Higher-derivative Wess-Zumino model in three dimensions

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We deform the well-known three-dimensional $\mathcal{N} = 1$ Wess-Zumino model by adding higher derivative operators (Lee-Wick operators) to its action. The effects of these operators are investigated both at the classical and quantum levels.

I. INTRODUCTION

Higher-derivative operators produce negative and positive effects in quantum field theories. Among the negative effects are the lack of a unitary S matrix, the presence of negative-norm states (ghosts), and the violation of the Lorentz invariance. The ghosts, strictly speaking, become evident by reformulating a given higher-derivative theory in terms of standard (lower-derivative) operators, i.e., by removing from it the higher-derivative operators by means of auxiliary fields. The lower-derivative theory obtained in this manner is a theory with an indefinite metric where the "auxiliary" fields play the role of ghost fields. In addition, this lower-derivative reformulation of a higher-derivative theory is a key step in its canonical quantization based on Ostrogradski's approach [1]. A positive effect, on the other hand, is the improvement of the ultraviolet (UV) behavior due to the exchange of both negative- and positive-norm states in the Feynman integrals. Another way to say this is that the propagators in theories with higher-derivative kinetic operators are more convergent at the UV limit $k^2 \rightarrow \infty$ than in the usual ones, implying a better UV behavior of the Feynman integrals (see Ref. [2] for a review on these issues). However, as reported in Ref. [3], such positive effect does not always occur due to subtle problems with the analytical continuation from the Minkowski to the Euclidean space.

To circumvent or eliminate the negative effects and take advantage of the positive ones, it is important to recognize that they have their origins in the additional degrees of freedom introduced by the higher-derivative operators. Therefore, to construct a quantum field theory with higher derivatives or with an indefinite metric which satisfies the

minimal physical requirements (unitarity, Lorentz invariance, and positive-energy spectrum), it is necessary to devise suitable mechanisms to get rid of "runaway" solutions or troublesome degrees of freedom. Evidently, one means is to impose constraints or boundary conditions on certain sectors of the theory.

A long time ago, Lee and Wick showed in Refs. [4,5] that it is possible to construct a quantum field theory with an indefinite metric (or in another language, with higher-derivative terms) in which the S matrix is relativistic and unitary. Specifically, they proposed a variant of quantum electrodynamics (QED), which is the result of introducing "heavy" negative-norm fields (heavy ghosts) in the gauge and fermion sectors of the original QED theory, free of divergences. The unitary problem, as a result of the presence of ghosts in Lee-Wick QED, is avoided by requiring (as a boundary condition) that ghosts do not belong to the asymptotic states of the S matrix, a condition that is possible only if the ghosts are unstable particles (i.e., if they have a nonvanishing decay width). On the other hand, as pointed out by Lee and Wick [6] afterward, the violation of Lorentz invariance overlooked in Lee-Wick QED (see, e.g., Ref. [2]) can also be avoided by adopting the prescription of Cutkosky *et al.* [7] in the choice of the Feynman contours. Nonetheless, in spite of these achievements, the Lee-Wick QED theory is still plagued by some residual ghost effects such as acausality [8].

Nowadays, higher-derivative operators are common in several branches of quantum physics and their negative effects are adequately dealt with by applying the Lee-Wick's ideas or any other killing-ghost mechanism, for instance, the method of "perturbative constraints" in nonlocal theories (see, e.g., the work by Simon in Ref. [9]). In lower-energy effective theories, for example, higher-derivative terms occur naturally as a result of integrating out the massive fields of a more fundamental

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theory and truncating its perturbative expansion, while in gravity [10] such terms (quadratic or higher powers of the curvature tensor) are generated dynamically by radiative corrections.

Higher-derivative operators were also studied in supersymmetry (SUSY)/SUGRA models, string theory, Randall-Sundrum models, cosmology, phase transitions, and Higgs models, and in other contexts (see Refs. [3,11] and references therein). More recently, the Lee-Wick's ideas were applied in the framework of the standard model in order to solve the hierarchy problem. Indeed, it was shown in Ref. [12] that all quadratic divergent radiative corrections to the Higgs mass are completely removed by introducing higher-derivative kinetic terms in each sector of the standard model.

In this paper, we bring together the good features (in relation to the improved UV behavior) of supersymmetry and higher-derivative operators. In particular, we show that a single higher-derivative kinetic operator inserted in the usual Wess-Zumino action is sufficient to remove, under a rather general assumption on the complex poles, all the SUSY remaining divergences of the two-loop scalar self-energy. Notice that in this kind of theory (where supersymmetry and higher-derivative properties are combined) two completely different mechanisms of removing UV divergences are involved. The cancellation of UV divergences in higher-derivative theories occurs due to the exchange of normal and ghost states, while in SUSY theories this cancellation is achieved by the exchange of virtual particles with opposite statistics. Here we show explicitly in connection with the two-loop self-energy how both mechanisms work together to give a finite result.

On the other hand, to obtain some insights into the vacuum structure of our higher-derivative Wess-Zumino model, we compute the effective potential at one loop in the superfield formalism. By implicitly imposing the SUSY condition of non-negative energy to throw away "runaway" solutions, we find that supersymmetry remains intact at one-loop order, while the rotational symmetry is (spontaneously) broken iff a specific condition for $\sigma_1 \neq 0$ is satisfied.

Our paper is organized as follows. In Sec. II we discuss in general terms the three-dimensional Wess-Zumino model and define our higher-derivative Wess-Zumino model (HWZ$_3$). This model is the result of introducing three types of higher-derivative operators in the usual Wess-Zumino action. In Sec. III we compute the effective potential at one loop and study its minima. As mentioned above, it is shown that supersymmetry remains intact at this order, while the rotational symmetry defines two phases of the theory. In Sec. IV we analyze the UV behavior of the scalar self-energy up to two loops. Here we explicitly show how the mutual cancellation of UV divergences happens. Finally, our conclusions are given in Sec. V.

II. HIGHER-DERIVATIVE WESS-ZUMINO MODEL

Our starting point is the three-dimensional $\mathcal{N} = 1$ Wess-Zumino model,

$$S = \int d^5 z \left\{ -\frac{1}{2} D^\alpha \bar{\Phi} D_\alpha \Phi + m \bar{\Phi} \Phi - U \right\}, \quad (1)$$

where Φ denotes a complex scalar superfield whose θ-Taylor expansion is

$$\Phi(x, \theta) = \varphi(x) + \theta^\alpha \psi_\alpha(x) - \theta^2 F(x). \quad (2)$$

Here $\varphi(x)$ and $F(x)$ represent bosonic fields and $\psi_\alpha(x)$ represents a fermionic field. Throughout the paper, we shall adopt the notation of Ref. [13].

The superpotential U in general involves terms of the form $(D_\alpha)^{N_D} (\bar{\Phi} \Phi)^{\frac{N_\Phi}{2}}$, where $D_\alpha \doteq \partial_\alpha + i\theta^\beta \partial_{\alpha\beta}$ denotes the SUSY covariant derivative, N_D and N_Φ are, respectively, the number of SUSY derivatives (D_α) and the number of complex scalar superfields in a typical interaction vertex.

Note that the rotational symmetry $\Phi'(z) = e^{i\alpha} \Phi(z)$, where α is a constant phase, and the Lorentz symmetry of the Wess-Zumino model (1) restrict the values of N_D and N_Φ. In fact, Lorentz invariance requires an even number of SUSY covariant derivatives, with their spinor indices completely contracted, while rotational invariance requires an even number of Φ and $\bar{\Phi}$ superfields. Consequently, N_D and N_Φ must be even numbers. In addition, N_Φ has to be greater than zero: $N_\Phi \geq 2$.

By imposing the power-counting renormalizability condition, it is possible to find the general form of U. To this end, we compute the superficial degree of divergence ω of a typical Feynman diagram. From the free superpropagator

$$\langle \Phi(k, \theta) \bar{\Phi}(-k, \theta') \rangle = -i \frac{D^2 - m^2}{k^2 + m^2} \delta^2(\theta - \theta'), \quad (3)$$

which is obtained by inverting the kernel of the Wess-Zumino action (1), and taking into account the Grassmann reduction procedure [13], one may show that

$$\omega = IV + 2 - \frac{E}{2} - \frac{n_D}{2}, \quad (4)$$

where I denotes the index of divergence

$$I \doteq \frac{N_D + N_\Phi}{2} - 2, \quad (5)$$

V is the number of vertices in a typical Feynman diagram, E is the number of external lines, and n_D is the number of SUSY derivatives transferred to the external lines.

Since the index I depends only on the form of the interaction vertex, a simple analysis of (4) shows that the Wess-Zumino theory is renormalizable when the condition $I \leq 0$ is satisfied. This in turn means that U can be of three types:

$$U \propto (\bar{\Phi} \Phi)^2, \qquad U \propto D^\alpha \bar{\Phi} D_\alpha \Phi, \qquad U \propto \bar{\Phi} \Phi; \quad (6)$$

thereby the only genuine vertex is the first one, while the others (kinetic- and masslike vertices) can be resummed in the quadratic part of the Wess-Zumino action (1) and then completely absorbed by suitably redefining the superfield Φ and the parameters of the theory.

Our goal now is to extend the Wess-Zumino model (1) by adding to it higher-derivative operators both at the kinetic and interaction parts. These operators must fulfill all the symmetries of the conventional theory, that is to say, they must be SUSY, Lorentz, and rotational invariants. Furthermore, all must be Hermitian. There is indeed an infinity of such operators, but in this work we shall restrict ourselves to the following ones:

$$D^2\bar{\Phi}D^2\Phi, \quad D^\alpha\bar{\Phi}\Box D_\alpha\Phi, \quad [\bar{\Phi}^2(D^2\Phi)^2 + \Phi^2(D^2\bar{\Phi})^2], \tag{7}$$

where the two formers modify the kinetic part of the Wess-Zumino model, while the latter modifies its interaction part.

Including these higher-derivative operators in (1), with $U = g(\bar{\Phi}\Phi)^2$, our higher-derivative Wess-Zumino model in three dimensions (HWZ$_3$) is described by

$$S = \int d^5z \left\{ -\frac{1}{2}D^\alpha\bar{\Phi}D_\alpha\Phi + aD^2\bar{\Phi}D^2\Phi \right.$$
$$- \frac{b}{2}D^\alpha\bar{\Phi}\Box D_\alpha\Phi + m\bar{\Phi}\Phi - g(\bar{\Phi}\Phi)^2$$
$$\left. + \lambda[\bar{\Phi}^2(D^2\Phi)^2 + \Phi^2(D^2\bar{\Phi})^2] \right\}. \tag{8}$$

It is very simple to check out that the mass dimensions of the coefficients in this action are $[a] = -1$, $[b] = -2$, $[m] = 1$, $[g] = 0$, and $[\lambda] = -2$. Thus we shall take $a = 1/M$ and $b = 1/\tilde{M}^2$, where M and $\tilde{M}$ are, in principle, arbitrary and different mass parameters. However, as we are regarding the higher-derivative operators as the residual (low-energy limit) effects of an underlying fundamental theory, it is important to keep in mind that a, b, and λ must be very small compared with the original parameters of the theory. In particular, M and $\tilde{M}$ must be of the Planck mass order. It should also be noted that the λ interaction is nonrenormalizable within the usual Wess-Zumino theory, according to the previous discussion.

In terms of the component fields, the HWZ$_3$ Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\rm bos} + \mathcal{L}_{\rm fer} + \mathcal{L}_{\rm int}, \tag{9}$$

where

$$\mathcal{L}_{\rm bos} = \bar{\varphi}\Box\varphi + m(\varphi\bar{F} + \bar{\varphi}F) + F\bar{F}$$
$$+ a(\varphi\Box\bar{F} + \bar{\varphi}\Box F) + b(\bar{\varphi}\Box^2\varphi + \bar{F}\Box F), \tag{10}$$

$$\mathcal{L}_{\rm fer} = \bar{\psi}i\partial\psi + m\bar{\psi}\psi + a\bar{\psi}\Box\psi + b\bar{\psi}\Box i\partial\psi, \tag{11}$$

and

$$\mathcal{L}_{\rm int} = -2g[\varphi^2\bar{\psi}^2 + \bar{\psi}^2\psi^2 + \bar{\varphi}\varphi(\varphi\bar{F} + \bar{\varphi}F + 2\bar{\psi}\psi)]$$
$$+ 2\lambda[\bar{\varphi}F(\bar{F}F + 2\bar{\psi}i\partial\psi) + \psi^2\bar{F}^2$$
$$+ \bar{\varphi}^2(F\Box\varphi - (\partial\psi)^2) + \text{H.c.}]. \tag{12}$$

Here the contraction of spinor indices follows the northwest rule ($\searrow$) and the square of a spinor includes a factor of $1/2$ in its definition. So $\psi^2 \doteq \frac{1}{2}\psi^\alpha\psi_\alpha$ and $\bar{\psi}i\partial\psi \doteq \bar{\psi}^\alpha i\partial_\alpha^\beta\psi_\beta$, for instance.

By setting up the F, $\bar{F}$ equations of motion of the free (i.e., switch off the g and λ couplings) part of (9),

$$(a\Box + m)\varphi + (b\Box + 1)F = 0,$$
$$(a\Box + m)\bar{\varphi} + (b\Box + 1)\bar{F} = 0, \tag{13}$$

one can easily see that F and $\bar{F}$ become dynamical fields only when b is different from zero. In fact, by taking $b = 0$ in (9), F and $\bar{F}$ play the roles of auxiliary Lee-Wick fields by introducing the higher-derivative operator for the scalar φ field only in the so-called on-shell case.

III. THE EFFECTIVE POTENTIAL AT ONE-LOOP

In what follows we compute the one-loop effective potential in order to study the vacuum effect of the higher-derivative operators. As is well known, the classical potential is given by the negative of the classical HWZ$_3$ action (9) evaluated at constant fields. Hence, writing the fields φ and F as

$$\varphi \doteq \frac{1}{\sqrt{2}}[\sigma_1 + i\pi_1], \qquad F \doteq \frac{1}{\sqrt{2}}[\sigma_2 + i\pi_2], \tag{14}$$

where σ_i and π_i are real constant fields, the classical potential $V_{\rm cl}$ is given by

$$V_{\rm cl} = -\frac{1}{2}(\sigma_2^2 + \pi_2^2) - (\sigma_1\sigma_2 + \pi_1\pi_2)[m - g(\sigma_1^2 + \pi_1^2)$$
$$+ \lambda(\sigma_2^2 + \pi_2^2)]. \tag{15}$$

Since the effective potential V possesses the rotational SO(2) symmetry, $\sigma_i' + i\pi_i' = \exp(i\omega)(\sigma_i + i\pi_i)$, that inherits from the classical HWZ$_3$ action, we shall take $\pi_i = 0$ in (15) to simplify the analysis of the vacuum structure of the theory. The function $V_{\rm cl}(\sigma_1, \sigma_2)$ is smooth and satisfies $V_{\rm cl}(0, 0) = 0$. Solving the Euler-Lagrange equation $\partial V_{\rm cl}/\partial\sigma_2 = 0$, one finds the solution for σ_2:

$$\sigma_2(\sigma_1) = \frac{-1 + \sqrt{1 + 12\lambda\triangle}}{6\lambda\sigma_1}, \tag{16}$$

where $\triangle = g\sigma_1^2(\sigma_1^2 - r)$, with $r = m/g$. As we shall see below, r is an order parameter related to the spontaneous breakdown of the rotational symmetry at the classical level. Eliminating the "auxiliary" field σ_2 of the classical potential $V_{\rm cl}$ by means of (16), one obtains
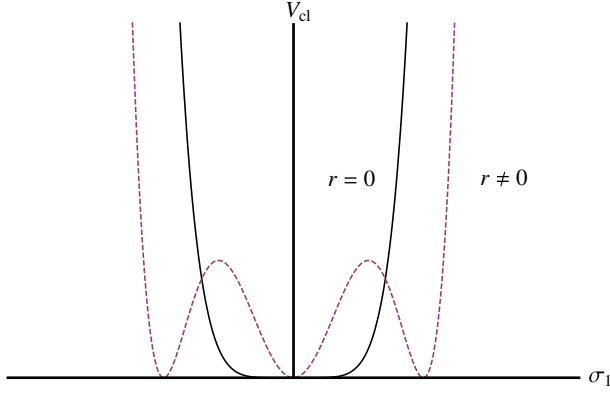
FIG. 1 (color online). Typical curves of V_{cl}, with $\lambda = 0$. The solid line represents the unbroken phase ($r = 0$), while the dashed line represents the broken phase ($r \neq 0$) of the rotational symmetry.

$$V_{\text{cl}}(\sigma_1) = \frac{1}{108\lambda^2\sigma_1^2}[-1 - 18\lambda \triangle +(1 + 12\lambda\triangle)^{\frac{3}{2}}]. \quad (17)$$

Expanding the potential V_{cl} around $\lambda = 0$,

$$V_{\text{cl}} = \frac{g^2\sigma_1^2}{2}(\sigma_1^2 - r)^2[1 - 2\lambda \triangle +9\lambda^2 \triangle^2 +\mathcal{O}(\lambda^3)], \quad (18)$$

we find out that V_{cl} is analytic at $\lambda = 0$ and positive definite ($V_{\text{cl}} \geq 0$), as required by supersymmetric grounds. Figure 1 displays two characteristic curves of V_{cl} for $r = 0$ and $r \neq 0$, setting $\lambda = 0$. They show us that there are two phases associated to the rotational symmetry: an unbroken phase of the rotational symmetry corresponding to $r = 0$ and an spontaneous broken phase corresponding to $r \neq 0$.

Classically, note also that supersymmetry remains intact at both phases. An analysis for the $\lambda \neq 0$ case leads us to the same conclusions.

Next we shall compute the one-loop contribution to the effective potential by employing the steepest-descent method [14] or, as it is also known, the Jackiw functional method [15], implemented in the superspace. For simplicity and technical reasons, from now on we will limit to the case $a \neq 0$ and $b = 0 = \lambda$. The one-loop contribution is given by

$$V_1 = -\frac{i}{2v} \ln \text{Det}(A - B) - \frac{i}{2v} \ln \text{Det}C, \quad (19)$$

where $v = \int d^3x$,

$$A(z, z') = [m + a\square - g(3\sigma_1^2 + \pi_1^2) + D^2$$
$$+ 2g(3\sigma_1\sigma_2 + \pi_1\pi_2)\theta^2]\delta^5(z - z'),$$

$$B(z, z') = \int dz_1 dz_2 H(z, z_1)C^{-1}(z_1, z_2)H(z_2, z'),$$

$$C(z, z') = [m + a\square - g(\sigma_1^2 + 3\pi_1^2) + D^2$$
$$+ 2g(\sigma_1\sigma_2 + 3\pi_1\pi_2)\theta^2]\delta^5(z - z'), \quad (20)$$

with

$$H(z, z') = [-2g\sigma_1\pi_1 + 2g(\sigma_1\pi_1 + \sigma_2\pi_2)\theta^2]\delta^5(z - z'). \quad (21)$$

The superspace functional determinants which appear in V_1 are evaluated using the ζ-functional method [16,17], taking advantage of the rotational SO(2) symmetry (i.e., setting $\pi_i = 0$). The result can be cast in the form

$$V_1 = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \left\{ \ln \left[\frac{a^2(k^2)^2 + \alpha(3)k^2 + \beta(3, 3)}{a^2(k^2)^2 + \alpha(3)k^2 + \beta(3, 0)} \right] \right.$$
$$\left. + \ln \left[\frac{a^2(k^2)^2 + \alpha(1)k^2 + \beta(1, 1)}{a^2(k^2)^2 + \alpha(1)k^2 + \beta(1, 0)} \right] \right\}, \quad (22)$$

where, defining $\omega(n) \doteq m - n \cdot g\sigma_1^2,$

$$\alpha(n) \doteq 1 - 2a\omega(n) \quad \text{and}$$
$$\beta(n_1, n_2) \doteq \omega(n_1)^2 + n_2 \cdot (2g\sigma_1\sigma_2). \quad (23)$$

Note that setting $a = 0$ apparently reduces V_1 to the usual case, where there are no ghosts in the theory, since the $(k^2)^2$ terms vanish. However, the behavior of the result of the integrals as a function of a depends on some extra assumptions to complete the definition of the quantum theory. In the first calculation of V_{eff} below, we consider the higher-derivative term literally. By this we mean that, if we were using a momentum UV cutoff to regularize the integrals, we would be integrating up to a cutoff $\Lambda \gg M = 1/a$ (in fact, we are regularizing by dimensional reduction, where this assumption is not clear). In the second calculation, we will be considering the higher-derivative term as a first approximation of a series of terms coming from the integration over high energy states in a more fundamental theory; as we will see, in this case, the result is very different from the first one.

In the following, let us focus on the case with $a \neq 0$. To solve the above integrals we express each numerator and denominator as the product of two binomial factors of the form $(k^2 + M_i^2)$ and employ the formulas in Ref. [18]. In this way, we get

$$V_1 = -\frac{1}{12\pi} \sum_{i=+,-} [M_i(3, 3)^3 - M_i(3, 0)^3$$
$$+ M_i(1, 1)^3 - M_i(1, 0)^3], \quad (24)$$

where

$$M_{\pm}(n_1, n_2) \doteq \frac{1}{\sqrt{2}a}\sqrt{1 - 2a\omega(n_1) \pm \sqrt{1 - 4a\omega(n_1) - n_2 \cdot (8a^2 g\sigma_1\sigma_2)}}. \qquad (25)$$

Adding (15), with $\pi_i = 0 = \lambda$, and (24), we get the one-loop effective potential:

$$V_{\text{eff}} = -\frac{\sigma_2^2}{2} - \sigma_1\sigma_2(m - g\sigma_1^2) - \frac{1}{12\pi}\sum_{i=+,-}[M_i(3,3)^3$$
$$- M_i(3,0)^3 + M_i(1,1)^3 - M_i(1,0)^3]. \qquad (26)$$

At first glance, this result seems fairly intricate. However, considering the σ_2-linear approximation for the one-loop contribution, we will get some light about its physical implications (in three dimensions $\mathcal{N} = 1$ SUSY, this approximation is enough to analyze the possibility of SUSY breaking [16,19]). Thus, the one-loop effective potential is given by

$$V_{\text{eff}} = -\frac{\sigma_2^2}{2} - \sigma_1\sigma_2[m - g\sigma_1^2 + X(a, \sigma_1)], \qquad (27)$$

where

$$X(a, \sigma_1) \doteq \frac{g}{4\pi}\left[\frac{M_-(1,0) - M_+(1,0)}{\sqrt{1 - 4a\omega(1)}}\right.$$
$$\left. + \frac{3[M_-(3,0) - M_+(3,0)]}{\sqrt{1 - 4a\omega(3)}}\right]. \qquad (28)$$

After eliminating the auxiliary field σ_2 by means of its equation of motion, one gets

$$V_{\text{eff}} = \frac{1}{2}\sigma_1^2[m - g\sigma_1^2 + X(a, \sigma_1)]^2. \qquad (29)$$

Notice that $V_{\text{eff}} \gtrsim 0$ and its minima occur at $\sigma_1 = 0$ or $\sigma_1 \neq 0$, provided that σ_1 is the solution to the equation:

$$g\sigma_1^2 = m + X(a, \sigma_1). \qquad (30)$$

This result means that SUSY is unbroken at one-loop order, while the rotational symmetry is spontaneously broken at this order only if there is a nontrivial solution $\sigma_1 \neq 0$ for the condition (30).

On the other hand, the series expansion for V_{eff} around $a = 0$,

$$V_{\text{eff}} \approx \frac{g^2\sigma_1^2}{2\pi^2 a^2} + \frac{g\sigma_1^2|m - g\sigma_1^2|}{\pi a}$$
$$+ \frac{1}{2}\sigma_1^2(m - g\sigma_1^2)^2 + \mathcal{O}(a^{10}), \qquad (31)$$

shows us that V_{eff} is singular at $a = 0$. This result shows that a theory with a higher-derivative term with a small coefficient (no matter how small it is), if considered literally, implies in a contribution to the effective action that leads to a nonanalytic behavior of the potential in the small parameter a. A similar result is presented in Ref. [3] for the $\lambda\phi^4$ model in $3 + 1$ dimensions. In models with higher derivatives, the higher-derivative term introduces a new degree of freedom (a ghost) represented by the pole M_+^2 in the propagators, or what is the same, a new zero in the arguments of the logarithmic terms which appear in the calculation of the effective potential [see Eq. (22)]. Let us consider one of the four logarithmic terms contributing to Eq. (22). In the above calculation, the effect of this ghost term was taken on the same grounds as that of the pole M_-^2, of the fundamental particle of the model, by calculating the integrals:

$$\int(dk)\ln(1/\triangle) \sim \int(dk)\ln[a^2(k^2)^2 + \alpha k^2 + \beta]$$
$$\sim \int(dk)\ln(k^2 + M_-^2)$$
$$+ \int(dk)\ln(k^2 + M_+^2), \qquad (32)$$

where $M_-^2(n_1, n_2) = (m^2 - 2gm\sigma_1^2 n_1 + 2g\sigma_1\sigma_2 n_2 + g^2\sigma_1^4 n_1^2)(1 + \mathcal{O}(m/M))$ and $M_+^2(n_1, n_2) = M^2(1 - 2\frac{m}{M} + 2g\frac{\sigma_1^2}{M}n_1 + \cdots)$ are the square masses of the particle and the ghost in the presence of non-null expectation values of the fields [see also Eq. (25) above]. In this formula, a term of the form $\int(dk)\ln(a^2)$, which cancels among the four logarithmic terms in (22), has been dropped.

Another way of looking at this model is as an effective theory, which comes from a more fundamental theory by the integration of the high energy modes $k^2 > M^2$. In this case, if we were using an UV cutoff Λ, instead of dimensional reduction (DR), we would be limiting the integrations in momenta up to a $\Lambda \ll M$. So, in the integral above, the integrand can be expanded [3,9,20] in powers of k^2/α^2, as

$$\int(dk)\ln(1/\triangle) \sim \int(dk)\ln[a^2(k^2)^2 + \alpha k^2 + \beta],$$
$$\sim \int(dk)\ln(k^2 + \mu^2)$$
$$+ \int(dk)\ln\left[1 + \frac{a^2 k^2}{\alpha}\frac{k^2}{k^2 + \mu^2}\right],$$
$$\sim \int(dk)\ln(k^2 + \mu^2)$$
$$+ \int(dk)\left[\frac{a^2 k^2}{\alpha}\frac{k^2}{k^2 + \mu^2} + \mathcal{O}\left(\frac{a^2 k^2}{\alpha}\right)^2\right], \qquad (33)$$

where $\mu^2 = \beta/\alpha = (m^2 + g^2\sigma_1^4 n_1^2 - 2mg\sigma_1^2 n_1)/(1 - 2a(m - g\sigma_1^2 n_1))$ is related to the particle mass in the presence of the expectation values of the fields and the ghost [here also, a term of the form $\int(dk)\ln(\alpha)$ has been dropped]. By doing the integrations, we get

$$V_1 = -\frac{1}{12\pi}[\mu(3,3)^3 - \mu(3,0)^3 + \mu(1,1)^3 - \mu(1,0)^3],$$

$$-\frac{1}{8\pi}a^2\left[\frac{\mu(3,3)^5}{\alpha(3)} - \frac{\mu(3,0)^5}{\alpha(3)} + \frac{\mu(1,1)^5}{\alpha(1)} - \frac{\mu(1,0)^5}{\alpha(1)}\right]$$

$$+ \mathcal{O}(a^4). \tag{34}$$

In the same approximation as above, one obtains

$$V_{\text{eff}} = \frac{1}{2}\left[m\sigma_1 - g\sigma_1^3 + \frac{3g\sigma_1}{8\pi}[(2\mu(3,0) + \mu(1,0)]\right.$$

$$\left. + \frac{5g\sigma_1}{8\pi}[3\mu^3(3,0) + \mu^3(1,0)]a^2 + \mathcal{O}(a^4)\right]^2, \tag{35}$$

which is analytic in a and goes to the effective potential of the usual theory, when $a \rightarrow 0$.

IV. ULTRAVIOLET ANALYSIS OF THE SELF-ENERGY UP TO TWO LOOPS

In this section, we analyze the UV behavior of the scalar self-energy for the HWZ$_3$ model. We show that the introduction of a higher-derivative operator in the Lagrangian (and consequently in the propagator) improves the structure of the divergences that appear in the momentum-space integrals, as compared with the usual Wess-Zumino model. In fact, we shall see that all the integrals appearing in these corrections are finite in the UV regime, and so we have a much better UV behavior when comparing with the usual case.

The Feynman diagrams which contribute to the scalar self-energy up to two loops are depicted in Fig. 2. As in the previous discussion about the effective potential at the one-loop order, here we set $b = 0 = \lambda$ as well. Hence the free propagator of the HWZ$_3$ model is given by

$$\Delta(k, \theta - \theta') = \langle 0|T\Phi(k, \theta)\bar{\Phi}(-k, \theta')|0\rangle,$$

$$= \frac{i}{(m - ak^2)^2 + k^2}[m - ak^2 - D^2]\delta^2(\theta - \theta'),$$

$$= i[A(k) + B(k)D^2]\delta^2(\theta - \theta'), \tag{36}$$

with

$$A(k) = \frac{m - ak^2}{(m - ak^2)^2 + k^2}, \tag{37}$$
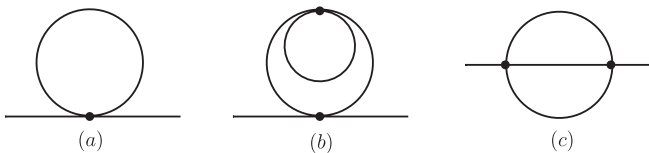
$$B(k) = -\frac{1}{(m - ak^2)^2 + k^2}.$$

FIG. 2. Self-energy radiative corrections up to two loops in the HWZ$_3$ model.

In three dimensions and within the dimensional reduction (DReD$_3$) scheme [21], the one-loop diagram of Fig. 2 is finite. This statement is very general and well known in lower-derivative theories [22] and is not spoiled by the introduction of higher-derivative operators [23]. Indeed, as far as the UV behavior of higher-derivative theories is concerned, the Wick rotation can be performed in the usual way disregarding all contributions of the complex poles in the complex energy plane. The reason is that in the majority of the theories with higher-derivative operators the residue contributions of the complex poles in the Cauchy's residue theorem are finite and so the form of the energy contour in the Feynman integrals is irrelevant. Note, however, that in order to construct a relativistic and unitary S matrix it is necessary to fix the Feynman contour according to the Cutkosky's prescription [7].

Bearing these remarks in mind we proceed with the UV analysis of the diagram (b) in Fig. 2. The expression for this diagram is written as

$$\mathcal{G}_{2b} = \mathcal{C}_{2b}\int d\tilde{p}d\tilde{k}d\tilde{q}\int d^2\theta_1 d^2\theta_2\Phi(\theta_1, p)\bar{\Phi}(\theta_1, -p)$$

$$\times \Delta(k, \theta_1 - \theta_2)\Delta(k, \theta_2 - \theta_1)\Delta(q, \theta_2 - \theta_2), \tag{38}$$

where p [with $d\tilde{p} = d^3 p/(2\pi)^3$ and so on] is the external momentum, k and q are the momenta appearing in the loops, and $\mathcal{C}_{2b}$ is a numerical constant and since we are concerned only with the divergent behavior of the diagram we do not worry about it. After carrying out the D algebra, the expression above takes the form

$$\mathcal{G}_{2b} \propto \int d\tilde{p}d\tilde{k}d\tilde{q}\int d^2\theta\{A(k)B(k)B(q)\bar{\Phi}(\theta, -p)\Phi(\theta, p)$$

$$+ [B(k)]^2 B(q)[D^2\bar{\Phi}(\theta, -p)\Phi(\theta, p)$$

$$+ D^\alpha\bar{\Phi}(\theta, -p)D_\alpha\Phi(\theta, p)$$

$$+ \bar{\Phi}(\theta, -p)D^2\Phi(\theta, p)]\}, \tag{39}$$

and as can be easily seen, the integrals in the momenta k and q are independent, and so by the same argument given above, each one of these one-loop integrals is finite in the DReD$_3$ scheme.

Now let us consider the last diagram in Fig. 2. Its mathematical expression is given by

$$\mathcal{G}_{2c} = \mathcal{C}_{2c}\int d\tilde{p}d\tilde{k}d\tilde{q}\int d^2\theta_1 d^2\theta_2\Phi(\theta_1, p)\bar{\Phi}(\theta_2, -p)$$

$$\times \Delta(k, \theta_1 - \theta_2)\Delta(r, \theta_1 - \theta_2)\Delta(q, \theta_2 - \theta_1), \tag{40}$$

where $r = p + k - q$ and $\mathcal{C}_{2c}$ is a numerical constant and we do not worry about it in the same way that we did with the constant $\mathcal{C}_{2b}$ in diagram (b). After the D algebra, one obtains

$$\mathcal{G}_{2c} \propto \int d\tilde{p}d\tilde{k}d\tilde{q} \int d^2\theta \{[A(r)B(k)B(q) + A(q)B(k)B(r)$$
$$+ A(k)B(q)B(r)]\bar{\Phi}(\theta, -p)\Phi(\theta, p)$$
$$+ B(k)B(q)B(r)D^2\bar{\Phi}(\theta, -p)\Phi(\theta, p)\}. \qquad (41)$$

The analysis of the UV behavior for this diagram is much more cumbersome. It presents the so-called "overlapping momenta," which did not appear in diagram (b). In order to isolate the divergent part from the finite part of the integral above, we must take its Taylor series expansion around $p^2 = 0$. Since each differentiation with respect to the external momentum improves the convergence of the integrand, the divergences will reside only in the first few

terms of its Taylor expansion. In our case, the Taylor expansion of (41) around $p^2 = 0$ is given by

$$\mathcal{G}_{2c} \propto \int d\tilde{p} \int d^2\theta [\mathcal{R}(p^2)D^2\bar{\Phi}(\theta, -p)\Phi(\theta, p)$$
$$+ \mathcal{S}(p^2)\bar{\Phi}(\theta, -p)\Phi(\theta, p)], \qquad (42)$$

where

$$\mathcal{R}(p^2) = \mathcal{R}_1 + p^2\mathcal{R}_2 + \mathcal{O}(p^4), \qquad (43)$$

$$\mathcal{S}(p^2) = \mathcal{S}_1 + p^2\mathcal{S}_2 + \mathcal{O}(p^4), \qquad (44)$$

with, defining $l \doteq k - q$ and $f(x) \doteq (m - ax)^2 + x$,

$$\mathcal{R}_1 = \int d\tilde{k}d\tilde{q} \frac{r_1}{f(k^2)f(q^2)f(l^2)},$$

$$\mathcal{R}_2 = \int d\tilde{k}d\tilde{q} \frac{r_{2a}l^2 + r_{2b}f(l^2) + r_{2c}l^2 f(l^2)}{f(k^2)f(q^2)[f(l^2)]^3},$$

$$\mathcal{S}_1 = \int d\tilde{k}d\tilde{q} \frac{s_{1a} + s_{1b}(k^2 + q^2 + l^2)}{f(k^2)f(q^2)f(l^2)},$$

$$\mathcal{S}_2 = \int d\tilde{k}d\tilde{q} \frac{[s_{2a}(l^2)^2 + (s_{2b} + s_{2c}l^2)(k^2 + q^2)]f(l^2) + [P(l^2) + k \cdot qQ(l^2)]l^2}{f(k^2)f(q^2)[f(l^2)]^3}.$$

Here r_i and s_i denote numerical constants which might depend on the parameters a and m, while P and Q are polynomials in l^2 of degrees 4 and 2, respectively.

At this point two comments are in order. First, since the two-loop self-energy in the usual Wess-Zumino model, i.e., without higher-derivative operators, involves merely logarithmic and linear divergences [see Eq. (4), setting $I = 0$], we focus our attention only to the first term in the Taylor expansions for $\mathcal{R}$ and $\mathcal{S}$. Clearly, these terms enclose all the divergences present in the two-loop self-energy of the HWZ$_3$ model. Second, we should point out that the (k, q) integrals associated with p^μ-linear terms in the Taylor expansions are identically zero, a result that is in agreement with the Lorentz invariance of the model.

As a result of the indefinite metric in Lee-Wick theories, in particular our HWZ$_3$ model, the $\mathcal{R}_i$ and $\mathcal{S}_i$ Feynman integrals turn out to be finite. This assertion can be proved explicitly by evaluating each integral and observing the mutual cancellation between the divergent contributions from positive- and negative-norm (ghost) states, or implicitly by examining the superficial degree of divergence of each diagram.

As the algebraic manipulations required to solve the $\mathcal{R}_i$ and $\mathcal{S}_i$ integrals are too lengthy to be shown here, we are going to illustrate the divergence cancellation by evaluating in detail only the simplest integral, namely, $\mathcal{R}_1$. Up to a constant, we first rewrite the integral as

$$\mathcal{R}_1 = \int d\tilde{k}d\tilde{q} \frac{1}{[(m - ak^2)^2 + k^2][(m - aq^2)^2 + q^2][(m - al^2)^2 + l^2]},$$
$$= \int d\tilde{k}d\tilde{q} \frac{1}{a^6(k^2 + \alpha)(k^2 + \beta)(q^2 + \alpha)(q^2 + \beta)(l^2 + \alpha)(l^2 + \beta)}, \qquad (45)$$

with

$$\alpha, \beta = \frac{1}{2a^2}(1 - 2am \mp \sqrt{1 - 4am}).$$

Next, we use the method of partial fraction decomposition to write

$$\frac{1}{(k^2 + \alpha)(k^2 + \beta)} = \frac{1}{\beta - \alpha}\left(\frac{1}{k^2 + \alpha} - \frac{1}{k^2 + \beta}\right),$$

and split the integrand into eight fractions, obtaining

$$\mathcal{R}_1 = \frac{1}{a^6(\beta - \alpha)^3} \int d\tilde{k} d\tilde{q} \left[\frac{1}{(k^2 + \alpha)(q^2 + \alpha)(l^2 + \alpha)} - \frac{1}{(k^2 + \alpha)(q^2 + \alpha)(l^2 + \beta)} - \frac{1}{(k^2 + \alpha)(q^2 + \beta)(l^2 + \alpha)} \right.$$
$$+ \frac{1}{(k^2 + \alpha)(q^2 + \beta)(l^2 + \beta)} - \frac{1}{(k^2 + \beta)(q^2 + \alpha)(l^2 + \alpha)} + \frac{1}{(k^2 + \beta)(q^2 + \alpha)(l^2 + \beta)}$$
$$+ \left. \frac{1}{(k^2 + \beta)(q^2 + \beta)(l^2 + \alpha)} - \frac{1}{(k^2 + \beta)(q^2 + \beta)(l^2 + \beta)} \right]. \tag{46}$$

Within the dimensional regularization scheme, each one of these integrals is calculated using the following formula [18]:

$$\mathcal{I}(x, y, z) = \int \frac{d^D k d^D q}{(2\pi)^{2D}} \frac{1}{(k^2 + x)(q^2 + y)(l^2 + z)} = \frac{\mu^{-2\epsilon}}{32\pi^2} \left\{ \frac{1}{\epsilon} - \gamma_E + 1 - \ln \left[\frac{(\sqrt{x} + \sqrt{y} + \sqrt{z})^2}{4\pi\mu^2} \right] \right\} + \mathcal{B}_{\text{res}}, \tag{47}$$

where $\epsilon = 3 - D$, with D labeling the dimension of the spacetime, $\gamma_E = 0.5772$ is Euler's constant, and $\mathcal{B}_{\text{res}}$ denotes the sum of residues of the integrand over all complex poles inside an energy contour appropriate for performing the "Wick rotation" (i.e., an energy contour which permits us to change the real integration axis to the imaginary axis by means of Cauchy's residue theorem). Strictly speaking, the analytical continuation from the Minkowski to the Euclidean space, which is common in conventional quantum field theories, is lost in the theories with higher-derivative operators (i.e., Lee-Wick theories) due to the presence of complex poles in the complex energy plane. This fact is reflected, for example, in the nonvanishing value of $\mathcal{B}_{\text{res}}$ that one finds in this sort of theory.

Using the formula (47), we express $\mathcal{R}_1$ in the form

$$\mathcal{R}_1 = \frac{1}{a^6(\beta - \alpha)^3} [\mathcal{I}(\alpha, \alpha, \alpha) - 3\mathcal{I}(\alpha, \alpha, \beta)$$
$$+ 3\mathcal{I}(\alpha, \beta, \beta) - \mathcal{I}(\beta, \beta, \beta)]. \tag{48}$$

From (47) and (48), and assuming without proof that $\mathcal{B}_{\text{res}}$ is finite, one can see that the divergent parts of the $\mathcal{I}$ terms cancel mutually, so that the integral $\mathcal{R}_1$ as a whole is in fact finite, as was claimed before. The same procedure can be used to calculate the other integrals and all of them happen to be finite. The finiteness of the self-energy in the HWZ$_3$ model is in contrast with the usual Wess-Zumino model, in which the diagram (c) in Fig. 2 gives a nonvanishing divergent contribution in the UV regime, showing us that a higher-derivative kinetic operator improves the behavior of the model in this regime.

There is a more elegant and general form to see why this better UV behavior is achieved. This is the study of the superficial degree of divergence ω of a diagram. Recalling Eq. (36), we can see that the two terms in the expression for the superpropagator, one proportional to $A(k)$ and the other proportional to $B(k)$, give different contributions to a given diagram, since $A(k)$ has power -2 and $B(k)$ has power -4 in the momentum k. Moreover, $B(k)$ is accompanied by a superspace derivative D^2, which is not the case for $A(k)$. We have therefore to consider that the complete propagator comprises two types of propagators in order to compute the

superficial degree of divergence. These two propagators are defined by the $A(k)$- and $B(k)$-terms in (36):

$$\Delta(k, \theta - \theta') = \Delta_A(k, \theta - \theta') + \Delta_B(k, \theta - \theta'), \tag{49}$$

where

$$\Delta_A(k, \theta - \theta') = \frac{i(m - ak^2)}{(m - ak^2)^2 + k^2} \delta^2(\theta - \theta'),$$

$$\Delta_B(k, \theta - \theta') = -\frac{iD^2}{(m - ak^2)^2 + k^2} \delta^2(\theta - \theta').$$

Taking into account the two types of propagators, it is straightforward to show that

$$\omega = 2 - x, \tag{50}$$

where

$$x = 2V + P_B + \frac{n_D}{2}, \tag{51}$$

and, as before, V and n_D are, respectively, the number of vertices and the number of SUSY derivatives transferred to the external lines, and P_B is the number of propagators of the type B in a given diagram. Since x is strictly an integral positive number ($x \geq 1$), our HWZ$_3$ model exhibits only logarithmic ($\omega = 0$) and linear ($\omega = 1$) divergences.

On the other hand, from Eqs. (39) and (41) we see that n_D assumes only two values, $n_D = 0$ and $n_D = 2$ and, for the two-loop diagrams in Fig. 2, we have that $V = 2$. Thus, by considering the worst case (i.e., $P_B = 0$) in both diagrams, one sees that $\omega < 0$, showing that these diagrams are finite.

V. SUMMARY AND CONCLUSIONS

In this work we investigated the classical (and quantum) effects of three types of higher-derivative operators introduced in the Lagrangian of the three-dimensional Wess-Zumino model. These operators respect all the symmetries of the original model, but the potential λ operator turns out to be nonrenormalizable by power-counting arguments. At the classical level, we show that these Lee-Wick operators

modify the structure of the equations of motion for the component fields. In particular, one finds that the Lee-Wick b operator promotes the component field $F(\bar{F})$ from an auxiliary field to a dynamical field.

We also considered the quantum aspects of the model in two distinct analyses. First, we computed the one-loop correction to the classical potential, which is singular at a null value for the higher-derivative parameter a. This fact was already expected [9] and shows that the higher-derivative term cannot be treated as a perturbation in the lower-derivative theory. After this, we analyze the UV behavior of the self-energy up to two loops. We showed by direct computation of one of the momentum-space integrals and also by calculating the superficial degree of divergence that the ''setting-sun'' diagram gives a finite contribution to the self-energy, in contrast with the usual Wess-Zumino model. This explicitly shows that the introduction of the higher-derivative operator improves the behavior of the theory in the UV regime.

In future efforts, we shall intend to analyze the features of the model presented here with b, $\lambda \neq 0$. This is a more involved work, due to the fact that the integrals appearing in the one-loop correction to the effective potential and in the correction to the self-energy are much more cumbersome and, besides that, the number of two-loop diagrams increases considerably. Also, we will consider the study of three-dimensional supersymmetric models with gauge fields within the framework of higher-derivative models, a very interesting task which deserves attention. The situation in this case is different from what we have in the present work, for the gauge potential multiplet is a spinorial function on superspace and the procedure to obtain the propagators, even without higher-derivative operators, is more intricate [19].

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