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A BURN-IN MODEL

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ABSTRACT. Many manufactured items, for example, electronic components, tend either to last a relatively long time or to fail very early. A technique used to screen out the items with short lifelengths before they are delivered to the customer is the so-called burn-in.Considering n components and given some cost structure such as costs for failure during and after the test and gains for unit time for released items, one problem relate to burn-in is to determine the optimal burn-in length time τ to minimizing the rate between the total cost and the total gain per unit time.

Keywords: Martingale methods in reliability theory, burn in, stopping problem.

INTRODUCTION

Burn-in is a widely used engineering method to eliminate defective components before they are put into operation. Electronic components tend either to last a relatively long time or to fail very early. Before released, the components are tested under typical, or even severe use conditions for a burn-in (random) length time τ. The component which survives such a burn-in test is put in operation.

Considering n components and given some cost structure such as costs for failure during and after the test and gains for unit time for released items, one problem relate to burn-in is to determine the optimal burn-in length time τ following some criteria. A basic criteria to find out an optimal burn-in time τ is to minimizing the rate between the total cost and the total gain per unit time. Based on the random information given by the life lengths of the items failing during the test we use a semimartingale approach to solving this optimal stopping problem.

1. MATHEMATICAL FORMULATION

We let $T_j, j = 1, ..., n$, be independent and finite random variables defined in a complete probability space $(\Omega, \Im, P)$ representing the life lengths of the components which are burned-in under severe conditions in the way that the failure rate during and after the burn-in procedure, denoted by $\lambda_j^0(t)$ and $\lambda_j^1(t)$, respectively, are such that $\lambda_j^0(t) \geq \lambda_j^1(t)$ for all $t \geq 0$.

The mathematical formulation of our observations is given by a family of sub σ-algebras of $\Im$, where $\mathbf{F} = (\Im_t)_{t\geq 0}$ and

$$\Im_t = \sigma\{I_{\{T_j > s\}}, 0 \leq s \leq t, j = 1, ..., n\}.$$

We assume that the life length T_j, of the j-th component admits the $\Im_t$-semimartingale representation :

Key words and phrases. Martingale methods in reliability theory, burn-in, stopping problem.

$$I_{\{T_j\leq t\}} = \int_0^t I_{\{T_j>s\}}\lambda_j^{Y_s}(s)ds + M_j(t), j = 1, ..., n, \tag{1.0.1}$$

where $Y_t = I_{\{\tau<t\}}, \tau$ is the burn-in $\Im_t$-stopping time and M_j is a bounded square integrable $\Im_t$-martingale.

In order to determine the optimal burn-in time we introduce the following cost and reward structure. There is a reward of $C > 0$ per unit operating time of released component and in addition a bonus $k_0 > 0$.There are costs for failures, $C_B > 0$ for a failure during burn-inn and $C_F > 0, C_F > C_B$, for a failure after the burn-in time τ.

We fix the burn-in time for a moment to $\tau = t$ and define the the total cost process

$$Z_t = C_B\sum_{j=1}^{n} I_{\{T_j\leq t\}} + C_F\sum_{j=1}^{n} I_{\{T_j>t\}} = (C_B - C_F)\sum_{j=1}^{n} I_{\{T_j\leq t\}} + nC_F,$$

with a smooth semimartingale (SSM) representation:

$$Z_t = nC_F + (C_B - C_F)\int_0^t \sum_{j=1}^{n} I_{\{T_j>s\}}\lambda_j^0(s)ds + \sum_{j=1}^{n} M_j(t),$$

and the reward process

$$\hat{X}_t = C\sum_{j=1}^{n}(T_j - t)^+ + k_0,$$

where $k_0 > 0$.

We note that the total cost process Z_t is $\Im_t$-adapted, however we cannot observe the reward process in $\Im_t$ and therefore we consider the process

$$X_t = E[\hat{X}_t|\Im_t] = C\sum_{j=1}^{n} I_{\{T_j>t\}}\mu_j(t) + k_0.$$

with $\mu_j(t) = E[(T_j - t)^+|T_j > t] = \dfrac{1}{\overline{F}_j(t)}\displaystyle\int_t^\infty \overline{F}_j(x)dx, j = 1, ..., n.$

Now $(X_t)_{t\geq0}$ and $(Z_t)_{t\geq0}$ are $\Im_t$-adapted , real and right continuous processes where $E[|X_T|] < \infty$, $E[Z_T] > -\infty$ with $T = \max\{T_1, ..., T_n\} > 0$ is a finite $\Im_t$-stopping time. Let:

$$C_T^{\mathbb{F}} = \{\tau : \tau \text{ is an } \Im_t - \text{stopping time}, \tau \leq T, E[Z_\tau] > -\infty \text{ and } E[|X_\tau|] < \infty\}.$$

For $\tau \in C_T^{\mathbb{F}}$ we consider the ratio $K_\tau = \dfrac{E[Z_\tau]}{E[X_\tau]}$. To solve the optimal stopping problem we have to find a stopping time $\sigma \in C_T^{\mathbb{F}}$, with:

$$K^* = K_\sigma = inf\{K_\tau : \tau \in C_T^{\mathbb{F}}\} \tag{1.0.2}$$

Remark 1.0.1. The representation (1.0.1) (see Aven and Jensen, 1999) can also be obtained by modelling the lifelength of the j-th item in the following way:

$$T_j = L_j \wedge \tau + R_j I_{\{L_j>\tau\}},$$

Where $L_j, R_j, j = 1, ..., n$, are independent random variable and $a \wedge b$ denotes the minimum of a and b; L_j is the lifelength of the item when it exposed to a higher stress level and R_j is the operating time of the item if it survived the burn-in phase τ. Let F_j be the lifelength distribution, H_j denote the distribution function of $L_j, j = 1, ..., n$, and let $H_j(0) = F_j(0) = 0, \overline{H}_j(t) = 1 - H_j(t), \overline{F}_j(t) = 1 - F_j(t)$. Furthermore, next section, we assume that H_j and F_j admit densities h_j and f_j, respectively. It is assumed that the operating time R_j follows the conditional survival distribution corresponding to F_j:

$$P(T_j \leq t+s|\tau = t < L_j) = P(R_j \leq s|\tau = t < L_j) = \frac{F_j(t+s) - F_j(t)}{\overline{F}_j(t)}, t, s \in \mathbb{R}_+.$$

To solve the problem above, we are going to use a condition called the monotone case, and the infinitesimal-look-ahead stopping rule, defined below:

Definition 1.0.2. (MON) Let Z a smooth semimartingale (SSM), with the following representation:

$$Z_t = Z_0 + \int_0^t f_s ds + M_t$$

Then the following condition

$$\{f_t \leq 0\} \subseteq \{f_{t+h} \leq 0\}, \forall t, h \in \mathbb{R}_+, \bigcup_{t \in \mathbb{R}_+} \{f_t \leq 0\} = \Omega$$

is said to be the monotone case and the stopping time

$$\sigma = inf\{t \in \mathbb{R}_+ : f_t \leq 0\}\}$$

is called the ILA-stopping rule (infinitesimal-look-ahead).

Besides, the ILA-stopping rule σ is a natural candidate to solve the maximization problem, since condition (MON) is satisfying, Jensen (1989) proved the following theorem:

Theorem 1.0.3. *Let Z be a smooth semimartingale (SSM)and σ the ILA-stopping rule. If the martingale M is uniformly integrable (u.i.), then in the monotone case (MON)*

$$EZ_\sigma = sup\{EZ_\tau : \tau \text{ stopping time}\}.$$

2. A PROBLEM SOLUTION

In order to determine the optimal burn-in time, we are looking for an $\Im$-stopping time $\sigma \in C_T^{\mathbb{F}}$ satisfying

$$K^* = K_\sigma = inf\{K_\tau : \tau \in C_T^{\mathbb{F}}\},$$

where $K_\tau = \dfrac{EZ_\tau}{EX_\tau}$.

In other words, at any time t the observer has to decide whether to stop or to continue with burn-in with respect to the available information up to time t.

Assuming that the mean residual $\mu_j(t) = \dfrac{1}{\overline{F}_j(t)} \displaystyle\int_t^\infty \overline{F}_j(x)dx, j = 1, ..., n$, has derivative with respect to $t, t \geq 0$, given by $\mu_j'(t) = -1 + \lambda_j^1(t)\mu_j(t)$, Aven and Jensen (1999) proved the next theorem

Theorem 2.0.4. *The process $X_t = C\sum_{j=1}^n I_{\{T_j>t\}}\mu_j(t) + k_0$ defined as before has a smooth semimartingale (SSM) representation:*

$$X_t = X_0 + C\int_0^t \sum_{j=1}^n I_{\{T_j>s\}}[-1-\mu_j(s)(\lambda_j^0(s)-\lambda_j^1(s))]ds + N_t,$$

where N_t is a martingale which is bounded in L^2, and $X_0 = C\sum_{j=1}^n \mu_j(0) + k_0$.

Proof. In order to obtain a semimartingale representation for X_t, we derive such a representation for $I_{\{T_j>t\}}\mu_j(t)$. Since $\mu_j(.)$ and $I_{\{T_j>.\}}$ are right-continuous and of bounded variation on $[0,t]$, we can use the integration by parts formula for Stieltjes integrals (pathwise) to obtain

$$\begin{aligned} I_{\{T_j>t\}}\mu_j(t) &= I_{\{T_j>0\}}\mu_j(0) + \int_0^t \mu_j(s^-)dI_{\{T_j>s\}} \\ &+ \int_0^t I_{\{T_j>s\}}d\mu_j(s). \end{aligned}$$

Substituting

$$I_{\{T_j>s\}} = 1 + \int_0^s (-I_{\{T_j>x\}}\lambda_j^0(x))dx + M_j(s)$$

in this formula and using the continuity of μ we obtain

$$\begin{aligned} I_{\{T_j>t\}}\mu_j(t) &= \mu_j(0) + \int_0^t [-\mu_j(s)I_{\{T_j>s\}}\lambda_j^0(s) + I_{\{T_j>s\}}\mu_j'(s)]ds \\ &+ \int_0^t \mu_j(s)dM_j(s) \\ &= \mu_j(0) + \int_0^t I_{\{T_j>s\}}[-1-\mu_j(s)(\lambda_j^0(s)-\lambda_j^1(s))]ds \\ &+ \widetilde{M}_j(t), \end{aligned}$$

where $\widetilde{M}_j$ is a martingale, which is bounded in L^2. This yields the following semimartingale representation for X:

$$X_t = C\sum_{j=1}^n \mu_j(0) + k_0 + C\int_0^t \sum_{j=1}^n I_{\{T_j>s\}}[-1-\mu_j(s)(\lambda_j^0(s)-\lambda_j^1(s))]ds + N_t$$

with a martingale bounded in L^2 (therefore uniformly integrable)

$$N = C\sum_{j=1}^n \widetilde{M}_j,$$

with $N_0 = 0$. □

Considering that all relations between real random variable hold (only) P-almost surely (P-a.s), the first step to solve the optimization problem is to establish bounds for K^* in (1.0.2), and this is given by proposition below:

Proposition 2.0.5. *Let*

$$q = inf\{\frac{(C_F-C_B)\sum_{j=1}^n I_{\{T_j(\omega)>t\}}\lambda_j^0(t)}{C\sum_{j=1}^n I_{\{T_j(\omega)>t\}}[1+\mu_j(t)(\lambda_j^0(t)-\lambda_j^1(t))]} : 0 \le t < T(\omega), \omega \in \Omega\}.$$

Then $b_l \leq K^* \leq b_u$, where

$$b_u = \frac{EZ_0}{EX_0} \quad \textit{and}$$

$$b_l = \begin{cases} \frac{EZ_0 - qEX_0}{EX_T} + q & \textit{if } E[Z_0 - qX_0] < 0 \\ \frac{EZ_0}{EX_0} & \textit{if } E[Z_0 - qX_0] \geq 0. \end{cases}$$

Proof. Because $0 \in C_T^{\mathbf{F}}$ only the lower bounder has to be shown. Since the M, N are uniformly integrable, the optimal sampling theorem yields $EM_\tau = EN_\tau = 0$ for all $\tau \in C_T^{\mathbf{F}}$, and with $q \geq 0$, we have

$$K_\tau \geq \frac{EZ_0 + qE[X_\tau - X_0]}{EX_\tau} = \frac{EZ_0 - qEX_0}{EX_\tau} + q.$$

The lower bound is derived observing that $EX_T \leq EX_\tau \leq EX_0$, which completes the proof. □

As an immediate consequence we obtain the following corollary:

Corollary 2.0.6. *If* $0 \leq q \leq \dfrac{nC_F}{\sum_{j=1}^n \mu_j(0) + k_0}$ *then*

$$K^* = \frac{nC_F}{\sum_{j=1}^n \mu_j(0) + k_0}$$

and $\sigma = 0$ is the optimal burn-in time.

It is a well- known technique to replace the minimization problem (1.0.2) by an equivalent maximization problem. Note that $K_\tau = \dfrac{EZ_\tau}{EX_\tau} \geq K^*$ is equivalent to $K^*EX_\tau - EZ_\tau \leq 0$ for all $\tau \in C_T^{\mathbf{F}}$, where equality holds for an optimal stopping time, one has the maximization problem:

(2.0.3) Find $\sigma \in C_T^{\mathbf{F}}$ with $EH_\sigma = sup\{EH_\tau : \tau \in C_T^{\mathbf{F}}\} = 0$, where
$H_t = K^*X_t - Z_t$ and $K^* = inf\{K_\tau : \tau \in C_T^{\mathbf{F}}\}$.

Therefore to solve the stopping problem (1.0.2) for ratio of expectations, we use solution of the simpler case in which we look for the maximum of the expectations EH_τ, where H is an SSM and $\tau \in C_T^{\mathbf{F}}$ (see Jensen (1989)). This new stopping problem can be solved by using the semimartingale representation of the process H_t for $t \in [t, T)$:

$$\begin{aligned} H_t &= K^*X_0 - Z_0 \\ &+ \int_0^t \sum_{j=1}^n I_{\{T_j > s\}}[K^*C(-1 - \mu_j(s)(\lambda_j^0(s) - \lambda_j^1(s)) + (C_F - C_B)\lambda_j^0(s)]ds \\ &+ R_t \end{aligned}$$

where the martingale $R = (R_t)_{t \in \mathbf{R}_+}$ is given by $R_t = K^*N_t - M_t$.

Without any conditions on the structure of the process H one cannot hope to find an solution of the stopping problem. The condition (MON) defined above is suggesting. Now the procedure as follows: If the integrand

$$h_s = \sum_{j=1}^n I_{\{T_j > s\}}[K^*C(-1 - \mu_j(s)(\lambda_j^0(s) - \lambda_j^1(s)) + (C_F - C_B)\lambda_j^0(s)]$$

fulfills the monotone case (MON) them theorem (1.0.3) yields that the ILA-stopping rule $\sigma = inf\{t \in \mathbb{R}_+ : h_t \leq 0\} \wedge T$ is optimal, since R_t is u.i.. Note, however, that this stopping time σ depends on the unknown value K^*, which can be determined from the equality $EH_\sigma = 0$. The main idea for solving the stopping problem is that:

Define

$$H_t^x = xX_t - Z_t, x \in [b_l, b_u], t \geq 0;$$

where X_t, Z_t, b_l and b_u are before.

Theorem 2.0.7. *Suppose that functions*

$$g_j(x,t) = -Cx - Cx\mu_j(t)(\lambda_j^0(t) - \lambda_j^1(t)) + (C_F - C_B)\lambda_j^0(t), \forall x \in [bl, b_u],$$

satisfy the following condition

$$\sum_{j\in\gamma} g_j(x,t) \leq 0 \Rightarrow g_j(x,s) \leq 0, \forall j \in \gamma \subseteq \{1,...,n\}, \forall s \geq t. \tag{2.0.4}$$

Then

(i)

$$\sigma_x = inf\{t \in \mathbb{R}_+ : \sum_{j=1}^n I_{\{T_j>t\}} g_j(x,t) \leq 0\} \wedge T$$

is an optimal burn-in time

$$EH_{\sigma_x}^x = sup\{EH_\tau^x : \tau \in C_T^{\mathbb{F}}\}, x \in [b_l, b_u];$$

(ii)

$\sigma = \sigma_{x^*}$, with $x^* = inf\{x \in [b_l, b_u] : xEX_{\sigma_x} - EZ_{\sigma_x} \geq 0\}$ is an optimal burn-in and $x^* = K^*$.

Proof. Since for all $\omega \in \Omega$ and all $t \in \mathbb{R}_+$, there exists some $\gamma \subseteq \{1,...,n\}$ such that:

$$\sum_{j=1}^n I_{\{T_j>t\}} g_j(x,t) = \sum_{j\in\gamma} g_j(x,t), \forall x \in [b_l, b_u],$$

the condition (2.0.4) in theorem ensures that the monotone case (MON) holds true, and therefore we get the desired result by theorem (1.0.3) and the proof (i) is complete.

In particular, for $x = K^*$ we have:

$$\sigma_{K^*} = inf\{t \in \mathbb{R}_+ : \sum_{j=1}^n I_{\{T_j>t\}} g_j(K^*,t) \leq 0\} \wedge T,$$

is optimal, with $EH_{\sigma_{K^*}} = sup\{EH_\tau : \tau \in C_T^{\mathbb{F}}\} = 0$. Besides, it is easy to see that $x^* = K^*$.

□

Firstly some observations about the theorem above:

Remark 2.0.8.

(i)**Burn-in forever.**

If $g(x,t) > 0$ for $(x,t) \in [b_l, b_u] \times \mathbb{R}_+, j = 1,...,n$, then $\sigma = T = max\{T_1,...,T_n\}$, i.e., burn-in until all items failed.

(ii) **No burn-in.**

If $g(x,0) \leq 0$ for $(x,0) \in [b_l, b_u] \times \mathbb{R}_+, j = 1, ..., n$, then $\sigma = 0$.
(iii) **Identical items.**
If failure rates coincide, i.e., $\lambda_1^0(t) = ... = \lambda_n^0(t)$ and $\lambda_1^1(t) = ... = \lambda_n^1(t)$ for all $t \geq 0$, then $g_j(x,t) = g_1(x,t)$ for all $j = 1, ..., n$ and condition (2.0.4) reduces to

$$g_1(x,s) \leq 0 \text{ for } \ s \geq t_1 = inf\{t \in \mathbb{R}_+ : g_1(x,t) \leq 0\}, x \in [b_l, b_u].$$

Remark 2.0.9. (Change of Information Level)
One of the advantages of semimartingale technique is the possibility of studying the random evolution of the stochastic process under different information levels. If we are considering a subfiltration $\mathbb{A} = (\mathcal{A}_t)_{t\in\mathbf{R}}$ of $\mathbb{F} = (\Im_t)_{t\in\mathbf{R}}$, then the projection theorem can be apply and the process H_t defined in (2.0.3), we become a new process $\mathcal{A}_t$-mensurable

$$\begin{aligned} \hat{H}_t &= K^* X_0 - Z_0 \\ &+ \int_0^t \sum_{j=1}^n E[I_{\{T_j > s\}} | \mathcal{A}_s][K^* C(-1 - \mu_j(s)(\lambda_j^0(s) - \lambda_j^1(s)) \\ &+ (C_F - C_B)\lambda_j^0(s)]ds + E[R_t | \mathcal{A}_t], \end{aligned}$$

and we can rewrite the theorem (2.0.7) changing

$$\sigma_x = inf\{t \in \mathbb{R}_+ : \sum_{j=1}^n I_{\{T_j > t\}} g_j(x,t) \leq 0\} \wedge T,$$

for

$$\hat{\sigma}_x = inf\{t \in \mathbb{R}_+ : \sum_{j=1}^n E[I_{\{T_j > t\}} | \mathcal{A}_t] g_j(x,t) \leq 0\}.$$

For example:
(i) If $\mathcal{A}_t = \{\Omega, \emptyset\}, \forall t \in \mathbb{R}_+$ then

$$\hat{\sigma}_x = inf\{t \in \mathbb{R}_+ : \sum_{j=1}^n P(T_j > t) g_j(x,t) \leq 0\}.$$

(ii) If $\mathcal{A}_t = \{\Omega, \emptyset\}$, for $0 \leq t < h$ and $\mathcal{A}_t = \Im_t$ for $t \geq h$, then

$$\begin{aligned} \hat{\sigma}_x &= \inf\{0 \leq t < h : \sum_{j=1}^n P(T_j > t) g_j(x,t) \leq 0\} \\ &\wedge \ \inf\{h \leq t : \sum_{j=1}^n I_{\{T_j > t\}} g_j(x,t) \leq 0\} \wedge (T \vee h). \end{aligned}$$

3. APPLICATION

We are considering now two example of application:,
Example 1: (For easiness, we will consider n =1.)
Let $\lambda^0(t), \lambda^1(t), \Im_t$-intensity of exponential with parameter λ^0, λ^1 respectively (therefore constant), then $\mu(t) = \lambda^1$ is constant, and $g(x,t)$ depends only on x.
If $\dfrac{(C_F - C_B)\lambda^0}{C(1 + \lambda^1(\lambda^0 - \lambda^1))} \leq \dfrac{C_F}{C\lambda^1 + k_0}$, then, by corollary (2.0.6), $K^* = \dfrac{C_F}{C\lambda^1 + k_0}$ and

$\sigma = 0$. Otherwise, by theorem (2.0.7), $\sigma_x = T$ for all $x \in [b_l, b_u]$, and $K^* = \dfrac{C_B}{k_0}$.

Example 2:
Now conside, $T_j, j = 1, ..., n$ i.i.d. and $\lambda_j^0(t) = \dfrac{\beta}{\theta_0}(\dfrac{t}{\theta_0})^{\beta-1}, \lambda_j^1(t) = \dfrac{\beta}{\theta_1}(\dfrac{t}{\theta_1})^{\beta-1}, \Im_t$-intensity of the Weibull distribution, with parameter $(\beta, \theta_0), (\beta, \theta_1), \theta_1 > \theta_0 > 0$, and $\beta > 1$, respectively. As $q = 0$, by corollary (2.0.6), $K^* = \dfrac{nC_F}{nC\mu_1(0) + k_0}$ and $\sigma = 0$.

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