

The surprising little effectiveness of cooperative algorithms in parallel problem solving

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Abstract. Biological and cultural inspired optimization algorithms are nowadays part of the basic toolkit of a great many research domains. By mimicking processes in nature and animal societies, these general-purpose search algorithms promise to deliver optimal or near-optimal solutions using hardly any information on the optimization problems they are set to tackle. Here we study the performances of a cultural-inspired algorithm – the imitative learning search – as well as of asexual and sexual variants of evolutionary algorithms in finding the global maxima of NK-fitness landscapes. The main performance measure is the total number of agent updates required by the algorithms to find those global maxima and the baseline performance, which establishes the effectiveness of the cooperative algorithms, is set by the blind search in which the agents explore the problem space (binary strings) by flipping bits at random. We find that even for smooth landscapes that exhibit a single maximum, the evolutionary algorithms do not perform much better than the blind search due to the stochastic effects of the genetic roulette. The imitative learning is immune to this effect thanks to the deterministic choice of the fittest string in the population, which is used as a model for imitation. The tradeoff is that for rugged landscapes the imitative learning search is more prone to be trapped in local maxima than the evolutionary algorithms. In fact, in the case of rugged landscapes with a mild density of local maxima, the blind search either beats or matches the cooperative algorithms regardless of whether the task is to find the global maximum or to find the fittest state within a given runtime.

1 Introduction

Today's web-enabled collective intelligence enterprises such as Google and Wikipedia [1] are outstanding implementations of the familiar notion that the solution of important real-world problems is beyond the capability of any single individual and requires the cooperative effort of many individuals. In fact, the benefits of cooperative work to tackle problems that endanger survival have long been explored by nature [2] and nature's diverse strategies have, in turn, been developed into a variety of general-purpose optimization algorithms [3,4].

Perhaps, the first and most popular of these bio-inspired algorithms are the evolutionary algorithms [5] that rely on the well-known biological processes of mutation, selection and recombination to drive a population towards global or near-global maxima of abstract fitness landscapes. In this line, there are also the more recent and less known cultural-inspired algorithms [6,7], where social learning

(imitation) replaces the biological processes of selection and recombination [8,9].

Both the evolutionary and the cultural algorithms are examples of parallel or distributed cooperative problem-solving systems [10,11] in which a number of equivalent agents seek to solve the same problem and the activities of a particular agent offer insight to others about the configuration of the problem space [12]. This is typically achieved through the exchange of information among the agents about their partial success (i.e., their states and their fitness at the current trial) towards the completion of the task.

Here we study the performances of the cultural-inspired imitative learning search [7,13] and of evolutionary algorithms [14,15] for the problem of finding the global maxima of NK-fitness landscapes [16]. The main advantage of using this problem is the possibility of tuning the ruggedness of the landscape, which is roughly determined by the number of local maxima. In addition, the implementations of the cultural and evolutionary algorithms to explore the landscape are straightforward since the state space of the NK-fitness landscapes are binary strings of

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length N . (Hence, in this paper we will use the terms agent and string interchangeably.) We take the blind search, in which the agents simply flip bits at random independently of each other, as a baseline to gauge the efficiency of the cooperative search algorithms. Although our main measure of performance is the number of agent updates required to find the global maxima, which is essentially the computational cost of the search, we considered also other measures such as the probability of finding the global maximum and the fittest string found for a fixed runtime.

The cooperative search algorithms we consider here have only two tunable parameters, viz., the population size M and the bitwise mutation probability u . In particular, in the imitative learning (IL) search the agents imitate the model agent – the fittest agent in the population at the generation – by copying one of its bits. The resulting string then goes through the mutation process where each bit is flipped with probability u . We consider two variants of the evolutionary algorithms, namely, the asexual variant (AGA) that accounts for mutation and selection, and the sexual variant (SGA) that accounts for recombination as well. The blind search corresponds to the choice $u = 1/2$ in any of these algorithms.

Surprisingly, we find that for simple problems in which the fitness landscapes are smooth and exhibit a single maximum, the evolutionary algorithms do not perform much better than the blind search. This is probably because the genetic roulette is not effective to select the fittest agent in the case the agents have similar fitness values. The genetic drift effect becomes stronger as the population size decreases and for small sizes the evolutionary algorithms typically perform worse than the blind search. This finding exposes the pitfall of general-purpose optimization algorithms that use little or no information regarding the optimization problem they are set to solve.

The IL search is immune to genetic drift because the model agent is always chosen as the fittest string in the population. The tradeoff is that IL is strongly affected by the trapping effects of the local maxima in rugged landscapes, so it performs much worse than the blind search in the case of low mutation probability or large population size. Nevertheless, tuning M and u independently for the three cooperative search algorithms indicates that IL is either superior or equivalent to the evolutionary algorithms for rugged landscapes. In addition, we find that already for mildly rugged landscapes, the blind search either outperforms or matches the cooperative algorithms. These conclusions hold true even when the task of the search algorithms is to find the fittest state within a relatively short runtime so that the chances of reaching the global maximum are negligible.

The rest of this paper is organized as follows. Section 2 presents the NK model of rugged fitness landscapes and Section 3 describes the three cooperative search algorithms, as well as the blind search, and introduces our definition of computational cost. Section 4 offers a comparison and discussion of the performances of the search algorithms on smooth and rugged fitness landscapes. Section 5 summarizes our main findings and offers our concluding remarks.

2 NK-fitness landscapes

The NK model [16] is the choice computational implementation of fitness landscapes that has been extensively used to study optimization problems in theoretical immunology, population genetics, developmental biology and protein folding [17]. Although the NK model was widely used to study adaptive evolution as walks on rugged fitness landscapes, its reputation went way beyond the (theoretical) biology realm. In fact, today the NK model is considered a paradigm for problem representation in management research [12,18,19], as it allows the manipulation of the difficulty of the problems and challenges posed to individuals and companies.

More pointedly, the NK model is defined in the space of binary strings of length N and so the parameter N determines the size of the state space, 2^N . The other parameter $K = 0, \dots, N-1$ determines the range of the epistatic interactions among the bits of the binary string and influences strongly the number of local maxima on the landscape. We recall that two bits are said to be epistatic whenever the combined effects of their contributions to the fitness of the binary string are not merely additive. In particular, for $K = 0$ the smooth and additive landscape has one single maximum whereas for $K = N-1$, the (uncorrelated) landscape has on the average $2^N / (N+1)$ maxima with respect to single bit flips [20]. Since the 2^N binary strings can be arranged in a N -dimensional hypercube, we can say that N is the dimensionality of the landscape.

In the NK model, each string $\mathbf{x} = (x_1, x_2, \dots, x_N)$ with $x_i = 0, 1$ has a fitness value $\Phi(\mathbf{x})$ that is given by the average of the contributions of each component i in the string, i.e.,

$$\Phi(\mathbf{x}) = \frac{1}{N} \sum_{i=1}^N \phi_i(\mathbf{x}), \quad (1)$$

where ϕ_i is the contribution of component i to the fitness of string \mathbf{x} . It is assumed that ϕ_i depends on the state x_i as well as on the states of the K right neighbors of i , i.e., $\phi_i = \phi_i(x_i, x_{i+1}, \dots, x_{i+K})$ with the arithmetic in the subscripts done modulo N . Hence K measures the degree of interaction (epistasis) among the components of the bit string. Here we assume, in addition, that the functions ϕ_i with $i = 1, \dots, N$ are distinct real-valued functions on $\{0, 1\}^{K+1}$ and, as usual, we assign to each ϕ_i a uniformly distributed random number in the unit interval [16]. Because of the randomness of ϕ_i , we can guarantee that $\Phi \in (0, 1)$ has a unique global maximum and that different strings have different fitness values.

There are many variants of the NK-model characterized by different interaction structures, i.e., different ways of choosing the K interaction partners of a site (see [21] for a recent review). Here we consider the adjacent neighborhood variant only, which is one of Kauffman's original choices. Whereas in many cases different interaction structures give rise to similar behavior, they affect the computational complexity, and hence the efficiency of searches, in a nontrivial way [22]. In particular, for the adjacent neighborhood variant there is a dynamic programming

algorithm that is polynomial in N and exponential in K . For the random version, where the K interaction partners are chosen randomly, the optimization of the NK-fitness landscapes for $K \geq 2$ is NP-complete [22], which means that the time required to solve some particular instances of the problem using any currently known deterministic algorithm increases exponentially fast with the length N of the strings [23].

As pointed out, for $K = 0$ there are no local maxima and the sole maximum of Φ is easily located by picking for each component i the state $x_i = 0$ if $\phi_i(0) > \phi_i(1)$ or the state $x_i = 1$, otherwise. Increase of the parameter K from 0 to $N - 1$ results in the decrease of the correlation between the fitness of neighboring strings (i.e., strings that differ at a single component) in the state space. For $K = N - 1$, those fitness values are uncorrelated so the NK model reduces to the house-of-cards landscape [20,24]. The simplest way to see this is to consider two neighboring configurations, say $\mathbf{x}^a = (0, 0, \dots, 0)$ and $\mathbf{x}^b = (1, 0, \dots, 0)$, and calculate explicitly the correlation between their fitness. This procedure yields

$$\text{corr}(\Phi(\mathbf{x}^a), \Phi(\mathbf{x}^b)) = 1 - \frac{K+1}{N}, \quad (2)$$

indicating thus that the increase of the dimensionality of the landscape N while the epistasis parameter K is kept fixed produces nearly flat fitness landscapes.

We note that since the functions ϕ_i are random, the ruggedness measures (e.g., the number of local maxima) of a particular realization of a NK landscape is not uniquely determined by the parameters N and K . In fact, the number of local maxima can vary considerably between landscapes characterized by the same values of N and $K > 0$ [16], which implies that the performance of any search algorithm based on the local correlations of the fitness landscape will depend on the particular realization of the landscape. Therefore, in order to produce a meaningful comparison between the search algorithms we must guarantee that they survey the same landscapes. To achieve that we generate and store a set of 100 landscape realizations for each value of N and K , which are then used to test the parallel search algorithms.

3 Parallel search algorithms

Here we describe the three cooperative parallel search algorithms we use to explore the NK-fitness landscapes, namely, the imitative learning (IL) search, the asexual genetic algorithm (AGA) and the sexual genetic algorithm (SGA). In order to render possible a fair comparison between these algorithms we implement a slight variant of the IL: instead of considering mutation and imitation as two independent processes as in the original version [7,13], here we tie these two processes together so that mutation takes place after imitation, thus mimicking the usual evolutionary view of mutations as copy errors. For the sake of completeness, we present also the blind independent search where the agents explore the fitness landscape flipping bits at random.

We consider a well-mixed population of M agents or binary strings of length N that explore the state space of an NK-fitness landscape searching for its unique global maximum. Initially, all binary strings are drawn at random with equal probability for the digits 0 and 1. The M agents are updated synchronously following the rules of the specific search algorithm and the search is halted when one of the agents finds the global maximum. We denote by t^* the time when this happens.

3.1 Imitative learning (IL)

In the imitative learning search, the synchronous update of the M agents proceeds as follows. At time t we first determine the model agent (i.e., the fittest agent in the population) and then we repeat the following update rule M times before incrementing the time to $t + 1$.

The update rule consists of selecting a string at random with uniform probability (the target string) which will then imitate the model string. More pointedly, the model string and the target string are compared and the different bits are singled out. Then one of the distinct bits in the target string is selected at random and flipped so that this bit is now the same in both strings. This imitation procedure is inspired by the mechanism used to model the influence of external media [25–27] in the celebrated agent-based model proposed by Axelrod to study the process of culture dissemination [28]. After imitation the target string goes through the mutation process: each of its N bits is flipped with probability u so that the mean number of flipped bits is Nu . This is the usual mutation operation of the evolutionary algorithms. The resulting string is then passed to the next generation.

As expected, imitation results in the increase of the similarity between the target and the model strings, which may not necessarily lead to an increase of the fitness of the target string. If the target string is identical to the model string, which is not an uncommon situation since the imitation process reduces the diversity of the population, then imitation does not occur and the target string changes due to mutation only. This means that for $u = 0$ the population rapidly becomes isogenic and the search for the global maximum is very likely to fail. We note that a same string may be chosen as target string more than once and, more importantly, that there is no guarantee that the model string will pass to the new generation, as opposed to the elitist selection strategy of some evolutionary algorithms [15].

3.2 Asexual genetic algorithm (AGA)

In the context of evolutionary algorithms, the parallel update of the agents amounts to the usual assumption of non-overlapping generations in which the offspring replace the parental population in a single time step. Accordingly, at time t we repeat the following update procedure M times and then increment the time from t to $t + 1$. As usual in evolutionary algorithms, the two operations – replication and mutation – are applied sequentially [15]. Explicitly, we select a string with probability proportional

to its fitness and then subject it to the mutation operation described before. The resulting string is then passed to the next generation. We note that the same string may be chosen more than once and that new strings are produced by the mutation operation only. The inefficiency of AGA to generate and maintain string diversity is the culprit for its poor performance reported in this paper.

3.3 Sexual genetic algorithm (SGA)

In this case the two operations that comprise the update rule are reproduction and mutation. Reproduction consists in selecting two strings (parents) without replacement and with probability proportional to their fitness. The single offspring of each mating is generated by applying the one point crossover operation: we pick one point $1 \leq n \leq N-1$ at random from each of parents' strings to form one offspring string by taking all bits from the first parent up to the crossover point n , and all bits from the second parent beyond the crossover point. Thus the offspring will always be a recombinant string. Next, the offspring is put through the mutation process as before and then passed to the next generation. This update procedure is repeated M times before incrementing the time variable by one unit. As in the previous algorithms, the same string can be selected more than once as a parent in the reproduction process. In the SGA, string diversity is generated both by the mutation and the crossover operations.

We use the word reproduction rather than replication in the SGA because the offspring may be different from their parents (it is equal in the case the parents are clones) in contrast to the AGA where, except for the bits flipped in the mutation operation, the offspring is always a clone of the parent.

3.4 Blind search (BS)

In the blind search the agents flip bits at random (i.e., $u = 1/2$) and so the ruggedness of the landscape has no effect whatsoever on their chances to find the global maximum, which depends only on the length of the strings N and on the population size M . Since the agents explore the fitness landscape independently of each other, the halting time of the search is given by

$$t^* = \min(t_1^*, \dots, t_M^*), \quad (3)$$

where t_i^* , $i = 1, \dots, M$, are identically distributed independent random variables distributed by the geometric distribution

$$f(t_i^*) = p(1-p)^{t_i^*-1}. \quad (4)$$

Here $p = 1/2^N$ is the success probability. (We recall that a NK-fitness landscape has a unique global maximum.) The probability distribution of the halting time t^* is also a geometric distribution [29] with success probability

$1 - (1-p)^M$, i.e.,

$$P_M(t^*) = \left[1 - (1-p)^M\right] (1-p)^{M(t^*-1)} \quad (5)$$

$$\approx Mp \exp(-Mpt^*), \quad (6)$$

where in the last step we have assumed that the system size is much smaller than the size of the state space, i.e., $Mp \ll 1$. The mean time to find the global maximum is then

$$\langle t^* \rangle = \frac{1}{1 - (1-p)^M} \quad (7)$$

$$\approx \frac{1}{Mp},$$

where, as before, the last step assumes that $Mp \ll 1$. It is also of interest to know the probability that the population of M agents finds the global maximum before or at time t , which is given by

$$\pi_M(t) = \sum_{t^*=1}^t P_M(t^*) = 1 - (1-p)^{Mt}. \quad (8)$$

3.5 Computational cost

We measure the computational cost of a search by the total number of string updates performed by the algorithms until they find the global maximum of the NK-fitness landscape. Hence we ignore the complexity of the update procedures which may greatly impact the actual running time of the algorithms in a computer. Needless to say, this performance measure is very unfavorable to the blind search, which has the simplest and fastest update rule. Of course, the total number of updates is simply Mt^* where t^* is the halting time of the algorithm. Since t^* is typically on the order of 2^N and in order to compare performances for landscapes of different dimensionality, we choose to define the computational cost C of a search as

$$C = Mt^*/2^N. \quad (9)$$

This quantity must be averaged over many searches in a same landscape and the result then averaged over an ensemble of landscapes with the same parameters N and K .

For the blind search, the mean computational cost is simply

$$\langle C \rangle = \frac{Mp}{\left[1 - (1-p)^M\right]}, \quad (10)$$

where the notation $\langle \dots \rangle$ stands for the average over independent searches on the same landscape. Since all landscapes with fixed N and a single global maximum are equivalent from the perspective of the blind search, there is no need to average over different landscapes in equation (10). In particular, for $Mp \ll 1$ we have $\langle C \rangle \approx 1$ and for $Mp \gg 1$ we have $\langle C \rangle \approx M/2^N$. The first and

more realistic regime is characterized by a mean computational cost that is independent of the population size M and corresponds to the case that the halting time t^* decreases linearly with increasing M . The second regime, where $\langle C \rangle$ increases linearly with M , corresponds to the situation $t^* \approx 1$, i.e., the population size is so large that the global maximum is likely to be found already during the assemblage of the initial population.

4 Results

As pointed out, the performances of the cooperative search algorithms are measured by the mean computational cost $\langle C \rangle$, which is estimated by averaging the computational cost defined in equation (9) over 10^3 searches on the same landscape realization. The resulting average cost is then further averaged over the set of 100 landscape realizations with the same values of the parameters N and K . Whereas the results for the cooperative search algorithms are obtained via simulations, the results for the blind search are given by the exact analytical expressions derived before, unless otherwise stated.

4.1 Smooth landscapes

Regardless of the dimensionality N , the NK-fitness landscape with $K = 0$ exhibits a single maximum. Hence, in principle, finding this maximum should be a very easy task to powerful search algorithms such as AGA and SGA. Somewhat surprisingly, this is not the case, as we will see next.

Figure 1 summarizes the performances of the three cooperative search algorithms, viz. IL search, AGA and SGA, as well as of the blind search, for smooth landscapes with fixed dimensionality $N = 12$. The performance of the blind search is used as a baseline to determine the usefulness of the cooperative algorithms. This figure reveals a few surprising results. Although the poor performance of the evolutionary algorithms for small population sizes was somewhat expected since the genetic drift (i.e., the stochastic effects due to the finitude of the population) overwhelms the selective pressure towards fitter strings, it comes as a surprise that those algorithms perform much worse than the blind search for small u . (The AGA does not appear in panel A of Figure 1 because its mean computational cost is greater than 2 for all M .) The IL search does not suffer from the drift effect since it always picks the fittest string to imitate, i.e., this choice is not probabilistic as in the evolutionary algorithms. All the cooperative search algorithms considered exhibit an optimum population size that minimizes the computational cost of the search. This is due to the duplication of work (i.e., the presence of multiple copies of a same string) that occurs for large M and reduces the efficiency of the search.

We note that for low dimensional landscapes the performances of the three cooperative search algorithms are not remarkably better than the baseline set by the blind search. More explicitly, by fine tuning the parameters M and u , the IL search yields a computational cost that is about 20 times lower than the baseline, whereas the

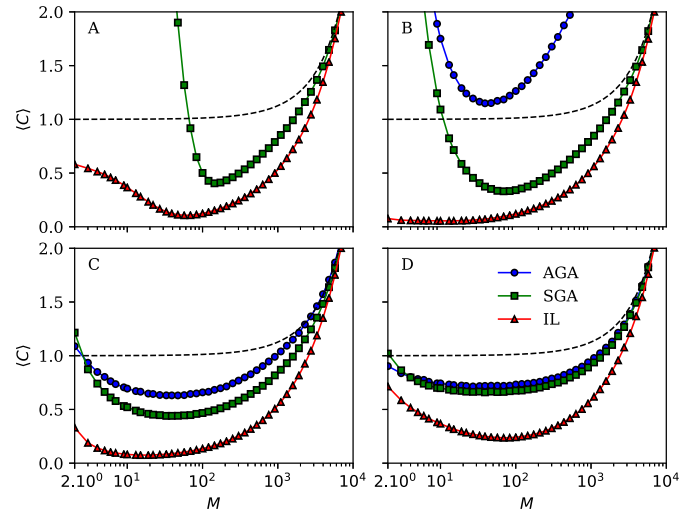


Fig. 1. Mean computational cost $\langle C \rangle$ as function of the population size M for smooth ($K = 0$) landscapes with dimensionality $N = 12$. The bitwise mutation probability is $u = 0.001$ (panel A), $u = 0.01$ (panel B), $u = 0.1$ (panel C), and $u = 0.2$ (panel D). The dashed curve is the analytical prediction for the blind search, equation (10), and the symbols are the simulation results for the imitative learning (IL) search, the asexual (AGA) and the sexual (SGA) genetic algorithms as indicated.

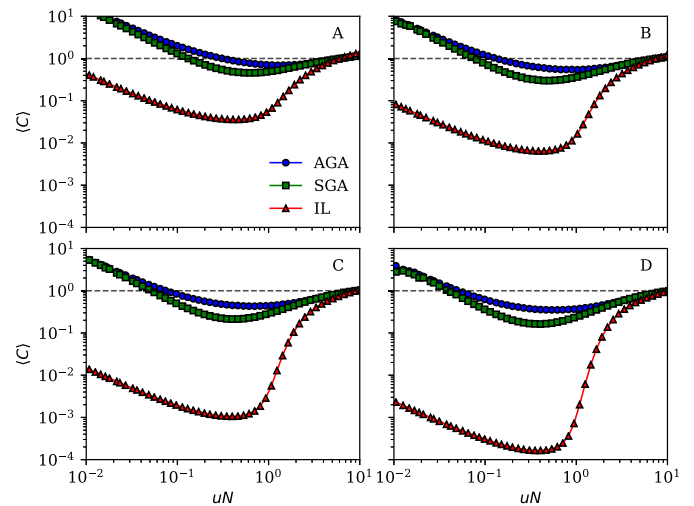


Fig. 2. Mean computational cost $\langle C \rangle$ as function of the mean number of mutations per string uN for smooth ($K = 0$) landscapes with dimensionality $N = 12$ (panel A), $N = 15$ (panel B), $N = 18$ (panel C) and $N = 21$ (panel D). The population size is $M = 10$. The dashed straight lines indicate the performance of the blind search and the symbols are the simulation results for the imitative learning (IL) search, the asexual (AGA) and the sexual (SGA) genetic algorithms as indicated.

SGA and AGA result in a twofold improvement over the baseline only. For purpose of comparison, we note that a greedy heuristics starting from a random string yields $\langle C \rangle = N/2^{N+1} \approx 0.0015$ for $N = 12$ that is about 750 times lower than the baseline.

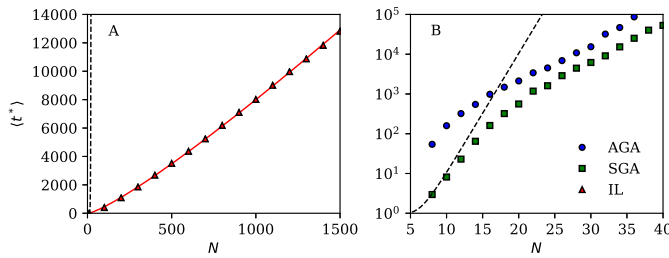


Fig. 3. Scaling of the mean halting time $\langle t^* \rangle$ with the (smooth) landscape dimensionality N for the imitative learning (IL) search (panel A) and for the asexual (AGA) and the sexual (SGA) genetic algorithms (panel B) as indicated. The solid curve fitting the results of the IL search is the function $aN + bN \ln N$ with $a = -2.30$ and $b = 1.49$. The dashed curves are the analytical prediction for the blind search (BS), equation (7). The mean number of mutations per string is $uN = 0.01$ and the population size is $M = 100$.

Figure 2 shows that the performance of the evolutionary algorithms scales very poorly with increasing dimensionality N , whereas the computational cost of the IL search decreases exponentially with increasing N , similarly to the greedy heuristics. In addition, this figure reveals that for each cooperative search algorithm there is a mean number of mutations per string uN that minimizes the computational cost for fixed population size M . This is expected since for $uN = N/2$, the performances of the cooperative search algorithms are identical to the baseline by construction, whereas for $uN \rightarrow 0$ they may not find the solution for some initial population settings, thus leading to the divergence of the mean computational cost.

In order to better quantify the effect of the landscape dimensionality on the performance of the algorithms, Figure 3 shows how the mean halting time $\langle t^* \rangle$ scales with N . For the IL search we find that $\langle t^* \rangle \propto N \ln N$ similarly to the findings for the random adaptive walk [30,31], whereas for the evolutionary algorithms our results are inconclusive due to the limited range of values of N considered. However, our guess is that $\langle t^* \rangle$ increases exponentially with increasing N for those algorithms as we will argue below.

The main reason for the superior performance of the IL over the evolutionary algorithms in finding the single maximum of NK landscapes with $K = 0$, especially for high-dimensional NK landscapes, is that the genetic roulette is very inefficient to pick up the fittest string in a situation where the strings exhibit similar fitness values. In fact, for large N , the fitness difference between states that differ by a few bits is vanishingly small so the evolutionary algorithms are essentially exploring a nearly flat landscape, hence our conjecture that their mean halting times increase exponentially with the landscape dimensionality. A similar result appears in the context of probabilistic adaptive walks, where it has been found that the selection strength must grow logarithmically with N in order that the walker reaches the fitness maximum efficiently. Otherwise, the walker cannot efficiently find the maximum, as the time required to reach it becomes exponential in N with overwhelming probability [32]. We stress, however,

that the evolutionary algorithms perform similarly to or poorer than the blind search for small N only. For large N , they greatly outperform the blind search even though $\langle t^* \rangle$ seems to grow exponentially with N (see Fig. 3). In the Appendix we show that these conclusions hold true for the Ising model of ferromagnetism as well. In particular, for the non-interacting version, where the landscape exhibits a single maximum, we find the same results as those described here for $K = 0$. For the ferromagnetic version, where the landscape exhibits two degenerate maxima, we find that all the cooperative search algorithms considered seem to be exponential in N .

In contrast to the evolutionary algorithms, the imitative learning search always selects the fittest string as the model string, since its selection criterion uses the fitness rank rather than the relative fitness. This is, of course, a huge leverage for smooth landscapes, as shown in Figures 2 and 3, because the fitness value is a reliable indicator of proximity to the global maximum in this case. Next we will see whether this leverage holds for rugged landscapes as well.

4.2 Rugged landscapes

Figure 4 exhibits the performances of the search algorithms for landscapes of fixed dimensionality $N = 12$ and increasing ruggedness, as determined by the increasing values of the epistasis parameter $K = 1, 3, 5, 9$. These results reveal a distinctive characteristic of the IL search on rugged landscapes, namely, the appearance of a peak on the computational cost for large population sizes [7,13]. In contrast, increase of the ruggedness of the fitness landscape does not produce qualitative changes on the dependence of the computational cost on the parameters M and u for the evolutionary algorithms.

The poor performance of the IL search for large population sizes is akin to the groupthink phenomenon of social psychology [33], which occur when everyone in a group starts thinking alike as the result of people putting unlimited faith in a leader (the model agent in our scenario). In the IL search, this phenomenon is due to the rapid loss of diversity of the population that occurs when the model string is a high fitness local maximum and the imitation process starts to produce too many clones of that string. (This explains why this effect does not appear for small M .) This extreme susceptibility to the presence of local maxima is the price that the IL search pays for its good performance on smooth landscapes. The groupthink-like phenomenon can be circumvented by limiting the influence of the model agent using, for instance, low connectivity networks [34] or by allowing the agents to move randomly in an two-dimensional space [35]. It is curious to note that a similar performance degradation was reported in models of Parkinson's law that show the lessening of bureaucratic efficiency when the size of administrative staff exceeds a certain number [36].

As expected, the performances of the cooperative search algorithms degrade gradually as the landscapes become more rugged. In fact, since the state space size is fixed in Figure 4, the density of local maxima increases with

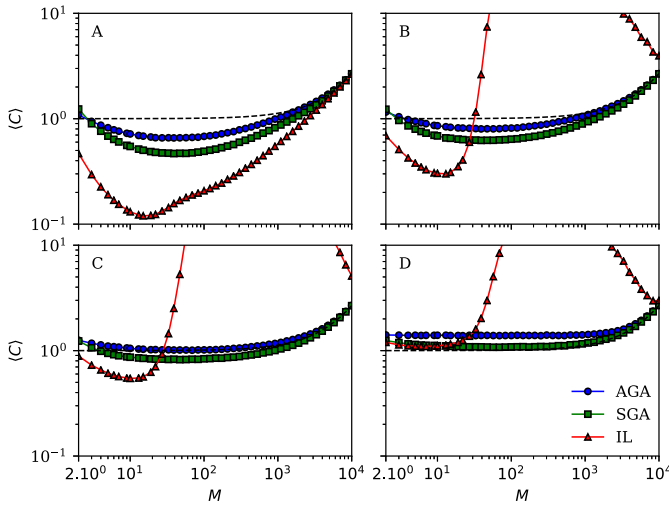


Fig. 4. Mean computational cost $\langle C \rangle$ as function of the population size M for rugged landscapes with fixed dimensionality $N = 12$ and epistasis parameter $K = 1$ (panel A), $K = 3$ (panel B), $K = 5$ (panel C) and $K = 9$ (panel D). The dashed curve is the analytical prediction for the blind search, equation (10), and the symbols are the simulation results for the imitative learning (IL) search, the asexual (AGA) and the sexual (SGA) genetic algorithms as indicated. The bitwise probability of mutation is $u = 0.1$.

increasing K . It is interesting that for the rugged landscapes with $N = 12$ and $K = 9$ (panel D of Fig. 4), for which the fitness correlation between neighboring strings is $1/6$, the blind search equals or outperforms the cooperative search algorithms for all population sizes.

Figure 5 shows the performances of the search algorithms when the parameters N and K increase such that the correlation between the fitness of neighboring strings, equation (2), is kept fixed to $2/3$. The results lay bare the reliance of the cooperative search algorithms on the mutation operation to produce diversity at the bit level and that the IL search is much more susceptible to lose diversity at that level and to get trapped in the local maxima. We recall that if all strings exhibit the same bit at a given position then, in the absence of mutations, this bit will be fixed in the population since neither the crossover nor the imitation operation can flip it. This is a major hurdle if the fixed bit is not the correct one, i.e., the corresponding bit in the global maximum, as indicated by the divergence of the computational cost as $u \rightarrow 0$ for all three cooperative algorithms.

Another interesting result exhibited in Figure 5 is that the task seems to become easier as the dimensionality of the landscape increases. This is so because the density of local maxima actually decreases as both N and K increase such that the ratio $(K + 1)/N$ is kept constant. For example, for the ensemble of 100 landscapes with $N = 12$ and $K = 3$ we find that the mean number of maxima is 27.18 and the density of maxima is 0.007, whereas for the landscapes with $N = 21$ and $K = 6$ the mean number of maxima is 3074.52, which corresponds to a density of 0.001 maxima per state. In fact, in the limits of large K and N such that the ratio $\alpha = K/N$ is

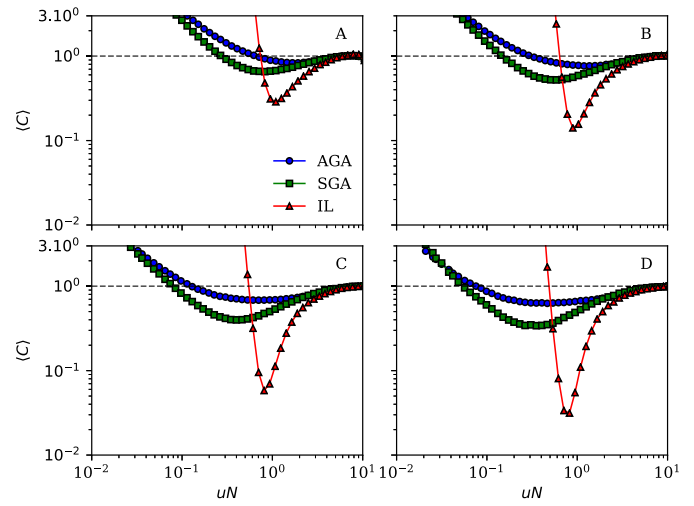


Fig. 5. Mean computational cost $\langle C \rangle$ as function of the mean number of mutations per string uN for rugged landscapes with parameters $N = 12, K = 3$ (panel A), $N = 15, K = 4$ (panel B), $N = 18, K = 5$ (panel C) and $N = 21, K = 6$ (panel D). The population size is $M = 10$. The dashed straight lines indicate the performance of the blind search and the symbols are the simulation results for the imitative learning (IL) search, the asexual (AGA) and the sexual (SGA) genetic algorithms as indicated.

fixed, the probability that a random string is a local maximum can be analytically shown to decay algebraically as $N^{-1/\alpha}$ [21], so that the density of local maxima must vanish for large N . This means that, as N and K increase, the number of paths (i.e., learning trajectories) leading to the global maximum that circumvent the local maxima increases very fast [37], explaining thus the qualitative similarity of the performances of the algorithms illustrated in Figure 5 for rugged landscapes to those in Figure 2 for smooth landscapes. In this line, it is interesting to note that the local maxima of the NK-fitness landscapes are strongly clustered in the state space [38].

One may argue that since in real-world problems the state space is very large, the odds that a general-purpose search algorithm reaches the optimal solution in all runs within a feasible runtime are negligible for NP-complete problems. Hence, since it is impractical to estimate the mean halting time $\langle t^* \rangle$ for problems with a very large state space, the computational costs exhibited in, say, Figure 4 may not be indicative of the performance of the cooperative search algorithms in the real-world scenario where the duration of the search is limited a priori. We can partially assess the significance of this claim by estimating the fraction of runs for which the global maximum is found when the duration of the search is fixed to t . In the case of the blind search, this fraction is nothing but the probability $\pi_M(t)$ that the population of M agents finds the global maximum before or at time t given in equation (5).

Accordingly, Figure 6 shows $\pi_M(t)$ for the challenging case exhibited in panel D of Figure 4 where the blind search either matches or outperforms all three cooperative search algorithms. The results indicate that this outcome holds true for searches of limited duration as well. We note

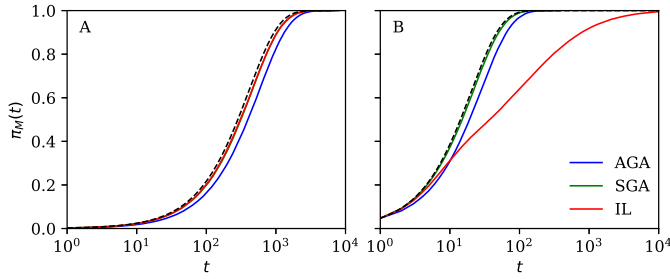


Fig. 6. Fraction of runs for which the global maximum is found before or at time t for the imitative learning (IL) search, the asexual (AGA) and the sexual (SGA) genetic algorithms as indicated. The dashed curve is the analytical prediction for the blind search, equation (8). The bitwise mutation probability is $u = 0.1$ and the population size is $M = 10$ (panel A) and $M = 200$ (panel B). The curves for the IL search and SGA are indistinguishable in panel A. The parameters of the NK-fitness landscapes are $N = 12$, $K = 9$ so that the mean number of mutations per string is $uN = 1.2$.

that, similarly to the observed in the study of searches of unrestricted duration, the short-time performance of the IL search is strongly dependent on the population size whereas the evolutionary algorithms are little influenced by this parameter.

In a similar vein, an argument that is usually put forth in support of general-purpose optimization algorithms is that they excel at finding solutions that are ‘sufficiently good’ for practical purposes within realistic runtimes. In this line of reasoning, finding optimal or near-optimal solutions were never the intent of the bio and cultural inspired algorithms. This is a fair point, though we think it is instructive, to say the least, to learn how those algorithms fare when their task is to find the optimal solution, as done in this paper. However, we can easily check the validity of this argument by measuring the expected value of the fittest solution found by the algorithms up to time t , which we denote by $\Phi_{\max}(t)$. More pointedly, for a fixed landscape we run 10^3 searches and record the fittest solution found up to time t for each run. Then we average over these fitness values to obtain $\Phi_{\max}(t)$ for a fixed landscape. Since we also need to average this result over different landscapes, we consider a more appropriate, landscape-independent measure given by the ratio $\Phi_{\max}/\Phi_{\text{global}} \leq 1$, where Φ_{global} is the fitness of the global maximum.

Accordingly, Figure 7 shows the ratio $\Phi_{\max}/\Phi_{\text{global}}$ for the challenging rugged landscapes of panel D of Figure 4. We note that, in this case, the results for the blind search were obtained by simulations as the analytical results of Section 3.4 deal with the distributions of the absorbing times only. These illuminating results show no evidence that the cooperative algorithms are more effective than the blind search in finding good solutions in searches of short duration. In fact, the very same conclusions hold true when the task changes from finding the global maximum in the minimum runtime to finding high fitness states within a fixed runtime. Since for large M there is a good chance of finding high fitness states already during the assemblage of the initial population, we have $\Phi_{\max}(0)/\Phi_{\text{global}} \rightarrow 1$ as M increases, which explains the

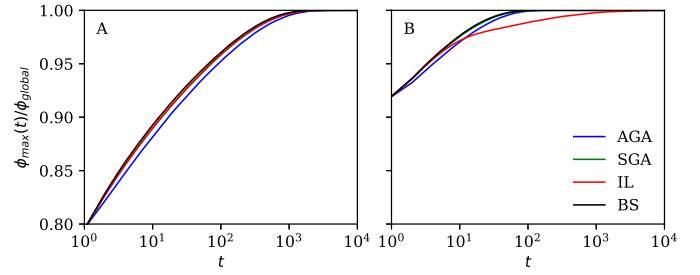


Fig. 7. Ratio between the fittest state found up to time t , Φ_{\max} , and the global optimum fitness, Φ_{global} , for the imitative learning (IL) search, the blind search (BS), the asexual (AGA) and the sexual (SGA) genetic algorithms as indicated. The bitwise mutation probability is $u = 0.1$ and the population size is $M = 10$ (panel A) and $M = 200$ (panel B). The curves for the IL search and SGA are indistinguishable in panel A, whereas the curves for the BS and SGA are indistinguishable in panel B. The parameters of the NK-fitness landscapes are $N = 12$, $K = 9$ so that the mean number of mutations per string is $uN = 1.2$.

great difference in the ranges of variation of the ratio $\Phi_{\max}/\Phi_{\text{global}}$ between the two panels of Figure 7. Therefore our results support the (actually) conservative stance that the algorithms that reach the global maxima more rapidly are also more likely to visit the fittest states within a fixed runtime.

5 Conclusion

The imitative learning (IL) search was introduced originally as a model to study quantitatively the potential of imitation as the underlying mechanism – the critical connector – of collective brains [7,13]. A natural baseline to assess that potential is a scenario where the agents explore the problem space independently of each other, performing a sort of blind search on that space. The finding that IL search performs much worse than the baseline for certain values of the control parameters and the apparent similarities between the IL and the popular evolutionary algorithms gave rise to the question of whether similar negative results could hold for those algorithms as well. The aim of this paper is to address this issue by challenging the cultural and biologically inspired cooperative search algorithms to beat the blind search in the task of finding the unique global maximum of NK-fitness landscapes [16].

In addition to the IL search, we consider two evolutionary algorithms, viz., the asexual (AGA) and the sexual (SGA) genetic algorithms. The difference between them is that the former lacks the crossover mechanism to generate diversity in the string population. The main performance measure considered here is the total number of agent updates required by the algorithms to find the global maximum of the fitness landscape. Our conclusion is that the cooperative search algorithms are only marginally superior to the blind search in the exploration of rugged NK-fitness landscapes. Moreover, the evolutionary algorithms do not exhibit the catastrophic performance of the

imitative learning search that is observed for certain population sizes and that has been likened to the groupthink phenomenon of social psychology [33].

Within a genetic perspective, the model string in the IL search may be thought of as a mandatory parent in all mates at a given generation, which contributes a single bit to the offspring. In the absence of mutations, this offspring is identical to the other parent, namely, the randomly chosen target string, except for the bit that comes from the model string. In addition, that bit is not random since it must differ from the original bit of the target string. In fact, since the imitation process was based on Axelrod's model of cultural dissemination [28], the IL search follows the rules of cultural, rather than of genetic, evolution.

Our definition of computational cost as, essentially, the total number of updates performed by the algorithms to find the optimal solution begets a humbler perspective on the power of general-purpose search algorithms (see [39] for another approach to this issue that leads to a similar conclusion). In particular, we find that even for simple problems with no local maxima (i.e., $K = 0$), the evolutionary algorithms are not much better than the blind search provided the landscape dimensionality is not too large. In this simple scenario, the IL exhibits the best performance since it always guarantees the selection of the fittest string as a model to be imitated, thus avoiding the genetic drift that hinders the performance of the evolutionary algorithms (see Figs. 2 and 3). The robustness of these findings are confirmed through the analysis of the Ising model of ferromagnetism offered in the Appendix.

The prospects of the cooperative search algorithms are somewhat gloomy in the case of rugged landscapes plagued with local maxima that may act as traps for the evolving population. In fact, even for mildly rugged landscapes (see panel D of Fig. 4) the blind search either outperforms or matches the cooperative search algorithms regardless of whether the duration of the search for the global maximum is limited a priori or not. This conclusion holds true also in the case the criterion to evaluate the algorithms is the fittest state found within a fixed runtime.

In many respects, the unenthusiastic performances of the cooperative search algorithms reported here, as compared with the baseline set by the blind search, are in line with the very notion of NP-completeness class [23] in the sense that we should not expect any algorithm to change substantially the scale of the time needed to find the optimal solution for problems in that class. (Here we assume without proof that the adjacent neighborhood variant of the NK-model with $\alpha = K/N > 0$ fixed is NP-complete.) Overall we find that if the population size M and the bit-wise mutation probability u are tuned so as to optimize the performance of each cooperative search algorithm separately, then the IL surpasses the evolutionary algorithms, especially for rugged landscapes characterized by a low density of local maxima as illustrated in Figure 5.

The somewhat unexpected conclusions of our study calls into question the efficiency of the mechanisms of selection and recombination to explore NK-fitness landscapes. This is probably due to a peculiarity of these

landscapes, viz., they become flatter as their dimensionality increase, thus greatly impairing the capacity of the genetic roulette to select the best agents for the next generation. We stress, however, that the performance advantage of the evolutionary algorithms over the blind search actually increases with the landscape dimensionality (see Fig. 3). As shown in the Appendix, the same conclusions hold true for a far more popular fitness landscape, the Ising model of ferromagnetism. We see the poor performance of the evolutionary algorithms on single-peak fitness landscapes as the price general-purpose algorithms have to pay to deliver good solutions using hardly any information on the optimization problems they are set to tackle. Like the blind search, for those algorithms the scaling on the landscape dimensionality of the time required to find the global maxima is not very sensitive to the topology of the landscape. This contrast with the imitative learning search that performs almost optimally for single-peak landscapes but exhibits a catastrophic performance for landscapes with local maxima if its parameters are not tuned properly. There is indeed no free lunch in optimization.

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Author contribution statement

All the authors contributed equally to the paper.

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Appendix A

In this Appendix we address the robustness of our findings about the performance of the cooperative search algorithms on smooth fitness landscapes. Although the NK-model of rugged landscapes is widely used in a variety of disparate disciplines, such as evolutionary biology, physics and economics, the wanting performance of the evolutionary algorithms reported in this paper may raise concern about the usefulness of the NK model to describe realistic landscapes. To dispel these doubts, here we briefly examine the performance of the cooperative search algorithms on a very popular fitness (or energy) landscape, viz., the Ising model of ferromagnetism [40].

We begin with the non-interaction version of the Ising model where the fitness (the opposite of the energy) of the system specified by the binary string $\mathbf{x} = (x_1, x_2, \dots, x_N)$

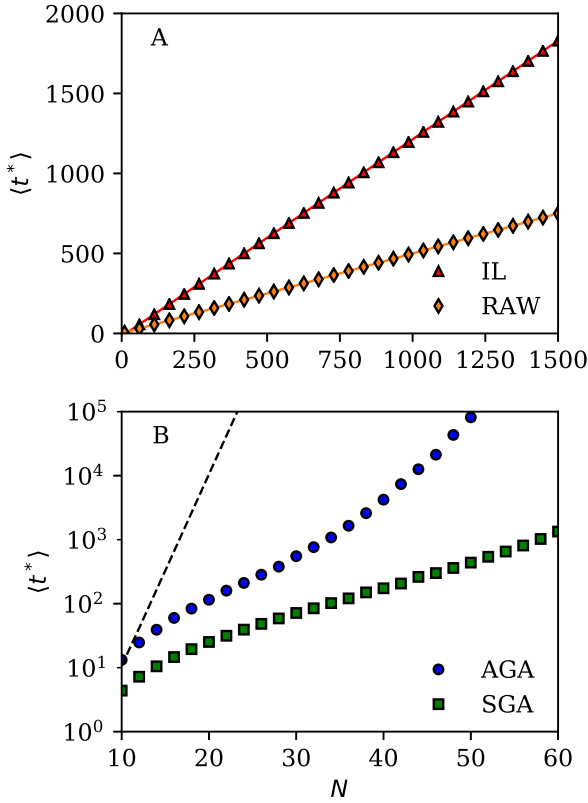


Fig. A.1. Scaling of the mean halting time $\langle t^* \rangle$ on the landscape dimensionality N for the non-interacting Ising model. The results for the imitative learning (IL) search and the random adaptive walk (RAW) are shown in panel A, whereas the results for the asexual (AGA) and the sexual (SGA) genetic algorithms are shown in panel B, as indicated. The solid curve fitting the result of the IL search is the function $aN + b$ with $a = 1.23$ and $b = -18.13$, whereas the solid curve fitting the result of the RAW is the function $N/2$. The dashed curve is the analytical prediction for the blind search (BS), equation (7). The mean number of mutations per string is $uN = 0.1$ and the population size is $M = 100$.

with $x_i = 0, 1$ is

$$F_{ni} = \sum_{i=1}^N (2x_i - 1) + N + 1. \quad (\text{A.1})$$

Here the spin variable is $s_i = (2x_i - 1) = \pm 1$ and we have added the factor $N + 1$ into the fitness definition so as to guarantee that it is positive for all configurations and hence that the genetic roulette behind the evolutionary algorithms can be applied straightaway. Clearly, this fitness function has a single maximum at $x_i = 1 \forall i$ and so it offers an alternative to the NK-fitness landscape with $K = 0$.

Figure A.1 shows how the mean halting time $\langle t^* \rangle$ scales with the landscape dimensionality N for the fitness function defined in equation (A.1). In addition to the four search algorithms discussed in the paper, this figure shows the performance of the random adaptive walk (RAW) as well [20,38]. This search algorithm considers a single agent

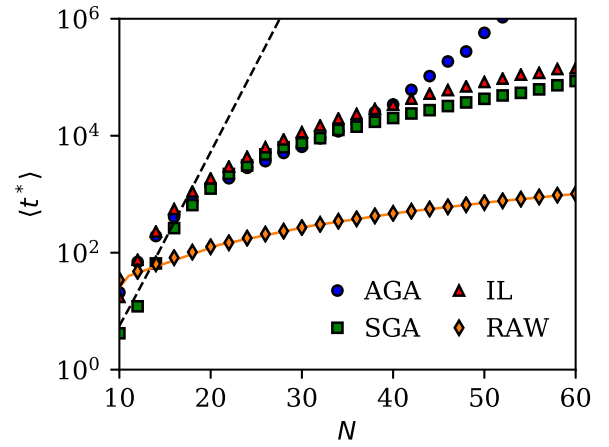


Fig. A.2. Scaling of the mean halting time $\langle t^* \rangle$ on the landscape dimensionality N for the one-dimensional ferromagnetic Ising model. The solid curve fitting the result of the RAW is the function $0.41N^2$ whereas the dashed curve is the analytical prediction for the blind search (BS), equation (7), with $p = 1/2^{N-1}$. The mean number of mutations per string is $uN = 0.1$ and the population size is $M = 100$.

(walker) that at each time step flips a randomly chosen bit that does not result in a fitness decrease. For the fitness function (A.1), this strategy is equivalent to the greedy heuristics, since all bit flips that do not decrease the fitness increase it by the same amount. Hence we have $\langle t^* \rangle = N/2$ for the RAW. Interestingly, we find that the mean halting time of the IL search also scales linearly with N , i.e., $\langle t^* \rangle \propto N$. Similarly to the findings summarized in Figure 3 for the case $K = 0$, the results for the evolutionary algorithms are inconclusive due to the limited range of values of N considered. Nonetheless, it is clear that the performances of those algorithms scale very poorly with the landscape dimensionality. Therefore, the conclusions drawn from the analysis of smooth NK-fitness landscapes hold true for the well-known case of the non-interacting Ising model landscape.

We consider now the one-dimensional ferromagnetic Ising model where the fitness of the system specified by the binary string \mathbf{x} is

$$F_f = \sum_{i=1}^N (2x_i - 1)(2x_{i+1} - 1) + N + 1 \quad (\text{A.2})$$

with $x_{N+1} \equiv x_1$. This fitness function has two degenerate maxima, viz., $x_i = 1 \forall i$ and $x_i = -1 \forall i$. Moreover, it has several plateaus where a given configuration and its N neighbors exhibit the same fitness. Actually, the reason we tolerate flips that do not increase fitness in the RAW is to allow the walker to escape those plateaus. The results summarized in Figure A.2 show the disruptive effect of the two degenerate maxima on the performance of all the cooperative search algorithms considered. Since the fitness function (A.2) has no local maxima, the reason for the poor performance of the IL search is probably the frequent alternation of the model agent between strings in the neighborhoods of the two degenerate maxima. The

back and forth motion between those regions, as well as the blind search on the fitness plateaus, are the reasons that the halting time of the RAW scales with N^2 , rather than with N , for this landscape.

Finally, we note that the reason the RAW performs better than the cooperative search algorithms on the two landscapes considered in this Appendix is due solely to the absence of local maxima on those landscapes. We stress that despite the somewhat wanting performances of the cooperative search algorithms regarding their scaling with the landscape dimensionality, they do much better than the blind search in the case of high dimensional landscapes.

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