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Dynamic Bayesian Network Modeling, Learning, and Inference: A Survey

PEDRO SHIGUIHARA[1], (Member, IEEE), ALNEU DE ANDRADE LOPES[2], AND DAVID MAURICIO[1]

[1]AI Group, Universidad Nacional Mayor de San Marcos, Lima 15081, Peru
[2]Institute of Mathematical and Computer Sciences (ICMC), University of Sao Paulo, Sao Carlos 13566-590, Brazil

Corresponding author: David Mauricio (dmauricios@unmsm.edu.pe)

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ABSTRACT Since the introduction of Dynamic Bayesian Networks (DBNs), their efficiency and effectiveness have increased through the development of three significant aspects: (i) modeling, (ii) learning and (iii) inference. However, no reviews of the literature have been found that chronicle their importance and development over time. The aim of this study is to provide a systematic review of the literature that details the evolution and advancement of DBNs, focusing in the period 1997-2019 that emphasize the aspects of modeling, learning and inference. While the literature presents temporal event networks, knowledge encapsulation, relational and time varying representations as the four predominant DBN modeling approaches, this work groups them as essential techniques within DBNs and help practitioners by associating each to various challenge that arise in pattern discovery and prediction in dynamic processes. Regarding learning, the predominant methods mainly focus on scoring with greedy search. Finally, our study suggests that the main methods used in DBN inference extend or adapt those used in static BNs, and are oriented to either optimize processing time or error rate.

INDEX TERMS Dynamic Bayesian networks, dynamic probabilistic graphical models, literature review, systematic literature review.

I. INTRODUCTION

Probabilistic Graphical Models (PGMs) use a graphical representation to compactly express probability distributions while at the same time explicitly represent large joint distributions, for transparent evaluation by specialists [1]. According to [2], PGMs can be classified into: (i) directed/undirected, (ii) static/dynamic, and (iii) probabilistic/decisional. The first group represents symmetric (undirected) or asymmetric (directed) dependency relationships. The second group represents a set of variables at a specific point in time (static) or across a period of time (dynamic). The third group uses random variables (probabilistic) or decision and utility variables (decisional). Among the different dynamic representative PGMs we have (1) Markov Chains, (2) Hidden Markov Models, (3) Markov Decision Processes (MDPs), (4) Partially Observable MDPs and (5) Dynamic Bayesian Networks (DBNs). Markov Chains [2] are (i) directed and (ii) probabilistic models that present discrete numbers of states and transitions that are stochastic. Hidden Markov Models are also (i) directed and (ii) probabilistic and are comprised of a double stochastic process where one set is underlying and unobservable and only revealed through a sequence of observations from the second set of processes [3]. Markov Decision Processes (MDPs) are (i) directed and (ii) decisional sequential decision models that evolve over time and are controlled by an agent [4]. Partially Markov Decision Processes are (i) directed and (ii) decisional but differ from MDPs as they are designed to address hidden or partial information concerning the state of the system [2]. Finally, Dynamic Bayesian Networks (DBNs) are extensions of Bayesian networks to model dynamic processes and consist of a series of time intervals that present the states of all variables at a given time and thus represent the evolution of a process over time [1]. As such, DBNs can be seen as a generalization of Markov Chains and Hidden Markov Models because they represent a space of states in a factorized way instead of as a single discrete random variable [5], and can be classified as (i) directed and (ii) probabilistic. Also, DBNs can represent a linear dynamical system such as Kalman filters, where the

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variables are all continuous and all of the dependencies are linear Gaussian [1].

DBNs are important as they capture and analyze information over time and fulfill two important functions in machine learning: classification and pattern discovery. Other classification algorithms, such as neural networks act as black boxes and make it difficult for a specialist to interpret the resulting model according to the domain of the problem, DBNs on the other hand are advantageous as they transparently encoded probability distributions over complex domains [1]. Some examples of applications that require the use of DBNs in order to capture their dynamic behavior are the pattern detection of human brain behavior [6], speech recognition [7], [8], medical diagnosis [7], [9], identification of regulatory gene networks [8], [10], target tracking [8], visual activities [11], [12], crime risk analysis [9], [12], sensor validation [9], client analysis [9], and video object tracking [12].

DBNs have three important aspects [2]: (1) modeling, (2) learning, and (3) inference. Regards to modeling, concepts from other domains have been merged with DBNs in order to present a better representation of the data and its behavior. To achieve this, studies have proposed Temporal Event Networks (TEN) [13] or relational ones [14]. Modeling, in general, has gone in the direction of specialization instead of generalization, resulting in useful models for specific contexts. Automatic learning has emerged as a response to the automation of the representation of DBNs, inspired particularly by static BNs. Search and parameter estimation algorithms typically found in BNs are also used in DBNs. As stated previously, the main challenge in highly dense network DBNs is to construct them efficiently [15]. One of the prevailing professional applications of DBN construction has been in the biomedical area [16], [17]. Inference allows the DBN to develop a diagnosis or prediction while maintaining efficient execution time and memory usage [18]. Automatic learning allows the DBN to build networks capable of representing relationships from data. Modeling allows the DBN to analyze dynamic behavior on a granular time scale. According to [1], there are two problems that arise when using DBN inference, as opposed to BN inference. One is that the BNs generated from the DBNs can have an arbitrarily big and complex structure. The second problem is that the temporal reasoning is often different from the reasoning required of a static model. This is especially common in networks with dense connectivity that pose problems for exact and approximate inference algorithms and thus require an algorithmic analysis. Variants of exact inference algorithms based on the variable elimination were proposed long before the development of probabilistic graphical models [1].

Despite the importance of DBNs and the diversity of studies and aspects developed, a Scopus or Web of Science (WoS) level literature review could not be found in the literature. While a Google Scholar search identifies two technical reports between 2001 and 2006 [19], [20], new solutions have been described to address the diversity of challenges associated with inference, automatic learning and modeling. To address this gap, this paper aims to answer the following question: What advances have been made with respect to modeling, automatic learning and inference in DBNs?

In the reminder of this paper, we carry out a literature review on DBNs with the following structure. In Section II, we present the background and theoretical basis of DBNs. In Section III, we present the methodology of the literature review. In Section IV, we analyze the different DBNs and present their approaches in detail. In Section V, we discuss the findings and important characteristics of the DBN studies. Finally, in Section VI, we detail the conclusions of the work.

II. BACKGROUND

Definition of DBNs are described in detail in [1], [5]. $X^t = \{X_1^t, \ldots, X_I^t\}$, denotes a set of random variables representing the state process at a given time t. A DBN is a pair $(\mathcal{G}, \theta)$, where $\mathcal{G}$ is the structure and θ is the set of parameters of a DBN. The DBN models a dynamic process, specifying a probability distribution for $X^0, \ldots, X^T$ with $P(X^0, \ldots, X^T \mid \mathcal{G}, \theta)$. $\mathcal{G}$ is a directed acyclic graph (DAG), whose nodes are the variables $X^0, \ldots, X^T$, whose edges follow a dynamic sequence defined as $X^t \to X^{t+1}$, where $t \in \{0, \ldots, T-1\}$, and cannot have edges of a future time pointing to a past time of type $X^t \leftarrow X^{t+1}$. θ is a set of parameters that contains a conditional probability distribution $P(X_i^t \mid Pa(X_i^t), \mathcal{G}, \theta)$ for each X_i^t given the set of parents $Pa(X_i^t)$ obtained from the $\mathcal{G}$ structure.

Modeling is based on $(\mathcal{G}, \theta)$ representation, a basic property of any model, having (1) the entities that constitute it, and (2) the relationships between these entities. All probabilistic network models are represented as graphs to define their structure and with local functions to describe their parameters. The difference between one model and another is the type of graph and the local functions used. The $\mathcal{G}$ structure is designated as a model and the θ distribution is designated as the parameters. In most cases, it is assumed that a DBN presents the same model at every time t. In that sense, this model is dynamic as its parameters vary over time and their distribution is estimated each time a new observation occurs.

Learning consists of building models in one of two manners: (1) by hand with the support of specialists, and (2) automatically from data. The current trend is to use automatic learning techniques. In many cases, estimates of $\mathcal{G}, \theta$ use automatic learning techniques, also known as Bayesian learning. This technique consists of calculating the probability of each hypothesis from the data. If $\mathcal{D}$ is a data set and h_i is the ith hypothesis, it is possible to estimate the probability of each hypothesis that maximizes $P(h_i \mid \mathcal{D})$, using the maximum a posteriori (MAP) estimate.

Inference consists of answering the probabilistic query according to the model and a set of evidence. The inference is a basic task to compute the posterior probability distribution for a sert of query nodes, given values for some evidence nodes which is called belief updating or probabilistic inference [21]. The representation $\mathcal{G}, \theta$ responds to queries through the intractable process of inference, and can be defined as the calculation of the probability distribution a posteriori of a set of query variables given a set of observed events. Let a query variable Q and a set of n evidence variables denoted by $\mathbf{E} = \{E_1, \ldots, E_n\}$, represent observable events $\mathbf{E} = \mathbf{e}$, where $\mathbf{e}$ is the evidence. To respond to the query, the conditional distribution $P(Q \mid \mathbf{e})$ is used.

DBNs intend to reveal patterns and temporal relationships by capturing the complexity and variable nature of a problem. DBNs model these dynamic processes, naturally establishing a compact structure capable of capturing the semantics of the temporal relationships between measured events within a dynamic system.

III. RESEARCH METHODOLOGY

Similar to [22], in order to query the literature for reviews on DBNs, we used the following keyword search strings in both the Scopus and WoS citation databases:

("dynamic bayesian network") AND ("review" OR "research synthesis" OR "research integration" OR "systematic overview" OR "systematic research synthesis" OR "integrative research"). No results were found. We performed a second search in Google Scholar with the query: *("dynamic bayesian network" AND ("survey" OR "review"))* . which retrieved two articles from the years 2001 and 2006.

In this literature review, the standard systematic review methods for software engineering area were considered [23], [24]. The final method was divided into three phases: (1) planning, (2) development, and (3) results. In the planning phase, the importance of reviewing the literature is discussed, the research questions are formulated, and the study protocol is presented. In the development phase, the primary studies are evaluated for potential inclusion and filtered for data extraction. In the results phase we present statistics and findings and answer the research questions posed in the first phase. The planning and development phases are described in this current section III, while the results phase consisting of statistics and answers to the research questions, are found in the following sections.

A. PLANING OF THE REVIEW

This study is designed to answer three specific research questions concerning the progress of DBNs (Table 1).

TABLE 1. Research questions about the review of DBNs.

ID	Research Question
RQ1	What advances have been made regarding the modeling aspect of DBNs?
RQ2	What advances have been made regarding the learning aspect of DBNs?
RQ3	What advances have been made regarding the inference aspect of DBNs?

The search process is carried out by designing a search string that queries a citation database (Table 2). For the design of this search string, it is important to use synonyms associated with DBN terms that best represent the research questions. This review covers the relevant research from 1997 to 2019, using the Scopus and Web of Science citation databases. The selection criteria are shown in Table 3.

TABLE 2. Search strings used to consult DBN articles in scopus and web of science (WoS) citation databases.

Source	Search String
Scopus	TITLE-ABS-KEY ("DYNAMIC BAYESIAN NETWORK" OR "DYNA- MIC BELIEF NETWORK" OR "DYNAMIC PROBABILISTIC GRAPHICAL MO- DEL" OR "DYNAMIC BAYESIAN MODEL" OR "DYNAMIC GENERA- TIVE MODEL" OR "TEMPORAL BAYESIAN NETWORK" OR "TEMPO- RAL BELIEF NETWORK" OR "TEMPORAL PROBABILISTIC GRAPHICAL MO- DEL" OR "TEMPORAL BAYESIAN MODEL" OR "TEMPORAL GENERATIVE MODEL") AND (LIMIT-TO(SRCTYPE, "J")) AND (LI- MIT TO (DOCTYPE , "AR ") OR LIMIT-TO (DOCTY- PE , "RE")) AND (LIMIT-TO (SUBJAREA , "COMP ")) AND (LIMITTO(LANGUAGE, "ENGLISH"))
WoS	(TS=("DYNAMIC BAYESIAN NETWORK" OR "DYNAMIC BELIEF NETWORK" OR "DYNAMIC PROBABILISTIC GRAPHICAL MODEL" OR "DYNAMIC BAYESIAN MODEL" OR "DYNAMIC GENERATIVE MODEL" OR "TEMPORAL BAYESIAN NETWORK" OR "TEMPORAL BELIEF NETWORK" OR "TEMPORAL PROBABILIS- TIC GRAPHICAL MODEL" OR "TEMPORAL BAYESIAN MODEL" OR "TEMPORAL GENERATIVE MODEL")) AND LANGUAGE: (ENGLISH) AND DOCU- MENT TYPES: (ARTICLE OR REVIEW)

TABLE 3. DBN selection criteria.

Selection Criteria
Studies that present new approaches to the aspects of inference, learning, and modeling that extend or enhance DBN functionality
Studies of indexed books, journals, and papers
Studies written in English

B. DEVELOPMENT OF THE REVIEW

Once the research questions and selection criteria were defined, the article search process was implemented.

IV. ANALYSIS

In this section, answers are given to the research questions posed in Section III-A.

A. RQ1: WHAT ADVANCES HAVE BEEN MADE WITH RESPECTO TO DBN MODELING?

Four types of DBN modeling were identified in the literature (Table 4): (1) Temporal Event Networks (TEN), (2) DBNs and Knowledge Encapsulation, (3) relational DBNs, and (4) time-varying DBNs.

TEN simplifies the DBNs into small dynamic processes. In traditional DBNs, a node represents the value of a variable at a certain time, while in TEN, a node represents when an

TABLE 4. DBN modeling types.

Types	References
Temporal Event Networks (TEN)	[9, 13, 25]–[27]
DBNs and Knowledge Encapsulation	[28, 29]
Relational DBMs (DBNR)	[11, 14, 30, 31]
Time-varying DBNs	[32]–[40]

event or change of state of a variable occurs, and thus it is simpler and more efficient than DBNs [2].

TNBN, TBNDTE and DTPEN are three subtypes of NET [13], [25] and are more efficient than DBNs for small problems. However, DTPEN has a disadvantage of having a single time granularity [26]. The studies involving these subtypes of TENs are shown in Table 5.

Two types of DBN representations were identified as a way to encapsulate knowledge (Table 6): Dynamic Influence Network and DBN (DIN-DBN), and Ordinary Differential Equations in DBNs (ODE-DBN). While Dynamic Influence Network (DIN) presents a compact modeling procedure that permits efficient managing of temporal restrictions [41], it does not assimilate updated information easily, a challenge which has produced new methods that would alternate between it and DBN inference in a single process [28]. This has also been achieved with differential equations (ODE-DBN) [29].

TABLE 5. Modeling of DBNs by type of event.

Modeling	Year	Description
Temporary Node Bayesian Network (TNBN) [13]	1999	Model for event detection through temporal reasoning.
Discrete Time Probabilistic Event Network (DTPEN) [25]	2002	Model for uncertain temporal reasoning in domains involving probabilistic events.
Temporary Bayesian Network of Discrete Time Events (TBNDTE) [26]	2005	RBTE allows the modeling of a problem using multiple time granularity, overcoming the single time granularity limitation of DTPENs.
Comparison between TNBN and TBNDTE [27]	2007	Comparison of both approaches in order to determine the main advantages and disadvantages of each method, observing that TBNDTE has better results than TNBN.
TNBN Learning [9]	2013	Data derived Method for TNBN learning to obtain its structure and time intervals.

TABLE 6. Knowledge encapsulation modeling of DBNs.

Modeling	Year	Description
DIN-RBD [28]	2009	Algorithm that transforms DIN to DBN in order to exploit the advantages of both models.
Ordinary differential equations in DBNs (EDO-DBN) [29]	2013	Methodology to encapsulate knowledge in the form of ordinary differential equations in DBNs.

TABLE 7. Relational DBM modeling.

Modeling	Year	Description
Dynamic Probabilistic Relational Models [14]	2003	Probabilistic time series models that take into consideration the relationships between objects.
Relational Dynamic Bayesian Networks [30]	2005	Relational Dynamic Bayesian Networks provide greater simplicity and expressiveness than DBNs.
Interval Temporal Bayesian network [11]	2013	A graphical model that combines Bayesian networks with interval algebra to explicitly model relationships over time.
Activator Dynamic Bayesian Networks (ADBNs) [31]	2017	Models capable of adapting quickly to contexts under a cautious use of time, anticipating indirect influences on a solid mathematical basis, and allowing the modeling of cyclical dependencies from local and causal perspectives.

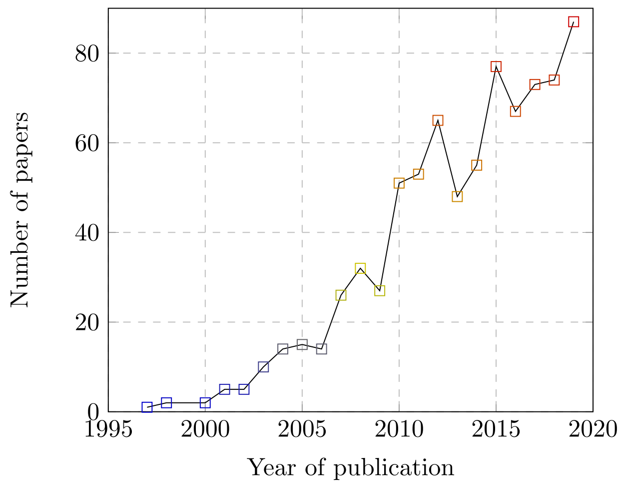

FIGURE 1. Distribution of publications by year from January 1997 to December 2019.

The modeling of relational DBNs has non-directed edges unlike traditional DBNs, which are applied to scenarios where relationships are bi-directional, such as friendship relationships in a social network. An inventory of relational DBN modeling is shown in Table 7.

One of the issues addressed in recent years concerns the intractable challenge of implementing algorithmic methods that allow DBN structures and parameters to evolve over time, since it involves making complicated update, change interval, and network structure design decisions, among others (Table 8).

Figure 4 illustrates the emergence of the DBN modeling types over time.

B. RQ2: WHAT ADVANCES HAVE BEEN MADE WITH RESPECT TO DBN LEARNING?

The literature presents many different approaches that describe effective learning methods (Table 9), all of them can be organized into four groups of strategies for efficient

TABLE 8. DBN modeling with a time varying structure.

Modeling	Year	Description
Non-stationary dynamic Bayesian networks [32]	2010	Graphic model where the conditional dependency structure of the underlying data generation process can change over time. This enables the study of problems where the network structure evolves over time.
Non-homogeneous dynamic Bayesian networks for continuous data [33]	2011	Model that combines a Bayesian network with linear conditional Gaussian probabilities with a Bayesian multiple change process, where the number and location of change points are sampled from a distribution a posteriori witdh MCMC.
Time varying dynamic Bayesian network (TVDBN) [34]	2011	Model that enables the analysis of non-stationary sequences. The changing parameters and structures in a TVDBN are treated as random processes whose values at each point of time establish a stationary DBN model. This model is then used to specify the distribution of sequence data across time.
Hidden Markov Model into a DBN (HMDBN) [35]	2015	Modeling that looks to manage the network's evolution in time by extending each hidden node of an HMM in a DBN.
Non-stationary Continuous Time Bayesian Networks [36]	2016	Method that allows the parent set of each node to change over time. Non-stationary continuous time Bayesian networks are trained from data under three scenarios: known transition times, number of known time periods, and number of unknown time periods.
Causal time-varying dynamic Bayesian network (cTVDBN) [37]	2016	Efficiently models pattern discovery for varying causal relationships over time, while looking to control overfitting.
Non-homogeneous DBN model [38]	2016	Model that seeks consensus between the mixed DBN model with free assignment and the DBN model with segmented changes points. In addition, the study focuses on non-homogeneous DBNs with HMM.
Hybrid Time Bayesian Networks [39]	2017	Method that combines discrete and continuous time Bayesian networks. The new approach enables natural modeling of dynamic systems with regular and irregular changing variables.
Partially Non-homogeneous Dynamic Bayesian Networks (NH-DBNs) [40]	2019	Method to train cellular networks from time series, based on Bayesian hierarchical regression models and partitioned design matrices. Presents the advantage of assuming that the parameters cannot be constant according to the conditions.

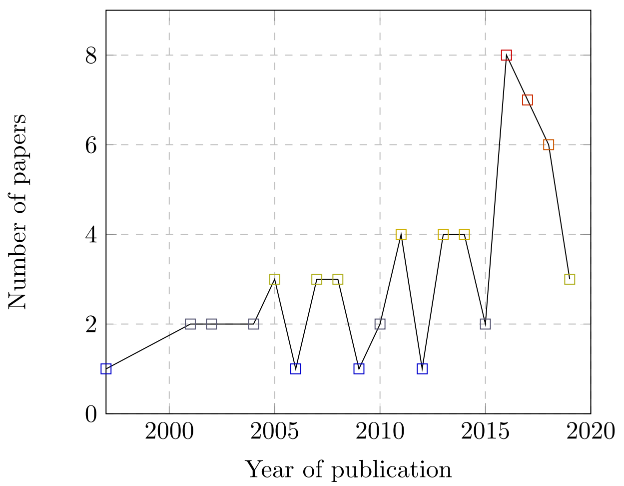

FIGURE 2. Distribution of selected publications by year from January 1997 to December 2019.

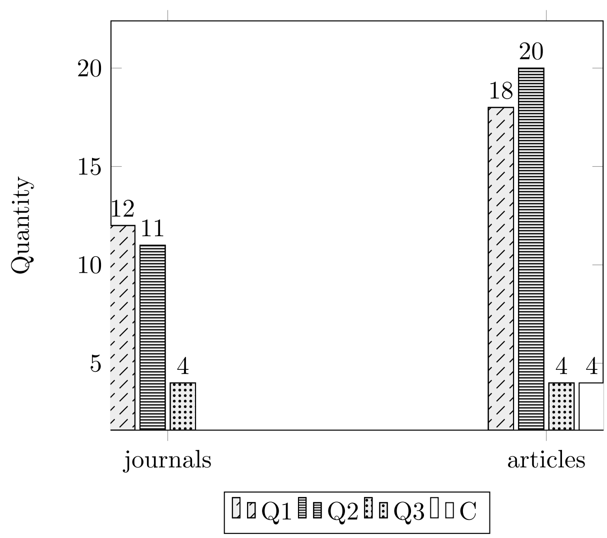

FIGURE 3. Distribution of publications by quartiles from January 1997 to December 2019.

building of DBNs (Table 10). The first strategy is scoring or greedy search, functions that use metrics to measure the quality of each structure in the structure search space. The second is constraints, a strategy that applies statistical techniques to restrict the use of edges within the graphical structure. Sampling is the third group, which allows for the generation of possible structures from a distribution. The final strategy is a posteriori probability, which generates structures after having validated their usefulness (see Table 11).

Concerning the construction of their structures, the reviewed studies present two important factors that affect learning and ultimately the DBN's proper performance. One is the quantity of random variables a factor that reduces performance as its quantity increases. Another key factor in DBN learning performance is the relationship between variables that must be filtered by causality methods. Therefore, managing hundreds of variables when building the structures of the networks requires very efficient algorithms. However, a large quantity of variables also requires a valid connection through edges by means of causality measures. If machine learning is used, this becomes even more complex, considering the search space of directed acyclic graphs.

Studies suggested that these learning methods are best evaluated using metrics that consider their structure, the structure generation time, the inference results using the learned structure, and the time intervals (Table 10). The evaluations considering structure look to determine the one that best represents the problem, derived from the data, and are mainly

TABLE 9. DBN learning methods.

ID	Learning Methods	Year	Description
M01	Evolutionary algorithm using time-series characteristics [15]	2001	Evolutionary algorithm that exploits time series characteristics to quickly generate structures.
M02	Search method for DBN structures with hidden nodes [42]	2004	Bayesian space-time network classifier that assumes dependencies between variables based on a neighborhood space, such as first order Cartesian coordinates, and requires a set of operators that exploit the spatial nature of the dataset to learn the network structures.
M03	BN Learning for biological data [10]	2004	Method to manage false-positive interactions in networks generated from limited data by combining first, moderate data interpolation with second, an influence scoring method that estimates both the sign (positive/negative) and relative magnitude of variable interactions.
M04	Cross-validation scoring criterion [43]	2005	DBN learning method that uses cross-validation as a scoring mechanism to select the best network.
M05	Multi-objective evaluation algorithm applied in genetics [44]	2007	Genetic algorithm to generate a network structure by applying a multi-objective assessment strategy based on scoring and structure complexity to model the causal relationships that explain a sequence.
M06	Variational Bayesian structural expectation maximization [45]	2007	Variational Bayesian Structural Expectation Maximization technique that learns how to estimate both the parameters and structure of a network, then estimates the probabilities of the network topology a posteriori with a simple Bayesian strategy that integrate two datasets.
M07	autoDBN [46]	2008	Adaptive learning method for training DBNs with changing structures derived from multivariate time series.
M08	Learning using steady state measurements [8]	2008	Method for generating learning DBN structures from a time series using steady state measurements through 2 methods: (1) based on approximation and (2) based on exact calculation.
M09	Temporal Qualitative Probabilistic Networks Learning [47]	2010	Method for learning Temporal Qualitative Probabilistic Networks (TQPN) from time series, using DBN learning methods based on Markov Chain Monte Carlo
M10	A Comparison of Learning Algorithms to maintain the global optimality guarantee [48]	2011	Study of learning algorithms that intend to reduce the time and memory costs of many known methods without losing the guarantee of global optimality and whose properties are based on different scoring criteria such as MDL, Akaike and BIC.
M11	Multi Objective DPSO [49]	2011	Method using a multi-target discrete particle swarm optimizer for training DBN structures, presenting a hierarchical structure, and finding effective DBN structures faster than when compared to conventional methods.
M12	Mapping dynamic bayesian networks to α-shapes [50]	2012	Method that helps build hierarchical structures assigned to a DBN, has a continuous representation of the traditional DBN as alpha shape , is more informative about the objects to be classified, and whose objects can be seen at different levels of detail within a hierarchy.
M13	Temporal nodes BN Learning [9]	2013	Algorithm for the construction of TNBN, with three phases: (1) approximate the intervals, (2) obtain the structure using algorithms and standard, and (3) refine each time node's intervals through a clustering algorithm.
M14	Continuous Time BN Learning [51]	2014	Algorithm based on conditional log-likelihood scoring functions to train continuous time Bayesian network classifiers that consider structural constraints to control model complexity.
M15	Structural prediction [52]	2015	Method that employs local information for DBN structural learning that considers the presence and absence of edges from previous information.
M16	Max-Min high-order DBN [53]	2016	Method that applies score-based search and efficiently models time lags by using constraints to limit the space of potential structures. Based on the max-min hill-climbing Bayesian network technique that was originally created to learn/train static BNs.
M17	High-order DBNs (HO-DBNs) [17]	2017	Method for reducing search space of DAGs structures using dynamic programming and properties of scoring functions to identify effective connectivity between brain regions from brain magnetic resonance imaging (fMRI) data.
M18	Restricted-Derestricted DBNs [16]	2019	Method that searches for structures for transcriptional regulatory networks in order to recover true relationships.

used to build regulatory networks in the biomedical field. The evaluations considering the structure generation time intend to measure how long it takes the method to build a DBN structure. The evaluations that consider the inference using the produced structure are based on its performance of tasks, such as classification and others. Finally, the evaluations regarding time intervals intend to measure the ability of the method to select the adequate time intervals and their corresponding time-slices for the generation of a DBN.

Regarding the types of datasets utilized, 15 works use synthetic datasets while 13 utilized real datasets, with biomedical predominating the studies with real datasets (Table 12).

C. RQ3: WHAT ADVANCES HAVE BEEN MADE WITH RESPECT TO DBN INFERENCE?

Six studies were found relating to the inference aspect of DBNs and collectively identify three types of inferences

TABLE 10. Evaluation criteria used in DBN learning methods.

Measurement	Methods
Structure	M01, M02, M03, M04, M05, M06, M08, M09, M11, M13, M15, M16, M17, M18,
Structure generation time	M01, M10, M14
Inference using the produced structure	M07, M12, M14
Time intervals	M13

(Table 13). One type, exact inference, intends to estimate a query in terms of the conditional probability $P(Y \mid \mathbf{X})$, where Y is the random variable to be estimated and $\mathbf{X}$ is the evidence and is considered a NP-hard problem that requires development in exponential time in the worst case [1]. In [54], a *structural interface* algorithm is presented that accelerates

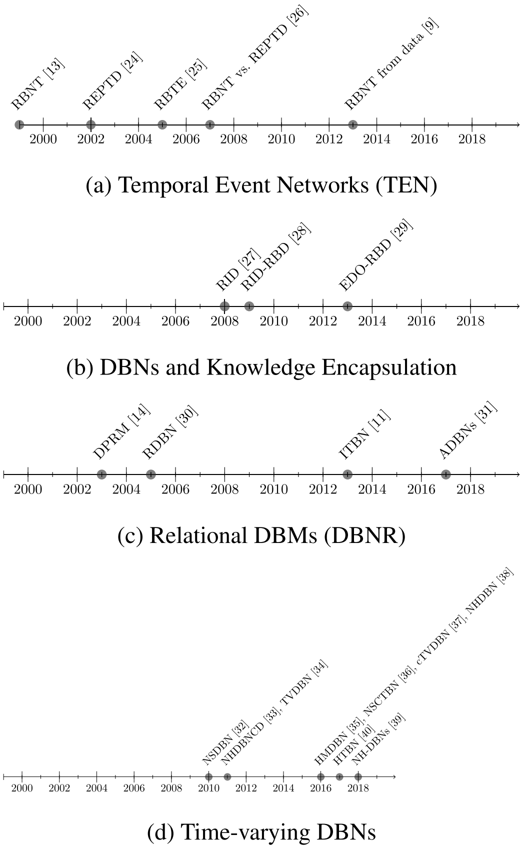

FIGURE 4. Emergence of DBN modeling procedures over time.

TABLE 11. Learning strategies for DBNs.

Learning Strategy	Methods
Scoring/Greedy search	M01, M02, M03, M04, M05, M10, M11, M13, M14, M16, M17, M18
Constraints	M07, M12, M15
Sampling	M08, M09
Probability a posteriori	M06

inference by exploiting the repeated and local structures as well as the conditional independences, thus improving their scalability for large and complex networks.

Approximate inference intends to find a successful solution in the shortest time possible. Its approach uses the DBN factors to estimate the joint distribution and, in many cases, use additional information to support the inference [55]. Carrying out inference on the different time-slices is a challenge, and requires important design considerations, such as whether to include supporting information and which DBN segment should be applied to perform the inference. Hybrid inferences are those that combine the characteristics of both exact and approximate inference, enabling them to develop selective updates about the belief factors from the network, and thus producing exact inference under certain assumptions, and approximate inference under others [56].

For optimum performance, inference methods look for efficiency in at least one of the following metrics: processing time, error rate (minimum certainty limit), coherence, and scalability. Coherence refers to the consistency of results from the logical use of data in the specific domain addressed, while scalability refers to the suitability of these methods for large networks, with processing time and error rate being the two metrics most commonly used (Table 13).

TABLE 12. Classification of the DBNs learning methods based on the types of datasets used, synthetic or real, and specific application/problem domain.

Type of dataset	Methods
Synthetic	M01, M02, M03, M04, M05, M06, M07, M08, M09, M11, M13, M14, M15, M16, M17
Real:	
cDNA microarray	M06
interest rates and stock price	M07
UCI datasets	M10
Gene Expression	M16, M18
fMRI	M17
Oil Refinery	M01, M02
Rust	M04
Energy plants	M13
PPI and drosophila muscle	M15
Stroke rehabilitation	M14
Age prediction through facial recognition	M12

TABLE 13. Advances in DBN inference.

Type	Method	Metric
Exact	*structural interface* algorithm [54]	Scalability
Approximate	Active prediction [55]	Error Rate
	Non-homogenous Inference [57]	Error Rate
	Inference with constraints and sliding window [12]	Time
	Inference with qualitative information [58]	Coherence
Hybrid	Inference with selective updates [56]	Time

V. DISCUSSION

The result of this systematic review is a catalog of factors that influence the building of DBNs. Researchers can use the different metrics, strategies and criteria presented herein to understand and determine the optimal approaches for their specific application. The relevance of this information is validated as 90% of the reviewed studies were from the first and second quartiles (Q1, Q2) journals and thus fortify the findings presented in this review. Each research question is discussed below.

A. RQ1

it was identified that, in general, inference in DBNs involves to estimate probabilities to answer queries from the representation, transition and observation of the network. Unlike static BNs, DBNs include the transition step, which allows the sequential transfer of probabilities between different time periods. The reviewed studies show a greater tendency to apply approximate inference as opposed to hybrid inference, possibly due to its low computational cost. Inference evaluation metrics are diverse, and include consistency, scalability, time, and error rate, with the last two being the most commonly used. However, there is no consensus on which is the most informative metric to use and few studies apply them all. Inference methods for DBNs are mainly adaptations of static BNs.

B. RQ2

Regarding the learning process, studies were found on the construction of the structure of the Bayesian network $\mathcal{G} =< V, E >$. They also showed that a majority of the technical studies on learning strategies used scoring and greedy search methods, while a minority used constraints, a posteriori probability, and sampling. In addition, these strategies are oriented to (1) machine learning and (2) experiential learning. Machine learning deals with problems such as scarcity of data, unhelpful relationships between variables that decrease performance, or very wide search spaces where it is not possible to achieve the convergence of an optimal model. These limitations suggest that further research on new learning methods that improve the performance of DBNs for both classification and pattern discovery is warranted. However, there are strategies that have not been applied to DBNs, nor even BNs. One is using GRASP instead of greedy search. Also, the evaluation approach was oriented towards optimizing the quality of the structure of the results, the construction time, and the time intervals for the structure. For a fixed number of instances, the quality of results declines as the quantity of instances increases, while the quality of results improves when the relationships are filtered by some measure of causality in E. However, no studies were identified that address both aspects at once.

C. RQ3

New approaches to DBN modeling have emerged, such as temporal event networks, knowledge encapsulation, relational, and time-varying. In time event networks, the objective is to simplify the construction of a DBN in order to evaluate events in small dynamic processes. In knowledge encapsulation, differential equations or dynamic influence networks are used to represent data. In relational modeling, the objective is to manage cyclicality in order to include it within the modeling of the DBMs, taking advantage of the changing processes in time without affecting the temporal reasoning. In time-varying models, the objective is to consider the evolution of the structure and parameters of the DBNs over time, with a clear interest in seeking DBNs that contribute to the monitoring of complex domains as time progresses. This is an important topic and is likely to continue receiving greater attention in the coming years due to its impact on monitoring applications.

VI. CONCLUSION

This work aimed to provide a systematic review of the literature related to new approaches of inference, learning and modeling of DBNs. Three research questions were proposed regarding the advances in inference (RQ1), learning (RQ2) and modeling (RQ3). The search was carried out in both the Scopus and Web of Science citation databases, selecting 42 out of 777 identified studies, with 42.8% of those studies addressing learning (RQ2) and 47.6% addressing modeling (RQ3). It is important to point out that 90% of the selected articles belong to journals from the first and second quartiles (Q1, Q2), which ensures that this study presents reliable results. With regard to modeling, it is important to emphasize that DBN modeling approaches are evolving, with four predominating: (1) temporal event networks, (2) knowledge encapsulation, (3) relational, and (4) time-varying. However, as no studies were identified that involve more than one approach at a time, future research should consider this as it could bring good results at low computational cost. Specifically regarding modeling, this work seeks to contribute to the practice by grouping these four modeling aspects as essential components of DBNs and associating each to the various challenges found in pattern discovery or prediction in dynamic processes to further advance the effectiveness of DBNs. With regard to learning, studies related to structure learning and its evaluation were identified. Learning methods are oriented to scoring with greedy search and the reviewed studies show that the quality of results declines as the number of variables increases while the quality of results improves as the number of edges increase. Future research should seek to understand the effect of these two aspects at the same time, as well as apply more advanced search strategies, such as GRASP. About inference, it involves the probability estimation to answer queries from the representation, transition and observation in the network. The identified methods are an extension or adaptation those used in static BNs and usually tend to consider the approximate approach over the exact and hybrid ones, possibly due to their lower computational cost. While time and error rate are the most commonly used metrics to evaluate the methods, no studies were identified that focused on optimizing both metrics at the same time. A limitation of the study is that it analyzed only Scopus or Web of Science studies, leaving aside conference articles and other sources that could shed more light on the benefits of Dynamic Bayesian Networks.

REFERENCES

[1] D. Koller and N. Friedman, *Probabilistic Graphical Models: Principles and Techniques—Adaptive Computation and Machine Learning*. Cambridge, MA, USA: MIT Press, 2009.

[2] L. E. Sucar, *Probabilistic Graphical Models: Principles and Applications*. London, U.K.: Springer, 2015.

[3] L. Rabiner and B. Juang, "An introduction to hidden Markov models," *IEEE ASSP Mag.*, vol. AM-3, no. 1, pp. 4–16, Jan. 1986.

[4] M. L. Puterman, *Markov Decision Processes: Discrete Stochastic Dynamic Programming*, 1st ed. New York, NY, USA: Wiley, 1994.

[5] K. P. Murphy, *Dynamic Bayesian Networks: Representation, Inference and Learning*. Berkeley, CA, USA: Univ. California, 2002.

[6] J. C. Rajapakse and J. Zhou, "Learning effective brain connectivity with dynamic Bayesian networks," *NeuroImage*, vol. 37, no. 3, pp. 749–760, 2007.

[7] T. Charitos, P. R. de Waal, and L. C. van der Gaag, "Convergence in Markovian models with implications for efficiency of inference," *Int. J. Approx. Reasoning*, vol. 46, no. 2, pp. 300–319, Oct. 2007.

[8] H. Lähdesmäki and I. Shmulevich, "Learning the structure of dynamic Bayesian networks from time series and steady state measurements," *Mach. Learn.*, vol. 71, nos. 2–3, pp. 185–217, Jun. 2008.
[9] P. Hernandez-Leal, J. A. Gonzalez, E. F. Morales, and L. E. Sucar, "Learning temporal nodes Bayesian networks," *Int. J. Approx. Reasoning*, vol. 54, no. 8, pp. 956–977, Oct. 2013.
[10] J. Yu, V. A. Smith, P. P. Wang, A. J. Hartemink, and E. D. Jarvis, "Advances to Bayesian network inference for generating causal networks from observational biological data," *Bioinformatics*, vol. 20, no. 18, pp. 3594–3603, Dec. 2004.
[11] Y. Zhang, Y. Zhang, E. Swears, N. Larios, Z. Wang, and Q. Ji, "Modeling temporal interactions with interval temporal Bayesian networks for complex activity recognition," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 35, no. 10, pp. 2468–2483, Oct. 2013.
[12] X.-G. Gao, J.-F. Mei, H.-Y. Chen, and D.-Q. Chen, "Approximate inference for dynamic Bayesian networks: Sliding window approach," *Appl. Intell.*, vol. 40, no. 4, pp. 575–591, Jun. 2014.
[13] G. Arroyo-Figueroa and L. E. Sucar, "A temporal Bayesian network for diagnosis and prediction," in *Proc. 15th Conf. Uncertainty Artif. Intell. (UAI)*, San Francisco, CA, USA. San Mateo, CA, USA: Morgan Kaufmann, 1999, pp. 13–20.
[14] S. Sanghai, P. Domingos, and D. Weld, "Dynamic probabilistic relational models," in *Proc. 18th Int. Joint Conf. Artif. Intell. (IJCAI)*, San Francisco, CA, USA. San Mateo, CA, USA: Morgan Kaufmann, 2003, pp. 992–997.
[15] A. Tucker, X. Liu, and A. Ogden-Swift, "Evolutionary learning of dynamic probabilistic models with large time lags," *Int. J. Intell. Syst.*, vol. 16, no. 5, pp. 621–645, 2001.
[16] E. S. Adabor and G. K. Acquaah-Mensah, "Restricted-derestricted dynamic Bayesian network inference of transcriptional regulatory relationships among genes in cancer," *Comput. Biol. Chem.*, vol. 79, pp. 155–164, Apr. 2019.
[17] S. Dang, S. Chaudhury, B. Lall, and P. K. Roy, "The dynamic programming high-order dynamic Bayesian networks learning for identifying effective connectivity in human brain from fMRI," *J. Neurosci. Methods*, vol. 285, pp. 33–44, Jun. 2017.
[18] A. Darwiche, "Constant-space reasoning in dynamic Bayesian networks," *Int. J. Approx. Reasoning*, vol. 26, no. 3, pp. 161–178, Apr. 2001.
[19] V. Mihajlovic and M. Petkovic, "Dynamic Bayesian networks: A state of the art," Univ. Twente, Enschede, The Netherlands, CTIT Tech. Rep. TR-CTIT-34, Oct. 2001.
[20] B. Ng, "Survey of Bayesian models for modelling of stochastic temporal processes," Lawrence Livermore Nat. Lab., Livermore, CA, USA, Tech. Rep. UCRL-TR-225272, 2006.
[21] K. B. Korb and A. E. Nicholson, *Bayesian Artificial Intelligence*, 2nd ed. Boca Raton, FL, USA: CRC Press, 2010.
[22] T. Tahir, G. Rasool, and C. Gencel, "A systematic literature review on software measurement programs," *Inf. Softw. Technol.*, vol. 73, pp. 101–121, May 2016.
[23] P. S. C. Vargas and D. Mauricio, "A review of literature about models and factors of productivity in the software factory," *Int. J. Inf. Technol. Syst. Approach*, vol. 11, no. 1, pp. 48–71, Jan. 2018.
[24] G. Arcos-Medina and D. Mauricio, "Aspects of software quality applied to the process of agile software development: A systematic literature review," *Int. J. Syst. Assurance Eng. Manage.*, vol. 10, no. 5, pp. 867–897, Oct. 2019.
[25] S. F. Galán and F. J. Díez, "Networks of probabilistic events in discrete time," *Int. J. Approx. Reasoning*, vol. 30, pp. 181–202, Sep. 2002.
[26] G. Arroyo-Figueroa and L. E. Sucar, "Temporal Bayesian network of events for diagnosis and prediction in dynamic domains," *Appl. Intell.*, vol. 23, pp. 77–86, Oct. 2005.
[27] S. F. Galán, G. Arroyo-Figueroa, F. J. Díez, and L. E. Sucar, "Comparison of two types of event Bayesian networks: A case study," *Appl. Artif. Intell.*, vol. 21, no. 3, pp. 185–209, Mar. 2007.
[28] S. Haider, "From dynamic influence nets to dynamic Bayesian networks: A transformation algorithm," *Int. J. Intell. Syst.*, vol. 24, no. 8, pp. 919–933, Aug. 2009.
[29] C. G. Enright, M. G. Madden, and N. Madden, "Bayesian networks for mathematical models: Techniques for automatic construction and efficient inference," *Int. J. Approx. Reasoning*, vol. 54, no. 2, pp. 323–342, Sep. 2013.
[30] S. Sanghai, P. Domingos, and D. Weld, "Relational dynamic Bayesian networks," *J. Artif. Intell. Res.*, vol. 24, pp. 759–797, Dec. 2005.
[31] A. Motzek and R. Möller, "Indirect causes in dynamic Bayesian networks revisited," *J. Artif. Intell. Res.*, vol. 59, pp. 1–58, May 2017.
[32] J. W. Robinson and A. J. Hartemink, "Learning non-stationary dynamic Bayesian networks," *J. Mach. Learn. Res.*, vol. 11, pp. 3647–3680, Dec. 2010.
[33] M. Grzegorczyk and D. Husmeier, "Non-homogeneous dynamic Bayesian networks for continuous data," *Mach. Learn.*, vol. 83, no. 3, pp. 355–419, Jun. 2011.
[34] Z. Wang, E. E. Kuruoglu, X. Yang, Y. Xu, and T. S. Huang, "Time varying dynamic Bayesian network for nonstationary events modeling and online inference," *IEEE Trans. Signal Process.*, vol. 59, no. 4, pp. 1553–1568, Apr. 2011.
[35] S. Zhu and Y. Wang, "Hidden Markov induced dynamic Bayesian network for recovering time evolving gene regulatory networks," *Sci. Rep.*, vol. 5, no. 1, pp. 1–17, Nov. 2016.
[36] S. Villa and F. Stella, "Learning continuous time Bayesian networks in non-stationary domains," *J. Artif. Intell. Res.*, vol. 57, pp. 1–37, Sep. 2016.
[37] V. W. Chu, R. K. Wong, F. Chen, S. Fong, and P. C. K. Hung, "Self-regularized causal structure discovery for trajectory-based networks," *J. Comput. Syst. Sci.*, vol. 82, no. 4, pp. 594–609, Jun. 2016.
[38] M. Grzegorczyk, "A non-homogeneous dynamic Bayesian network with a hidden Markov model dependency structure among the temporal data points," *Mach. Learn.*, vol. 102, no. 2, pp. 155–207, Feb. 2016.
[39] M. Liu, A. Hommersom, M. van der Heijden, and P. J. F. Lucas, "Hybrid time Bayesian networks," *Int. J. Approx. Reasoning*, vol. 80, pp. 460–474, Jan. 2017.
[40] M. S. Kamalabad, A. M. Heberle, K. Thedieck, and M. Grzegorczyk, "Partially non-homogeneous dynamic Bayesian networks based on Bayesian regression models with partitioned design matrices," *Bioinformatics*, vol. 35, no. 12, pp. 2108–2117, Jun. 2018.
[41] S. Haider and A. H. Levis, "Modeling time-varying uncertain situations using dynamic influence nets," *Int. J. Approx. Reasoning*, vol. 49, no. 2, pp. 488–502, Oct. 2008.
[42] A. Tucker and X. Liu, "A Bayesian network approach to explaining time series with changing structure," *Intell. Data Anal.*, vol. 8, no. 5, pp. 469–480, Oct. 2004.
[43] J. M. Peña, J. Björkegren, and J. Tegnér, "Learning dynamic Bayesian network models via cross-validation," *Pattern Recognit. Lett.*, vol. 26, no. 14, pp. 2295–2308, Oct. 2005.
[44] B. J. Ross and E. Zuviria, "Evolving dynamic Bayesian networks with multi-objective genetic algorithms," *Int. J. Speech Technol.*, vol. 26, no. 1, pp. 13–23, Jan. 2007.
[45] I. T. Luna, Y. Huang, Y. Yin, D. P. R. Padillo, and M. C. C. Perez, "Uncovering gene regulatory networks from time-series microarray data with variational Bayesian structural expectation maximization," *EURASIP J. Bioinf. Syst. Biol.*, vol. 2007, p. 71312, Jun. 2007.
[46] K. Wang, J. Zhang, F. Shen, and L. Shi, "Adaptive learning of dynamic Bayesian networks with changing structures by detecting geometric structures of time series," *Knowl. Inf. Syst.*, vol. 17, no. 1, pp. 121–133, Oct. 2008.
[47] Y. Lv and S. Liao, "Integration of multiple temporal qualitative probabilistic networks in time series environments," *Int. J. Intell. Eng. Syst.*, vol. 3, no. 2, pp. 9–17, Jun. 2010.
[48] C. P. D. Campos and Q. Ji, "Efficient structure learning of Bayesian networks using constraints," *J. Mach. Learn. Res.*, vol. 12, no. 3, pp. 663–689, 2011.
[49] K. Shibata, H. Nakano, and A. Miyauchi, "A learning method for dynamic Bayesian network structures using a multi-objective particle swarm optimizer," *Artif. Life Robot.*, vol. 16, no. 3, pp. 329–332, Dec. 2011.
[50] D. Bouchaffra, "Mapping dynamic Bayesian networks to α-shapes: Application to human faces identification across ages," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 23, no. 8, pp. 1229–1241, Aug. 2012.
[51] D. Codecasa and F. Stella, "Learning continuous time Bayesian network classifiers," *Int. J. Approx. Reasoning*, vol. 55, no. 8, pp. 1728–1746, Nov. 2014.
[52] A. Maiti, R. Reddy, and A. Mukherjee, "Structural prediction of dynamic Bayesian network with partial prior information," *IEEE Trans. Nanobiosci.*, vol. 14, no. 1, pp. 94–102, Oct. 2015.
[53] Y. Li, H. Chen, J. Zheng, and A. Ngom, "The max-min high-order dynamic Bayesian network for learning gene regulatory networks with time-delayed regulations," *IEEE/ACM Trans. Comput. Biol. Bioinf.*, vol. 13, no. 4, pp. 792–803, Jul. 2016.

[54] J. Vlasselaer, W. Meert, G. V. den Broeck, and L. De Raedt, "Exploiting local and repeated structure in dynamic Bayesian networks," *Artif. Intell.*, vol. 232, pp. 43–53, Mar. 2016.

[55] C. Komurlu, J. Shao, B. Akar, E. S. Bayrak, E. M. Brey, A. Cinar, and M. Bilgic, "Active inference for dynamic Bayesian networks with an application to tissue engineering," *Knowl. Inf. Syst.*, vol. 50, no. 3, pp. 917–943, Mar. 2017.

[56] S. V. Albrecht and S. Ramamoorthy, "Exploiting causality for selective belief filtering in dynamic Bayesian networks," *J. Artif. Intell. Res.*, vol. 55, pp. 1135–1178, Apr. 2016.

[57] P. Wu, M. J. Caley, G. A. Kendrick, K. Mcmahon, and K. Mengersen, "Dynamic Bayesian network inferencing for non-homogeneous complex systems," *J. Roy. Stat. Soc. C, Appl. Statist.*, vol. 67, no. 2, pp. 417–434, Feb. 2018.

[58] R. Chang, M. Stetter, and W. Brauer, "Quantitative inference by qualitative semantic knowledge mining with Bayesian model averaging," *IEEE Trans. Knowl. Data Eng.*, vol. 20, no. 12, pp. 1587–1600, Dec. 2008.

PEDRO SHIGUIHARA (Member, IEEE) received the master's degree from the University of São Paulo, in 2013. He is currently pursuing the Ph.D. degree with the National University of San Marcos. He has been a Coordinator of the Computer Science program with Peruvian University of Applied Sciences, Lima, Peru. He is also a Professor with the Pontifical Catholic University of Peru.

ALNEU DE ANDRADE LOPES received the degree in civil engineering from the Federal University of Mato Grosso do Sul, Brazil, in 1985, the master's degree in computer science and computational mathematics from the University of São Paulo, in 1995, and the Ph.D. degree in computer science from the University of Porto, Portugal, in 2001. Since 2002, he has been with the Faculty of Computer Science, University of São Paulo. His research interests include artificial intelligence, machine learning, data mining, and complex network mining.

DAVID MAURICIO received the bachelor's degree in computer science from the National University of San Marcos, and the Master of Science degree in applied mathematics and the Ph.D. degree in systems and computer engineering from the Federal University of Rio de Janeiro, Brazil. He was a Professor with North Fluminense State University, Brazil, from 1994 to 1998. Since 1998, he has been a Professor with the National University of San Marcos. His research interests include mathematical programming, artificial intelligence, software engineering, and entrepreneurship.