. They require less restrictive conditions and allow the possibility of the derivative of V to be positive in some regions. The advantage of these results is that it is easier to find the function V and some quite complicated problems can be treated as well. The first result obtained is:

Theorem II.2.(The Extended Invariance Principle): Let $V: R^n \rightarrow R$ *and* $f: R^n \rightarrow R^n$ *be* C^1 *functions. Let* $L \in R$ *be a constant such that* $\Omega_L = \{x \in R^n : \dot{V}(x) < L\}$ *is bounded. Let* $C := \{x \in \Omega_L : \dot{V}(x) > 0\}$, *suppose that* $\sup_{x \in C} V(x) = l < L$. *Define* $\overline{\Omega}_l = \{x \in R^n : V(x) \leq l\}$ *and* $E := \{x \in \Omega_L : \dot{V}(x) = 0\} \cup \overline{\Omega}_l$. *Let B be the largest invariant set of (1) contained in E. Then every solution of (1) starting in* Ω_L *converges to the invariant set B, as* $t \rightarrow \infty$.



(a)



(b)

Figure 1- Lorenz System (a) Attractor Estimation (b) Derivate of Energy

Moreover if $x_0 \in \overline{\Omega}_l$ *then* $\varphi(t, x_0) \in \overline{\Omega}_l$ *for every* $t \geq 0$ *and* $\varphi(t, x_0)$ *tends to the largest invariant set of (1) contained in* $\overline{\Omega}_l$.

For a proof and more details see [8] and [9].

Remark II.1: The $\sup_{x \in \overline{C}} \dot{V}(x)$ is always attained in the boundary ∂C of C. If in particular $\overline{C}$ is a convex set and V is a convex function, then the Lagrange technique is very useful for the calculus of this supremum.

Remark II.2: In the above theorem if we assume that $V: R^n \rightarrow R_+$ is such that $V(x) \rightarrow \infty$, as $\|x\| \rightarrow \infty$, then a global stability result can be established.

Example II.1: Attractor Estimate of the Lorenz System. In order to illustrate the application of the extended Invariance Principle, consider the following interesting Lorenz system :

$$\begin{cases} \dot{x} = -\sigma x + \sigma y \\ \dot{y} = -y - xy = rx \\ \dot{z} = -bz = xy \end{cases}$$

where, σ =10, r=28 and b=8/3. Let

$$V(x,y,z)=rx^2+4\sigma y^2+4\sigma (z-5r/4)^2 \qquad (2)$$

be a Liapunov function for the previous system. It is easy to see that this function satisfies the conditions established in remark II.2, therefore the extended Invariance Principle will be used to estimate the global Lorenz attractor. The derivative of V is given by:

$$\dot{V}\ (x,y,z)=-2\sigma\ (rx^2+4y^2+4bz^2-5rbz)$$

The set C is given by $C:=\{x \in R^3: rx^2+ y^2 + bz^2 -2rbz < 0\}$ and it is easy to see that the boundary of C is an ellipsoid centered at (x=0,y=0,z=5r/8). As C is a convex set and the Liapunov function V is a convex function, the Lagrange multipliers technique will be used to calculate the $\sup_{x \in C}$ V(x). Using the Lagrange function:

$$F(x,y,z)=rx^2+4\sigma y^2+4\sigma (z-5r/4)^2+ \lambda\ (rx^2+4y^2+4bz^2-5rbz)$$

the following extreme conditions are obtained:

$$\begin{cases} \frac{\partial F}{\partial x} = 2rx(1+\lambda) = 0 \\ \frac{\partial F}{\partial y} = 8y(\sigma+\lambda) = 0 \\ \frac{\partial F}{\partial z} = 8(\sigma+b\lambda)z-(10\sigma+5\lambda)r = 0 \\ \frac{\partial F}{\partial \lambda} = rx^2+4y_2+4bz_2-5rbz = 0 \end{cases}$$

The solution of the previous system is $\lambda=-\sigma$, x=0, $z=\frac{5r(2-b)}{8(1-b)}$ and $y^2=\frac{25b^2r^2(b-2)}{64(b-1)^2}$. Substituting these values in the expression of V, the number l is obtained:

$$l=\sup_{x \in C} V(x)=\frac{25b^2r^2\sigma}{16(b-1)}=\frac{156800}{3}<52267$$

The set $\overline{\Omega}_l$ is the ellipsoid: $\{(x,y,z) \in R^3: rx^2+4\sigma y^2 +4\sigma\ (z-5r/4)^2 \le \frac{156800}{3}\}$

The set in which $\dot{V}$ =0 is contained in $\overline{\Omega}_l$ and so every solution converges to the largest invariant set contained in $\overline{\Omega}_l$. The set $\overline{\Omega}_l$ is an estimate of the attractor. Figure 1a shows the estimate of the attractor. In this case it is important to note that the derivative of V keeps changing of sign after the solution enters in $\overline{\Omega}_l$. A graphic of $\dot{V}$ (x(t),y(t),z(t)) is shown in Figure 1b.

3. LIAPUNOV FUNCTION FOR POWER SYSTEMS WITH TRANSMISSION LOSSES

A. *Single-Machine-Infinite-Bus System*

Consider the SMIB system of Figure 2 where a synchronous machine is connected to the infinite bus through a transmission line with losses.

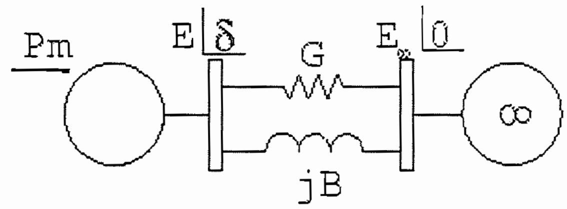


Figure 2- Single Machine infinite Bus System

Modeling the generator as a constant electromotive force behind the transient reactance, this system can be mathematically described by the following pair of differential equations:

$$\begin{cases} \dot{\delta} = \omega \\ M\dot{\omega} = P_m - E^2G + EE_\infty B\sin\delta + EE_\infty G\cos\delta - T\omega \end{cases} \qquad (3)$$

where δ and ω are respectively the rotor angle and the generator frequency deviation from the synchronous frequency, P_m is the input mechanical power, E is the electromotive force, E_∞ the voltage modulus at the infinite bus, T is the damping coefficient and $G+jB$ is the admittance of the equivalent transmission line. For notation simplicity, let us rewrite the SMIB differential equations as:

$$\begin{cases} \dot{\delta} = \omega \\ M\dot{\omega} = P - C\sin\delta - D\cos\delta - T\omega \end{cases} \qquad (4)$$

where $P = P_m - E^2G, C = -EE_\infty B$ and $D = -EE_\infty G$.

Although this model incorporates the line losses, this system has a general Liapunov function in the usual sense given by:

$$V(\delta,\omega) := M\frac{\omega^2}{2} - P\delta - C\cos\delta + D\sin\delta + cte \qquad (5)$$

It is easy to show that the derivative of V along the orbits is given by:

$$\dot{V} = -T\omega^2 \qquad (6)$$

which is a negative semi-definite function. The function V satisfies the requirements of the usual Invariance Principle and as consequence this function can be used to study the stability of this system in the usual way. In spite of that a new energy function will be proposed in what follows and the extended Invariance Principle will be used to study the stability of this system.

Our purpose is to ilustrate the application of the extension of the Invariance Principle and to prepare the ideas to solve the multi-machine problem which does not present a Liapunov function in the usual sense when the tranfer conductances are not neglected in the model.

With this purpose consider the following energy function:

$$W(\delta,\omega):=M\frac{\omega^2}{2}-P\delta-C\cos\delta- \\ -\beta\omega(P-Csin\delta+D\cos\delta) \quad (7)$$

where β is a parameter to be adjusted. Our goal is to show that this function satisfies the requirements of Theorem II.2.

Calculating the derivative of W along the orbits we obtain:

$$\dot{W}:=-(T-\beta(C\cos\delta-Dsin\delta))\omega^2+\beta T(P-Csin\delta- \\ -D\cos\delta)\omega-\beta(P-Csin\delta-D\cos\delta)-D\cos(\delta)\omega \quad (8)$$

which is equivalent to:

$$-\dot{W}:=\begin{bmatrix}P_l(\delta)\\ \omega\end{bmatrix}^T\begin{bmatrix}\beta & \frac{-\beta T}{2}\\ \frac{-\beta T}{2} & T-\beta(C\cos\delta-Dsin\delta)\end{bmatrix}\begin{bmatrix}P_l(\delta)\\ \omega\end{bmatrix} \quad (9)$$
$$+D\cos(\delta)\omega$$

where $P_l(\delta):=P-Csin\delta-D\cos\delta$. Note that this function is composed by a quadratic term plus the term $D\cos(\delta)\omega$. Note that the parameter β can be chosen in order to make the quadratic term positive definite. Applying the Silvester's Criteria one can easily find that this is certainly guaranteed if

$$\beta<\frac{T}{C+D+\frac{T^2}{4}}$$

In this way, only the term $D\cos(\delta)\omega$ will be responsible for generating regions where the derivative of W is positive.

Example III.1: Consider the SMIB system of Figure 2 with $P_l=1.0$, $C=2.0$, $D=0.05$, $T=0.15$ and M=0.5. The level curves of W are depicted in Figure 3. Note that the regions where the derivative of W is positive are small bounded sets. One of them is close to the stable equilibrium point and this set corresponds to the set C in Theorem II.2. The another region is close to the unstable equilibrium point. The maximum value of W in $\overline{C}$ defines the set $\overline{\Omega}_l$ which is an attractor estimate, i.e, all the solutions starting into the stability region will enter in this attractor estimate in a finite time. To estimate the stability region or attraction area of the attractor we must choose the largest number L such that the conditions of Theorem II.2 are satisfied. In practice,

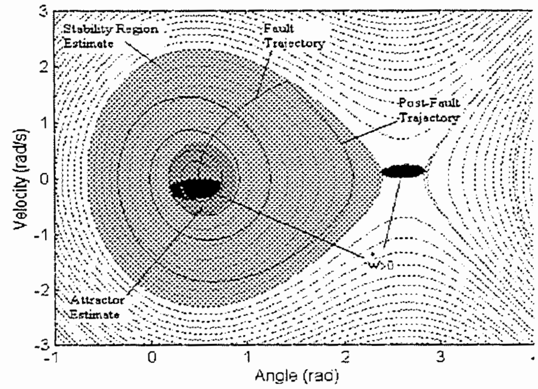


Figure-3. Level Curves of W

we must guarantee that $\overline{\Omega}_L$ does not intercept the region close to the unstable equilibrium point where the derivative is positive.

Figure 3 illustrates the attractor and stability region estimates.

The critical clearing time obtained by simulation for a solid three-phase-short-circuit at the machine bus belongs to the interval (1.07,1.08s). The estimated critical clearing time obtained with this new energy function belongs to the interval (1.0,1.01s). As expected, this estimate is a little conservative because the stability region estimate is contained into the real stability region. In spite of that, it is not much more conservative than the estimated clearing time obtained with the conventional Liapunov function V which belongs to the interval (1.03,1.04s). Figure 3 shows the trajectories of the fault and post-fault system for a clearing time equal to 1.0s.

B: Two-machine versus infinite bus system

Before considering the general multi-machine case, let us firstly consider the two-machine versus infinite bus system of 4.

The following differential equations:

$$\begin{cases}\dot{\delta}_1=\omega_1\\ M_1\dot{\omega}_1=P_1-C_1sin\delta_1-D_1\cos\delta_1-C_{12}sin(\delta_1- \\ \quad -\delta_2)-D_{12}\cos(\delta_1-\delta_2)-T_1\omega_1\\ \dot{\delta}_2=\omega_2\\ M_2\dot{\omega}_2=P_2-C_2sin\delta_2-D_2\cos\delta_2-C_{12}sin(\delta_2- \\ \quad -\delta_1)D_{12}\cos(\delta_2-\delta_1)-T_2\omega_2\end{cases} \quad (10)$$

describe the dynamical behaviour of this system. When the transfer conductances are neglected in the model (D_{12}=0), there exist a general Liapunov function in the usual sense which can be used to study the stability of this system. This function can be easily found by a traditional integration process and it is given by:

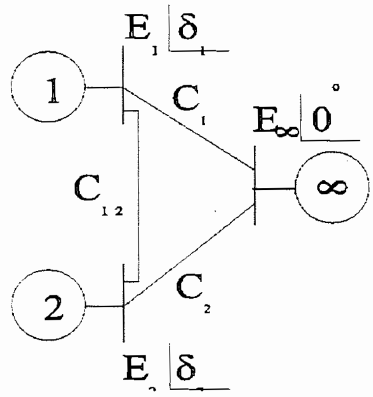


Figure 4- Two-machine versus infinite bus system

$$V(\delta_1,\omega_1,\delta_2,\omega_2) = M_1\frac{\omega_1^2}{2} - P_1\delta_1 - C_1\cos\delta_1 + \\ + D_1 sin\delta_1 + M_2\frac{\omega_2^2}{2} - P_2\delta_2 - C_2\cos\delta_2 + D_2 sin\delta_2 - \\ - C_{12}\cos(\delta_1-\delta_2) + cte \tag{11}$$

However when $D_{12} \neq 0$ the integration process yields a path dependant integral and it is impossible to prove that its derivative, along the trajectories, is semi-negative definite.

In order to solve this problem we propose a new energy function and we will use the extension of the Invariance Principle to study the stability of this system. It will be shown that this new energy function is a Liapunov function in the sense of the extension of the Invariance Principle if the tranfer conductance D_{12} is small. With that in mind consider the following energy function:

$$W(\delta_1,\omega_1,\delta_2,\omega_2) = M_1\frac{\omega_1^2}{2} - P_1\delta_1 - C_1\cos\delta_1 + \\ + D_1 sin\delta_1 - \beta_1\omega_1[P_1 - C_1 sin\delta_1 - D_1\cos\delta_1 - \\ - C_{12} sin(\delta_1-\delta_2) - D_{12}\cos(\delta_1-\delta_2)] + M_2\frac{\omega_2^2}{2} - \\ - P_2\delta_2 - C_2\cos\delta_2 + D_2 sin\delta_2 - \beta_2\omega_2[P_2 - C_2 \\ sin\delta_2 - D_2\cos\delta_2 - C_{12} sin(\delta_2-\delta_1) - D_{12}\cos(\delta_2 \\ - \delta_1)] - C_{12}\cos(\delta_1-\delta_2) + cte \tag{12}$$

where β_1 and β_2 are parameters to be determined.

Calculating the derivative of this function along the system orbits one finds:

$$-\dot{W} = \begin{bmatrix} P_{l1}(\delta_1,\delta_2) \\ \omega_1 \\ P_{l2}(\delta_1,\delta_2) \\ \omega_2 \end{bmatrix}^T B \begin{bmatrix} P_{l1}(\delta_1,\delta_2) \\ \omega_1 \\ P_{l2}(\delta_1,\delta_2) \\ \omega_2 \end{bmatrix} \\ + D_{12}\cos(\delta_1-\delta_2)(\omega_1+\omega_2) \tag{13}$$

where

$$B = \left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{12} & B_{22} \end{array}\right]$$

and

$$B_{11} = \left[\begin{array}{c|c} \beta_1 & -\frac{\beta_1 T_1}{2} \\ \hline -\frac{\beta_1 T_1}{2} & \begin{array}{l} T_1 + \beta_1[-C_1\cos\delta_1 + D_1 sin\delta_1 \\ - C_{12}\cos(\delta_1-\delta_2) + D_{12} sin(\delta_1-\delta_2)] \end{array} \end{array}\right]$$

$$B_{22} = \left[\begin{array}{c|c} \beta_2 & -\frac{\beta_2 T_2}{2} \\ \hline -\frac{\beta_2 T_2}{2} & \begin{array}{l} T_2 + \beta_2[-C_2\cos\delta_2 + D_2 sin\delta_2 \\ - C_{12}\cos(\delta_2-\delta_1) + D_{12} sin(\delta_2-\delta_1)] \end{array} \end{array}\right]$$

$$B_{12} = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & \begin{array}{l} \frac{\beta_1}{2}[C_{12}\cos(\delta_1-\delta_2) - D_{12} sin(\delta_1-\delta_2)] + \\ \frac{\beta_2}{2}[C_{12}\cos(\delta_2-\delta_1) - D_{12} sin(\delta_2-\delta_1)] \end{array} \end{array}\right]$$

Note again that the derivative of W is composed by a quadratic term plus the term D_{12} $\cos(\delta_1-\delta_2)(\omega_1+\omega_2)$. Parameters β_1 and β_2 can be chosen in order to make the quadratic term positive definite. In this way, only the term $D_{12}\cos(\delta_1-\delta_2)(\omega_1+\omega_2)$ will be responsible for generating regions where the derivative of W is positive.

Example III.2: Consider the system of Figure 4 with P_1=1.25, P_2=1.5, C_1=1.7, C_2=2.0, D_1=D_2=0.1, C_{12}=0.5, D_{12}=0.04, T_1=T_2=0.1 and M_1=M_2=0.5. The level curves of W are depicted in Figure 5. Note that the region where the derivative of W is positive is a small bounded region close to the stable equilibrium point. As consequence the extended Invariance Principle can be used to study the stability of this system. Figure 5 shows the projection of the stable attractor estimate.

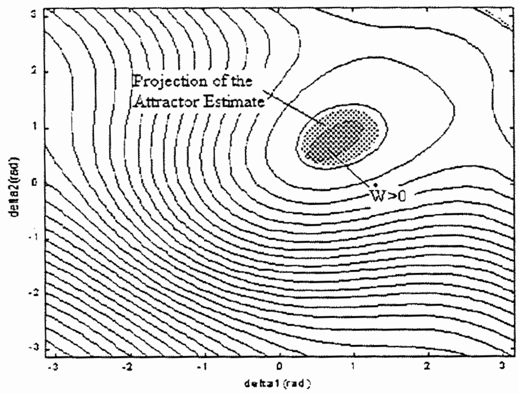


Figure 5- Level curves of W

C.Multimachine-systems

Consider a system composed by n machines where the n^{th} -machine is an infinite bus. One can show, similarly to the case of two-machines, that the following energy function

$$
\begin{aligned}
W = \sum_{i=1}^{n-1} \{ & M_i \frac{\omega_i^2}{2} - P_i\delta_i - C_i \cos\delta_i + D_i sin\delta_i - \\
& - \beta_i \omega_i [P_i - C_i sin\delta_i - D_i \cos\delta_i - \\
& - \sum_{\substack{j=1 \\ j\neq i}}^{n-1} C_{ij} sin(\delta_i - \delta_j) - \sum_{\substack{j=1 \\ j\neq i}}^{n-1} D_{ij} \cos(\delta_i - \delta_j)] \\
& - \sum_{j=i+1}^{n-1} C_{ij} \cos(\delta_i - \delta_j) + cte \}
\end{aligned} \qquad (14)
$$

is a Liapunov function in the sense of the extended Invariance Principle if the transfer conductances are small. Consequently it can be used for power system transient stability analysis.

4. CONCLUSIONS

In this paper, a more general version of the Invariance Principle, which seems to be very interesting in the field of stability of non-linear systems, was used to study the existence of general Liapunov functions for power systems with transmission losses. In this version, less restrictive conditions than the conditions of the Classical Invariance Principle were used in order to allow their application in a larger class of problems. Basically, the derivative of the Liapunov function is allowed to be positive in some bounded regions. In this way, many complex problems of physics and engineering, such as systems with chaotic behavior, can now be more easily treated. The theorem was successfully applied in this paper to support theoretically the proposal of a new general energy function which is a general Liapunov function in a wider sense(its derivative can assume positive values) for power systems with transmission losses.

In this paper we gave a contribution in the search for general Liapunov functions for power systems considering only the problem of transfer conductances. However the range of applications of the extension of the Invariance Principle is very large and we expect in the future researchers would have solved other problems as the incorporation of more realistic generator models as well as effects of some regulators allowing the application of energetic methods to the assessment of transient stability with more realistic models.

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