



Production of fully-heavy tetraquark states through the double parton scattering mechanism in pp and pA collisions

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Abstract The production of fully-heavy tetraquark states in proton–proton (pp) and proton–nucleus (pA) collisions at the center-of-mass energies of the Large Hadron Collider (LHC) and at the Future Circular Collider (FCC) is investigated considering that these states are produced through the double parton scattering mechanism. We estimate the cross sections for the T_{4c} , T_{4b} and T_{2b2c} states and present predictions for pp , pCa and pPb collisions considering the rapidity ranges covered by central and forward detectors. We demonstrate that the cross sections for pA collisions are enhanced in comparison to the pp predictions scaled by the atomic number. Moreover, our results indicate that a search of these exotic states is, in principle, feasible in the future runs of the LHC and FCC.

Over the last years, the LHCb [1], ATLAS [2] and CMS [3] Collaborations have observed a sharp peak in the di- J/ψ channel consistent with a narrow resonance at $M = 6.9$ GeV, which is a viable candidate for a fully-charm tetraquark state (for reviews see, e.g. Refs. [4–6]). Such data have motivated a series of studies that propose the existence of a large number of new exotic states, composed only by charm and/or bottom quarks, denoted fully-heavy tetraquark states T_{4Q} (See e.g. Refs. [7–26]). Although the mass spectra and decay properties of these states are reasonably well understood, the production mechanism of fully-heavy tetraquark states is still a theme of intense debate (See e.g. Refs. [27–40]). In particular, Ref. [29] has proposed that a fully-charm tetraquark state can be generated from the hadronization of $c\bar{c}$ pairs, which

are largely produced in double parton scatterings present in pp collisions at LHC energies [41–43]. Such an idea was elaborated in more detail in Ref. [33], which confirmed that this mechanism is one of the more promising ways to probe the T_{4c} state. One of the goals of this letter is the update of Ref. [29] in two aspects: (a) by assuming that the T_{4c} state has a mass equal to 6.9 GeV instead of 5.4 GeV used in [29]; and (b) by considering a more recent parametrization for the gluon distribution function. Another goal is to extend the model for the fully-bottom tetraquark state, T_{4b} , and for the $c\bar{c}b\bar{b}$ state, denoted T_{2b2c} hereafter, and present predictions for the associated cross sections derived considering pp collisions at the LHC and FCC energies and assuming the typical rapidity ranges covered by central ($-2.5 \leq Y \leq +2.5$) and forward ($+2.0 \leq Y \leq +4.5$) detectors. Finally, our third and main goal, is to present for the first time the predictions for the T_{4Q} production in proton–nucleus (pA) collisions via the double scattering mechanism. Our analysis is strongly motivated by the recent LHCb results [44], which observed the enhancement of the double scattering mechanism in pA collisions predicted in Refs. [45–49]. Assuming two distinct nuclei, we will estimate the energy dependence of the total cross sections and present predictions for future pA collisions at the LHC and FCC. As we will demonstrate in what follows, our results indicate that if the fully-heavy tetraquark states are produced via the double parton scattering mechanism, the search of these states in proton–nucleus collisions is highly recommended.

Initially, let us present a brief review of the model proposed in Ref. [29] for the T_{4Q} production in pp collisions, which is represented in Fig. 1. At the LHC energy, the high density of partons in the hadron wave function implies the increasing

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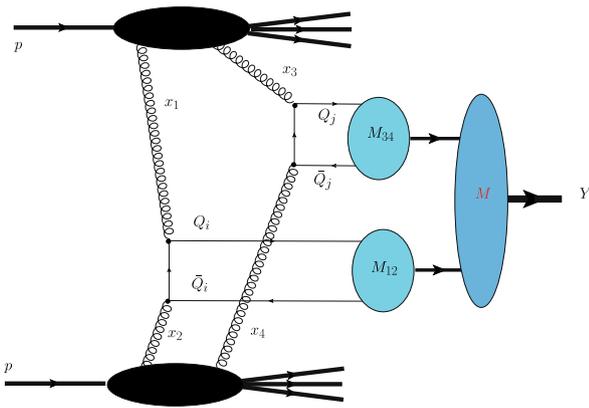


Fig. 1 Production of fully-heavy tetraquark states via the double parton scattering mechanism in pp collisions

of the probability that two or more hard partonic scatterings in a single hadron–hadron collision can take place. Such theoretical expectation has been confirmed by distinct experimental collaborations at the LHC, considering different final states (see e.g. Ref. [50–54]). In particular, the LHCb collaboration has observed a large number of events with four charm quarks ($c\bar{c}c\bar{c}$) in the same event [55], which indicated a large contribution of double parton scatterings (DPS). In Ref. [29], the authors have proposed that a final state composed by two heavy quark pairs, $Q_i\bar{Q}_iQ_j\bar{Q}_j$ can be generated by two independent gluon–gluon scatterings,¹ i.e. two times the reaction $gg \rightarrow Q_i\bar{Q}_i$. These two pairs have invariant masses M_{12} and M_{34} , and are assumed to form a system with mass $M = M_{12} + M_{34}$ when the rapidities of the two $Q\bar{Q}$ pairs are equal, i.e. $y_{12} = y_{34} = Y$. The formation of the color neutral T_{4Q} state is described by the Color Evaporation Model (CEM) [56,57], which assumes that the color

¹ The contribution of the $q\bar{q} \rightarrow Q_i\bar{Q}_i$, where q is a light quark, is subleading at the LHC energies.

neutralization occurs through the emission of soft gluons and that the bound state is formed when the invariant mass of the $Q_i\bar{Q}_iQ_j\bar{Q}_j$ system, M , is of the order of $M_{T_{4Q}}$. The probability for the $Q_i\bar{Q}_iQ_j\bar{Q}_j \rightarrow T_{4Q}$ transition is described in the CEM by a nonperturbative parameter $F_{\mathcal{T}}$. As detailed in Ref. [29], these assumptions imply that the production cross section can be expressed as follows

$$\begin{aligned} \sigma_{T_{4Q}}(\sqrt{s}) &= \frac{F_{\mathcal{T}}}{\sigma_{eff,pp}} \left[\frac{1}{s} \int dy_{12} \int dM_{12}^2 g(\bar{x}_1, \mu^2) \right. \\ &\quad \times g(\bar{x}_2, \mu^2) \sigma_{g_1g_2 \rightarrow Q_i\bar{Q}_i} \left. \right] \\ &\quad \times \left[\frac{1}{s} \int dy_{34} \int dM_{34}^2 g(\bar{x}_3, \mu^2) g(\bar{x}_4, \mu^2) \sigma_{g_3g_4 \rightarrow Q_j\bar{Q}_j} \right] \\ &\quad \times \Theta(1 - \bar{x}_1 - \bar{x}_3) \Theta(1 - \bar{x}_2 - \bar{x}_4) \\ &\quad \times \Theta(M_{12}^2 - 4m_{Q_i}^2) \Theta(M_{34}^2 - 4m_{Q_j}^2) \\ &\quad \times \delta(y_{34} - y_{12}) \end{aligned} \tag{1}$$

where $\sigma_{eff,pp} = 15$ mb is a constant extracted from data analysis, $g(x, \mu^2)$ is the gluon distribution in the proton with a fractional momentum x at the factorization scale μ^2 and $\sigma_{gg \rightarrow Q_i\bar{Q}_i}$ is the elementary cross section for the $gg \rightarrow Q_i\bar{Q}_i$ subprocess. The step functions $\Theta(1 - x_1 - x_3)$ and $\Theta(1 - x_2 - x_4)$ enforce momentum conservation in the projectile and in the target, while the step functions $\Theta(M_{12}^2 - 4m_{Q_i}^2)$ and $\Theta(M_{34}^2 - 4m_{Q_j}^2)$ guarantee that the invariant masses of the gluon pairs 12 and 34 are large enough to produce two heavy quark pairs. Moreover, the delta function implements the condition that the two heavy quark pairs are in the same rapidity. The Bjorken variables of the four gluons in the initial state are given by

$$\bar{x}_1 = \frac{M_{12}}{\sqrt{s}} e^{y_{12}}, \bar{x}_2 = \frac{M_{12}}{\sqrt{s}} e^{-y_{12}},$$

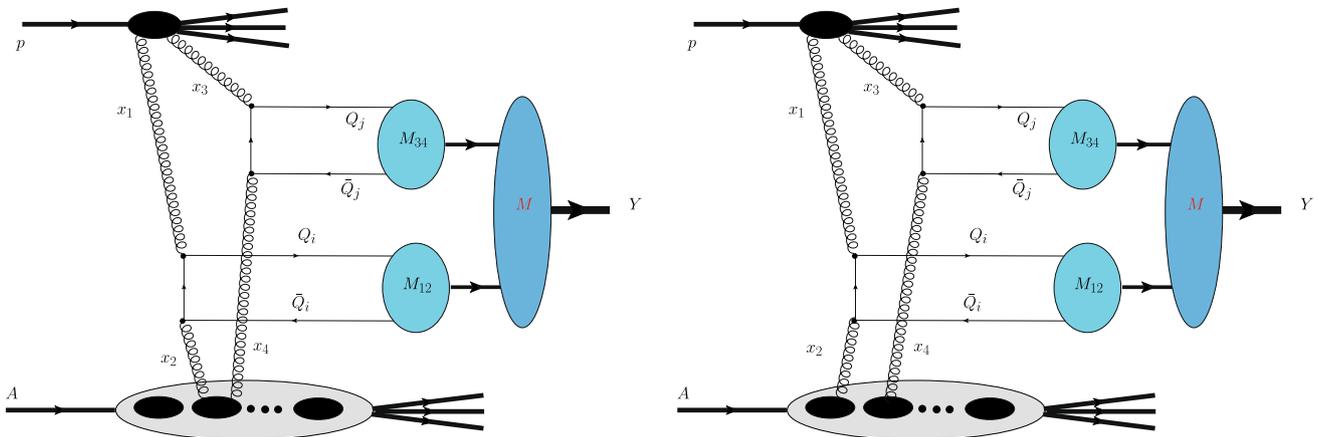


Fig. 2 Representation of the two diagrams that contribute for the production of fully-heavy tetraquark states via the double parton scattering mechanism in pA collisions

$$\bar{x}_3 = \frac{M_{34}}{\sqrt{s}} e^{y_{34}}, \bar{x}_4 = \frac{M_{34}}{\sqrt{s}} e^{-y_{34}}. \tag{2}$$

In our calculations we will assume $m_c = 1.5$ GeV, $m_b = 4.5$ GeV and that the hard scale μ is equal to the invariant mass of the $Q\bar{Q}$ system. Moreover, we will consider the CT14 parametrization [58] for the gluon distribution function. The last ingredient needed to estimate the T_{4Q} production in pp collisions is the value of $F_{\mathcal{T}}$ for the T_{4c} , T_{4b} and T_{2b2c} states. As in Ref. [29], its value for the T_{4c} production will be estimated in terms of the cross section for the $X(3872)$ production measured by the CMS collaboration [59] in pp collisions at $\sqrt{s} = 7.0$ TeV. The value of $F_{\mathcal{T}}$ is determined by imposing that $\sigma_{T_{4c}} = 0.12\sigma_X$, which implies $F_{\mathcal{T}} = 0.00119$. In what follows, we will assume this same value in the calculation of the T_{4b} and T_{2b2c} production cross sections. Such assumption is motivated by the results obtained e.g. in Refs. [60,61], which have indicated that the value of this nonperturbative factor is similar for different quarkonium states.

In order to generalize the model for proton–nucleus collisions, we should take into account that the parton flux associated to the nucleus is enhanced by a factor $\propto A$ and that the two gluons associated to the proton can interact with two gluons coming from the same nucleon from the nucleus or with two gluons coming from different nucleons from the nucleus. Both possibilities are represented in the left and right panels of the Fig. 2. In what follows, we will denote the cross sections associated to these two contributions by $\sigma_{pA}^{DPS,1}$ and $\sigma_{pA}^{DPS,2}$, respectively. Following Refs. [45,46], one has that $\sigma_{pA}^{DPS,1} = A \cdot \sigma_{pp}^{DPS}$. Moreover, cross section associated to the second contribution will be given by $\sigma_{pA}^{DPS,2} = \sigma_{pp}^{DPS} \cdot \sigma_{eff,pp} \cdot F_{pA}$, with $F_{pA} = [(A - 1)/A] \int T_{pA}^2(\mathbf{r})d^2r$, where \mathbf{r} is the impact parameter between the colliding proton and nucleus and T_{pA} is the nuclear thickness function. Assuming that the nucleus has a spherical form (with uniform nucleon density) of radius $R_A = r_0A^{1/3}$, and $r_0 = 1.25$ fm, the integral of the nuclear thickness factor becomes $F_{pA} = 9A(A - 1)/(8\pi R_A^2)$. As a consequence, the formalism proposed in Ref. [45] implies that the cross section for the production of fully-heavy tetraquark states via the double parton scattering mechanism in pA collisions can be expressed as follows

$$\begin{aligned} \sigma_{pA \rightarrow T_{4Q}}^{DPS} &= \sigma_{pA \rightarrow T_{4Q}}^{DPS,1} + \sigma_{pA \rightarrow T_{4Q}}^{DPS,2} \\ &= A\sigma_{pp \rightarrow T_{4Q}}^{DPS} \left[1 + \frac{1}{A}\sigma_{eff,pp}F_{pA} \right], \end{aligned} \tag{3}$$

where $\sigma_{pp \rightarrow T_{4Q}}^{DPS}$ is given by Eq. (1). In our calculations, we will consider two distinct nuclei ($A = 40$ and 208).

In Fig. 3 we present our predictions for the energy dependence of the cross sections for the production of fully-heavy tetraquark states via the double parton scattering mechanism

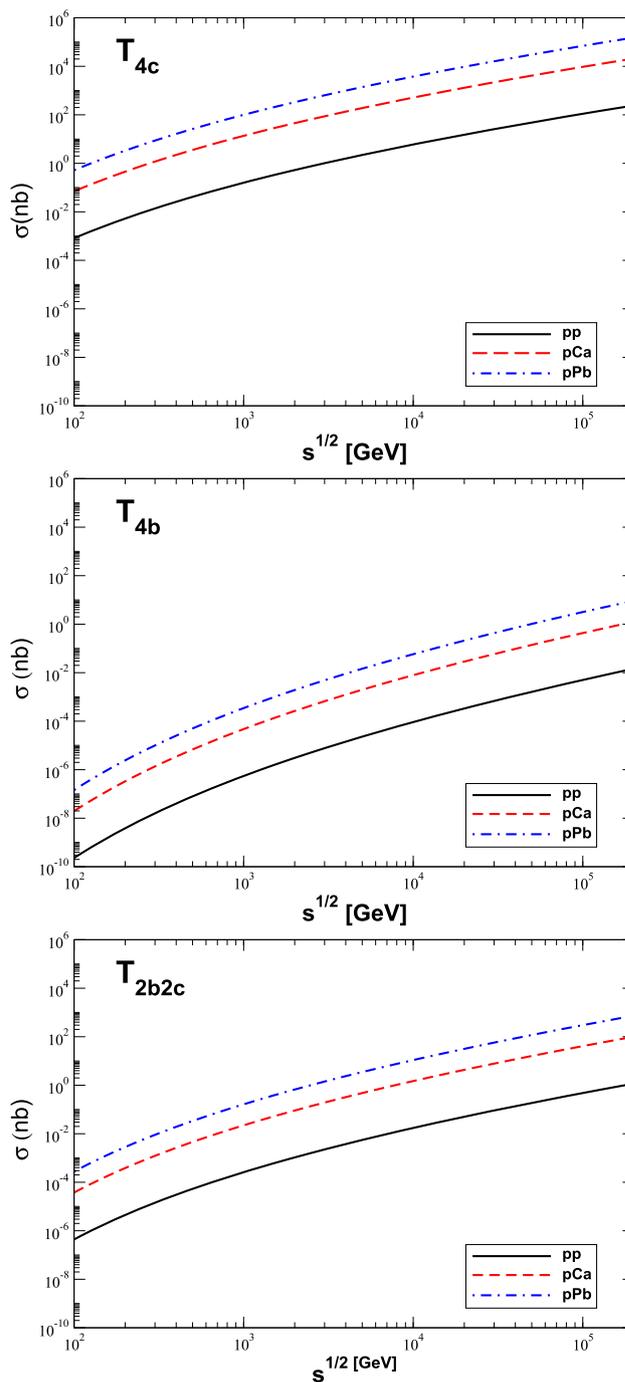


Fig. 3 Energy dependence of the cross sections for the production of fully-heavy tetraquark states via the double parton scattering mechanism in pp , pCa and pPb collisions

in pp , pCa and pPb collisions. A strong increasing with the energy is predicted, which is directly associated with the fact that in our model for the T_{4Q} production, the cross section is proportional to $g(x, \mu^2)^4$. In contrast, in models based on the single parton scattering mechanism, one has $\sigma_{T_{4Q}}^{SPS} \propto g(x, \mu^2)^2$. Therefore, we can expect that for large

Table 1 Cross sections in nb for the T_{4Q} production in pp and pA collisions at the LHC calculated considering the rapidity ranges covered by a typical central ($-2.5 \leq Y \leq +2.5$) and forward ($+2.0 \leq Y \leq +4.5$) detectors

	pp ($\sqrt{s} = 14.0$ TeV)		pCa ($\sqrt{s} = 8.1$ TeV)		pPb ($\sqrt{s} = 8.1$ TeV)	
	Central	Forward	Central	Forward	Central	Forward
T_{4c}	6.05	1.86	520.22	159.74	3816.44	1171.86
T_{4b}	0.00033	0.000078	0.028	0.0067	0.21	0.049
T_{2b2c}	0.021	0.0055	1.80	0.47	13.23	3.46

Table 2 Cross sections in nb for the T_{4Q} production in pp and pA collisions at the FCC calculated considering the rapidity ranges covered by a typical central ($-2.5 \leq Y \leq +2.5$) and forward ($+2.0 \leq Y \leq +4.5$) detectors

	pp ($\sqrt{s} = 100.0$ TeV)		pCa ($\sqrt{s} = 63.0$ TeV)		pPb ($\sqrt{s} = 63.0$ TeV)	
	Central	Forward	Central	Forward	Central	Forward
T_{4c}	57.27	21.74	4918.35	1867.03	36081.93	13696.89
T_{4b}	0.0081	0.0026	0.70	0.22	5.11	1.63
T_{2b2c}	0.27	0.095	23.18	8.16	170.11	59.85

energies one will have $\sigma_{T_{4Q}}^{DPS} > \sigma_{T_{4Q}}^{SPS}$. The results derived in [41, 43] for the four charm production indicate that DPS charm production is already comparable to SPS production at LHC energies. As a consequence, our predictions for the T_{4Q} production can be considered a lower bound for the number of events in hadronic collisions at the LHC and FCC.

For the T_{4c} production (upper panel), one has that our predictions for pp collisions are similar to those derived in Ref. [29], which is directly associated to the fact that in both studies the normalization was fixed using the cross section for the $X(3872)$ production. On the other hand, our prediction for pCa collisions is a factor ≈ 85 larger than for the pp case, i.e. it is not a simple A scaling of the pp prediction. Such result is expected due to contribution of $\sigma_{pA}^{DPS,2}$ (right panel in Fig. 2). Similarly, the predictions for pPb collisions are enhanced by a factor ≈ 630 , i.e. $\approx 3A$, in agreement with the results derived in Ref. [48], where the double heavy quark pair production in pA collisions has been estimated for the first time.

Our predictions for the T_{4b} and T_{2b2c} production are presented in the middle and lower panels of Fig. 2, respectively. One has that the energy behaviour is similar to that predicted for the T_{4c} case and that the pCa and pPb cross sections are enhanced by a similar factor in comparison to the pp predictions. The main difference is in the magnitude of the cross sections. In Tables 1 and 2 we present our predictions for the T_{4Q} production cross sections at the LHC and FCC energies, respectively, considering the typical rapidity ranges covered by a central ($-2.5 \leq Y \leq +2.5$) and forward ($+2.0 \leq Y \leq +4.5$) detectors. We predict cross sections of the order of few nb for the T_{4c} production in pp collisions at the LHC. In contrast, for the T_{4b} (T_{2b2c}) case, the values predicted are smaller by four (two) orders of magnitude. At the FCC, we predict an increasing in the cross

sections of one order of magnitude, which is directly associated to the fact that $\sigma_{T_{4Q}}^{DPS} \propto xg^4$. For pA collisions, one has the enhancement discussed in the previous paragraph. These results indicate that the cross sections for the T_{4Q} production in pp and pA collisions are large and that a future experimental analysis is, in principle, feasible.

A comment is in order before summarize our main results. In this letter, we have focused on the production of two $Q\bar{Q}$ pairs via the DPS mechanism, which occurs in a given hadron–hadron collision. However, extra hadronic interactions per bunch crossing, usually called pile-up, can occur in high luminosity collisions at the LHC. As a consequence, it is natural to ask what is the contribution of these overlapping events for the production of two $Q\bar{Q}$ pairs, with each $Q\bar{Q}$ being created in a different hadron–hadron collision. For pA runs, as the pile-up is very small, the contribution of the overlapping events is negligible. On the other hand, in pp runs, especially for the future high luminosity LHC (HL-LHC) run, the pile-up is expected to be very high, with an average amount of pile-up interaction per bunch crossing of the order of 200. In order to cope with these conditions, new detectors and triggers with high granularity and precision timing have been developed and installed, dedicated to the separation of the tracks coming from different primary vertex, associated to pile-up events. The current studies [62–64] indicate that these new detectors will allow reducing the contribution of overlapping events in the HL-LHC run to values similar to those present at current luminosities. Therefore, in principle, we do not expect a large contribution of overlapping events for the T_{4Q} production in pp collisions. Surely, this topic deserves a more detailed analysis, which we postpone for a future study.

As a summary, in this letter we have investigated the production of fully-heavy tetraquark states through the double

parton scattering (DPS) mechanism in pp and pA collisions at the LHC and FCC. Such contribution is expected to be, at least, of the same order of the contribution associated to the T_{4Q} production via the single parton scattering, which implies that our predictions can be considered a lower bound for the magnitude of the total cross sections. In our analysis, one has updated the predictions for the T_{4c} production in pp collisions presented in Ref. [29] and extended the model for the production of the T_{4b} and T_{2b2c} states. Our results indicated that the cross sections for these states are, respectively, four and two orders of magnitude smaller than the T_{4c} predictions. However, considering the large luminosity expected in the future runs of the LHC, one predict a large number of events, which makes possible to search for these states in the forthcoming years. In addition, in this letter one has presented, for the first time, the predictions for the T_{4Q} production in pCa and pPb collisions. We have demonstrated that the DPS cross sections are enhanced by a factor larger than the expected A scaling predicted by a model based on the simple superposition of proton–nucleon collisions. In particular, we predict that the pCa (pPb) cross section will be enhanced by a factor $\approx 2A$ ($3A$) in comparison to the pp result for the same center-of-mass energy. Such result indicates that a future experimental analysis of the T_{4Q} production in pA collisions can be useful to probe the existence of these states, as well to improve our understanding of the double parton scattering mechanism.

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Code Availability Statement My manuscript has no associated code/software. [Author’s comment: Code sharing not applicable to this article, as no code was generated during the current study.]

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