RT-MAE 9525

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Key words: Asymptotic distribution, hypotheses testing, reliability ratio, variances ratio, one sided test.

Classificação AMS: 62F05, 62J05, 62J12, 62F03.
(AMS Classification)

- Outubro de 1995 -

ON RESTRICTED HYPOTHESIS IN STRUCTURAL REGRESSION MODELS

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Abstract

This paper discusses inference about the slopes of two or more populations when the explanatory variables are measured with error. It is derived the asymptotic null distribution of three asymptotically equivalent statistics for two situations of general one-sided hypotheses. For some particular cases such as those of simple order for the slope parameters, the asymptotic null distribution reduces to the well known chi-bar-squared distribution. Consequently, the approximations developed for this distribution, specially for four or more constraints can be applied for the above cases. Some simulation studies illustrate the behavior of the proposed statistics.

1. Introduction

The main object of this paper is to consider inference on the slope parameter of structural regression models with measurement errors. We consider s such populations, so that for population i, there is a bivariate random variable (y_i, x_i), such that

$$y_{ij} = \alpha_i + \beta_i x_{ij}, \tag{1.1}$$

$j = 1, \ldots, n_i$, $i = 1, \ldots, s$. The parameters α_i and β_i, $i = 1, \ldots, s$ are to be estimated and x_{ij} and y_{ij} are not observed directly. We can observe only

$$Y_{ij} = y_{ij} + e_{ij} \quad \text{and} \quad X_{ij} = x_{ij} + u_{ij}, \tag{1.2}$$

with the typically made assumption

$$\begin{pmatrix} e_{ij} \\ u_{ij} \\ x_{ij} \end{pmatrix} \sim N_3(\begin{pmatrix} 0 \\ 0 \\ \mu_x \end{pmatrix}; \begin{pmatrix} \sigma^2_{ei} & 0 & 0 \\ 0 & \sigma_{ui} & 0 \\ 0 & 0 & \sigma^2_{xi} \end{pmatrix}), \tag{1.3}$$

$j = 1, \ldots, n_i$ and $i = 1, \ldots, s$. The main idea behind equations (1.1)-(1.2) is that (x_{ij}, y_{ij}) is not observed and estimation has to be based on (Y_{ij}, X_{ij}), which is observed, $j = 1, \ldots, n_i$ and $i = 1, \ldots, s$. It is well known that when both σ^2_{ei} and σ^2_{ui} are unknown, β_i is not identifiable. Commom assumptions to circumvent this problem is to take the ratio of variances $\lambda_{ei} = \sigma^2_{ei}/\sigma^2_{ui}$ or the coefficient $k_{xi} = \sigma^2_{xi}/(\sigma^2_{xi} + \sigma^2_{ui})$ as known. Notice that we can take $\lambda_{ei} = 1$, $i = 1, \ldots, s$ without loss of generality. Moreover, k_{xi} is typically known as the reliability ratio (Fuller, 1987) and knowing k_{xi} is equivalent to knowing $\lambda_{xi} = \sigma^2_{xi}/\sigma^2_{ui}$, $i = 1, \ldots, s$.

The paper is organized as follows. Section 2 discusses orthogonal parametrizations for both assumptions and maximum likelihood estimation, presenting also the behavior of $\mathbf{I}_{Fi}/n$, as $n \to \infty$, with $\mathbf{I}_{Fi}$ being the information matrix under the structural model. Section 3 presents a revision of maximum likelihood estimation under the model specified by assumptions (1.1)-(1.3). Section 4 discusses the asymptotic null distribution of three asymptotically equivalent statistics under general one-sided hypotheses for both cases, λ_{xi} and λ_{ei} known, $i = 1, \ldots, s$. Particular situations where the asymptotic null distribution reduces to the well known chi-bar-squared distribution are presented. A simulation study illustrating the behavior of the test statistics under the null hypothesis are provided in Section 5.

2. An orthogonal parametrization

Note that we can write model (1.1)-(1.3) as

$$\mathbf{Z}_{ij} = \mathbf{g}_{ij} + \epsilon_{ij}, \tag{2.1}$$

where

$$\mathbf{Z}_{ij} = \begin{pmatrix} Y_{ij} \\ X_{ij} \end{pmatrix}, \quad \mathbf{g}_{ij} = g(x_{ij}) = \begin{pmatrix} \alpha_i + \beta_i x_{ij} \\ x_{ij} \end{pmatrix} \quad \text{and} \quad \epsilon_{ij} = \begin{pmatrix} e_{ij} \\ u_{ij} \end{pmatrix},$$

$j = 1, \ldots, n_i$ and $i = 1, \ldots, s$. Thus, $\mathbf{Z}_{ij}$, $j = 1, \ldots, n_i$ and $i = 1, \ldots, s$ are independent and identically distributed with

$$\mathbf{Z}_{ij} \sim N_2(\mu_i; \Sigma_i),$$

where

$$\mu_i = E[\mathbf{Z}_{ij}] = \begin{pmatrix} \mu_{Yi} \\ \mu_{Xi} \end{pmatrix} = \begin{pmatrix} \alpha_i + \beta_i \mu_{xi} \\ \mu_{xi} \end{pmatrix}, \tag{2.2}$$

and

$$\Sigma_i = Cov[\mathbf{Z}_{ij}] = \begin{cases} \begin{pmatrix} \lambda_{xi}\beta_i^2\sigma^2_{ui} + \sigma^2_{ei} & \lambda_{xi}\beta_i\sigma^2_{ui} \\ \lambda_{xi}\beta\sigma^2_{ui} & (\lambda_{xi}+1)\sigma^2_{ui} \end{pmatrix}, & \text{if } \lambda_{xi} \text{ is known,} \\ \begin{pmatrix} \beta_i^2\sigma^2_{xi} + \lambda_{ei}\sigma^2_{ui} & \beta_i\sigma^2_{xi} \\ \beta_i\sigma^2_{xi} & \sigma^2_{xi} + \sigma^2_{ui} \end{pmatrix}, & \text{if } \lambda_{ei} \text{ is known.} \end{cases} \tag{2.3}$$

Further, it can be shown that

$$|\Sigma_i| = \begin{cases} [\lambda_{xi}\beta_i^2\sigma^2_{ui} + (\lambda_{xi}+1)\sigma^2_{ei}]\sigma^2_{ui}, & \text{if } \lambda_{xi} \text{ is known,} \\ [\lambda_{ei}\sigma^2_{ui} + (\beta_i^2 + \lambda_{ei})\sigma^2_{xi}]\sigma^2_{ui}, & \text{if } \lambda_{ei} \text{ is known.} \end{cases} \tag{2.4}$$

The log-likelihood function may be written as

$$l \propto -\frac{1}{2}\sum_{i=1}^{s} n_i \log|\Sigma_i| - \frac{1}{2}\sum_{i=1}^{s}\sum_{j=1}^{n_i}(\mathbf{Z}_{ij}-\mu_i)'\Sigma_i^{-1}(\mathbf{Z}_{ij}-\mu_i), \tag{2.5}$$

where μ_i e Σ_i are as given in (2.2) and (2.3), respectively. It can be shown that the information matrix corresponding to the log-likelihood (2.5) is not diagonal and not so simple to obtain (Arellano-Valle and Bolfarine, 1995), which makes it hard to obtain large sample inference for β_i, particularly testing statistics. One way of aliviating this difficulty is to consider an orthogonal transformation of θ_i, that is, transforming

$$\theta_i = \begin{cases} (\alpha_i, \mu_{xi}, \sigma^2_{ei}, \sigma^2_{ui}, \beta_i)', & \text{if } \lambda_{xi} \text{ is known},\\ (\alpha_i, \mu_{xi}, \sigma^2_{xi}, \sigma^2_{ui}, \beta_i)', & \text{if } \lambda_{ei} \text{ is known}, \end{cases}$$

into $\phi_i = (\phi_{1i}, \phi_{2i}, \phi_{3i}, \phi_{4i}, \beta_i)'$ so that ϕ_{ij}, $i = 1,2,3,4$, are orthogonal to β_i, $i = 1,\ldots,s$. In the case when λ_{ei} is known, the orthogonal parametrization is given in Wong (1989) and when λ_{xi} is known, a solution is given in Bolfarine and Cordani (1993).

The solution presented in Wong (1989) and Bolfarine and Cordani (1993) may be written as

$$\phi_{1i} = \alpha_i + \beta_i\mu_{xi}, \quad \phi_{2i} = \mu_{xi}, \quad \phi_{4i} = \sigma^2_{ui}, \tag{2.6}$$

and

$$\phi_{3i} = \begin{cases} \lambda_{xi}\beta_i^2\sigma^2_{ui} + (\lambda_{xi}+1)\sigma^2_{ei}, & \text{if } \lambda_{xi} \text{ is known},\\ (\beta_i^2 + \lambda_{ei})\sigma^2_{xi} + \lambda_{ei}\sigma^2_{ui}, & \text{if } \lambda_{ei} \text{ is known}, \end{cases} \tag{2.7}$$

$i = 1,\ldots,s$. Considering the parametrization $\phi_i = (\phi_{1i}, \phi_{2i}, \phi_{3i}, \phi_{4i}, \beta_i)$ given above, we have that

$$\mu_i = \mu_i(\phi) = \begin{pmatrix} \phi_{1i} \\ \phi_{2i} \end{pmatrix}$$

and
(2.8)

$$\Sigma_i = \Sigma_i(\phi) = \begin{cases} (\lambda_{xi}+1)^{-1}\begin{pmatrix} \phi_{3i} + (\lambda_{xi}\beta_i)^2\phi_{4i} & (\lambda_{xi}+1)\lambda_{xi}\beta_i\phi_{4i} \\ (\lambda_{xi}+1)\lambda_{xi}\beta_i\phi_{4i} & (\lambda_{xi}+1)^2\phi_{4i} \end{pmatrix}, & \text{if } \lambda_{xi} \text{ is known},\\ (\beta_i^2+\lambda_{ei})^{-1}\begin{pmatrix} \beta_i^2\phi_{3i} + \lambda_{ei}^2\phi_{4i} & \beta_i(\phi_{3i} - \lambda_{ei}\phi_{4i}) \\ \beta_i(\phi_{3i} - \lambda_{ei}\phi_{4i}) & \phi_{3i} + \beta_i^2\phi_{4i} \end{pmatrix}, & \text{if } \lambda_{ei} \text{ is known}, \end{cases}$$

$i = 1,\ldots,s$. Note that, in both cases,

$$|\Sigma_i| = \phi_{3i}\phi_{4i}, \tag{2.9}$$

$i = 1,\ldots,s$.

3. Maximum likelihood estimators and the information matrix

By differentiating the log-likelihood (2.5) given in terms of the orthogonal parameters ϕ and equating the derivatives to zero, we obtain the maximum likelihood estimators, which can be written, when λ_{xi} is known, as

$$\hat{\phi}_{3i} = (\lambda_{xi}+1)S_{YYi} - 2(\lambda_{xi}\hat{\beta}_i)S_{YXi} + (\lambda_{xi}\hat{\beta}_i^2)\hat{\phi}_{4i},$$

$$\hat{\phi}_{4i} = \frac{S_{XXi}}{\lambda_{xi}+1},$$

$$\hat{\beta}_i = (\frac{\lambda_{xi}+1}{\lambda_{xi}})\frac{S_{YXi}}{S_{XXi}},$$

$i = 1,\ldots,s$, where $S_{XXi} = \sum_{j=1}^{n_i}(X_{ij}-\bar{X}_i)^2/n_i$, $S_{YYi} = \sum_{j=1}^{n_i}(Y_{ij}-\bar{Y}_i)^2/n_i$, $S_{YXi} = \sum_{j=1}^{n_i}(Y_{ij}-\bar{Y}_i)^2/n_i$, $\bar{X}_i = \sum_{j=1}^{n_i}X_{ij}/n_i$ and $\bar{Y}_i = \sum_{j=1}^{n_i}Y_{ij}/n_i$, $i = 1,\ldots,s$.

Replacing $\hat{\phi}_{4i}$ and $\hat{\beta}_i$ in $\hat{\phi}_{3i}$, we have that

$$\hat{\phi}_{3i} = (\lambda_{xi}+1)S_{YY.Xi},$$

where

$$S_{YY.Xi} = S_{YYi} - S_{XXi}^{-1}S_{YXi}^2 = S_{YYi}(1-r_{YXi}^2),$$

and

$$r_{YXi} = \frac{S_{YXi}}{(S_{YYi}S_{XXi})^{1/2}},$$

$i = 1,\ldots,s$. When λ_{ei} is known, it follows that

$$\hat{\phi}_{3i} = \frac{\hat{\beta}_i^2 S_{YYi} + 2\lambda_{ei}\hat{\beta}_i S_{YXi} + \lambda_{ei}^2 S_{XXi}}{\hat{\beta}_i^2 + \lambda_{ei}},$$

$$\hat{\phi}_{4i} = \frac{S_{YYi} - 2\hat{\beta}_i S_{YXi} + \hat{\beta}_i^2 S_{XXi}}{\hat{\beta}_i^2 + \lambda_{ei}},$$

$$\hat{\beta}_i = (\frac{S_{YYi} - \lambda_{ei}S_{XXi}}{2S_{YXi}}) + \{(\frac{S_{YYi} - \lambda_{ei}S_{XXi}}{2S_{YXi}})^2 + \lambda_{ei}\}^{1/2}.$$

It can also be shown that the maximum likelihood estimator of $\phi_{Si} = (\phi_{3i}, \phi_{4i}, \beta_i)'$ is given by the solution of the equation

$$\Sigma_i(\hat{\phi}_{Si}) = \Sigma_i,$$

where $\Sigma_i(\hat{\phi}_{Si})$ is as given in (2.8), with ϕ_{Si} replaced by $\hat{\phi}_{Si}$, $i = 1,\ldots,s$. Some estimators may also be given alternative expressions as, for example,

$$\hat{\phi}_{3i} = \begin{cases} (\lambda_{xi}+1)S_{YYi} - (\lambda_{xi}\hat{\beta}_i)^2\hat{\phi}_{4i}, & \text{if } \lambda_{xi} \text{ is known,} \\ \lambda_{ei}S_{YYi} - \hat{\beta}_i S_{YXi}, & \text{if } \lambda_{ei} \text{ is known.} \end{cases}$$

Note also that

$$\hat{\phi}_{3i}\hat{\phi}_{4i} = |\hat{\Sigma}_i| = S_{YY.Xi}S_{XXi}, \tag{3.1}$$

$i = 1, \ldots, s$.

It can be shown that the information matrix corresponding to subpopulation i can be written as (Arellano-Valle and Bolfarine, 1995)

$$\mathbf{K}_i = \begin{pmatrix} \mathbf{K}_{Li} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{Si} \end{pmatrix},$$

that is, $\mathbf{K}_i$ is a block diagonal matrix with

$$\mathbf{K}_{Li} = n_i\mathbf{\Sigma}_i^{-1}$$

and,

$$\mathbf{K}_{Si} = n_i \begin{pmatrix} \frac{1}{2\phi_{3i}^2} & 0 & 0 \\ 0 & \frac{1}{2\phi_{4i}^2} & 0 \\ 0 & 0 & \frac{1}{\sigma_{\beta i}^2} \end{pmatrix},$$

where

$$\sigma_{\beta i}^2 = \begin{cases} \frac{\phi_{3i}}{\lambda_{xi}^2\phi_{4i}}, & \text{if } \lambda_{xi} \text{ is known,} \\ (\frac{\beta_i^2+\lambda_{ei}}{\phi_{3i}-\lambda_{ei}\phi_{4i}})^2\phi_{3i}\phi_{4i}, & \text{if } \lambda_{ei} \text{ is known,} \end{cases}$$

$i = 1, \ldots, s$. Assuming that $n_i/n \to a_i > 0$, $i = 1, \ldots, s$, it follows that

$$\mathbf{K}_i \longrightarrow \mathbf{J}_i = a_i \begin{pmatrix} \mathbf{\Sigma}_i & 0 & 0 & 0 \\ 0 & \frac{1}{2\phi_{3i}^2} & 0 & 0 \\ 0 & 0 & \frac{1}{2\phi_{4i}^2} & 0 \\ 0 & 0 & 0 & \frac{1}{\sigma_{\beta i}^2} \end{pmatrix},$$

$i = 1, \ldots, s$. It is easily checked that the matrix $\mathbf{J}_i$ is positive definite, $i = 1, \ldots, s$. Thus, for the case of s populations, it follows under standard regularity conditions that

$$\sqrt{n}(\hat{\theta} - \theta) \xrightarrow{D} N(0, \mathbf{V}_\theta^{-1}),$$

with $\theta' = (\theta_1, \ldots, \theta_s)$, $\theta_i' = (\phi_i, \beta_i)$ and $\mathbf{V}_\theta = \text{diag}(\mathbf{J}_1, \ldots, \mathbf{J}_s)$, so that, in particular,

$$\sqrt{n}(\hat{\beta} - \beta) \xrightarrow{D} N(0, \mathbf{V}_\beta^{-1}),$$

with $\beta = (\beta_1, \ldots, \beta_s)$, and

$$\mathbf{V}_\beta^{-1} = \text{diag}(\frac{\sigma_{\beta_1}}{a_1}, \ldots, \frac{\sigma_{\beta_s}}{a_s}),$$

where the notation $\text{diag}(d_1, \ldots, d_s)$ stands for an sxs diagonal matrix with diagonal elements given by $d_1, \ldots, d_s$.

4. One-sided hypothesis

Define $h(\theta) = (h_1(\theta), \ldots, h_p(\theta))'$, where $p \leq 5s$, and assume that $H(\theta) = \{(\partial/\partial\theta)h(\theta)\}$ is a continuous function of θ so that $H(\theta)$ has full rank. Then, as $n \to \infty$, we have that

$$\sqrt{n}(h(\hat{\theta}) - h(\theta)) \xrightarrow{D} N(0, H(\theta)V_\theta^{-1}H(\theta)'). \tag{4.1}$$

Let now the vector h be divided into two vectors, h_1 and h_2, consisting of the first p_1 and the remaining $(p-p_1)$ elements of h, respectively. The main interest is to perform a test of the hypothesis $H_0 : h_1(\theta) = 0$, $h_2(\theta) = 0$ against the hypothesis $H_1 : h_1(\theta) \neq 0, h_2(\theta) \geq 0$, with at least one strict inequality in H_1 when $p_1 = 0$. Then, using (4.1), it follows from Gourieux and Monfort (1989) (see also Wolak, 1989), for $c \geq 0$, the null distribution

$$\lim_{n\to\infty} P_r[W \geq c] = \sum_{l=0}^{q} Q(k, l; \Sigma)P[W \geq c],$$

where $q = p - p_1$, W is the likelihood ratio or some asymptotically equivalent statistics such as the Wald or the score statistics, $Q(q, l; \Sigma)$ are the level probabilities, $\Sigma = H_2(\theta)V_\theta H_2(\theta)'$ and $H_2(\theta) = (\partial/\partial\theta)h_2(\theta)$. The null coefficients $Q(q, l; \Sigma)$ depend on the null correlation coefficients associated with the qxq matrix Σ. Closed form expressions for them may be available up to $q \leq 3$. In particular, for $q = 4$, analytical forms are available for $Q(4, l; \Sigma)$, $l = 1, 2, 3$, as given, for example, in Wolak (1987). However, $Q(4, 4; \Sigma)$ must be obtained by numerically integrating a multivariate distribution function. For $q = 3$, these coefficients become

$$Q(3, 0; \Sigma) = \frac{1}{2} - Q(3, 2; \Sigma),$$

$$Q(3, 1; \Sigma) = \frac{1}{2} - Q(3, 3; \Sigma),$$

$$Q(3, 2; \Sigma) = \frac{1}{2}\pi^{-1}\{3\pi - cos^{-1}(\rho_{12.3}) - cos^{-1}(\rho_{13.2}) - cos^{-1}(\rho_{23.1})\}$$

and

$$Q(3, 3; \Sigma) = \frac{1}{4}\pi^{-1}\{2\pi - cos^{-1}(\rho_{12}) - cos^{-1}(\rho_{23})\},$$

where ρ_{ij} is the element of the correlation matrix associated to the matrix Σ_i and $\rho_{ij.t}$ are the corresponding partial correlations, which are defined by

$$\rho_{ij.t} = \frac{\rho_{ij} - \rho_{it}\rho_{jt}}{\sqrt{(1 - \rho_{it}^2)(1 - \rho_{jt}^2)}}.$$

For $q \geq 5$, the probabilities $Q(q, l; \Sigma)$ become intractable as the number of restrictions increase, which has motivated the development of various null approximations for them, particularly in problems related to computing orthant probabilities, in q-variate normal distributions. A Fortran subroutine for computing normal orthant probabilities is provided by Sun (1988).

4.1. Known reliability ratio

We consider in this section the case where $k_{xi} = \sigma_{xi}^2/(\sigma_{xi}^2 + \sigma_{ui}^2)$ is known. Let $\theta_i = (\phi_i, \beta_i)$, $\mathbf{I}_\theta = \text{diag}(\mathbf{I}_1, \ldots, \mathbf{I}_s)$, and assume that $n_i/n \to a_i > 0$, as $n \to \infty$, $i = 1, \ldots, s$. Hence,

$$\mathbf{V}_\theta = \lim_{n\to\infty} \frac{\mathbf{I}_\theta}{n} = \text{diag}(\mathbf{J}_1, \ldots, \mathbf{J}_s),$$

is a positive definite matrix with

$$\mathbf{J}_i = a_i \text{diag}(\Sigma_i, \mathbf{D}),$$

where

$$\mathbf{D} = \text{diag}\left(\frac{1}{2\phi_{3i}^2}, \frac{1}{2\phi_{4i}^2}, \frac{\lambda_{xi}^2 \phi_{4i}}{\phi_{3i}}\right).$$

Thus, from Section 3 it follows that

$$\sqrt{n}(\hat{\beta} - \beta) \xrightarrow{D} N(0, \mathbf{V}_\beta^{-1}),$$

where

$$\mathbf{V}_\beta^{-1} = \text{diag}(\mathbf{J}_1^{-1}(\beta), \ldots, \mathbf{J}_s^{-1}(\beta)),$$

with

$$\mathbf{J}_i^{-1}(\beta) = \frac{\phi_{3i}}{a_i \lambda_{xi}^2 \phi_{4i}},$$

$i = 1, \ldots, s$.

Simple order

The case $s = 4$ is considered first. For testing the simple order, that is, $H_0 : \beta_1 = \ldots = \beta_4$ against $H_1 : \beta_1 \leq \ldots \leq \beta_4$, with at least one strict inequality, which can be written as $H_0 : \mathbf{R}\beta = 0$ against $H_1 : \mathbf{R}\beta \geq 0$, with $\beta = (\beta_1, \ldots, \beta_4)'$ and

$$\mathbf{R} = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix},$$

so that

$$\mathbf{R}\mathbf{V}_\beta^{-1}\mathbf{R}' = \begin{pmatrix} \frac{\phi_{31}}{a_1\phi_{41}\lambda_{x1}^2} + \frac{\phi_{32}}{a_2\phi_{42}\lambda_{x2}^2} & -\frac{\phi_{32}}{a_2\phi_{42}\lambda_{x2}^2} & 0 \\ -\frac{\phi_{32}}{a_2\phi_{42}\lambda_{x2}^2} & \frac{\phi_{32}}{a_2\phi_{42}\lambda_{x2}^2} + \frac{\phi_{33}}{a_3\phi_{43}\lambda_{x3}^2} & -\frac{\phi_{33}}{a_3\phi_{43}\lambda_{x3}^2} \\ 0 & -\frac{\phi_{33}}{a_3\phi_{43}\lambda_{x3}^2} & \frac{\phi_{33}}{a_3\phi_{43}\lambda_{x3}^2} + \frac{\phi_{34}}{a_4\phi_{34}\lambda_{x4}^2} \end{pmatrix},$$

from where it follows that

$$\rho_{ij} = \frac{-1}{\{(1 + \frac{a_j\phi_{3i}\phi_{4j}\lambda_{xj}^2}{a_i\phi_{3j}\phi_{4i}\lambda_{xi}^2})(1 + \frac{a_j\phi_{3,i+2}\phi_{4j}\lambda_{xj}^2}{a_{i+2}\phi_{3j}\phi_{4,i+2}\lambda_{x,i+2}^2})\}^{1/2}}, \tag{4.2}$$

when $|i-j|=1$, $\rho_{ij}=1$, $i=j$ and $\rho_{ij}=0$, $|i-j|>1$, $i=1,\dots,s-2$. It follows from expression (4.2) that when ϕ_{3i} and ϕ_{4i} are constant over the s populations, that is,

$$\phi_{3i}=\phi_3, \quad \text{and} \quad \phi_{4i}=\phi_4, \tag{4.3}$$

$i=1,\dots,s$, then the correlations ρ_{ij} depend only on the weights $a_1^*=a_1\lambda_{x1}^2,\dots,a_s^*=a_s\lambda_{xs}^2$, which are known. In this case, the asymptotic null distribution for the test statistic W for the purpose of testing the homogeneity of the means versus a simple order reduces to the well known chi-bar-squared distribution

$$\lim_{n\to\infty} Pr[W\geq c]=\sum_{l=1}^{s} P(l,g;\mathbf{a}^*)Pr\{\chi_{l-1}^2\geq c\}, \tag{4.4}$$

where $P(l,s;\mathbf{a}^*)=Q(s-1,l-1;\Sigma)$ and $\mathbf{a}^*=(a_1^*,\dots,a_s^*)'$ are the weights. In particular, if we have equal weights, $a_1^*=\dots=a_s^*$, the null probabilities $P(l,s;a^*)$ take the recursive formula $P_S(1,s)=1/g$, $P_S(s,s)=1/s!$ and

$$P_S(i,s)=\frac{1}{s}P_S(i-1,s-1)+\frac{g-1}{s}P_S(i,s-1), \tag{4.5}$$

$i=2,\dots,s-1$. Robertson et al. (1988) have studied the equal-weights null distribution of restricted statistics when the sample sizes are not too different. Their conclusion is to suggest the approximation when the ratio of the largest weight to the smallest weight does not exceed 3.5.

Simple tree order

Another situation of practical importance is that of the simple tree order, namely, $H_0:\beta_1=\dots=\beta_s$ against $H_0:\beta_1\leq[\beta_2,\dots,\beta_s]$, with at least one strict inequality in H_1. In this case, for $s=4$, with $\beta'=(\beta_1,\dots,\beta_4)$ and

$$\mathbf{R}=\begin{pmatrix} -1 & 1 & 0 & 0\\ -1 & 0 & 1 & 0\\ -1 & 0 & 0 & 1\end{pmatrix},$$

it follows that

$$\mathbf{R}\mathbf{V}_\beta^{-1}\mathbf{R}'=\frac{\phi_{31}}{a_1\phi_{41}\lambda_{x1}^2}\begin{pmatrix} 1+\frac{a_1\phi_{32}\phi_{41}\lambda_{x1}^2}{a_2\phi_{31}\phi_{42}\lambda_{x2}^2} & 1 & 1\\ 1 & 1+\frac{a_1\phi_{33}\phi_{41}\lambda_{x1}^2}{a_3\phi_{31}\phi_{43}\lambda_{x3}^2} & 1\\ 1 & 1 & 1+\frac{a_1\phi_{34}\phi_{41}\lambda_{x1}^2}{a_4\phi_{31}\phi_{44}\lambda_{x4}^2}\end{pmatrix}, \tag{4.6}$$

which leads to

$$\rho_{ij}=\frac{1}{\{(1+\frac{a_1\phi_{3,i+1}\phi_{41}\lambda_{x1}^2}{a_{i+1}\phi_{31}\phi_{4,i+1}\lambda_{x,i+1}^2})(1+\frac{a_1\phi_{3,j+1}\phi_{41}\lambda_{x1}^2}{a_{j+1}\phi_{31}\phi_{4,j+1}\lambda_{x,j+1}^2})\}^{1/2}}, \tag{4.7}$$

$i, j = 1, \ldots, s-1$. For testing the simple tree order we have from (4.3) and (4.7) that when ϕ_{3i} and ϕ_{4i} are constant over the s populations, the correlations ρ_{ij} depend only on the weights $a_1\lambda_{x1}^2, \ldots, a_s\lambda_{xs}^2$. In this case, given the complexity of the matrix (4.6), the null asymptotic distribution of W is not chi-bar-squared. However, the asymptotic null distribution is unique. Thus, it is more complicated to implement hypotheses testing of the tree order type in the structural measurement error model considered above.

4.2. Known variances ratio

We consider in this section the case where the variances ratio $\lambda_{ei} = \sigma_{ei}^2/\sigma_{xi}^2$ is known, $i = 1, \ldots, s$. Let $\theta_i = (\phi_i, \beta_i)$, $\mathbf{I}_\theta = \mathrm{diag}(\mathbf{I}_1, \ldots, \mathbf{I}_s)$ and assume that $n_i/n \to a_i > 0$ as $\to \infty$, $i = 1, \ldots, s$. Thus,

$$\mathbf{V}_\theta = \lim_{n\to\infty} \frac{\mathbf{I}_\theta}{n} = \mathrm{diag}(\mathbf{J}_1, \ldots, \mathbf{J}_s)$$

is a positive definite matrix with

$$\mathbf{J}_i = a_i \mathrm{diag}(\mathbf{\Sigma}_i, \mathbf{D}),$$

where

$$\mathbf{D} = \mathrm{diag}\left(\frac{1}{2\phi_{3i}^2}, \frac{1}{2\phi_{4i}^2}, \frac{(\phi_{3i}-\phi_{4i})^2}{\phi_{3i}\phi_{4i}(\beta_i^2+1)^2}\right),$$

$i = 1, \ldots, s$. Thus, it follows that

$$\mathbf{V}_\beta^{-1} = \mathrm{diag}(\mathbf{J}_1^{-1}(\beta), \ldots, \mathbf{J}_s^{-1}(\beta)),$$

where

$$\mathbf{J}_i^{-1}(\beta) = \frac{(\beta_i^2+1)^2\phi_{3i}\phi_{4i}}{(\phi_{3i}-\phi_{4i})^2 a_i},$$

$i = 1, \ldots, s$.

Simple order

In the case of simple order and $s = 4$, that is, $H_0 : \beta_1 = \ldots = \beta_4$ against $H_1 : \beta_1 \leq \ldots \leq \beta_4$, with at least one strict inequality, it follows that

$$\mathbf{R}\mathbf{V}_\beta^{-1}\mathbf{R}$$

$$= \begin{pmatrix} \frac{(\beta_1^2+1)^2\phi_{31}\phi_{41}}{a_1(\phi_{31}-\phi_{41})^2} + \frac{(\beta_2^2+1)^2\phi_{32}\phi_{42}}{a_2(\phi_{32}-\phi_{42})^2} & -\frac{(\beta_2^2+1)^2\phi_{32}\phi_{42}}{a_2(\phi_{32}-\phi_{42})^2} & 0 \\ -\frac{(\beta_2^2+1)^2\phi_{32}\phi_{42}}{a_2(\phi_{32}-\phi_{42})^2} & \frac{(\beta_2^2+1)^2\phi_{32}\phi_{42}}{a_2(\phi_{32}-\phi_{42})^2} + \frac{(\beta_3^2+1)^2\phi_{33}\phi_{43}}{a_3(\phi_{33}-\phi_{43})^2} & -\frac{(\beta_3^2+1)^2\phi_{33}\phi_{43}}{a_3(\phi_{33}-\phi_{43})^2} \\ 0 & -\frac{(\beta_3^2+1)^2\phi_{33}\phi_{43}}{a_3(\phi_{33}-\phi_{43})^2} & \frac{(\beta_3^2+1)^2\phi_{33}\phi_{43}}{a_3(\phi_{33}-\phi_{43})^2} + \frac{(\beta_4^2+1)^2\phi_{34}\phi_{44}}{a_4(\phi_{34}-\phi_{44})^2} \end{pmatrix}$$

from where it follows that

$$\rho_{ij} = -\frac{1}{\left\{\left(1 + \frac{a_j(\beta_i^2+1)^2\phi_{3i}\phi_{4i}(\phi_{3j}-\phi_{4j})^2}{a_i(\phi_{3i}-\phi_{4i})^2(\beta_j^2+1)^2\phi_{3j}\phi_{4j}}\right)\left(1 + \frac{a_j(\beta_{i+2}^2+1)^2\phi_{3,i+2}\phi_{4,i+2}(\phi_{3j}-\phi_{4j})^2}{a_{i+2}(\phi_{3,i+2}-\phi_{4,i+2})^2(\beta_j^2+1)^2\phi_{3j}\phi_{4j}}\right)\right\}^{1/2}}$$

if $|i-j| = 1$, $\rho_{ij} = 1$, if $i = j$ and $\rho_{ij} = 0$, if $|i-j| > 1$. Note that under assumption (4.5), the correlations under H_0 depend only on the weights $a_1, \ldots, a_s$. Thus, the distribution of W reduces to the chi-bar-squared distribution given by (4.3). In particular, if the weights a_i are equal, the null probabilities can be computed from (4.4).

Simple tree order

We discuss now the case of the simple tree order with $s = 4$, that is, $H_0 : \beta_1 = \ldots = \beta_4$ against $H_1 : \beta_1 \leq [\beta_2, \beta_3, \beta_4]$, with at least one strict inequality. In this case, it can be shown that

$$\mathbf{RV}_{\beta}^{-1}\mathbf{R}' = \frac{(\beta_1^2+1)^2\phi_{31}\phi_{41}}{a_1(\phi_{31}-\phi_{41})^2}$$

$$\cdot \begin{pmatrix} 1+\frac{a_1(\beta_2^2+1)\phi_{32}\phi_{42}(\phi_{31}-\phi_{41})^2}{a_2(\phi_{32}-\phi_{42})^2(\beta_1^2+1)^2\phi_{31}\phi_{41}} & 1 & 1 \\ 1 & 1+\frac{a_1(\beta_3^2+1)^2\phi_{33}\phi_{43}(\phi_{31}-\phi_{41})^2}{a_3(\phi_{33}-\phi_{43})^2(\beta_1^2+1)^2\phi_{31}\phi_{41}} & 1 \\ 1 & 1 & 1+\frac{a_1(\beta_4^2+1)^2\phi_{34}\phi_{44}(\phi_{31}-\phi_{41})^2}{a_4(\phi_{34}-\phi_{44})^2(\beta_1^2+1)^2\phi_{31}\phi_{41}} \end{pmatrix}.$$

Using the above results, we can show that

$$\rho_{ij} = \frac{1}{\{(1+\frac{(\beta_{i+1}+1)^2\phi_{3,i+1}\phi_{4,i+1}}{a_{i+1}(\phi_{3,i+1}-\phi_{4,i+1})^2}\gamma)(1+\frac{(\beta_{j+1}^2+1)^2\phi_{3,j+1}\phi_{4,j+1}}{a_{j+1}(\phi_{3,j+1}-\phi_{4,j+1})^2}\gamma)\}^{1/2}},$$

if $|i-j| = 1$ and $\rho_{ij} = 1$, if $i = j$, with

$$\gamma = \frac{a_1(\phi_{31}-\phi_{41})^2}{(\beta_1^2+1)^2\phi_{31}\phi_{41}}.$$

Note that also in this case, the null distribution of W, although unique, can not be simplified under the assumption that $\phi_{3i} = \phi_3$ and $\phi_{4i} = \phi_4$, $i = 1, \ldots, s$.

5. A simulation study

In order to investigate the use of the equal weights probabilities $P_S(l, s)$ as an approximation to $Q(s-1, l-1; \Sigma)$, when the weights $a_1, \ldots, a_s$ are not too different and the differences $\Delta\phi_3 = \max\{\phi_{3i}\} - \min\{\phi_{3i}\}$ and $\Delta\phi_4 = \max\{\phi_{4i}\} - \min\{\phi_{4i}\}$ are not too much greater than zero, we have performed a simulation study whose results are reported in the sequel.

Without loss of generality we have assumed four groups ($s = 4$) and ϕ_{3i} and ϕ_{4i}, $i = 1, \ldots, 4$, as randomly generated variables following an exponential distribution. In the case of known reliability ratios, we have assumed $\lambda_{xi} = 1$, $i = 1, \ldots, 4$. Four possible situations were studied, namely, equal-weights, different weights, $w = (a, 3a, a, 3a)$, U-shaped weights, $w = (3a, a, a, 3a)$ and ∩-shaped weights, $w = (a, 3a, 3a, a)$.

(Tables 1 and 2 about here)

Tables 1-2 report the results for the case of a known reliability ratio, while Tables 3 and 4 present the results when the variances ratio is known. It was computed the frequency distribution of

$$C_{01}(t;\Sigma) = \sum_{l=0}^{3} Q(3,l;\Sigma)Pr\{\chi^2_{l-1} \geq t\},$$

for $t = e_{01}(0.05)$, the equal-weights 5% critical value. 10,000 pseudo random sets of $(\phi_{31},\ldots,\phi_{34},\phi_{41},\ldots,\phi_{44})$ were generated. For each value of $\Delta\phi_3$ and $\Delta\phi_4$, a value of $C_{01}(e_{01}(0.05);\Sigma)$. Those 10,000 probabilities were classified according to the third column of Tables 1-4. Typically, we have found the ranges $0.017 \leq C_{01}(t;\Sigma) \leq 0.077$ and $0 \leq \Delta\phi_3, \Delta\phi_4 \leq 15$.

(Tables 3-4 about here)

From Table 1, we can note an adequate performance of the equal-weights approximation, particularly for $\Delta\phi_3$ and $\Delta\phi_4$ closer to zero. No substantial difference was noticed between the equal-weights and the different weights cases. For $\cup$-shaped-weights (Table 2) the probabilities underestimated, in general, the true probability value. The majority of the cases fell in the interval [0.017, 0.060]. For $\cap$-shaped-weights (Table 2), in contrast with the $\cup$-shaped-weights case, the true probability values are overestimated. The majority of the cases are between 0.040 and 0.077. Finally, for the case of known variances ratio (Tables 3 and 4), we have similar tendencies as those reported in Tables 1 and 2. However, as noted from the tables, good approximations are not necessarily associated with small values of $\Delta\phi_3$ and $\Delta\phi_4$ for the cases of equal and different weights.

To sumarize the results reported above, we can conclude that the equal-weights approximation may be recommended for the cases of equal and different weights, under the assumption that ϕ_{3i} and ϕ_{4i} follow an exponential distribution. This conclusion seems to hold equally weel for both cases, variances ratio and reliability ratio known. However, for the cases of $\cup$-shaped and $\cap$-shaped weights, the approximation does not appear to be adequate.

Acknowledgments. The authors acknowledge partial finantial suport from CNpq and CAPES-Brasil and Colciencias-Colombia.

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Table 1. Frequency distribution for $C_{01}(e_{01}(0.05), \Sigma)$, for 10,000 random generated sets of $(\phi_{31}, \ldots, \phi_{34}, \phi_{41}, \ldots, \phi_{44})$ in each case and by assuming known reliability ratio.

			$\Delta\phi_3$		
Pattern	$\Delta\phi_4$	Interval	0-1	1-3	> 3
		0.01-0.04	0.1424	0.1909	0.2108
	0-1	0.04-0.06	0.6985	0.6117	0.5486
		0.06-0.08	0.1591	0.1973	0.2405
			(597)	(1566)	(370)
		0.01-0.04	0.1928	0.2075	0.2207
Equal-Weights	1-3	0.04-0.06	0.5991	0.5653	0.5506
		0.06-0.08	0.2081	0.2272	0.2287
			(1504)	(3644)	(870)
		0.01-0.04	0.2283	0.2201	0.2090
	> 3	0.04-0.06	0.5598	0.5442	0.5819
		0.06-0.08	0.2120	0.2356	0.2090
			(368)	(904)	(177)
		0.01-0.04	0.1505	0.1670	0.1761
	0-1	0.04-0.06	0.7330	0.6597	0.6132
		0.06-0.08	0.1165	0.1734	0.2107
			(618)	(1569)	(318)
		0.01-0.04	0.1879	0.2035	0.1801
Weights (a,3a,a,3a)	1-3	0.04-0.06	0.6398	0.5911	0.5748
		0.06-0.08	0.1723	0.2054	0.2451
			(1538)	(3661)	(816)
		0.01-0.04	0.2101	0.1989	0.2017
	> 3	0.04-0.06	0.5710	0.5709	0.5708
		0.06-0.08	0.2188	0.2302	0.2275
			(352)	(895)	(233)

Table 2. Frequency distribution for $C_{01}(e_{01}(0.05), \Sigma)$, for 10,000 random generated sets of $(\phi_{31}, \ldots, \phi_{34}, \phi_{41}, \ldots, \phi_{44})$ in each case and by assuming known reliability ratio.

Pattern	$\Delta\phi_4$	Interval	$\Delta\phi_3$ 0-1	1-3	> 3
		0.01-0.04	0.4819	0.4264	0.4299
	0-1	0.04-0.06	0.4680	0.5156	0.4627
		0.06-0.08	0.0501	0.0579	0.1075
			(579)	(1536)	(335)
		0.01-0.04	0.4147	0.4194	0.3945
U-Shaped	1-3	0.04-0.06	0.5278	0.4906	0.4829
Weights		0.06-0.08	0.0575	0.0900	0.1227
			(1548)	(3665)	(905)
		0.01-0.04	0.3869	0.3865	0.4201
	> 3	0.04-0.06	0.5089	0.4994	0.4703
		0.06-0.08	0.1042	0.1140	0.1096
			(336)	(877)	(219)
		0.01-0.04	0.0363	0.0506	0.1373
	0-1	0.04-0.06	0.5310	0.5395	0.4741
		0.06-0.08	0.4327	0.4100	0.3886
			(661)	(1444)	(386)
		0.01-0.04	0.0593	0.0741	0.1071
∩-Shaped	1-3	0.04-0.06	0.5422	0.5192	0.4647
Weights		0.06-0.08	0.3985	0.4068	0.4282
			(1551)	(3700)	(850)
		0.01-0.04	0.1243	0.1016	0.1117
	> 3	0.04-0.06	0.5000	0.4584	0.4681
		0.06-0.08	0.3757	0.4400	0.4202
			(354)	(866)	(188)

Table 3. Frequency distribution for $C_{01}(e_{01}(0.05), \Sigma)$, for 10,000 random generated sets of $(\phi_{31}, \ldots, \phi_{34}, \phi_{41}, \ldots, \phi_{44})$ in each case and by assuming known variance ratio.

Pattern	$\Delta\phi_3$	Interval	$\Delta\phi_4$ 0-1	$\Delta\phi_4$ 1-3	$\Delta\phi_4$ > 3
Equal-Weights	0-1	0.01-0.04	0.2233	0.2382	0.2151
		0.04-0.06	0.5047	0.5192	0.5587
		0.06-0.08	0.2720	0.2426	0.2263
			(636)	(1562)	(358)
	1-3	0.01-0.04	0.2206	0.2398	0.2577
		0.04-0.06	0.5297	0.4860	0.4879
		0.06-0.08	0.2497	0.2741	0.2544
			(1482)	(3615)	(908)
	> 3	0.01-0.04	0.2147	0.2645	0.2105
		0.04-0.06	0.4810	0.4915	0.4895
		0.06-0.08	0.3043	0.2440	0.3000
			(368)	(881)	(190)
Weights (a,3a,a,3a)	0-1	0.01-0.04	0.2289	0.2136	0.2548
		0.04-0.06	0.5261	0.5366	0.4931
		0.06-0.08	0.2349	0.2498	0.2521
			(664)	(1517)	(361)
	1-3	0.01-0.04	0.2288	0.2321	0.2228
		0.04-0.06	0.5284	0.5268	0.5365
		0.06-0.08	0.2427	0.2411	0.2407
			(1512)	(3671)	(835)
	> 3	0.01-0.04	0.2240	0.2347	0.2240
		0.04-0.06	0.5246	0.5306	0.4948
		0.06-0.08	0.2514	0.2347	0.2812
			(366)	(882)	(192)

Table 4. Frequency distribution for $C_{01}(e_{01}(0.05), \Sigma)$, for 10,000 random generated sets of $(\phi_{31}, \ldots, \phi_{34}, \phi_{41}, \ldots, \phi_{44})$ in each case and by assuming known variance ratio.

			$\Delta\phi_3$		
Pattern	$\Delta\phi_4$	Interval	0-1	1-3	> 3
U-Shaped Weights	0-1	0.01-0.04	0.4041	0.4241	0.4091
		0.04-0.06	0.4308	0.4415	0.4519
		0.06-0.08	0.1651	0.1344	0.1390
			(636)	(1495)	(374)
	1-3	0.01-0.04	0.3793	0.3866	0.3961
		0.04-0.06	0.4764	0.4563	0.4611
		0.06-0.08	0.1443	0.1571	0.1429
			(1587)	(3629)	(861)
	> 3	0.01-0.04	0.4392	0.3968	0.4118
		0.04-0.06	0.4332	0.4618	0.4363
		0.06-0.08	0.1276	0.1414	0.1520
			(337)	(877)	(204)
∩-Shaped Weights	0-1	0.01-0.04	0.1186	0.1089	0.1408
		0.04-0.06	0.4876	0.4916	0.4648
		0.06-0.08	0.3937	0.3995	0.3944
			(607)	(1487)	(355)
	1-3	0.01-0.04	0.1184	0.1367	0.1067
		0.04-0.06	0.4723	0.4647	0.4904
		0.06-0.08	0.4094	0.3986	0.4030
			(1622)	(3658)	(881)
	> 3	0.01-0.04	0.1175	0.1140	0.1336
		0.04-0.06	0.4536	0.4870	0.4608
		0.06-0.08	0.4290	0.3990	0.4055
			(366)	(807)	(217)

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