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# An Experiment with Images on Galilean Invariance to Throw Light on the Symmetry of Newton's Laws

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Higher-education textbooks[1–4] state that forces as physical entities are independent of the frame of reference, whenever it is inertial. It can be shown that the acceleration and Newton's laws are invariant by Galilean transformations, but this fact is hardly addressed beyond the formalism. We designed an instructional experiment, based on images from videos of gliders sliding on an air track, to contextualize the conditions of application of Newton's laws, the meaning of Galilean transformations, and the rationale of the second law. The interpretation provided here focuses on possible cause–effect relationships and the assumption of time simultaneity, and is aimed at teachers of introductory-physics courses.

The invariance of acceleration with the inertial frame of reference is approached in physics classes with deductions of expressions showing it; the algebraic simplicity of the "transformations" involved hides the profundity of the concepts and the importance of Galilean invariance in the architecture of Newtonian mechanics. DiSalle[5] addresses the historical evolution of these concepts, which are quite intricate; they involve the principles of relativity, causality, isotropy, and other properties of space and time.[6,7] The effort to elucidate them traces to the ancient Greeks and received contributions from many philosophers and scientists throughout the prehistory and history of physics and mathematics, extending to the present.[5–7]

Mechanics courses often aim for students to internalize these ideas by the resolution of exercises and/or laboratory experiments. According to Izquierdo,[8] the specific objectives of these activities are seldom completely achieved; it is then usual for students not to be able to solve problems that require the transference of these concepts to a diversity of contexts.

Our proposal uses images and a road map in order to contextualize Galilean transformations and provide evidence of a rationalist interpretation of Newton's second law. When dealing with a real system, filmed or not, both the question of uncertainties and the purpose of applying a relevant model need to be faced.[9,10] Reality is not an obstacle to understanding the behavior of physical systems with the use of constructions such as basic laws, but makes the road larger.

The Mechanical EXperiments with Images project (MEXI), http://www.fep.if.usp.br/~fisfoto/, uses information and communication digital technologies to develop online experiments that aim to contextualize topics covered in physics classes.[11,12] The experiment described here was created with the intention of promoting the understanding of coordinate transformations in inertial frames of reference. The images and text needed to perform the experiment can be found in found in the supplementary material,[13] as well as an Appendix with details on the Galilean transformations, the making of the laboratory, and the statistical analysis. Some objectives of this activity are general, explored in all MEXI experiments:

- Obtaining and tabulating experimental data, plotting the corresponding graphs, analyzing them, and inferring results, all of which are expected skills in the curricula of physics courses, whose centrality and the difficulties encountered with them by students have been described by Laverty and Kortemeyer.[14]
- Interpreting graphs, as reported by McDermott et al.[15]; this aim requires instructing students to overcome the difficulties in developing an authentic literacy in the skills of associating graphical representations both with physical concepts and with the physical world, whose acquisition is not spontaneous.
- Contextualizing physical laws, since learning is contextual as emphasized by Bowden et al.,[16] who point at the importance of developing different ways of approaching concepts, so that students interpret them qualitatively, without resorting to the direct application of formulas.

Two specific goals regarding the physical content of the activity described here are:

- Inviting students to the mental exercise of relating the displacements, velocities, and accelerations of a movement in two different frames of reference.
- Contributing to the understanding of the role of Newton's second law as the basic rule of the general and fundamental theoretical model on the movement of bodies.

This last, crucial point requires students to connect interactions (which should characterize relationships between bodies regardless of the chosen reference frame) with a quantity that is also invariant with the frame of reference. When only inertial frames of reference are considered, acceleration fills this requirement.

In order to determine the acceleration of a body in two different frames of reference, we recorded the motion of two gliders moving on an air track, one under the action of a retarding force and the other sliding freely. A measuring tape fixed to the air track and a ruler attached to the free glider serve as the stationary and movable frames of reference, respectively. The positions of both gliders were obtained from the measuring tape, and the position of the accelerated glider was also read in the ruler. It can be checked that an external action (a *retarding* force) produces the same effect—the same acceleration—in both frames of reference. This experimental result helps students to internalize a model of independence of an assumed cause–effect relationship with the frame of reference, as already pointed out by Galileo.[17]

We begin by highlighting the results of the Galilean transformation concerned with this experiment, followed by its description, and then the activities proposed to the students, with an example of the obtained results. The discussion and conclusion sections complete this text.

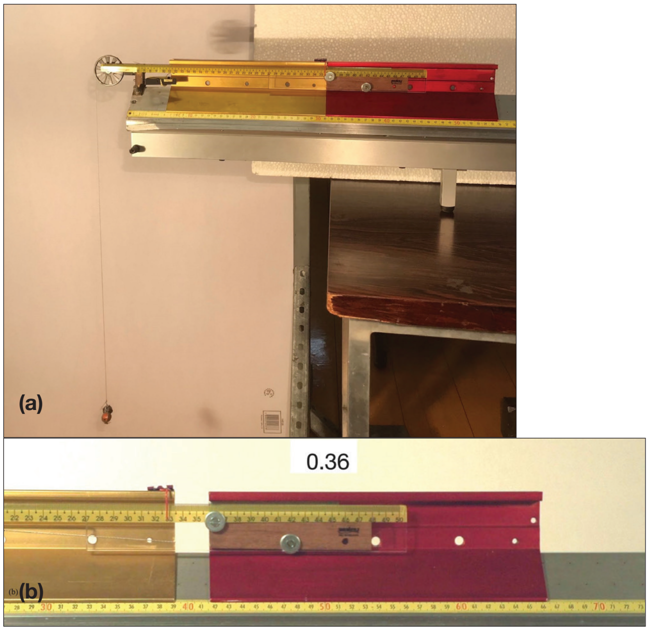


**Fig. 1. Experimental arrangement: (a) distant view; (b) close-up, showing the time code, *t* = 0.36 s. The glider on the right (R) slides freely, and the one on the left (L) accelerates because it is connected to a weight that falls vertically. A complete set of images can be found in the Supplementary Material.[13]**

## Galilean transformations

The Newtonian conception of the motion of bodies in absolute three-dimensional space and time has been classically modeled with the help of vector algebra. From the properties of this chosen mathematical modeling, it follows that the *invariance* of the acceleration with respect to the frame of reference can be "proved" or "deduced." The usual procedure is detailed in section C of the Appendix.[13]

In brief, the Galilean transformation relates the position vectors $\boldsymbol{r}_{\mathrm{P(O)}}$ = and $\boldsymbol{r}_{\mathrm{P(O')}}$ of an object P in reference frames with origins at points O and O′, which are origins of frames of reference stationary and in motion with a constant velocity, respectively ($\boldsymbol{r}_{\mathrm{P(O)}} = \boldsymbol{r}_{\mathrm{P(O')}} + \boldsymbol{r}_{\mathrm{O'(O)}}$). Including the transformation of the times in these frames, $t = t'$, it is algebraically straightforward to arrive at [Appendix,[13] Eq. (C5)]

$$\boldsymbol{a}_{\mathrm{P(O')}} = \boldsymbol{a}_{\mathrm{P(O)}}, \tag{1}$$

where $\boldsymbol{a}$ represents the acceleration. When the frame of reference with the origin at O is inertial, it is possible to use Newton's second law to calculate the acceleration $\boldsymbol{a}_{\mathrm{P(O)}}$, which will then be the same in all inertial frames of reference through out the (classical) universe.

## The experiment

### *Producing a MEXI experiment*

In section A of the Appendix,[13] a more detailed description of the online laboratory can be found, including its objectives and how the experiments are produced. In brief, from videos of a body moving alongside an instrument that allows the measurement of its position, we extract a set of frames from a selected snippet and insert a digital time code in each frame to act as a chronometer. Students read the positions of the body and times in all the images in the set.

As detailed in section B of the Appendix, the following can also be found in the Supplementary Material[13]: the set of the images analyzed here; videos taken from points of view that allow the viewer to understand the experimental arrangement; a spreadsheet; and the laboratory guide, which includes a guiding question.

### *Acceleration and inertial frames*

The arrangement consists of two gliders: one on the left, L, and the other on the right, R, that slide on a horizontal air track. Their positions are read on the measuring tape fixed to the rail, L on the lower right corner and R on the lower left corner, as can be seen in Fig. 1; these are coordinates in the frame of reference of the laboratory (S, since it is stationary). The glider R can slide freely and carries a ruler that gives the position in the movable reference frame (M, attached to the glider R), to measure the position of the glider L independently. A string with a weight that hangs vertically passes over a pulley and is tied to glider L.

At the beginning of the recording, the gliders were at rest with respect to the laboratory and not fixed together, but leaning against each other. They were manually pushed to the left, against an elastic band attached to one end of the air track. After the release, the two gliders start a movement to the right; R continues in uniform movement, while L presents an accelerated movement.

From position-against-time data, speeds are determined as a function of time in the reference systems S and M. From the corresponding velocity graphs, it can be seen that R has constant velocity (therefore, the ruler's frame of reference M is inertial), and also that the acceleration of L is the same in both frames of reference, despite its back-and-forth motion in frame S.

## Students' activities

A different set of images to measure and analyze is given to each team of students, http://www.fep.if.usp.br/~fisfoto/translacao/cinRefIn/. The laboratory guide (at the same link and also in the supplementary material[13]) starts with a guiding question, which does not need to be answered immediately, but it is expected to induce students to reflect on the results they will obtain and the objective of the experiment. After that, it directs students to observe the phenomenon, take and analyze data, and prepare a report. Below, we present details of some of the steps.

### *Measurements and graphics*

The positions of the L and R gliders in the S laboratory frame of reference, $x_{\mathrm{L(S)}}$ and $x_{\mathrm{R(S)}}$, respectively, and of L in the mobile reference system M, $x_{\mathrm{L(M)}}$, are read and recorded in a spreadsheet, as well as the respective times, $t_i$. The standard deviation of the glider position $\sigma_x \approx 0.05$ cm is evaluated from the limit error (LE) that the student assigns to his/her reading as [Appendix[13] Eq. (D1)]

$$\sigma_x \approx \frac{\mathrm{LE}}{2}, \tag{2}$$

a procedure explained in the online Appendix,[13] section D. Students then plot position-vs.-time graphs, which represent a situation similar to the guiding question.

With this data, it is possible to calculate the speeds. For example, the speed of the glider L in the frame of reference S in an instant $t_i$ is estimated by

$$v_{L(S)}(t_i) = \frac{x(t_{i+1}) - x(t_{i-1})}{t_{i+1} - t_{i-1}}, \quad (3)$$

which corresponds to the average speed in the interval $[t_{i-1}, t_{i+1}]$, which closely approximates the velocity at the average instant,

$$\bar{t} = \frac{t_{i-1} + t_{i+1}}{2} = t_i;$$

this last equality results from the selection of frames separated by the same time interval. The standard deviation in the positions, propagated to the speed, is given by

$$\sigma_v = \frac{\sqrt{2}\sigma_x}{t_{i+1} - t_{i-1}},$$

which results in $\sigma_v = 0.7$ cm/s, equal for all times, since the time interval between successive frames is always the same.

Students plot the velocity as a function of time, for both gliders and frames of reference, and determine the trend lines with the respective equations. Figure 2 presents an example of these graphs, in which the measured values of the velocities $v_{R(S)}$, $v_{L(S)}$, and $v_{L(M)}$ can be observed with the respective trend lines.

The standard deviation of the acceleration is calculated using the formula [Appendix,[13] Eq. (E7)]

$$\sigma_a \approx \frac{\sigma_v}{t_f - t_0}\sqrt{\frac{12}{N}},$$

where $t_0$ and $t_f$ are the initial and final instants in the data set, respectively, and $N$ the number of points. This formula is an approximation of the usual expression obtained using the least-squares method when the abscissas are equally spaced, and is given in section E of the Appendix. Substituting the obtained value for $\sigma_v$, the result is $\sigma_a \approx 0.5$ cm/s$^2$.

### *Interpretation and report*

The focus of students' expected interpretation of the experiment is put on realizing that the frame of reference defined by the moving ruler is inertial and that the accelerations of the glider L in the two frames of reference are compatible. This will correspond, within typical uncertainties of the measurement, to the results of most of the teams. Note that it is important to consider the statistical fluctuation of the data, which makes the measurement of a zero acceleration for the mobile glider markedly improbable. In the same line, measurements of the accelerations of glider L in the two frames of reference will give slightly different values. Thus, the consideration of standard deviations is essential for students to arrive at an interpretation of the results obtained as corresponding to those expected under the assumption of Galilean invariance.

The report required from students focuses on the comparison between expected and obtained results, the organized presentation of data in the form of graphs and tables, and the discussion about the very conception of an inertial frame.

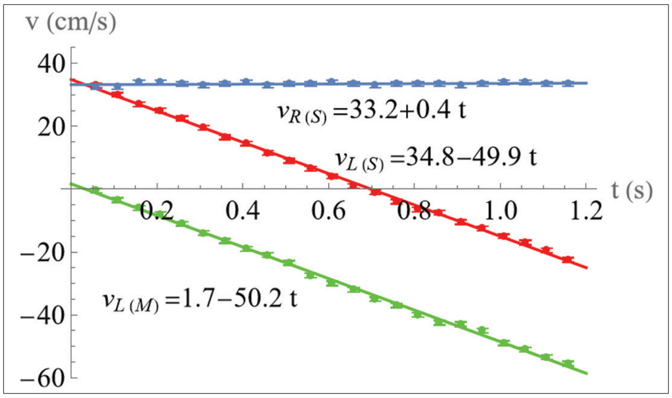


**Fig. 2. Points represent the experimental values of velocities $v_{R(S)}$, $v_{L(S)}$, and $v_{L(M)}$, where the uncertainty bars correspond to one standard deviation. The respective trend lines are the continuous curves. The time equations give the velocity in cm/s when time is in seconds.**

It is intended to call students' attention to the effect that the weight hanging from L is producing; accordingly, a *causal* interpretation of Galilean invariance is suggested, as follows. Velocity of glider L could not be the effect that the weight produces, since it changes with the frame of reference considered. It is then the acceleration of L, which is the same in relation to the reference frames attached to glider R and stationary in the laboratory, that should be considered a measurable *manifestation* of the interaction: the effect of a force that is supposed to be the same in any inertial frame of reference.

## Discussion

Students are expected to measure the time and position coordinates of the gliders, calculate their speeds, construct the corresponding graphs, analyze the trend lines (as in the graph in Fig. 2), and verify that the movement of the glider R is compatible with a uniform movement relative to the laboratory frame of reference. In this respect, the reality of the experiment requires understanding that the measured value of a null quantity will probably not be zero, but smaller than a few standard deviations, which is a necessary competence in the interpretation of many experiments in physics.

Measurements of the positions of glider L in relation to the frames of reference S and M are evidently independent for students. It is then possible to relate the external observation of the experimental situation to the mental exercise of placing oneself in the moving frame of reference and seeing the other glider moving away. The possibility of doing this is a key to understand the scope of Newton's laws. This thought experiment *maps numerous everyday life situations*.

Glider L, when observed in the laboratory frame of reference S, starts its course with positive speed, stops, and turns to the left, due to the weight attached to it; when viewed from frame M, invariably moves away from glider R, and in the arrangement used, its velocity is always negative. Compatibility of the values obtained for the slopes of the lines representing the acceleration to which glider L is subjected confirms that the acceleration of an object, when measured in different inertial frames of reference, has the same value.

The experiment can be conceptualized from the epistemological point of view of *cause–effect* relationships: it is then explained by inferring that weight as a force is the cause of changes in the movement of glider L. Classically, this cause should not depend on which of the two reference systems is adopted to describe motion. And since there is a single cause, a single effect must follow as well. Interaction is considered to be independent of the *kinematics* used to describe movements: its effect, the rate of change in velocity, should, by the same reasoning, be independent of the reference frame and any apparatus adopted for its description, such as physical units.

In order to explore what happens, it is necessary to draw attention to three aspects: i) the velocities and accelerations of L determined in the two reference systems come from completely independent measurements of position, although for simultaneous instants of time; ii) glider R, which carries the ruler, has no connection with the hanging weight; thus, the weight acting on L cannot exert any action on R, where the reference frame M is fixed; and iii) glider L has no connection with glider R once set in motion. The velocities of L are different in the two frames of reference, so this kinematic quantity cannot indicate an *effect* of a constant cause. Assuming (and finding) that glider L's acceleration measured in the two frames of reference is the same suggests that it is reasonable to relate the force on the glider to this kinematic quantity, which indicates changes in the state of motion. Finding that the hanging weight—which is independent of kinematic considerations in the classical frame of reference—produces the same acceleration in both frames provides strong evidence that Newton's second law, with the associated mathematical formalism, constitutes a satisfactory model for the motion of bodies, *completely adequate to the philosophical context* in which it was proposed.

In providing rationalist interpretations, some extra care should be taken with the use of the term *symmetry* with first-year students, which, in natural language, is a quality assigned to an object that does not change its shape when rotated or displaced—under an *active* transformation. These operations are equivalent to transformations of the system of coordinates used to describe the object—a *passive* transformation. Physics extends the definition of symmetry with recourse to the passive view—it is an attribute of any physical *feature* that does not change with a given coordinate transformation. In accordance with this, Galilean symmetry entails the invariance of Newtons' laws when applying a Galilean transform between two different inertial reference frames. In this formal interpretation, which will eventually reappear and expand in more advanced courses, acceleration is termed a physical *invariant.*

Most textbooks limit themselves to discussing Galilean transforms and the invariance of acceleration with the inertial frame as *consequences* of the rules of use of Newton's laws, but do not delve into the fundamental justification of their form. Kleppner and Kolenkow[4] and Kittel et al.[3] proceed in this way when they introduce Newton's laws, but, after exploring their applications, return to the subject in order to discuss special relativity. They show that the Lorentz's transformations connected to relativistic invariance, which overcomes Galilean invariance, reduce to the Galilean transformations when the velocity of light is set to infinite, ensuring the applicability of the classical model to a wide range of situations with macroscopic objects on or near Earth. However, with the focus on relativity and drawing attention to the fact that Newton's laws fail in other situations, it is difficult to discuss in depth the fundamental role of Galilean invariance in the *very formulation* of classical mechanics. Thus, in our opinion, the opportunity is lost to emphasize the importance of the principles of symmetry in the formulation of physical laws at the beginning of an introductory physics course, which is an important moment in the professional training of scientists.

## Conclusion

The experiment presented here serves as a focus to conceptualize what an inertial frame of reference is and to inquire about the invariance of acceleration within such frames. These are, according to our arguments, some of the key points in understanding Newton's laws and in their proper use when modeling. The instructional approach that we suggest to classical mechanics wants to explore the reach of the ideas of invariance and symmetry in the formulation of the laws associated with classical physics models.

Execution of this experiment in introductory physics courses would, in our opinion, contribute to the difficult mental exercise of locating oneself in another reference system, and would help students develop the necessary skills to measure, plot, and interpret values of physical quantities and to consider uncertainties in those values.

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### References

1. R. A. Serway and R. J. Beichner, *Physics for Scientists and Engineers*, 5th ed. (Saunders College Publishing, Philadelphia, 2000).
2. S. Thorton and J. Marion, *Classical Dynamics of Particles and Systems,* 5th ed. (Cengage Learning, New Delhi, 2012).
3. C. Kittel, W. D. Knight, and M. A. Ruderman, *Mechanics* (Berkeley course v 1, McGraw Hill Book, 1965).
4. D. Kleppner and R. Kolenkow, *An Introduction to Mechanics,* 2nd ed. (Cambridge University Press, 2014).
5. R. DiSalle, "Space and time: Inertial frames," *The Stanford Encyclopedia of Philosophy* (Winter 2020 Edition), edited by Edward N. Zalta, https://plato.stanford.edu/archives/win2020/entries/spacetime-iframes/, accessed Sept. 29, 2022.
6. K. M. Browne, "Galilei proposed the principle of relativity, but not the Galilean transformation," *Am. J. Phys*. **88**, 207–213 (2020).
7. P. Moylan, "Velocity reciprocity and the relativity principle," *Am. J. Phys.* **90**, 126–134 (2022).
8. M. Izquierdo, "Enseñar ciencias, una nueva ciencia," *Enseñanza Cienc. Soc. Rev. Invest.* **2007**, 125–138 (2007).
9. R. N. Giere, *Explaining Science: A Cognitive Approach* (University of Chicago Press, Chicago,1988).
10. R. N. Giere, "An agent-based conception of models and scientific representation," *Synthese* **172**, 269–281 (2010).
11. Marcos L. Leite et al., "Teaching friction at university level

through an experiment with images," *Eur. J. Phys.* **42,** 055017 (2021).
12. Nora L. Maidana et al., "The rolling with slipping experiment in the virtual physics laboratory—context-based teaching material," *Phys. Educ.* **51,** 045001 (2016).
13. Readers can access the Supplementary Material at *TPT Online* at https://doi.org/10.1119/5.0128828, in the "Supplementary Material" section.
14. J. Laverty and G. Kortemeyer, "Function plot response: A scalable system for teaching kinematics graphs," *Am. J. Phys.* **80**, 724–733 (2012).
15. L. C. McDermott, M. L. Rosenquist, and E. H. van Zee, "Student difficulties in connecting graphs and physics: Examples from kinematics," *Am. J. Phys.* **55**, 503–513 (1987).
16. J. Bowden et al., "Displacement, velocity, and frames of reference: Phenomenographic studies of students' understanding and some implications for teaching and assessment," *Am. J. Phys.* **60**, 262–269 (1992).
17. S. Ducheyne, "Galileo's interventionist notion of 'cause,'" *J. Hist. Ideas* **67**, 443–464 (2006).

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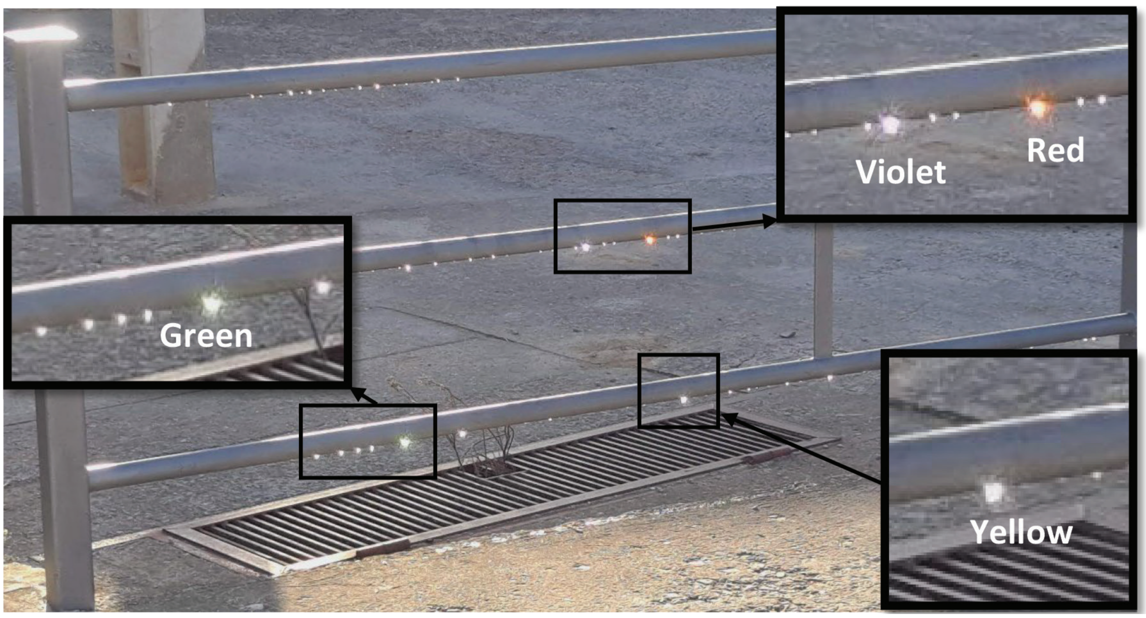


**Fig. 1. Dew drops hanging on a fence with the Sun shining on them from behind.**

# Colorful dew drops

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Dew is a meteorological phenomenon in which the humidity in the air condenses in the form of drops due to a sudden decrease in temperature or contact with cold surfaces. It typically occurs at night and may reveal captivating images during sunrise when observed properly. In Fig. 1, I present a photograph of dew drops with the sun shining from behind. As observed, some of them display colors, a result of light refraction and observation angle, akin to the phenomenon seen in a rainbow.