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Original Article

Maximizing profits while reducing environmental impacts in silvopastoral systems for Nellore: an idealized, simplified farm-level mathematical model

Maximizando lucros enquanto reduz impactos ambientais em sistemas silvipastoris para Nelore: um modelo matemático idealizado e simplificado ao nível da fazenda

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Abstract

This study focuses on maximizing profits and reducing environmental impacts in silvopastoral systems for Nellore cattle through an idealized farm-level mathematical model. The research team from various Brazilian universities and institutions aimed to develop a framework using linear programming to optimize resource allocation, integrate different plant and animal species, and minimize costs. The model considers parameters such as plant density, animal nutrition, and greenhouse gas emissions. Results indicate that a well-managed silvopastoral system can significantly improve profitability (R$ 867,712.72) and sequester substantial amounts of CO_2 (1,341.39 tons CO_2-equivalent). The study highlights the potential of integrated farming systems to enhance economic sustainability while mitigating environmental impacts.

Keywords: ecological sustainability, economic feasibility, GAMS CPLEX®, linear programming, operations research.

Resumo

Este estudo foca em maximizar lucros e reduzir impactos ambientais em sistemas silvipastoris para gado Nelore através de um modelo matemático idealizado ao nível da fazenda. A equipe de pesquisa de várias universidades e instituições brasileiras visou desenvolver um quadro usando programação linear para otimizar a alocação de recursos, integrar diferentes espécies de plantas e animais e minimizar custos. O modelo considera parâmetros como densidade de plantas, nutrição animal e emissões de gases de efeito estufa. Os resultados indicam que um sistema silvipastoril bem gerido pode melhorar significativamente a rentabilidade (R$ 867.712,72) e sequestrar quantidades substanciais de CO_2 (1.341,39 toneladas de CO_2-equivalente). O estudo destaca o potencial dos sistemas agrícolas integrados para melhorar a sustentabilidade econômica enquanto mitiga os impactos ambientais.

Palavras-chave: sustentabilidade ecológica, viabilidade econômica, GAMS CPLEX®, programação linear, pesquisa operacional.

1. Introduction

Agriculture has evolved from traditional, with hand tools, to modern, with machines and chemical inputs, progressing to precision agriculture with GPS (Global Positioning System) and sensors, and smart agriculture with IoT (internet of things) and AI (artificial intelligence). Finally, it has reached sustainable and regenerative agriculture, which prioritizes ecological practices, restores soil health, reduces the use of industrial fertilizers, promotes biodiversity and increases climate resilience (Ruttan, 2002; Mazoyer and Roudart, 2006; Borlaug, 2000; Pingali, 2012;

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Pierce and Nowak, 1999; Zhang et al., 2002; Wolfert et al., 2017; Bronson, 2019; Rhodes, 2017; Lal, 2020).

This makes it necessary to use sustainable alternatives for animal production that transform current production paradigms, since sustainable intensification of animal production systems can be achieved through silvopastoral systems (Pérez Márquez et al., 2021). More than 40% of the Earth's surface is occupied by ecologically managed pastures, characterized by native plants and their main use for grazing. They provide around 75% of forage for domestic animals and can contribute significantly to GDP (SRM, 2002; Brown and Thorpe, 2008). In addition, grasslands provide ecosystem services such as food, fiber, water, recreation, minerals and medicinal plants for rural and urban populations (Havstad et al., 2007).

However, Brazil's thriving livestock sector faces challenges due to the deterioration of strategic assets such as land, labor and technology. In order to guarantee the competitiveness and sustainability of the production chain, it is necessary to develop new solutions that reconcile efficiency, environmental preservation and the reduction of inequalities (ABRAFRIGO, 2024; EMBRAPA, 2024; Hötzel and Vandresen, 2022). Agroecological management strategies and the redesign of pasture-based livestock production systems are promising tools for achieving these goals, boosting the resilience, self-sufficiency, productivity and efficiency of national livestock farming (Pereira et al., 2024; Bonaudo et al., 2014). Thus, through mathematical modeling it is possible to create a framework to guide the redesign of productive landscapes (Gomes Lobo et al., 2023) in integrated systems which allow the insertion of different species of trees and shrubs (Lobo et al., 2024).

Mathematical linear programming is a powerful tool for modeling, analyzing and solving problems related to the allocation of scarce resources, with the aim of minimizing costs or maximizing profits. This tool is widely used in this context, making it possible to make efficient decisions in independent activities, based on available sources, and it is also possible to obtain an optimal solution using the simplex method in a linear model.

This structure is a deterministic, robust model that combines plant and animal species in a more suitable arrangement and at a lower cost (Marques, 2021), in which a benchmark is established to assess the effectiveness of management practices and proposed mitigation strategies. In addition, optimization is the mathematical process of finding the best solution within a set of options, especially necessary in complex situations where intuition is not enough.

As a way of taking advantage of the resources for nutrient cycling in the soil-plant-animal system, benefiting the farmer's profit margin and the sustainability of the environment in which he produces, the aim of this paper is to create a modelling framework, based on experimental parameters from the related literature, which can guide the redesign of productive landscapes for ruminants in tropical conditions, moving from the conventional to the sustainable scenario that estimate the potential economic and environmental gains to be obtained from integrating livestock farming with crop production.

2. Material and Methods

This empirical (*in silico*) study focuses on the application of a mathematical model to represent the reality of a beef cattle farm in the growing and finishing phases, characterized by the integration of cattle farming and plant production, mainly aimed at producing food for the animals, in a silvopastoral system. The formal description of the mathematical model includes the indices of the sets, the parameters, the variables and the equations that make it up. Next, the parameters to be used in the model are surveyed and defined, presenting the basis on which the information was obtained, both from databases and from existing literature. Finally, the proposed methods for estimating the gains obtained from adopting integrated production activities are described, which are applied to the results obtained from processing the proposed optimization model.

2.1. Study description

Linear Programming (LP) stands out as a fundamental technique in the field of Operations Research (OR) (Hillier and Lieberman, 2006). Its popularity stems from its ability to offer relatively simple mathematical modelling to solve complex problems, along with the availability of a variety of algorithms and the promise of finding optimal and unique solutions, when feasible.

The core of the approach is solving deterministic models composed of linear equations. These models have a linear objective function to be optimized (either maximized or minimized), subject to a set of linear constraints, as the name suggests. In general, when applying the Linear Programming model to farm planning, it can be represented as:

$$\text{Maximize}\, Z = c\ \ x$$

$$\text{subject to}\ A\ \ x \leq b$$

In this context, *Z* represents the farm's profit, *c* is a vector that expresses the net profits of each activity, ***x*** is a vector that describes the quantities of the respective activities, *A* is a matrix of technical coefficients and *b* is a vector of resources (*inputs*) available on the farm. The technique makes it possible to determine the values of the variables *x* that maximize *Z*, while complying with all the established restrictions.

2.2. Location and tools

The work was conducted in silico, in the Department of Biotechnology and Plant and Animal Production, of the Federal University of São Carlos, Araras city, whose geographical coordinates are 22° 21' 28" South, 47° 23' 6" West, state of São Paulo, located in Brazil. This experience does not need to go through an ethics committee, as its indirect work involves people and animals in the system.

The tools objects used were a laptop, meetings interview with experts on the subject to define the model and parameters, data from the literature with searches on database access sites such as Web of Science®, Google Scholar® and the use of software such as CiteSpace® to analyze emerging trends and critical changes in scientific literature and VOSviewer® for constructing and visualizing bibliometric networks. In addition, to obtain the model's answer, it was used the software GAMS/CPLEX® Version 46.4.0, which will be described in the methodology.

All in all, the software GAMS/CPLEX® looks for values for the variables that maximize the margin or profit, respecting all the "sine qua non" constraints, finding an optimal value for the variable (s) "X".

GAMS (General Algebraic Modelling System) is a *high-level* modelling system designed to solve complex mathematical programming problems, including linear, nonlinear and integer programming. It is widely used in areas such as economics, engineering, operations research and social sciences, providing a flexible platform for formulating and solving mathematical models. CPLEX, on the other hand, is an optimization solver developed by IBM that is recognized for its efficiency in solving large linear programming (LP), mixed integer programming (MIP), and quadratic programming (QP) problems. CPLEX is often used in conjunction with modelling systems such as GAMS to provide a robust and efficient solution to complex optimization problems. When integrated, GAMS and CPLEX offer a powerful combination for modelling and solving optimization problems. GAMS allows users to model problems in a declarative and intuitive way, while CPLEX provides the advanced algorithms needed to solve these problems efficiently (IBM, 2024a).

2.3. Study design

The text describes several key elements essential for building a decision-making model, focusing particularly on deterministic modelling in the context of reshaping productive landscapes for ruminants in tropical conditions. The core is to work within a silvopastoral system, at farm level, with tree (*Gliricidia sepium*) and shrub (*Tithonia diversifolia*) species, grazing *Urochloa brizantha* cv. Marandu, containing Nellore animals in the growing and finishing (fattening) phases.

A synthetic population of beef cattle was study *in silico*, which is a predictive tool applied to data analysis for fundamental understanding of the underlying theory (Madden et al., 2020), to be applied in practice and speed up the decision-making process (Shaker et al., 2021).

We are considering the rainy (animals in the growing phase) and dry (animals in the finishing phase) seasons of the year, as well as the nutritional requirements of the animals, where we want to obtain answers to the following questions like, *what is the margin, in reais (R$)? What is the number of animals, in head or number of animals? What is the plant area, in hectare? How much supplement, in tons? How much mineral supplement in the rainy season, in tons? How much protein-energy supplement in the dry season, in tons? How much greenhouse gases (GHG) are emitted or sequestered, in t of* CO_2*-equivalent? How much fertilizer, in tons? How much manure is produced, in tons? How much manure is used as fertilizer, in tons?*

2.4. Model approach

Developed at UFSCAR-ARARAS, this is a farm-level model capable of creating a modeling framework to guide the redesign of productive landscapes for ruminants in tropical conditions. It allows us to know how the plant and animal characteristics of ruminants fed diets with the addition of alternative forages, mineral supplementation during the wet season and minimal supplementation during the dry season behave; and this allows the use of alternative forages in the diet to contribute to the development of sustainable management strategies. This model combines factors capable of predicting what the redesign of a rural property would look like, having proposed productive, ecosystem and environmental impact mitigation characteristics and positive margins in this multifunctional landscape with the insertion of these new species.

2.4.1. Model features

In operations research (OR), simulations are often developed to solve practical problems of complex systems (Sinuany-Stern, 2023). Linear programming modelling is a technique within operations research used to find the optimal answer to a given problem, from which decisions can be made beyond all the coefficients of the objective function and the constraints being deterministic, as follows, a decision with certainty. And that, the simplistic model is subject to trade-offs, i.e. as simplification increases, the model's representation decreases and vice versa, which is why the model needs to contain some variables from reality. That said, when new approaches are inserted into the model, the solutions found are often conflicting, generating trade-offs in decisions (Péra, 2022).

Therefore, a toy-model is a deliberately simplistic model with many details removed so that it can be used to explain a mechanism concisely and is useful in a more complete description of the model (Blanchard, 2018). With this premise in mind, philosophically, the applicability of the toy-model in mathematical modelling is done by restricting the number of variables, being a deliberately simplistic model so that it can be used to explain a mechanism concisely, whose formal setting may be irrelevant (Frigg and Nguyen, 2020; Reutlinger et al., 2018; Koutsoyiannis, 2006).

2.4.2. Sustainable production system traits

For the operational research study, it was first necessary to define the spatial and temporal limits (Gameiro et al., 2010). The spatial limit was defined at farm level as being from growth to fattening or finishing, with Nellore bulls from 8 to 30 months of age (Trocóniz et al., 1991) and the temporal limit, considering a lifetime of a whole year, considering seasons such as wet and drought, taking into account a silvopastoral system with an intensive stocking rate (Lobo, 2023).

It is possible to characterize the forage plants, since this is a grazing system:

2.4.2.1. Urochloa brizantha cv. Marandu

It is a cespitose, robust plant that can reach 1.5-2.5 m (Nunes et al., 1984). The forage qualities of the *Urochloa brizanta* cv. Marandu ecotype make it an excellent alternative for the silvopastoral system (Oliveira et al., 2021), since it has been shown to be resistant to the pasture leafhopper (Jesus et al., 2023), with good nutritional value within the silvopastoral system (Gomes et al., 2022), high biomass production (Oliveira et al., 2022) and no cases of photosensitization.

2.4.2.2. Gliricidia sepium

A member of the Fabaceae family, commonly known as gliricidia, coyote, and mother of cocoa, reaches heights of 12 to 15 meters (Drumond and Carvalho Filho, 1999). Used in silvopastoral systems, it is pruned initially one year after planting and then every six months (Fontes et al., 2016). The species is notable for its deep roots and high drought tolerance, enduring up to eight months of drought with a minimum annual rainfall of 500 mm (Lima Filho et al., 2023; Fontes et al., 2016; Andrade et al., 2015). Gliricidia excels in nitrogen fixation, nutrient cycling, and biomass production, with 56 to 74% of its aerial nitrogen from biological fixation (Silva, 2022). It effectively reduces soil density and compaction and thrives in low-fertility soils (Andrade et al., 2015; Conceição, 2017).

2.4.2.3. Titonia diversifolia

Belonging to the Asteraceae family and commonly known in Brazil as titonia, Mexican sunflower, or margaridão, is a drought-tolerant shrub that grows between 1.5 and 4.0 meters tall (Reis et al., 2015; Radomski and Oliveira, 2018). It is used in silvopastoral systems, attracting pollinators and exhibiting nematicidal and allelopathic effects on weeds. This plant serves as green manure, aids in recovering degraded soils, and enhances nutrient cycling due to its deep root system and nutrient-rich aerial biomass (Radomski and Oliveira, 2018). Applying 7.5 $t.ha^{-1}$ of titonia biomass can significantly increase soil humidity by reducing soil temperature and water evaporation (Opala, 2020). Though it does not fix atmospheric nitrogen, titonia effectively cycles nitrogen from deeper soil layers, accumulating large amounts in its biomass (Jama et al., 2000). It efficiently utilizes nutrients in low-fertility soils, producing biomass high in N, P, K, Ca, and Mg, and is effective for erosion control due to its dense, deep roots and mulching capability (Reis et al., 2015; Radomski and Oliveira, 2018; Jama et al., 2000; Opala, 2020).

2.4.3. Mathematical connotation

The simulation model was used to generate *in silico* experiments to evaluate the following aspects: sets, parameters, response variables and equations like constraints and objective function, which will be described:

Sets - v represents the set of plant types, including tithonia (*Tithonia diversifolia*), gliricidia (*Gliricidia sepium*) and pasture (*Urochloa brizantha*, Marandu); l represents the set of cattle types, including growing and fattening; f represents the set of fertilizer types, including nitrogen, phosphorus, potassium, calcium and magnesium; s represents the set of feed types, including mineral supplement and protein supplement; ***Parameters*** **-** $densityv(v)$ is the density of plants per hectare for each type of plant; $pricel(l)$ is the price per head of cattle for each type of cattle; $costv(v)$ is the implementation cost per hectare for each type of plant; $costl(l)$ is the cost per head of cattle for each type of cattle; $costf(f)$ is the cost per ton of fertilizer for each type of fertilizer; $costs(s)$ is the cost per ton of food for each type of food; $livestockweight(l)$ is the weight per head of cattle for each type of cattle; productivRainy(v) is the biomass productivity in tons of dry matter per hectare for each type of plant during the rainy season; $produtivDry(v)$ is the biomass productivity in tons of dry matter per hectare for each type of plant during the dry season; $drymatterrequirement(l)$ is the daily dry matter requirement in kg for each type of livestock; $period(l)$ is the period in days for each type of livestock; drymattercompositionrainy(v) is the dry matter composition in percent for each type of plant during the rainy season; $drymattercompositiondry(v)$ is the dry matter composition in percent for each type of vegetable during the dry season; $drymattercompositionfeed(s)$ is the dry matter composition in percent for each type of food; $proteinrequirement(l)$ is the daily protein requirement in kg for each type of livestock; $proteincontentrainy(v)$ is the protein content in percentage of dry matter for each type of vegetable during the rainy season; $proteincontentdry(v)$ is the protein content as a percentage of dry matter for each type of vegetable during the dry season; $proteincontentfeed(s)$ is the protein content as a percentage of dry matter for each type of food; energyrequirement(l) is the daily energy requirement in Mcal for each type of livestock; $energycontentrainy(v)$ is the energy content in Mcal per ton of dry matter for each type of plant during the rainy season; $energycontentdry(v)$ is the energy content in Mcal per ton of dry matter for each type of vegetable during the dry season; $energycontentfeed(s)$ is the energy content in Mcal per ton of dry matter for each type of food; totaldigestiblen utrientsrequirement(l) is the daily requirement of total digestible nutrients in kg for each type of livestock; $to taldigestiblenutrientscontentrainy(v)$ is the content of total digestible nutrients as a percentage of dry matter for each type of plant during the rainy season; $totaldi gestiblenutrientscontentdry(v)$ is the content of total digestible nutrients as a percentage of dry matter for each type of vegetable during the dry season; $totaldi gestiblenutrientscontentfeed(s)$ is the content of total digestible nutrients as a percentage of dry matter for each type of feed; manureproduction(l) is the daily manure production in tons per head of cattle for each type of cattle; $costgee$ is the cost per ton of greenhouse gas emissions; $totalarea$ is the total area in hectares; $mortalityrate$ is the mortality rate in percent; $stockingrate$ is the stocking rate in hectares per head of cattle; $slaughterweight$ is the slaughter weight in kg; $minimumtithonia$, $minimumgliricidia$, and $minimumpasture$ are the minimum area coverage requirements for tithonia, gliricidia and pasture, respectively, in percentage of the total area; $DMIG$ and $DMIF$ are the dry matter intake values in kg for growth and fattening, respectively; $co2eqvegetal(v)$ is the amount of carbon dioxide equivalent in tons per hectare of each type of plant; $co2eqfertilizer(f)$ is the amount of carbon dioxide equivalent in tons per ton of each type of fertilizer; $co2eqlivestock(l)$ is the amount of carbon dioxide equivalent in tons per head of cattle of each type of cattle; $manurenutrients(f)$ is the percentage of manure weight that consists of nutrients for each type of nutrient in the manure; $FertilizerRequirement(v,f)$ is the amount of fertilizer needed in tons per hectare for each type of plant and fertilizer; **Variables -** M is the margin; $xlivestock(l)$ is the number of head of cattle for each type of cattle; $xvegetal(v)$ is the area of plants in hectares for each type of plant; $xfeed(s)$, $xfeedrainy(s)$ and $xfeeddry(s)$ are the amounts of supplement in tons for each type of food and climate condition; $xgee$ is the amount of greenhouse gases in tons; $xfertilizer(f)$ is the amount of fertilizer in tons for each type of fertilizer; $xdrymatterintake$, $xtotaldigestiblenutrients$, $xenergy$ and $xprotein$ are the intakes of dry matter, total digestible nutrients, energy and protein, respectively, in kg; $xmanure$ and $xmanurefertilizer\ (f)$ are the amounts of manure in tons and the amount of manure used as fertilizer for each type of nutrient in the manure; **Equations** – Objective Function: Maximizes the margin (Profit = Income – Cost); Restrictions: Imposes restrictions on area, livestock, nutrient intake, greenhouse gas emissions, manure production, fertilizer consumption, and minimum area coverage requirements for each type of plant.

2.4.4. Equations details

A solution is considered optimal in the Pareto sense, or not dominated, if there is no other solution in the viable space that improves any of the objectives without harming at least one other criterion (Zavala et al., 2014). Take note that the objective function serves as a response variable that optimizes the profit margin when seeking to maximize gains, considering all imposed restrictions (IBM, 2024b), since the degree of precision in evaluating the objective function is controlled by constraints (Larreal Herrera, 2023). Therefore, we have the objective function:

$$M = \text{pricelivestock}(\text{'fattening'}) x X_{\text{livestock}}\left(\text{'fattening'}\right) x \left(\frac{\text{slaughterwelght}}{15}\right) - \sum_{v} \text{costlivestock}(l) x X_{\text{livestock}}(l) - \sum_{v} \text{costvegetal}(v) x X_{\text{vegetal}}(v) - \sum_{v} costfeed(s) x X_{\text{feed}}(s) - \sum_{v} costfertilizer(f) x X_{\text{fertilizer}}(f) - costgeex X_{\text{gee}},$$

where,

$pricelivestock('fattening')$: price per unit of fattening cattle, in reais.

$X_{livestock}('fattening')$: quantity of fattening cattle, in head.

$\left(\frac{slaughterweight}{15}\right)$: weight of cattle at slaughter, considering a carcass yield of 50%, in kg.

$\sum_{v} costlivestock(l)$: cost associated with each cattle l, in reais.

$X_{livestock}(l)$: quantity of cattle l, in head.

$\sum_{v} costlvegetal(v)$: cost associated with each vegetable v, in reais.

$X_{vegetal}(v)$: area of plant v, in hectare.

$\sum_{v} costlfeed(s)$: cost associated with supplement s, in reais.

$X_{feed}(s)$: quantity of supplement s, in kg.

$\sum_{v} costlfertilizer(f)$: cost associated with each fertilizer f, in reais.

$X_{fertilizer}(f)$: quantity of fertilizer f, in tons.

$costgee$: cost per unit of greenhouse gas emissions, in reais per ton.

X_{gee} : quantity of greenhouse gas emissions, in tons.

The formula expresses profit M as the difference between the revenue generated by the sale of fattened cattle and the various operating costs involved in the process.

Informations used to construct the constraint equations for the model's response variables or outputs.

1. $\sum_{v} X_{vegetal}(v) = totalarea$
2. $X_{livestock}('fattening') = (1 - mortalityrate) x X_{livestock}('growing')$

3. $X_{livestock}('fattening')\, x\, stockingrate = totalarea$

4. $produtivRainy(v)\, x\, X_{vegetal}(v)\, x\, drymattercompositionrainy(v)$

$+\ drymattercompositionfeed('mineralsuple')\, x\, X_{feedrainy}('mineralsuple')$

$\geq drymatterrequirement('growing')\, x\ \ period('growing')\, x\, X_{livestock}('growing')/1000$

5. $\sum_{v} produtivDry(v)\ x\ X_{vegetal}(v)\, x\, drymattercompositiondry(v) + drymattercompositionfeed('protsuple')\, x\, X_{feeddry}('protsuple') \geq$

$drymatterrequirement('fattening')\, x \times period('fattening')\, x\, X_{livestock}('fattening')/1000$

6. $drymattercompositionfeed('mineralsuple')\, x\, X_{feedrainy}('mineralsuple')$

$-0.25 \times period('growing')\, x\, X_{livestock}('growing') \leq 0$

7. $drymattercompositionfeed(protsuple')\, x\, X_{feeddry}('protsuple')$

$-0.50 \times period('fattening')\, x\, X_{livestock}('fattening') \leq 0$

8. $0.90\, x \left(\sum_{v} produtivDry(v)\, x\ X_{vegetal}(v)\, x\, drymattercompositiondry(v) \right)$

$-0.10\, x \big(drymattercompositionfeed('protsuple')\, x\, X_{feeddry}('protsuple') \big) \geq 0$

9. $\sum_{v} X_{vegetal}(v) \leq 0.10\, x\, total\ area$

10. $\sum_{v} produtivRainy(v)\, x\, X_{vegetal}(v)\, x\, xdrymattercompositionrainy(v)\, x\ proteincontentrainy(v)$

$\geq proteinrequirement('growing')\, x\ period('growing')\, x\, X_{livestock}('growing')/1000$

11. $\sum_{v} produtivDry(v)\, x\, X_{vegetal}(v)\, x\, xdrymattercompositiondry(v)\, x\ proteincontentdry(v) \geq proteinrequirement('fattening')\, x\ period('fattening')\, x\, X_{livestock}('fattening')/1000$

12. $\sum_{v} produtivRainy(v)\, x\, X_{vegetal}(v)\, x\ \ xdrymattercompositionrainy(v)\, x\, energycontentrainy(v)$

$= energyrequirement('growing')\, x\ period('growing')\, x\, X_{livestock}('growing')$

13. $\sum_{v} produtivDry(v)\, x\, X_{vegetal}(v)\, x\ \ xdrymattercompositiondry(v)\, x\, energycontentdry(v)$

$= energyrequirement('fattening')\, x\ period('fattening')\, x\, X_{livestock}('fattening')$

14. $\sum_{v} produtivRainy(v)\, x\, X_{vegetal}(v)\, x\ \ xdrymattercompositionrainy(v)\, x$

$totaldigestiblenutrientscontentrainy(v) =$

$totaldigestiblenutrientsrequirement('growing')\, x\ period('growing')\, x\, X_{livestock}('growing')$

15. $\sum_{v} produtivDry(v)\, x\, X_{vegetal}(v)\ x\ \ xdrymattercompositiondry(v)\, x\, totaldigestiblenutrientscontentdry(v) = totaldigestiblenutrientsrequirement('fattening')\, x\ period('fattening')\, x\, X_{livestock}('fattening')$

16. $X_{gee} = \sum_{v} CO2eqvegetal(v)\, x\, X_{vegetal}(v) + \sum_{f} CO2eqfertilizer(f)\, x\, X_{fertilizer}(f) + \sum_{l} CO2eqlivestock(l)\, x\, X_{livestock}(l)$

17. $X_{manure} = \sum_{l} manureproduction(l)\, x\ period(l)\, x\, X_{livestock}(l)$

18. $X_{fertilizer}(f) + X_{manurefertilizer}(f) = \sum_{v} FertilizerRequirement(v, f)\, x\, X_{vegetal}(v)$

19. $X_{manurefertilizer}(f) \leq X_{manure}\, x\, manurenutrients(f)$

20. $X_{vegetal}\left(tithonia\right) \geq minimum\_tithonia \times DMI\_G$

21. $X_{vegetal}\left(gliricidia\right) \geq minimum\_gliricidia \times DMI\_G$

22. $X_{vegetal}\left(pasture\right) \geq minimum\_pasture \times DMI\_G$

23. $X_{vegetal}\left(tithonia\right) \geq minimum\_tithonia \times DMI\_F$

24. $X_{vegetal}\left(gliricidia\right) \geq minimum\_gliricidia \times DMI\_F$

25. $X_{vegetal}\left(pasture\right) \geq minimum\_pasture \times DMI\_F$

where:

2. Area Restriction: ensures that the area of planted vegetables does not exceed the total available area.

3. Livestock Restriction: controls the proportion of fattening cattle in relation to growing ones, considering the survival rate.

4. Capacity Restriction: limits the area for fattening cattle based on stocking rate and available area.

5. Restriction of Dry Matter Intake in the Rainy Season: ensures that dry matter intake by growing animals in the rainy season meets daily needs, considering plant productivity and dry matter composition.

6. Restriction of Dry Matter Intake in the Dry Season: similar to the previous restriction, but for fattening animals during the dry season.

7. Mineral Restriction in the Rainy Season: restricts the amount of mineral supplement for growing animals in the rainy season.

8. Protein Supplement Restriction in the Dry Season: restricts the amount of protein supplement for fattening animals in the dry season.

9. Forage and Concentrate Restriction in the Dry Season: balances forage and concentrate in the animals' diet in the dry season, ensuring that at least 90% of the diet is made up of forage.

10. Forage Presence Restriction: limits the area occupied by forage to a maximum of 10% of the total available area.

11. Restriction of Protein Intake in the Rainy Season: certifies adequate protein for growing animals in the rainy season, considering plant biomass, dry matter and protein concentration.

12. Restriction of Protein Intake in the Dry Season: confirms sufficient protein for fattening animals in the dry season according to nutritional requirements.

13. Energy Intake Restriction in the Rainy Season: ensures that energy intake by growing animals in the rainy season meets daily needs.

14. Restriction of Energy Intake in the Dry Season: protects that the energy consumed by fattening animals in the dry season corresponds to the daily requirement.

15. Restriction of Total Digestible Nutrients (TDN) Intake in the Rainy Season: endorses that the intake of TDN by growing animals in the rainy season meets their daily needs.

16. Restriction of Total Digestible Nutrients (TDN) Intake in the Dry Season: confirms that animals fattening in the dry season consume sufficient TDN.

17. Emissions Restriction: controls greenhouse gas emissions in production.

18. Manure Quantification Restriction: calculates the total amount of manure produced by animals.

19. Fertilizer Consumption Restriction: controls the use of fertilizers, limiting the amount consumed for each type of vegetable, including manure.

20. Restriction of Maximum Use of Manure-Derived Fertilizers: limits the amount of fertilizer coming from manure so that it does not exceed the total proportion.

21. Minimum Area Restriction for vegetal sets: sustains minimum proportion for tithonia, gliricidia and pasture in the total area.

22 to 26. Minimum Value Restriction for vegetal sets to growing and fattening animals: ensures minimum areas for tithonia, gliricidia and pasture regardless of daily needs.

3. Results

To analyze the results of the model and its sensitivity, we will consider the optimal solution found for the decision variables and how it relates to the constraints and parameters of the model.

3.1. Optimal decision variables

Decision variables provide information on how resources are allocated in the optimal solution, including the objective function response. For example, the amount of area dedicated to each type of plant, the number of animals at each stage, the amount of food supplements consumed, as described:

$x$vegetal(v): Area dedicated to the cultivation of each type of vegetable (tithonia, gliricidia, pasture).

$x$livestock(l): Number of animals at each stage (growth, fattening).

$x$feed(s): Amount of feed supplement consumed (mineral, protein).

$x$feedrainy(s): Amount of feed supplement consumed during the rainy season.

$x$feeddry(s): Amount of feed supplement consumed during the dry season.

$x$fertilizer(f): Amount of fertilizer used for each type of nutrient (nitrogen, phosphorus, potassium, calcium, magnesium).

$x$gee: Greenhouse gas emissions.

$x$manure: Total amount of manure produced by animals.

$x$manurefertilizer(f): Amount of manure used as fertilizer for each type of nutrient.

The numerical responses of the model for the response variables are shown below:

- What is the profit, in R$?
  R.: R$ 867,712.72
- What is the number of animals (head)?
  A.: Growing: 139.82 and Fattening: 124.44
- What is the plant area (ha)?
  A.: Tithonia = 3.30
  Gliricidia = 46.10
  Pasture = 6.60
- How much supplement, in t?
  A.: mineralsuple = 0
  protsuple = 0
  Note: the nutritional requirement is only being met by the vegetables.
- *What is the amount of mineral supplement in the rainy season, in t?*
  A.: mineralsuple = 8,474.25
  Note: the nutritional requirement is only being met by vegetables, and supplementation with macro and microminerals and salt (sodium chloride) is necessary.
- What is the amount of protein-energy supplement in the dry season, in t?
  A.: protsuple = 0
  Note: the nutritional requirement is only being met by vegetables.
- How much greenhouse gas (GHG) is emitted or sequestered, in t of CO_2-equivalent?
  R.: -1,341.39
  Note: the silvopastoral system sequesters the equivalent of the above amount.
- How much fertilizer, in t?
  A.: nitrogen = 2.61
  phosphorus = 20.18
  potassium = 9.56
  calcium = 4.16
  magnesium = 1.30
- How much manure is produced, in t?
  R.: 871.95
- How much manure is used as fertilizer, in t?
  A.: nitrogen = 5.23
  phosphorus = 45.34
  potassium = 21.80
  calcium = 8.72
  magnesium = 2.62

The mathematical analysis provided by the report generated by GAMS/CPLEX® makes it possible to understand not only the model's optimal solution, but also how this solution can change in response to alterations in the system's parameters and constraints.

3.2. Results analysis

The decision variables indicate how resources (land, animals, supplements, fertilizers) are allocated to optimize the model's margin, taking into account prices, costs and associated constraints. Thus, the solution may indicate that a certain amount of area is dedicated to growing tithonia, a certain number of animals are in the fattening stage, and a specific amount of feed supplement is being consumed during the rainy and dry seasons. The amounts of fertilizer and manure used will also be determined by the optimal solution, considering the nutritional requirements of the plants and the availability of nutrients in the manure. Greenhouse gas emissions will be minimized according to the optimal solution, considering the emissions associated with growing different types of vegetables and using fertilizers.

3.3. Parameters table and simulated scenarios

Below is a self-explanatory table (Table 1) with the parameters and scalar values used to construct the constraint equations for the model's response variables or outputs, as well as examples of production scenarios in agriculture that combine different elements in the system and can be an integrated sustainable agricultural production system, referring to this present model (Figure 1a, b).

Means of parameters taken from the literature under meticulous scanning.

*If the growing sales is included in the revenue. For this model, only the revenue from fattening animals was taken into account.

Table 1. Parameters and scalar values used to construct the constraint equations for the model's response variables or *outputs*.

Parameters	Values	Units balanced	Literary domain	Meaning	Sets
densidadev(v) - *Tithonia diversifolia*	14100	plants.ha^{-1}	Rivera et al. (2015), Vázquez (2023), Chará et al. (2017), Mauricio et al. (2019)	plant density	vegetal
densidadev(v) - *Gliricidia sepium*	2800	plants.ha^{-1}	Herrera et al. (2021), Oliveira et al. (2018b), Costa et al. (2021), Palma (2009)	plant density	vegetal
densidadev(v) - *Urochloa brizantha*	2700000	plants.ha^{-1}	Santos (2022)	plant density	vegetal
pricel(l) – growing	239.2	R$.@$^{-1}$	USP (2024)	animal price	livestock
pricel(l) - fattening	287.52	R$.@$^{-1}$	USP (2024)	animal price	livestock
costv(v) - *Tithonia diversifolia*	2295	R$.ha^{-1}	AGRO EM QUESTÃO: Revista de Iniciação Científica da Faculdade CNA (2017)	implantation cost	vegetal
costv(v) - *Gliricidia sepium*	1180	R$.ha^{-1}	Costa et al. (2004)	implantation cost	vegetal
costv(v) - *Urochloa brizantha*	1300	R$.ha^{-1}	Scot Consultoria (2015)	implantation cost	vegetal
costl(l) - growing	1812.3	R$.head^{-1}	Santos and Grzebieluckas (2014)	animal cost	livestock
costl(l) - fattening	1223.16	R$.head^{-1}	Santos and Grzebieluckas (2014)	animal cost	livestock
costf(f) - nitrogen	933	R$.t^{-1}	Nativa Agronegócios (2024)	fertilizer cost	fertilizer
costf(f) - phosphorus	934	R$.t^{-1}	Nativa Agronegócios (2024)	fertilizer cost	fertilizer
costf(f) - potassium	933	R$.t^{-1}	Nativa Agronegócios (2024)	fertilizer cost	fertilizer
costf(f) - calcium	70.46	R$.t^{-1}	Mato Grosso do Sul (2022)	fertilizer cost	fertilizer
costf(f) - magnesium	110	R$.t^{-1}	Mato Grosso do Sul (2022)	fertilizer cost	fertilizer
costs(s) - mineralsuple	3723.3	R$.t^{-1}	Matsuda (2024)	feed cost	feed
costs(s) - protsuple	3186.4	R$.t^{-1}	Matsuda (2024)	feed cost	feed
livestockweight(l) - growing	210	kg.animal^{-1}	Barbero et al. (2021)	animal weight	livestock
livestockweight(l) - fattening	520	kg.animal^{-1}	Barbero et al. (2021)	animal weight	livestock
produtivRainy(v) - *Tithonia diversifolia*	31.5	t of NM.ha^{-1}	Soares (2021)	biomass productivity	vegetal
produtivRainy(v) - *Gliricidia sepium*	20	t of NM.ha^{-1}	Silva et al. (2020)	biomass productivity	vegetal
produtivRainy(v) - *Urochloa brizantha*	33.5	t of NM.ha^{-1}	Barbosa (2020)	biomass productivity	vegetal
produtivDry(v) - *Tithonia diversifolia*	24.7	t of NM.ha^{-1}	Calsavara et al. (2016)	biomass productivity	vegetal
produtivDry(v) - *Gliricidia sepium*	9.7	t of NM.ha^{-1}	Silva et al. (2022)	biomass productivity	vegetal
produtivDry(v) - *Urochloa brizantha*	13.4	t of NM.ha^{-1}	Barbosa (2020), Rodrigues (2004)	biomass productivity	vegetal
drymatterrequirement(l) - growing	5	kg.day^{-1}	NASEM (2016)	dry matter requirement	livestock
drymatterrequirement(l) - fattening	11	kg.day^{-1}	NASEM (2016)	dry matter requirement	livestock
period(l) - growing	240	days	Oliveira et al. (2023)	season length	livestock
period(l) - fattening	120	days	Oliveira et al. (2023)	season length	livestock
drymattercompositionrainy(v) - *Tithonia diversifolia*	0.15	%	Odedire and Oloidi (2014)	vegetals dry matter during the rainy	vegetal
drymattercompositionrainy(v) - *Gliricidia sepium*	0.22	%	Valadares Filho et al. (2018)	vegetals dry matter during the rainy	vegetal
drymattercompositionrainy(v) - *Urochloa brizantha*	0.33	%	Valadares Filho et al. (2018)	vegetals dry matter during the rainy	vegetal
drymattercompositiondry(v) - *Tithonia diversifolia*	0.15	%	Odedire and Oloidi (2014)	vegetals dry matter during dry	vegetal
drymattercompositiondry(v) - *Gliricidia sepium*	0.22	%	Valadares Filho et al. (2018)	vegetals dry matter during dry	vegetal
drymattercompositiondry(v) - *Urochloa brizantha*	0.33	%	Valadares Filho et al. (2018)	vegetals dry matter during dry	vegetal
drymattercompositionfeed(s) - mineralsuple	0.99	%	Valadares Filho et al. (2018)	feed dry matter	feed
drymattercompositionfeed(s) - protsuple	0.92	%	Valadares Filho et al. (2018)	feed dry matter	feed
proteinrequirement(l) - growing	0.44	kg.day^{-1}	NASEM (2016)	crude protein nutritional requirement	livestock
proteinrequirement(l) - fattening	0.861	kg.day^{-1}	NASEM (2016)	crude protein nutritional requirement	livestock
proteincontentrainy(v) - *Tithonia diversifolia*	0.21	% of DM	Odedire and Oloidi (2014)	vegetals crude protein during the rainy	vegetal
proteincontentrainy(v) - *Gliricidia sepium*	0.17	% of DM	Valadares Filho et al. (2018)	vegetals crude protein during the rainy	vegetal
proteincontentrainy(v) - *Urochloa brizantha*	0.08	% of DM	Valadares Filho et al. (2018)	vegetals crude protein during the rainy	vegetal
proteincontentdry(v) - *Tithonia diversifolia*	0.21	% of DM	Odedire and Oloidi (2014)	vegetals protein during dry	vegetal

Table 1. Continued...

Parameters	Values	Units balanced	Literary domain	Meaning	Sets
proteincontentdry(v) - *Gliricidia sepium*	0.17	% of DM	Valadares Filho et al. (2018)	vegetals protein during dry	vegetal
proteincontentdry(v) - *Urochloa brizantha*	0.08	% of DM	Valadares Filho et al. (2018)	vegetals protein during dry	vegetal
proteincontentfeed(s) - mineralsuple	0	% of DM	Not applicable	feed crude protein	feed
proteincontentfeed(s) - protsuple	0.3	% of DM	Matsuda (2024)	feed crude protein	feed
energyrequirement(l) - growing	4.5	Mcal.day^{-1}	NASEM (2016)	nutritional requirement for metabolizable energy	livestock
energyrequirement(l) - fattening	8.2	Mcal.day^{-1}	NASEM (2016)	nutritional requirement for metabolizable energy	livestock
energycontentrainy(v) - *Tithonia diversifolia*	2360	Mcal.t^{-1}.DM^{-1}	Val (2019)	vegetals metabolizable energy during the rainy	vegetal
energycontentrainy(v) - *Gliricidia sepium*	2340	Mcal.t^{-1}.DM^{-1}	Valadares-Filho et al. (2016)	vegetals metabolizable energy during the rainy	vegetal
energycontentrainy(v) - *Urochloa brizantha*	1960	Mcal.t^{-1}.DM^{-1}	Valadares-Filho et al. (2016)	vegetals metabolizable energy during the rainy	vegetal
energycontentdry(v) - *Tithonia diversifolia*	1850	Mcal.t^{-1}.DM^{-1}	Val (2019)	vegetals metabolizable energy during dry	vegetal
energycontentdry(v) - *Gliricidia sepium*	2340	Mcal.t^{-1}.DM^{-1}	Valadares-Filho et al. (2016)	vegetals metabolizable energy during dry	vegetal
energycontentdry(v) - *Urochloa brizantha*	1960	Mcal.t^{-1}.DM^{-1}	Valadares-Filho et al. (2016)	vegetals metabolizable energy during dry	vegetal
energycontentfeed(s) - mineralsuple	0	Mcal.t^{-1}.DM^{-1}	Not applicable	feed metabolizable energy	feed
energycontentfeed(s) - protsuple	3480	Mcal.t^{-1}.DM^{-1}	Valadares-Filho et al. (2018)	feed metabolizable energy	feed
totaldigestiblenutrientsrequirement (l) - growing	3	kg.day^{-1}	NASEM (2016)	TDN nutritional requirement	livestock
totaldigestiblenutrientsrequirement (l) - fattening	6	kg.day^{-1}	NASEM (2016)	TDN nutritional requirement	livestock
totaldigestiblenutrientscontentrainy (v) - *Tithonia diversifolia*	0.61	% of DM	Silva et al. (2018)	vegetals TDN during the rainy	vegetal
totaldigestiblenutrientscontentrainy (v) - *Gliricidia sepium*	0.61	% of DM	Valadares-Filho et al. (2016)	vegetals TDN during the rainy	vegetal
totaldigestiblenutrientscontentrainy (v) - *Urochloa brizantha*	0.56	% of DM	Valadares-Filho et al. (2016)	vegetals TDN during the rainy	vegetal
totaldigestiblenutrientscontentdry(v) - *Tithonia diversifolia*	0.61	% of DM	Silva et al. (2018)	TDN of vegetables during dry	vegetal
totaldigestiblenutrientscontentdry(v) - *Gliricidia sepium*	0.61	% of DM	Valadares-Filho et al. (2016)	TDN of vegetables during dry	vegetal
totaldigestiblenutrientscontentdry(v) - *Urochloa brizantha*	0.56	% of DM	Valadares-Filho et al. (2016)	TDN of vegetables during dry	vegetal
totaldigestiblenutrientscontentfeed (s) - mineralsuple	0	% of DM	Not applicable	feed TDN	feed
totaldigestiblenutrientscontentfeed (s) - protsuple	0.57	% of DM	Matsuda (2024)	feed TDN	feed
manureproduction(l) - growing	0.0125	t.animal-1.day^{-1}	Pagliari et al. (2020)	manure produced	livestock
manureproduction(l) - fattening	0.0303	t.animal-1.day^{-1}	Pagliari et al. (2020)	manure produced	livestock
CO_2eqvegetal(v) - *Tithonia diversifolia*	-26.6	t.ha^{-1}	Montagnini et al. (2013)	ton of CO_2 equivalent per hectare of vegetal	vegetal
CO_2eqvegetal(v) - *Gliricidia sepium*	-26.6	t.ha^{-1}	Montagnini et al. (2013)	ton of CO_2 equivalent per hectare of vegetal	vegetal
CO_2eqvegetal(v) - *Urochloa brizantha*	-26.5	t.ha^{-1}	Eri et al. (2020)	ton of CO_2 equivalent per hectare of vegetal	vegetal
CO_2eqfertilizer(f) - nitrogen	0.65	t.ha^{-1}	Gissi (2017)	ton of CO_2 equivalent per ton of fertilizer (100 kg.N.ha-1)	fertilizer
CO_2eqfertilizer(f) - phosphorus	0.71	t.ha^{-1}	Kumar et al. (2012)	ton of CO_2 equivalent per ton of fertilizer	fertilizer
CO_2eqfertilizer(f) - potassium	0.46	t.ha^{-1}	Kumar et al. (2012)	ton of CO_2 equivalent per ton of fertilizer	fertilizer
CO_2eqfertilizer(f) - calcium	1.4	t.ha^{-1}	Silva et al. (2014)	ton of CO_2 equivalent per ton of fertilizer	fertilizer
CO_2eqfertilizer(f) - magnesium	0.13	t.ha^{-1}	Silva et al. (2014)	ton of CO_2 equivalent per ton of fertilizer	fertilizer

Table 1. Continued...

Parameters	Values	Units balanced	Literary domain	Meaning	Sets
CO_2eqlivestock(l) - growing	0.27	t.animal^{-1}	Lobo (2023)	ton of CO_2 equivalent per head (during the cycle 240 days)	livestock
CO_2eqlivestock(l) - fattening	0.67	t.animal^{-1}	Méo Filho (2020)	ton of CO_2 equivalent per head	livestock
manurenutrients(f) - nitrogen	0.006	%	Sandoval Paixão et al. (2020)	amount of manure weight in nutrients	fertilizer
manurenutrients(f) - phosphorus	0.052	%	Sandoval Paixão et al. (2020)	amount of manure weight in nutrients	fertilizer
manurenutrients(f) - potassium	0.025	%	Sandoval Paixão et al. (2020)	amount of manure weight in nutrients	fertilizer
manurenutrients(f) - calcium	0.01	%	Sandoval Paixão et al. (2020)	amount of manure weight in nutrients	fertilizer
manurenutrients(f) - magnesium	0.003	%	Sandoval Paixão et al. (2020)	amount of manure weight in nutrients	fertilizer
FertililzerRequirement(v,f) - *Tithonia diversifolia* - nitrogen	0.14	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Tithonia diversifolia* - phosphorus	1.17	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Tithonia diversifolia* - potassium	0.56	t.ha^{-1}	Eifediyi et al., 2023, Sandoval Paixão et al., 2020	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Tithonia diversifolia* - calcium	0.23	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Tithonia diversifolia* - magnesium	0.07	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Gliricidia sepium* - nitrogen	0.135	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Gliricidia sepium* - phosphorus	1.17	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Gliricidia sepium* - potassium	0.562	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Gliricidia sepium* - calcium	0.225	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Gliricidia sepium* - magnesium	0.0675	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Urochloa brizantha* - nitrogen	0.135	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Urochloa brizantha* - phosphorus	1.17	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Urochloa brizantha* - potassium	0.562	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Urochloa brizantha* - calcium	0.225	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
FertililzerRequirement(v,f) - *Urochloa brizantha* - magnesium	0.0675	t.ha^{-1}	Eifediyi et al. (2023), Sandoval Paixão et al. (2020)	fertilizer consumption per plant in tons of fertilizer per hectare of plant	vegetal,fertilizer
costgee	100	R$.t^{-1}	DATAGRO (2024)	methane cost in CO_2eq	scalar
totalarea	56	ha	Rivera-Acosta and Xiuchuan (2023), Simioni et al. (2022a, b), Resende et al. (2020)	farm-level area	scalar
mortalityrate	11.05	%	Portes et al. (2020), Marín-Garzon et al. (2021)	animal mortality rate	scalar
stockingrate	0.45	ha.animal^{-1}	Neves (2020)	animal stocking rate	scalar
slaughterweight	520	kg	Kul et al. (2020)	slaughter weight	scalar
minimum_tithonia	0.3	%	Pazla et al. (2021)	minimum amount of Tithonia diversiflora in DMI	scalar
minimum_gliricidia	0.1	%	Tahuk et al. (2022)	minimum amount of *Gliricidia sepium* in DMI	scalar
minimum_pasture	0.6	%	Marsetyo et al. (2021)	minimum amount of *Urochloa brizantha* in DMI	scalar
DMI_G	5	kg	NASEM (2016)	dry matter intake by growing animals	scalar
DMI_F	11	kg	NASEM (2016)	dry matter intake by fattening animals	scalar

Figure 1. Simulated scenarios transposed from the information generated by the GAMS® software. Software used: Adobe Illustrator® 2024 (Adobe Inc., 2024) to vectorize the images and Adobe Photoshop® CC 2019 (Adobe Inc., 2019) to join the vectors.

4. Discussion

The discussion revolves around the hypotheses, which in this case are the issues that plague the commitment of the work. These questions are fundamental to assessing the validity of initial assumptions and identifying areas that require further investigation. By analysing these hypotheses, we can recognize the strengths and weaknesses of the commitment made, allowing adjustments and improvements to ensure the effectiveness of the project. Therefore, these questions are essential to guide the research and ensure that the work achieves its objectives in a solid and well-founded way, as shown in the mind map below (Figure 2).

4.1. Maximizing profit

Implementing this mathematical model for beef cattle in tropical conditions will make it possible to maximize profits by optimizing available resources and boosting herd productivity.

Implementing the 'GETAP_model' for beef cattle in tropical conditions effectively maximizes profits by optimizing the use of resources and increasing herd productivity, as evidenced by the impressive profit maximization of R$ 867,712.72. This result is corroborated by several studies in the literature. For example, Oliveira et al. (2018a) demonstrated a 15% reduction in operating costs through the inclusion of alternative forages, while Santos et al. (2021) reported a 20% increase in farm profitability with the application of optimization models. In addition, the development of selection indices and management strategies, as highlighted by Portes et al. (2020) and Bomfim (2023), further reinforces the economic benefits, with net profitability figures of R$ 213,637.55 and R$ 184,550.15, respectively. In addition, Ogawa et al. (2021) and Pahmeyer and Britz (2020) showed substantial profits (R$ 598,403.80 and R$ 568,000.00, respectively) from improved carcass production and optimized dairy farming practices, emphasizing the broad applicability and financial advantages of these optimization models in agriculture. These findings collectively highlight the significant potential to increase economic sustainability and profitability through advanced modelling and management techniques in agricultural systems, including payment for carbon credits.

4.2. Minimizing environmental impacts

The adoption of sustainable practices in beef cattle management is imperative for minimizing environmental impacts, ensuring the conservation of natural resources and long-term sustainability, as shown in model. This discussion evaluates the nutritional management of animals within such a system, the need for supplementation and the environmental impact of the system, with a particular focus on greenhouse gas (GHG) emissions.

In the system studied, the animal population includes 139.82 head in the growing phase and 124.44 head in the fattening phase. The plant area is divided into Tithonia (3.30 ha), Gliricidia (46.10 ha) and pasture (6.60 ha). These figures are comparable to those found in the literature, which highlights the importance of efficient management of the planted area to maintain animal productivity (Nair and Garrity, 2012). The cattle's nutritional needs are mainly met by the vegetation present in the system. Supplement intake is minimal, with no need for mineral or energy-protein supplements during the dry season due to the adequate availability of forage. However, during the rainy season, 8,474.25 tons of mineral supplements are needed to meet the animals' requirements, highlighting the need for supplementation with macro and microminerals and salt (sodium chloride) during this period. This supplementation pattern is consistent with studies that indicate seasonal variability in nutrient availability and the need for additional supplementation during periods of higher nutritional demand (Murgueitio et al., 2011).

A significant environmental benefit of the silvopastoral system is its ability to sequester carbon dioxide. The system sequesters 1,341.39 tons of CO_2-equivalent, demonstrating its potential to mitigate climate change by absorbing more GHGs than it emits. This sequestration

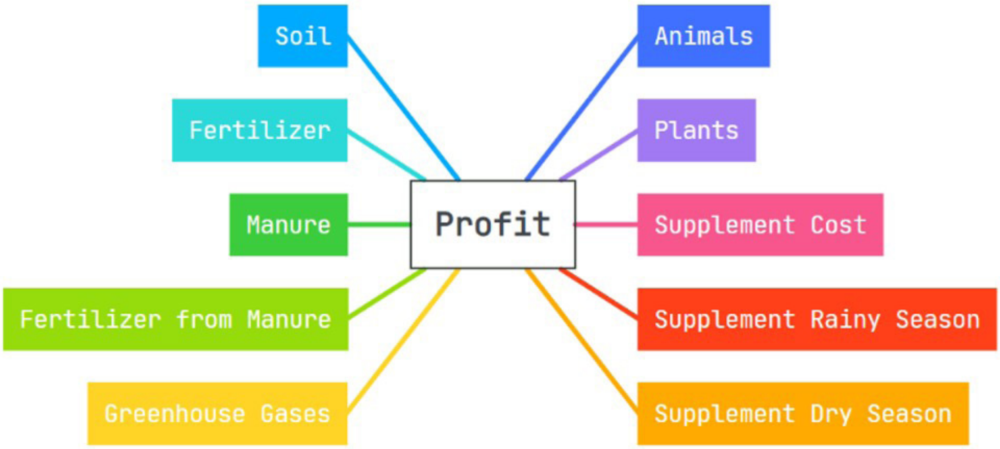

Figure 2. Organization of information in relation to model response variables to encourage critical thinking and problem solving.

capacity is corroborated by the literature, which highlights silvopastoral systems as effective strategies for mitigating climate change (Smith et al., 2014). Fertilizer use includes 2.61 tons of nitrogen, 20.18 tons of phosphorus, 9.56 tons of potassium, 4.16 tons of calcium and 1.30 tons of magnesium. The manure produced, totaling 871.95 tons, plays a crucial role in recycling nutrients within the system. Specifically, the manure used as fertilizer contributes 5.23 tons of nitrogen, 45.34 tons of phosphorus, 21.80 tons of potassium, 8.72 tons of calcium and 2.62 tons of magnesium, thus reducing the need for synthetic fertilizers and improving soil fertility. These values for reusing manure as fertilizer are higher than those often found in the literature, which suggests that efficient management can significantly reduce dependence on synthetic fertilizers and promote agricultural sustainability (Nair and Garrity, 2012).

Thus, the silvopastoral system effectively meets the nutritional needs of cattle through its diverse vegetation, minimizing dependence on external supplements. It also offers substantial environmental benefits by sequestering significant amounts of COand promoting sustainable nutrient management practices. The integration of trees, forage and livestock not only supports animal health and productivity, but also contributes to mitigating climate change and improving soil health. So, as a complement, due to the scarcity of current works in this context, this work serves as a literary reference, supported by evidence in relation to rigor and quality of writing, as well as meeting the interest to a general audience due to the overall quality of the content.

5. Conclusion

Optimization models derived from linear programming have emerged as valuable tools in the planning and management of agricultural farms, offering a strategic approach to dealing with operational complexity. Furthermore, these models are useful for evaluating the possible benefits arising from the implementation of integrated production systems, which combine animal husbandry and cultivation activities on a single property.

The technical synergies between the proposed activities, especially those related to livestock and agriculture, represent clear opportunities for economies of scale. The prospect of complete recycling of nitrogen, phosphorus and potassium contained in animal waste could increase when moving from scenarios with less diversification to those with greater diversification.

In addition to economic gains, running integrated systems brings benefits to the environment, especially through the reuse of resources that, once discarded, would cause negative externalities to the environment. Furthermore, the estimated volume of CO_2-equivalent sequestered could increase from the lowest to the highest diversification scenario.

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Data Availability Statement

The entire data set that supports the results of this study was published in the article itself.

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