

**Drell-Yan production in universal theories beyond dimension-six SMEFT**Tyler Corbett<sup>1,\*</sup>, Jay Desai<sup>2,†</sup>, O. J. P. Éboli<sup>3,‡</sup>, M. C. Gonzalez-Garcia<sup>2,4,5,§</sup>,  
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We study Drell-Yan production in universal theories consistently including effects beyond dimension six in the Standard Model effective field theory (SMEFT). Within universal SMEFT and with  $C$  and  $P$  conservation we find that 11 dimension-eight operators contribute in addition to the six contributing at dimension six. We first work in an operator basis in which operators with higher derivatives of the bosonic fields have been rotated by equations of motion in favor of combinations of operators involving SM fermion currents. We derive the general form of the amplitudes consistently in the expansion to  $\mathcal{O}(\Lambda^{-4})$  and identify eight combinations of the 17 Wilson coefficients which are physically distinguishable by studying the invariant mass distribution of the lepton pairs produced. We then introduce an extension of the parametrization of universal effects in terms of oblique parameters obtained by linearly expanding the self-energies of the electroweak gauge bosons to  $\mathcal{O}(q^6)$ . It contains 11 oblique parameters of which only eight are generated within SMEFT at dimension eight:  $\hat{S}$ ,  $\hat{T}$ ,  $W$ ,  $Y$ ,  $\hat{U}$ ,  $X$ , plus two additional which we label  $W'$  and  $Y'$  and show how they match at linear order with the eight identified combinations of operator coefficients. We then perform a combined analysis of a variety of LHC data on the neutral- and charged-current Drell-Yan processes with the aim of constraining the eight combinations. We compare and combine the LHC bounds with those from electroweak precision  $W$  and  $Z$  pole observables which can only provide constraints in four directions of the eight-parameter space. We present the results in terms of limits on the eight effective Wilson coefficients as well as on the eight oblique parameters. In each case, we study the dependence of the derived constraints on the order of the expansion considered.

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**I. INTRODUCTION**

The large statistics collected by the CERN Large Hadron Collider (LHC) in its different runs have allowed for precise tests of the Standard Model (SM) predictions as well as

searches for new physics. A particular place in this quest is held by dilepton production, the so-called Drell-Yan (DY) process [1], which can proceed either via neutral current (NC) or charged current (CC)

$$pp \rightarrow \ell^+ \ell^- \quad \text{and} \quad pp \rightarrow \ell^\pm \nu_\ell$$

with  $\ell = e, \mu$ . Involving only leptons in the final state, this process provides a clean environment for both experimental studies and theoretical predictions. The LHC experimental collaborations have taken advantage of this to perform precision SM tests [2–6] and searches for new resonances [7–10]. Presently there is no data that is at variance with the SM, ergo there may exist a mass gap between the electroweak and the new physics scales. In such a scenario, hints on the new physics can first manifest through

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deviations from the SM predictions. In this case, it is natural to employ effective field theory (EFT) as a model-independent approach to analyze the experimental results.

Under the minimal assumption that the scalar particle observed in 2012 [11,12] is, in fact, part of an electroweak doublet, the  $SU(2)_L \otimes U(1)_Y$  symmetry can be linearly realized and the resulting EFT is the so-called Standard Model EFT (SMEFT). In this framework, deviations from the SM predictions are parametrized as higher order operators

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{n>4,j} \frac{f_{n,j}}{\Lambda^{n-4}} \mathcal{O}_{n,j}, \quad (1.1)$$

where  $\Lambda$  is a characteristic energy scale and  $\mathcal{O}_{n,j}$  are higher dimension operators. At the LHC the first sizable contributions are of dimension six, i.e.  $\mathcal{O}(\Lambda^{-2})$ . It is well known that there are 59 independent dimension-six operators [13] leading to 2499 arbitrary Wilson coefficients when flavor is taken into account [14]. The situation becomes close to untreatable when we consider the next order [ $\mathcal{O}(\Lambda^{-4})$ ] in the expansion that exhibits 44,807 possible operators [15]. As a consequence in the most general scenario, the number of dimension-eight operators contributing to the present observables is prohibitively large, which precludes a complete analysis including all effects at that order. In this context, Drell-Yan processes have been studied in the SMEFT framework at  $\mathcal{O}(\Lambda^{-2})$  [16–27] and partially at order  $\mathcal{O}(\Lambda^{-4})$  [28–39] where due to the large number of operators contributing the majority of the studies considers just one or at most a few operators at one time.

Identifying physically motivated hypotheses to be able to capture a large class of beyond the standard model (BSM) theories while reducing the number of relevant operators becomes mandatory for general studies. One such well-motivated hypothesis is that of *universality*, which in brief refers to BSM scenarios where the new physics either dominantly couples to the bosons of the Standard Model or if it couples to fermions, couples via SM currents allowing one to express the EFT exclusively in terms of bosonic operators. At  $\mathcal{O}(\Lambda^{-2})$  under the assumption of  $C$  and  $P$  conservation, this universal SMEFT (herein USMEFT) contains 16 dimension-six independent operators [40]. Recently, Ref. [41] presented the basis for USMEFT at  $\mathcal{O}(\Lambda^{-4})$  which, without imposing  $C$  or  $P$  symmetries, contains 175 dimension-eight operators.

In this work, we perform a complete study of the neutral and charged-current Drell-Yan processes and electroweak precision observables (EWPO) in the framework of  $C$  and  $P$  conserving universal new physics beyond  $\mathcal{O}(\Lambda^{-2})$ . Within the USMEFT framework there are 17 operators that contribute to the EWPO and Drell-Yan. They are presented both in the purely bosonic form and in the *rotated* form in which the bosonic operators with higher derivatives are traded for fermionic operators involving the SM gauge

currents. We identify eight combinations of the 17 Wilson coefficients which are physically distinguishable in the EWPO and Drell-Yan analyses. Furthermore, we introduce an extension of the parametrization of universal effects in terms of oblique parameters obtained by linearly expanding the self-energies of the electroweak gauge bosons to  $\mathcal{O}(q^6)$ . It contains 11 oblique parameters of which only eight are generated within the USMEFT at  $\mathcal{O}(\Lambda^{-4})$ .

In these studies, we compare and combine the constraints derived from the analysis of LHC DY processes with those from the  $W$  and  $Z$  pole observables which can only provide bounds in four directions of the eight-parameter space. We present the results in terms of constraints on the eight testable effective Wilson coefficients as well as on the eight oblique parameters. In each case, we study the dependence of the derived constraints with the order of the expansion considered. Our results show that the present data, which favors no deviation from the SM predictions, can robustly constrain six of the eight parameters while strong correlations and cancellations are still present for three of the combinations.

The outline of the paper is as follows. In Sec. II we present the part of the USMEFT basis up to  $\mathcal{O}(\Lambda^{-4})$  employed in this work. This section is complemented with Appendix A where the full USMEFT at  $\mathcal{O}(\Lambda^{-4})$  basis and its properties under  $C$  and  $P$  are listed. Section III contains some analytic expressions of the corrections of the Drell-Yan amplitudes as derived in the rotated basis, which is most convenient for numerical implementation in phenomenological studies, consistently accounting for the effect induced by the renormalization of the SM inputs at  $\mathcal{O}(\Lambda^{-4})$ . In Sec. IV we introduce an extension of the oblique parameters to take into account  $\mathcal{O}(q^6)$  contributions and this section is complemented with Appendix B where we show the relation between those eight oblique parameters and the eight effective combinations identified in Sec. III. We then proceed to make a combined analysis of the experimental results presented in Sec. V. The quantitative results of the analysis are presented in Sec. VI, while Sec. VII contains our summary.

## II. OPERATOR BASIS

Within the SMEFT predictions for observables at order  $1/\Lambda^4$  require evaluating the SM contributions, the interference between the  $1/\Lambda^2$  amplitude ( $\mathcal{M}^{(6)}$ ) with the SM amplitude, the square of the dimension-six amplitude, as well as the interference of the  $1/\Lambda^4$  amplitude with the SM, which we represent as

$$|M_{\text{SM}}|^2 + \mathcal{M}_{\text{SM}}^* \mathcal{M}^{(6)} + |\mathcal{M}^{(6)}|^2 + \mathcal{M}_{\text{SM}}^* \mathcal{M}^{(6,2)} + \mathcal{M}_{\text{SM}}^* \mathcal{M}^{(8)}. \quad (2.1)$$

$\mathcal{M}^{(8)}$  includes amplitudes with one dimension-eight operator coefficient while  $\mathcal{M}^{(6,2)}$  includes the contribution of

TABLE I.  $C$  and  $P$  conserving dimension-six operators for universal theories and their respective Wilson coefficients.  $H$  stands for the SM Higgs doublet and  $W_{\mu\nu}^I$  and  $B_{\mu\nu}$  are the  $SU(2)_L$  and  $U(1)_Y$  field strength tensors respectively.  $\tau^I$  stands for the Pauli matrices.

Bosonic basis		Coefficient	Rotated basis		Coefficient
$Q_{\Phi,1}$	$(D_\mu H^\dagger H)(H^\dagger D^\mu H)$	$b_{\Phi,1}$	$Q_{\Phi,1}$	$(D_\mu H^\dagger H)(H^\dagger D^\mu H)$	$c_{\Phi,1}$
$Q_{WW}$	$H^\dagger W_{\mu\nu}^I W^{I,\mu\nu} H$	$b_{WW}$	$Q_{WW}$	$H^\dagger W_{\mu\nu}^I W^{I,\mu\nu} H$	$c_{WW}$
$Q_{BB}$	$H^\dagger B_{\mu\nu} B^{\mu\nu} H$	$b_{BB}$	$Q_{BB}$	$H^\dagger B_{\mu\nu} B^{\mu\nu} H$	$c_{BB}$
$Q_{BW}$	$H^\dagger B_{\mu\nu} \tau^I W^{I,\mu\nu} H$	$b_{BW}$	$Q_{BW}$	$H^\dagger B_{\mu\nu} \tau^I W^{I,\mu\nu} H$	$c_{BW}$
$\mathcal{R}_{2W}$	$-\frac{1}{2}(D^\nu W_{\mu\nu}^I)^2$	$r_{2W}$	$Q_{2JW}$	$J_{W\mu}^I J_W^{I\mu}$	$c_{2JW}$
$\mathcal{R}_{2B}$	$-\frac{1}{2}(D^\nu B_{\mu\nu})^2$	$r_{2B}$	$Q_{2JB}$	$J_{B\mu} J_B^\mu$	$c_{2JB}$

the insertion of two dimension-six Wilson coefficients in the amplitude.

In order to perform our analyses we must choose a basis of independent operators. Universal theories in the context of the SMEFT refer to BSM models for which the low-energy effects can be parametrized in terms of operators involving exclusively the SM bosons, herein referred to as bosonic operators [40]. In the EFT framework not all operators at a given order are independent. In USMEFT, integration by parts and Bianchi identities allow for the selection of a basis of independent operators still involving only bosonic fields. In what follows we refer to this as the bosonic basis. Generically some of these operators contain higher derivatives of the bosonic fields. As is widely known, operators connected by the use of the classical equations of motion (EOM) of the SM fields lead to the same  $S$ -matrix elements [42–45]. Furthermore, for the top-down approach, the low-energy effects of a UV model are encoded by the Wilson coefficients of some effective operators. Additionally, some extra care is needed since trading a given operator for another in the basis using field redefinitions gives rise to Wilson coefficients of the same and higher orders of the rotated operator [46]. While for the bottom-up approach, the truncation of the low-energy expansion at a given order carried out by equations of motion spans the space of  $S$ -matrix elements, and so at each order the basis of operators is complete. Thus, it is possible to trade those bosonic operators with higher derivatives for operators involving fermions in the form of combinations of SM currents herein called fermionic operators. We will refer to this basis as the rotated basis.

### A. Dimension-six basis

The complete list of universal dimension-six operators is presented in Ref. [40]. Assuming that the fermion masses (Yukawa couplings) are negligible as well as requiring  $C$  and  $P$  conservation one can identify six independent bosonic operators contributing to the weak boson propagators, hence contributing to EWPO and/or Drell-Yan processes, of which two can be rotated into fermionic operators by the EOM. Thus we have the bosonic and

rotated basis of operators relevant for Drell-Yan listed in Table I.

In the rotated basis the operators involve the SM fermion currents

$$J_B^\mu = g' \sum_{f \in \{q,l,u,d,e\}} \sum_a Y_f \bar{f}_a \gamma^\mu f_a,$$

$$J_W^{I\mu} = \frac{g}{2} \sum_{f \in \{q,l\}} \sum_a \bar{f}_a \gamma^\mu \tau^I f_a \quad (2.2)$$

with  $Y_f$  standing for the fermion  $f$  hypercharge,  $q$  and  $l$  are the quark and lepton doublets and  $u$ ,  $d$ , and  $e$  represent the fermion singlets, and the sum over  $a$  is over generations. The  $SU(2)_L$  and  $U(1)_Y$  gauge couplings are  $g$  and  $g'$ .

As mentioned above, the coefficients in both bases are related by EOM as

$$c_{\Phi,1} = b_{\Phi,1} + \frac{g^2}{2} r_{2B},$$

$$c_{WW} = b_{WW} - \frac{g^2}{4} r_{2W},$$

$$c_{BB} = b_{BB} - \frac{g^2}{4} r_{2B},$$

$$c_{BW} = b_{BW} - \frac{gg'}{4} (r_{2B} + r_{2W}),$$

$$c_{2JW} = -\frac{1}{2} r_{2W},$$

$$c_{2JB} = -\frac{1}{2} r_{2B}. \quad (2.3)$$

These relations are derived by identifying operator relations from the EOM and reducing the set of equations.

### B. Dimension-eight operators

The full basis of dimension-eight operators for universal theories was presented in Ref. [41] and contains 175 operators. It consists of 89 bosonic operators already included in Murphy's basis [47] and 86 additional bosonic operators with higher derivatives which can be rotated into

TABLE II. Independent dimension-eight  $C$  and  $P$  conserving USMEFT operators relevant for Drell-Yan.

Bosonic basis		Coefficient		Rotated basis		Coefficient	
$Q_{H^6}^{(2)}$	$(H^\dagger H)(H^\dagger \tau^I H)(D_\mu H)^\dagger \tau^I D^\mu H$	$b_{H^6}^{(2)}$	$Q_{H^6}^{(2)}$	$(H^\dagger H)(H^\dagger \tau^I H)(D_\mu H)^\dagger \tau^I D^\mu H$	$c_{H^6}^{(2)}$		
$Q_{WBH^4}^{(1)}$	$(H^\dagger H)(H^\dagger \tau^I H)W_{\mu\nu}^I B^{\mu\nu}$	$b_{WBH^4}^{(1)}$	$Q_{WBH^4}^{(1)}$	$(H^\dagger H)(H^\dagger \tau^I H)W_{\mu\nu}^I B^{\mu\nu}$	$c_{WBH^4}^{(1)}$		
$Q_{W^2H^4}^{(3)}$	$(H^\dagger \tau^I H)(H^\dagger \tau^J H)W_{\mu\nu}^I W^{J\mu\nu}$	$b_{W^2H^4}^{(3)}$	$Q_{W^2H^4}^{(3)}$	$(H^\dagger \tau^I H)(H^\dagger \tau^J H)W_{\mu\nu}^I W^{J\mu\nu}$	$c_{W^2H^4}^{(3)}$		
$R_{B^2D^4}^{(1)}$	$D^\rho D^\alpha B_{\alpha\mu} D_\rho D_\beta B^{\beta\mu}$	$r_{B^2D^4}^{(1)}$	$Q_{\psi^4D^2}^{(2)}$	$D^\alpha J_B^\mu D_\alpha J_{B\mu}$	$c_{\psi^4D^2}^{(2)}$		
			$Q_{\psi^2H^2D^3}^{(1)}$	$i(D^\mu J_B^\nu + D^\nu J_B^\mu) \times (D_{(\mu} D_{\nu)} H^\dagger H - H^\dagger D_{(\mu} D_{\nu)} H)$	$c_{\psi^2H^2D^3}^{(1)}$		
$R_{W^2D^4}^{(1)}$	$D^\rho D^\alpha W_{\alpha\mu}^I D_\rho D^\beta W_\beta^{I,\mu}$	$r_{W^2D^4}^{(1)}$	$Q_{\psi^4D^2}^{(3)}$	$D^\alpha J_W^{I\mu} D_\alpha J_{W\mu}^I$	$c_{\psi^4D^2}^{(3)}$		
			$Q_{\psi^2H^2D^3}^{(2)}$	$i(D^\mu J_W^{I\nu} + D^\nu J_W^{I\mu}) \times (D_{(\mu} D_{\nu)} H^\dagger \tau^I H - H^\dagger \tau^I D_{(\mu} D_{\nu)} H)$	$c_{\psi^2H^2D^3}^{(2)}$		
$R_{BH^4D^2}^{(1)}$	$i(D^\mu W_{\mu\nu}^I)(H^\dagger \overleftrightarrow{D}^{\nu I} H)(H^\dagger H)$	$r_{BH^4D^2}^{(1)}$	$Q_{\psi^2H^4D}^{(1)}$	$iJ_B^\mu (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$c_{\psi^2H^4D}^{(1)}$		
$R'_{WH^4D^2}^{(2)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)D^\nu (H^\dagger \tau^J H)(D^\mu W_{\mu\nu}^K)$	$r'_{WH^4D^2}^{(2)}$	$Q_{\psi^2H^4D}^{(2)}$	$iJ_W^{I\mu} [(H^\dagger \overleftrightarrow{D}_\mu^I H)(H^\dagger H) + (H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger \tau^I H)]$	$c_{\psi^2H^4D}^{(2)}$		
$R_{BH^4D^2}^{(1)}$	$i(D_\alpha B^{\alpha\mu})(H^\dagger \overleftrightarrow{D}_\mu H)(H^\dagger H)$	$r_{BH^4D^2}^{(1)}$	$Q_{\psi^2H^4D}^{(4)}$	$\epsilon^{IJK} J_W^{I\mu} (H^\dagger \tau^J H) D_\mu (H^\dagger \tau^K H)$	$c_{\psi^2H^4D}^{(4)}$		
$R_{B^2H^2D^2}^{(9)}$	$(D^\mu B_{\mu\alpha})(D_\nu B^{\nu\alpha})(H^\dagger H)$	$r_{B^2H^2D^2}^{(9)}$	$Q_{\psi^4H^2}^{(4)}$	$J_B^\mu J_{B\mu}(H^\dagger H)$	$c_{\psi^4H^2}^{(4)}$		
$R_{W^2H^2D^2}^{(9)}$	$(D^\mu W_{\mu\alpha}^I)(D_\nu W^{\nu\alpha I})(H^\dagger H)$	$r_{W^2H^2D^2}^{(9)}$	$Q_{\psi^4H^2}^{(5)}$	$J_W^{I\mu} J_{W\mu}^I (H^\dagger H)$	$c_{\psi^4H^2}^{(5)}$		
$R_{BWH^2D^2}^{(13)}$	$(D^\mu B_{\mu\alpha})(D_\nu W^{\nu\alpha I})(H^\dagger \tau^I H)$	$r_{BWH^2D^2}^{(13)}$	$Q_{\psi^4H^2}^{(7)}$	$J_W^{I\mu} J_{B\mu} (H^\dagger \tau^I H)$	$c_{\psi^4H^2}^{(7)}$		

fermionic operators by the EOM as shown in Ref. [41]. We list in Appendix A the full list of 175 operators in the bosonic basis together with their  $C$  and  $P$  properties. From those we identify 11 independent bosonic operators contributing to the weak boson propagators of which eight can

be rotated into combinations involving 10 fermionic operators by the EOM. We list the final bosonic and rotated basis of independent operators relevant for Drell-Yan in Table II. The coefficients of the two bases are related by the EOM as

$$\begin{aligned}
c_{H^6}^{(2)} &= b_{H^6}^{(2)} + \frac{g^2 g'^2}{4} r_{B^2D^4}^{(1)} + g^2 g'^2 r_{W^2D^4}^{(1)} + \frac{g'^2}{2} r_{B^2H^2D^2}^{(9)} + \frac{gg'}{2} r_{BWH^2D^2}^{(13)} - g' r_{BH^4D^2}^{(1)} + g r_{WH^4D^2}^{(2)}, c_{BWH^4}^{(1)} = b_{BWH^4}^{(1)} - \frac{g^3 g'}{2} r_{W^2D^4}^{(1)}, \\
c_{W^2H^4}^{(3)} &= b_{W^2H^4}^{(3)}, \\
c_{\psi^2H^4D}^{(1)} &= g' g^2 r_{W^2D^4}^{(1)} - r_{BH^4D^2}^{(1)} + g' r_{B^2H^2D^2}^{(9)} + \frac{g}{2} r_{BWH^2D^2}^{(13)}, \\
c_{\psi^2H^4D}^{(2)} &= \frac{g^2 g'}{4} r_{B^2D^4}^{(1)} + \frac{g^3}{4} r_{W^2D^4}^{(1)} - \frac{1}{2} r_{WH^4D^2}^{(1)} + \frac{g}{2} r_{W^2H^2D^2}^{(9)} + \frac{g'}{4} r_{BWH^2D^2}^{(13)}, \\
c_{\psi^2H^4D}^{(4)} &= \frac{g^3}{4} r_{W^2D^4}^{(1)} - \frac{1}{2} r_{WH^4D^2}^{(1)} - r'_{WH^4D^2}^{(2)} + \frac{g}{2} r_{W^2H^2D^2}^{(9)} - \frac{g'}{4} r_{BWH^2D^2}^{(13)}, \\
c_{\psi^4H^2}^{(4)} &= r_{B^2H^2D^2}^{(9)}, \\
c_{\psi^4H^2}^{(5)} &= r_{W^2H^2D^2}^{(9)}, \\
c_{\psi^4H^2}^{(7)} &= r_{BWH^2D^2}^{(13)}, \\
c_{\psi^4D^2}^{(2)} &= r_{B^2D^4}^{(1)}, \\
c_{\psi^4D^2}^{(3)} &= r_{W^2D^4}^{(1)}.
\end{aligned} \tag{2.4}$$

In addition the EOM imply two relations connecting the coefficients of the following operators in the rotated basis

$$c_{\psi^2 H^2 D^3}^{(1)} = -\frac{g'}{2} c_{\psi^4 D^2}^{(2)}, \quad c_{\psi^2 H^2 D^3}^{(2)} = -\frac{g}{2} c_{\psi^4 D^2}^{(3)}. \quad (2.5)$$

### III. CORRECTIONS TO THE DRELL-YAN AMPLITUDES

In total, we have identified six dimension-six and 11 dimension-eight operators entering the Drell-Yan production in USMEFT. As mentioned above, one can choose to work with the fully bosonic basis of operators containing some with higher derivatives of the bosonic fields, or with the rotated basis in which these operators have been rotated into fermionic operators. In order to consistently include the new effects into realistic simulations of the experimental observables up to order  $\mathcal{O}(\Lambda^{-4})$ , we numerically evaluate the corresponding event rates with standard numerical tools: MadGraph5\_aMC@NLO [48] with UFO files generated with FeynRules [49,50] including all relevant operators. For this purpose it is more convenient to work with the operators in the rotated basis including in addition the indirect effects induced by the finite renormalization of the SM parameters [51].

In this work, we adopt as input parameters  $\{\hat{\alpha}_{\text{em}}, \hat{G}_F, \hat{M}_Z\}$  and consider the following three relations to define the renormalized parameters:

$$\begin{aligned} \hat{e} &= \sqrt{4\pi\hat{\alpha}_{\text{em}}}, \\ \hat{v}^2 &= \frac{1}{\sqrt{2}\hat{G}_F}, \\ \hat{c}^2\hat{s}^2 &= \frac{\pi\hat{\alpha}_{\text{em}}}{\sqrt{2}\hat{G}_F\hat{M}_Z^2}, \end{aligned} \quad (3.1)$$

where  $\hat{s}$  ( $\hat{c}$ ) is the sine (cosine) of the weak mixing angle  $\hat{\theta}$ .

For convenience we parametrize the contributions of fermionic operators to the muon decay width as

$$\left[2\langle H^\dagger H \rangle - \frac{1}{\sqrt{2}\hat{G}_F}\right]_{\text{fermionic}} \equiv \frac{\hat{v}^4}{\Lambda^2} \Delta_{4F} + \frac{\hat{v}^6}{\Lambda^4} \Delta_{4F}^{(8)}, \quad (3.2)$$

where  $\Delta_{4F}$  ( $\Delta_{4F}^{(8)}$ ) contains the dimension-six (-eight) contributions. In USMEFT there is just one fermionic dimension-six operator that contributes:

$$\Delta_{4F} = -\frac{\hat{e}^2}{2\hat{s}^2} c_{2JW}. \quad (3.3)$$

#### A. Z and W couplings

After finite renormalization of the SM inputs and accounting for the direct contribution from the fermionic dimension-eight operators containing two fermion fields we can parametrize the Z coupling to fermion pairs  $\bar{f}f$  as

$$\begin{aligned} \frac{\hat{e}}{\hat{s}\hat{c}} \left[ \hat{g}_{L,R}^f \left( 1 + \Delta\bar{g}_1 + \Delta g_1^\square + \frac{p^2}{\hat{M}_Z^2} \Delta g_1' \right) \right. \\ \left. + Q^f \left( \Delta\bar{g}_2 + \Delta g_2^\square + \frac{p^2}{\hat{M}_Z^2} \Delta g_2' \right) \right], \end{aligned} \quad (3.4)$$

and the W coupling to left-handed fermions as

$$\frac{1}{\sqrt{2}} \frac{\hat{e}}{\hat{s}} \left( 1 + \Delta\bar{g}_W + \Delta g_W^\square + \frac{p^2}{\hat{M}_W^2} \Delta g_W' \right), \quad (3.5)$$

where  $\hat{g}_L^f = T_3^f - \hat{s}^2 Q^f$ ,  $\hat{g}_R^f = -\hat{s}^2 Q^f$ ,  $T_3^f$  is the fermion's third component of isospin, and  $Q^f$  is its charge. In the expressions above  $p^2$  is the square of the four-momentum of the corresponding gauge boson.

The  $\Delta\bar{g}_{1,2,W}$  pieces contain corrections which are dimension-six at leading order with additional contributions from either  $Q_{WW}$  and  $Q_{BB}$ <sup>1</sup> or dimension-eight operators which are only resolvable with Higgs observables. They read

$$\Delta\bar{g}_1 = -\frac{1}{4} [2\bar{\Delta}_{4F} + \bar{c}_{\Phi,1}] \frac{\hat{v}^2}{\Lambda^2}, \quad (3.6)$$

$$\Delta\bar{g}_2 = -\frac{\hat{s}_2}{8\hat{c}_2} [\hat{s}_2(2\bar{\Delta}_{4F} + \bar{c}_{\Phi,1}) + 4\bar{c}_{BW}] \frac{\hat{v}^2}{\Lambda^2}, \quad (3.7)$$

$$\Delta\bar{g}_W = -\frac{1}{4\hat{c}_2} [2\hat{s}_2\bar{c}_{BW} + 2\hat{c}_2^2\bar{\Delta}_{4F} + \hat{c}_2^2\bar{c}_{\Phi,1}] \frac{\hat{v}^2}{\Lambda^2} - \frac{1}{2} \bar{c}_{W^2H^4}^{(3)} \frac{\hat{v}^4}{\Lambda^4}, \quad (3.8)$$

with  $\hat{c}_n = \cos(n\hat{\theta})$  and  $\hat{s}_n = \sin(n\hat{\theta})$ . In addition we have introduced the effective coupling combinations

$$\bar{\Delta}_{4F} = -\frac{\hat{e}^2}{2\hat{s}^2} c_{2JW} \left( 1 - 2\frac{\hat{v}^2}{\Lambda^2} c_{WW} \right) - \frac{\hat{e}^2}{4\hat{s}^2} c_{\psi^4 H^2}^{(5)} \frac{\hat{v}^2}{\Lambda^2}, \quad (3.9)$$

$$\begin{aligned} \bar{c}_{BW} &= c_{BW} \left( 1 - \frac{\hat{v}^2}{\Lambda^2} (c_{WW} + c_{BB}) \right) \\ &+ \frac{1}{2} \left[ c_{WBH^4}^{(1)} + \frac{\hat{e}}{2\hat{s}} c_{\psi^2 H^4 D}^{(1)} + \frac{\hat{e}}{\hat{c}} c_{\psi^2 H^4 D}^{(2)} \right] \frac{\hat{v}^2}{\Lambda^2}, \end{aligned} \quad (3.10)$$

$$\bar{c}_{\Phi,1} = c_{\Phi,1} + \left[ c_{H^6}^{(2)} - \frac{\hat{e}}{\hat{c}} c_{\psi^2 H^4 D}^{(1)} - \frac{\hat{e}}{\hat{s}} \left( c_{\psi^2 H^4 D}^{(2)} - c_{\psi^2 H^4 D}^{(4)} \right) \right] \frac{\hat{v}^2}{\Lambda^2}, \quad (3.11)$$

<sup>1</sup>The coefficients of operators  $Q_{BB}$  and  $Q_{WW}$  induce an overall renormalization of the  $W^I$  and  $B$  field wave functions that can be absorbed by a redefinition of the gauge boson coupling constants at all orders. However, this does not apply to dimension-six operators that involve powers of the gauge couplings without corresponding powers of the  $W^I$  and/or  $B$  gauge fields. In particular, for  $Q_{2JW}$ ,  $Q_{2JB}$ , and  $Q_{BW}$  the field redefinitions give rise to  $\mathcal{O}(\Lambda^{-4})$  terms proportional to  $c_{2JW}c_{WW}$ ,  $c_{2JB}c_{BB}$ , and  $c_{WB}(c_{WW} + c_{BB})$ , respectively.

$$\bar{c}_{W^2H^4}^{(3)} = c_{W^2H^4}^{(3)} + \frac{\hat{e}}{2\hat{s}} \left( c_{\psi^2H^4D}^{(2)} - c_{\psi^2H^4D}^{(4)} \right). \quad (3.12)$$

We collect in  $\Delta g_{1,2}^\square$  the additional contributions which are quadratic in the dimension-six Wilson coefficients:

$$\Delta g_1^\square = \frac{1}{32} \frac{1}{\hat{c}_2} [16\hat{s}^2(\Delta_{4F})^2 + 3\hat{c}_2(4(\Delta_{4F})^2 + (c_{\Phi,1})^2) + 4\Delta_{4F}c_{\Phi,1} + 16\hat{s}_2\Delta_{4F}c_{BW}] \frac{\hat{v}^4}{\Lambda^4}, \quad (3.13)$$

$$\begin{aligned} \Delta g_2^\square &= \frac{\hat{s}_2^2}{128\hat{c}_2^3} [32\hat{s}^2\hat{c}_2(\Delta_{4F})^2 + (1 + 3\hat{c}_4)(4(\Delta_{4F})^2 + (c_{\Phi,1})^2) - 32(c_{BW})^2] \frac{\hat{v}^4}{\Lambda^4} \\ &+ \frac{\hat{s}_2}{8\hat{c}_2^3} [4(-1 + \hat{c}^2\hat{s}_2^2)\Delta_{4F}c_{BW} - \hat{s}_2^2c_{\Phi,1}c_{BW} - \hat{s}^2\hat{s}_2\Delta_{4F}c_{\Phi,1}] \frac{\hat{v}^4}{\Lambda^4}, \end{aligned} \quad (3.14)$$

$$\begin{aligned} \Delta g_W^\square &= \frac{1}{32\hat{c}_2^3} [4\hat{s}_2^2\hat{c}_2(\Delta_{4F})^2 + \hat{c}^4(5\hat{c}_2 - 2)(4(\Delta_{4F})^2 + (c_{\Phi,1})^2) + 4(-3\hat{c}_2^3 + (\hat{c}_2 - 2))(c_{BW})^2 \\ &+ 8\hat{s}^2\hat{s}_2(\hat{c}_2 - 2)\Delta_{4F}c_{BW} - 4\hat{s}^2\hat{s}_2(\hat{c}_2 + 2)c_{BW}c_{\Phi,1} + (\hat{c}_2 + 1)((5\hat{c}_2 - 2) - \hat{c}_2^2)\Delta_{4F}c_{\Phi,1}] \frac{\hat{v}^4}{\Lambda^4}. \end{aligned} \quad (3.15)$$

Furthermore  $\Delta g'_{1,2,W}$  contain the coefficients of momentum dependent corrections to the gauge boson couplings arising from  $Q_{\psi^2H^2D^3}^{(1),(2)}$ . Their coefficients are related with those of  $\psi^4D^2$  operators by the universality condition as Eq. (2.5) so

$$\Delta g'_1 = -\frac{\hat{e}^3}{4\hat{c}_2^2\hat{s}^2} \left( \frac{1}{\hat{c}} c_{\psi^2H^2D^3}^{(1)} + \frac{1}{\hat{s}} c_{\psi^2H^2D^3}^{(2)} \right) \frac{\hat{v}^4}{\Lambda^4}, \quad = \frac{\hat{e}^4}{8\hat{c}_2^4\hat{s}^4} \left( \hat{s}^2 c_{\psi^4D^2}^{(2)} + \hat{c}^2 c_{\psi^4D^2}^{(3)} \right) \frac{\hat{v}^4}{\Lambda^4}, \quad (3.16)$$

$$\Delta g'_2 = -\frac{\hat{e}^3}{4\hat{c}_2^2\hat{s}^2} \left( \hat{s} c_{\psi^2H^2D^3}^{(2)} - \hat{c} c_{\psi^2H^2D^3}^{(1)} \right) \frac{\hat{v}^4}{\Lambda^4}, \quad = \frac{\hat{e}^4}{8\hat{c}_2^2\hat{s}^2} \left( c_{\psi^4D^2}^{(3)} - c_{\psi^4D^2}^{(2)} \right) \frac{\hat{v}^4}{\Lambda^4}, \quad (3.17)$$

$$\Delta g'_W = -\frac{\hat{e}^3}{4\hat{s}^3} c_{\psi^2H^2D^3}^{(2)} \frac{\hat{v}^4}{\Lambda^4}, \quad = \frac{\hat{e}^4}{8\hat{s}^4} c_{\psi^4D^2}^{(3)} \frac{\hat{v}^4}{\Lambda^4}. \quad (3.18)$$

Lastly the  $W$  mass correction is

$$\begin{aligned} \frac{\Delta M_W}{\hat{M}_W} &= -\frac{1}{4\hat{c}_2} \frac{\hat{v}^2}{\Lambda^2} [2\hat{s}_2\bar{c}_{BW} + 2\hat{s}^2\bar{\Delta}_{4F} + \hat{c}^2\bar{c}_{\Phi,1}] - \frac{1}{2} \frac{\hat{v}^4}{\Lambda^4} \bar{c}_{W^2H^4}^{(3)} + \frac{(\Delta M_W)^\square}{\hat{M}_W}, \\ \frac{(\Delta M_W)^\square}{\hat{M}_W} &= \frac{1}{32\hat{c}_2^3} \frac{\hat{v}^4}{\Lambda^4} [16\hat{s}^4\hat{c}_2(\Delta_{4F})^2 - 4\hat{s}^4(3\hat{c}_2 + 2)(\Delta_{4F})^2 + \hat{c}^4(5\hat{c}_2 - 2)(c_{\Phi,1})^2 + 4(-3\hat{c}_2^3 + (\hat{c}_2 - 2))(c_{BW})^2 \\ &- 4\hat{s}^2\hat{s}_2(\hat{c}_2 + 2)c_{BW}c_{\Phi,1} - 4\hat{c}^2(7 - 19\hat{c}_2^2 + 14\hat{c}_2^4)\Delta_{4F}c_{\Phi,1} - 8\hat{s}_2(6 - 17\hat{c}_2^2 + 14\hat{c}_2^4)\Delta_{4F}c_{BW}], \end{aligned} \quad (3.19)$$

where  $\hat{M}_W = \frac{\hat{e}\hat{v}}{2\hat{s}}$ .

## B. Four-fermion contact amplitudes

In addition to the corrections to the couplings of  $Z$  and  $W$  bosons to fermion pairs, there are seven contact contributions to four-fermion amplitudes in the rotated basis:

- (i) two at dimension six:  $c_{2JW}$  and  $c_{2JB}$ ; and
- (ii) five at dimension eight:  $c_{\psi^4D^2}^{(2)}$ ,  $c_{\psi^4D^2}^{(3)}$ ,  $c_{\psi^4H^2}^{(4)}$ ,  $c_{\psi^4H^2}^{(5)}$ , and  $c_{\psi^4H^2}^{(7)}$ .

However, in Drell-Yan processes the contribution of  $c_{2JB}$  and  $c_{\psi^4H^2}^{(4)}$  ( $c_{2JW}$  and  $c_{\psi^4H^2}^{(5)}$ ) always enter together in the same combination. Furthermore, as before, the dimension-six Wilson coefficients  $c_{BB}$  and  $c_{WW}$  induce a shift on the gauge coupling constants which results in an associated shift on the four-fermion operator coefficients. Hence, we find that the four-fermion contact amplitudes depend upon the two combinations:

$$\bar{c}_{2JW} \equiv c_{2JW} \left( 1 - 2 \frac{\hat{v}^2}{\Lambda^2} c_{WW} \right) + c_{\psi^4 H^2}^{(5)} \frac{\hat{v}^2}{2\Lambda^2} = -\frac{2\hat{s}^2}{\hat{e}^2} \bar{\Delta}_{4F}, \quad (3.20)$$

$$\bar{c}_{2JB} \equiv c_{2JB} \left( 1 - 2 \frac{\hat{v}^2}{\Lambda^2} c_{BB} \right) + c_{\psi^4 H^2}^{(4)} \frac{\hat{v}^2}{2\Lambda^2}. \quad (3.21)$$

In addition  $Q_{\psi^4 H^2}^{(7)}$  contributes to a different combination of the momentum independent four-fermion contact amplitudes. Moreover,  $Q_{\psi^4 D^2}^{(2)}$  and  $Q_{\psi^4 D^2}^{(3)}$  generate distinctive momentum-dependent four-fermion vertices.

Altogether we can parametrize the four-fermion contact amplitudes in Drell-Yan NC and CC processes with  $p^2$ , the partonic center-of-mass energy, as

$$\begin{aligned} \mathcal{M}_{\text{Cont}}^{\text{DY,NC}} = & -\hat{e}^2 \frac{1}{\hat{M}_Z^2} \left[ \frac{1}{\hat{s}^2 \hat{c}^2} (j_Z^f)^\mu (j_Z^{f'})_\mu \left( \bar{\mathcal{N}}_{ZZ} + \mathcal{N}_{ZZ}^\square + \frac{p^2}{\hat{M}_Z^2} \mathcal{N}'_{ZZ} \right) \right. \\ & + \frac{1}{\hat{s} \hat{c}} \left( (j_Z^f)^\mu (j_Q^{f'})_\mu + (j_Q^f)^\mu (j_Z^{f'})_\mu \right) \left( \bar{\mathcal{N}}_{\gamma Z} + \mathcal{N}_{\gamma Z}^\square + \frac{p^2}{\hat{M}_Z^2} \mathcal{N}'_{\gamma Z} \right) \\ & \left. + (j_Q^f)^\mu (j_Q^{f'})_\mu \left( \bar{\mathcal{N}}_{\gamma\gamma} + \mathcal{N}_{\gamma\gamma}^\square + \frac{p^2}{\hat{M}_Z^2} \mathcal{N}'_{\gamma\gamma} \right) \right], \end{aligned} \quad (3.22)$$

$$\mathcal{M}_{\text{Cont}}^{\text{DY,CC}} = -\frac{\hat{e}^2}{2\hat{s}^2} \frac{1}{\hat{M}_W^2} (j_W^f)^\mu (j_W^{f'})_\mu \left( \bar{\mathcal{N}}_{WW} + \mathcal{N}_{WW}^\square + \frac{p^2}{\hat{M}_W^2} \mathcal{N}'_{WW} \right), \quad (3.23)$$

where we have made use of the following currents:

$$(j_Z^f)^\mu = \hat{g}_L^f \bar{f}_L \gamma^\mu f_L + \hat{g}_R^f \bar{f}_R \gamma^\mu f_R, \quad (3.24)$$

$$(j_Q^f)^\mu = Q^f \bar{f} \gamma^\mu f, \quad (3.25)$$

$$(j_W^f)^\mu = f_{uL} \gamma^\mu f_{dL}. \quad (3.26)$$

Keeping our conventions as before, we defined the  $\bar{\mathcal{N}}$  pieces containing corrections which are dimension-six at leading order with additional irresolvable contributions

$$\bar{\mathcal{N}}_{\gamma\gamma} = -\frac{\hat{e}^2}{2\hat{s}^2 \hat{c}^2} (\hat{s}^2 \bar{c}_{2JW} + \hat{c}^2 \bar{c}_{2JB}) \frac{\hat{v}^2}{\Lambda^2} + \frac{\hat{e}^2}{4\hat{s} \hat{c}} c_{\psi^4 H^2}^{(7)} \frac{\hat{v}^4}{2\Lambda^4}, \quad (3.27)$$

$$\bar{\mathcal{N}}_{ZZ} = -\frac{\hat{e}^2}{2\hat{s}^2 \hat{c}^2} (\hat{c}^2 \bar{c}_{2JW} + \hat{s}^2 \bar{c}_{2JB}) \frac{\hat{v}^2}{\Lambda^2} - \frac{\hat{e}^2}{4\hat{s} \hat{c}} c_{\psi^4 H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4}, \quad (3.28)$$

$$\bar{\mathcal{N}}_{\gamma Z} = -\frac{\hat{e}^2}{2\hat{s} \hat{c}} (\bar{c}_{2JW} - \bar{c}_{2JB}) \frac{\hat{v}^2}{\Lambda^2} + \hat{c}_2 \frac{\hat{e}^2}{8\hat{s}^2 \hat{c}^2} c_{\psi^4 H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4}, \quad (3.29)$$

$$\bar{\mathcal{N}}_{WW} = -\frac{\hat{e}^2}{2\hat{s}^2} \bar{c}_{2JW} \frac{\hat{v}^2}{\Lambda^2}. \quad (3.30)$$

In addition  $\mathcal{N}^\square$  contains the terms quadratic in the Wilson coefficients of the dimension-six operators

$$\mathcal{N}_{\gamma\gamma}^\square = -\frac{\hat{e}^2}{4\hat{s}^2 \hat{c}^2} \frac{1}{\hat{c}_2} \left[ \hat{e}^2 \hat{c}^2 (c_{2JW})^2 - c_{2JW} (\hat{s}^2 \hat{c}^2 c_{\Phi,1} + 4\hat{c} \hat{s}^3 c_{BW} - 4\hat{s}^2 \hat{c}_2 c_{WW}) \right] \quad (3.31)$$

$$+ \hat{c}^2 c_{2JB} (-\hat{e}^2 c_{2JW} - 4\hat{c}_2 c_{BB} + 4\hat{c} \hat{s} c_{BW} + \hat{s}^2 c_{\Phi,1}) \frac{\hat{v}^4}{\Lambda^4}, \quad (3.32)$$

$$\mathcal{N}'_{ZZ} = -\frac{\hat{e}^2}{4\hat{s}^2\hat{c}^2\hat{s}^2\hat{c}^2} [\hat{e}^2\hat{c}^2(c_{2JW})^2 - \hat{s}^2c_{2JW}(\hat{c}^2c_{\Phi,1} + 4\hat{c}^2\hat{c}_2c_{WW} + \hat{e}^2\hat{s}^2c_{2JB} + 2\hat{s}_2\hat{c}^2c_{BW})] \quad (3.33)$$

$$+ \hat{s}^4c_{2JB}(2\hat{s}_2c_{BW} + \hat{s}^2c_{\Phi,1} - 4\hat{c}_2c_{BB}) \frac{\hat{v}^4}{\Lambda^4}, \quad (3.34)$$

$$\mathcal{N}'_{YZ} = \frac{\hat{e}^2}{4\hat{s}^2\hat{c}^2\hat{s}^2\hat{c}^2} [-\hat{e}^2\hat{c}^2(c_{2JW})^2 + \hat{s}^2c_{2JW}(\hat{c}^2c_{\Phi,1} + 4\hat{c}_2c_{WW} + 2\hat{s}_2c_{BW} - \hat{e}^2c_{2JB}), \quad (3.35)$$

$$+ \hat{s}^2c_{2JB}(2\hat{s}_2c_{BW} + \hat{s}^2c_{\Phi,1} - 4\hat{c}_2c_{BB})] \frac{\hat{v}^4}{\Lambda^4}, \quad (3.36)$$

$$\mathcal{N}'_{WW} = \frac{\hat{e}^2}{4\hat{s}^2\hat{s}^2\hat{c}^2} [-\hat{e}^2\hat{c}^2(c_{2JW})^2 + \hat{e}^2\hat{s}^2c_{2JW}(\hat{c}^2c_{\Phi,1} + 2\hat{c}_2c_{WW} + 2\hat{s}_2c_{BW})] \frac{\hat{v}^4}{\Lambda^4}, \quad (3.37)$$

and  $\mathcal{N}'$  contains the coefficients of the momentum-dependent four-fermion couplings:

$$\mathcal{N}'_{\gamma\gamma} = -\frac{\hat{e}^4}{8\hat{s}^4\hat{c}^4} (\hat{s}^2c_{\psi^4D^2}^{(3)} + \hat{c}^2c_{\psi^4D^2}^{(2)}) \frac{\hat{v}^4}{\Lambda^4}, \quad (3.38)$$

$$\mathcal{N}'_{ZZ} = -\frac{\hat{e}^4}{8\hat{s}^4\hat{c}^4} (\hat{c}^2c_{\psi^4D^2}^{(3)} + \hat{s}^2c_{\psi^4D^2}^{(2)}) \frac{\hat{v}^4}{\Lambda^4} = -\Delta g'_1, \quad (3.39)$$

$$\mathcal{N}'_{YZ} = -\frac{\hat{e}^4}{8\hat{s}^3\hat{c}^3} (c_{\psi^4D^2}^{(3)} - c_{\psi^4D^2}^{(2)}) \frac{\hat{v}^4}{\Lambda^4} = -\frac{1}{\hat{c}\hat{s}} \Delta g'_2, \quad (3.40)$$

$$\mathcal{N}'_{WW} = -\frac{\hat{e}^4}{8\hat{s}^4} c_{\psi^4D^2}^{(3)} \frac{\hat{v}^4}{\Lambda^4} = -\Delta g_W, \quad (3.41)$$

where, in the rightmost equivalence, we used Eqs. (3.16)–(3.18).

In summary, at the linear level neglecting quadratic [i.e. proportional to (dimension-six)<sup>2</sup>] coefficients, we have identified eight combinations of Wilson coefficients which can be distinguished by studying the invariant mass distribution of the lepton pairs produced in Drell-Yan processes. They can be chosen to be the five combinations  $\bar{\Delta}_{4F}$ ,  $\bar{c}_{BW}$ ,  $\bar{c}_{\Phi,1}$ ,  $\bar{c}_{W^2H^4}^{(3)}$ , and  $\bar{c}_{2JB}$  in Eqs. (3.9)–(3.12) and (3.21), together with the coefficients of the dimension-eight operators  $c_{\psi^4H^2}^{(7)}$ ,  $c_{\psi^4D^2}^{(2)}$ , and  $c_{\psi^4D^2}^{(3)}$ . In what follows we refer to this set of variables as overline coefficients.

### C. Corrections to the Z- and W-pole observables

In our analyses we include the constraints from EWPO on the Z- and W-pole. The predictions for these observables can be obtained from Eq. (3.4) with  $p^2 = \hat{M}_Z^2$  and (3.5) with  $p^2 = \hat{M}_W^2$  respectively, with the quadratic pieces remaining the same. We can conveniently write these couplings at the linear level as

$$\Delta g_1^{Z\text{pole}} = \Delta \bar{g}_1 + \Delta g'_1 = -\frac{1}{4} [2\bar{\Delta}_{4F} + \bar{c}_{\Phi,1}] \frac{\hat{v}^2}{\Lambda^2}, \quad (3.42)$$

$$\Delta g_2^{Z\text{pole}} = \Delta \bar{g}_2 + \Delta g'_2 = -\frac{\hat{s}_2}{8\hat{c}_2} [\hat{s}_2(2\bar{\Delta}_{4F} + \bar{c}_{\Phi,1}) + 4\bar{c}_{BW}] \frac{\hat{v}^2}{\Lambda^2}, \quad (3.43)$$

$$\Delta g_W^{\text{pole}} = \Delta \bar{g}_W + \Delta g'_W = -\frac{1}{4\hat{c}_2} [2\hat{s}_2\bar{c}_{BW} + 2\hat{c}^2\bar{\Delta}_{4F} + \hat{c}^2\bar{c}_{\Phi,1}] \frac{\hat{v}^2}{\Lambda^2} - \frac{1}{2} \bar{c}_{W^2H^4}^{(3)} \frac{\hat{v}^4}{\Lambda^4}, \quad (3.44)$$

with

$$\begin{aligned} \bar{\Delta}_{4F} &= \bar{\Delta}_{4F} - \frac{\hat{e}^4}{4\hat{s}^4} c_{\psi^4D^2}^{(3)} \frac{\hat{v}^2}{\Lambda^2}, \\ \bar{c}_{BW} &= \bar{c}_{BW} + \frac{\hat{e}^4}{8\hat{s}^3\hat{c}^3} (c_{\psi^4D^2}^{(2)} + c_{\psi^4D^2}^{(3)}) \frac{\hat{v}^2}{\Lambda^2}, \\ \bar{c}_{\Phi,1} &= \bar{c}_{\Phi,1} - \frac{\hat{e}^4}{2\hat{s}^2\hat{c}^4} (c_{\psi^4D^2}^{(2)} + \hat{c}^2c_{\psi^4D^2}^{(3)}) \frac{\hat{v}^2}{\Lambda^2}, \\ \bar{c}_{W^2H^4}^{(3)} &= \bar{c}_{W^2H^4}^{(3)} + \frac{\hat{e}^4}{4\hat{s}^2\hat{c}^2} c_{\psi^4D^2}^{(3)}, \end{aligned} \quad (3.45)$$

while it still holds that

$$\begin{aligned} \frac{\Delta M_W}{\hat{M}_W} &= -\frac{1}{4\hat{c}_2} \frac{\hat{v}^2}{\Lambda^2} [2\hat{s}_2\bar{c}_{BW} + 2\hat{s}^2\bar{\Delta}_{4F} + \hat{c}^2\bar{c}_{\Phi,1}] \\ &\quad - \frac{1}{2} \frac{\hat{v}^4}{\Lambda^4} \bar{c}_{W^2H^4}^{(3)} + \frac{(\Delta M_W)^{\square}}{\hat{M}_W}. \end{aligned} \quad (3.46)$$

In summary, the corrections to the Z and W pole observables to order  $\mathcal{O}(\Lambda^{-4})$  which are dominant can be expressed in terms of four combinations (herein referred to as *tilde coefficients*)  $\bar{c}_{BW}$ ,  $\bar{c}_{\Phi,1}$ ,  $\bar{c}_{W^2H^4}^{(3)}$ , and  $\bar{\Delta}_{4F}$  which

involve six of the eight *overline coefficients*:  $\bar{c}_{BW}$ ,  $\bar{c}_{\Phi,1}$ ,  $\bar{c}_{W^2H^4}^{(3)}$ ,  $\bar{\Delta}_{4F}$ ,  $c_{\psi^4D^2}^{(2)}$ , and  $c_{\psi^4D^2}^{(3)}$ .

#### IV. PARAMETRIZATION IN TERMS OF OBLIQUE PARAMETERS

Corrections to electroweak observables from universal theories can be described in terms of the so-called oblique parameters defined in terms of the corrections to the gauge boson self-energies [40,52,53] by expanding the vacuum-polarization amplitudes as

$$\begin{aligned} \Pi_{VV''}(q^2) &= \Pi_{VV''}(0) + \Pi'_{VV''}(0)q^2 + \frac{1}{2}\Pi''_{VV''}(0)q^4 \\ &+ \frac{1}{6}\Pi'''_{VV''}(0)q^6 + \dots \end{aligned} \quad (4.1)$$

To order  $q^4$  one can define seven independent such oblique parameters for the weak gauge bosons

$$\hat{S} = -\frac{\hat{c}}{\hat{s}}\Pi_{3B}(0), \quad (4.2)$$

$$\hat{T} = \frac{1}{\hat{M}_W^2}[\Pi_{WW}(0) - \Pi_{33}(0)], \quad (4.3)$$

$$\hat{U} = \Pi'_{33}(0) - \Pi'_{WW}(0), \quad (4.4)$$

$$V = \frac{\hat{M}_W^2}{2}[\Pi''_{WW}(0) - \Pi''_{33}(0)], \quad (4.5)$$

$$X = -\frac{\hat{M}_W^2}{2}\Pi''_{3B}(0), \quad (4.6)$$

$$Y = -\frac{\hat{M}_W^2}{2}\Pi''_{BB}(0), \quad (4.7)$$

$$W = -\frac{\hat{M}_W^2}{2}\Pi'''_{33}(0). \quad (4.8)$$

Within the dimension-six SMEFT only  $\hat{S}$ ,  $\hat{T}$ ,  $W$ , and  $Y$  are generated of which only three combinations enter in the electroweak gauge boson pole observables.

Because the corrections to the amplitudes are expressed in terms of the oblique parameters only at linear order, in the context of the USMEFT such approach is only consistent with the operator expansion at first order, i.e. at  $\mathcal{O}(\Lambda^{-2})$ . Nevertheless, the parametrization in terms of oblique parameters can be considered more general than USMEFT as it can be applied also to other expansions like the HEFT as well as specific universal theories. With this in mind one can extend the formalism by expanding the self-energies to order  $q^6$ . This involves four additional parameters which can be defined as

$$V' = \frac{\hat{M}_W^4}{6}[\Pi'''_{WW}(0) - \Pi'''_{33}(0)], \quad (4.9)$$

$$X' = -\frac{\hat{M}_W^4}{6}\Pi'''_{3B}(0), \quad (4.10)$$

$$Y' = -\frac{\hat{M}_W^4}{6}\Pi'''_{BB}(0), \quad (4.11)$$

$$W' = -\frac{\hat{M}_W^4}{6}\Pi'''_{33}(0). \quad (4.12)$$

Including the above self-energy contributions one can write the Drell-Yan amplitudes in terms of the modified propagators for the NC and CC interactions extending the expressions in Ref. [24] to include the additional oblique parameters as

$$\left( \begin{array}{c} \left( \frac{1}{p^2 - \hat{M}_Z^2} + \frac{2\Delta g_1^O}{p^2 - \hat{M}_Z^2} - \frac{(\varepsilon_{ZZ} + \frac{2}{\hat{c}^2}\varepsilon'_{ZZ})}{\hat{M}_W^2} - \frac{\varepsilon'_{ZZ}}{\hat{M}_W^4} p^2 \right) \frac{\Delta g_2^O}{\hat{s} \hat{c} (p^2 - \hat{M}_Z^2)} - \frac{(\varepsilon_{Z\gamma} + \frac{1}{\hat{c}^2}\varepsilon'_{Z\gamma})}{\hat{M}_W^2} - \frac{\varepsilon'_{Z\gamma}}{\hat{M}_W^4} p^2 \\ * \\ \frac{1}{p^2} - \frac{\varepsilon_{\gamma\gamma}}{\hat{M}_W^2} - \frac{\varepsilon'_{\gamma\gamma}}{\hat{M}_W^4} p^2 \end{array} \right), \quad (4.13)$$

$$\frac{1}{p^2 - \hat{M}_W^2} + \frac{2\Delta g_W^O}{p^2 - \hat{M}_W^2} - \frac{\varepsilon_{WW} + 2\varepsilon'_{WW}}{\hat{M}_W^2} - \frac{\varepsilon'_{WW}}{\hat{M}_W^4} p^2. \quad (4.14)$$

Written in this form one can identify the pieces of the modified propagators exhibiting  $(p^2 - M_V^2)^{-1}$  as generated by the amplitude with the standard model  $V$  propagator and with modified vertices to the fermions. In terms of oblique parameters they read

$$\Delta g_1^O = \frac{1}{2} \left[ \hat{T} - \left( W + \frac{2}{\hat{c}^2} W' \right) + \frac{2\hat{s}}{\hat{c}} \left( X + \frac{2}{\hat{c}^2} X' \right) - \frac{\hat{s}^2}{\hat{c}^2} \left( Y + \frac{2}{\hat{c}^2} Y' \right) \right], \quad (4.15)$$

$$\Delta g_2^O = \frac{\hat{s}^2}{\hat{c}^2} \left[ \hat{c}^2 \hat{T} - \hat{S} + \hat{s}^2 \left( W + \frac{1}{\hat{c}^2} W' \right) - \frac{1 - 2\hat{s}^2 \hat{c}^2}{\hat{s} \hat{c}} \left( X + \frac{1}{\hat{c}^2} X' \right) + \hat{c}^2 \left( Y + \frac{1}{\hat{c}^2} Y' \right) \right], \quad (4.16)$$

$$\Delta g_W^O = \frac{1}{2\hat{c}_2} \left[ \hat{c}^2 \hat{T} - 2\hat{s}^2 \hat{S} - (1 - 3\hat{s}^2)(W + 2W') + \hat{s}^2 \left( Y + \frac{1}{\hat{c}^2} Y' \right) - 2\hat{s} \hat{c} \left( X + \frac{1}{\hat{c}^2} X' \right) \right] - \frac{1}{2} \hat{U} + V + \frac{3}{2} V'. \quad (4.17)$$

Moreover, the pieces proportional to  $M_V^{-2}$  ( $q^2/M_V^4$ ) are linear combinations of the  $W$ ,  $Y$ ,  $X$ , and  $V$  ( $W'$ ,  $Y'$ ,  $X'$ , and  $V'$ ) parameters,

$$\varepsilon_{\gamma\gamma}^{(\prime)} = \hat{s}^2 W^{(\prime)} + \hat{c}^2 Y^{(\prime)} + 2\hat{s} \hat{c} X^{(\prime)}, \quad (4.18)$$

$$\varepsilon_{ZZ}^{(\prime)} = \hat{c}^2 W^{(\prime)} + \hat{s}^2 Y^{(\prime)} - 2\hat{s} \hat{c} X^{(\prime)}, \quad (4.19)$$

$$\varepsilon_{Z\gamma}^{(\prime)} = (\hat{c}^2 - \hat{s}^2) X^{(\prime)} + \hat{s} \hat{c} (W^{(\prime)} - Y^{(\prime)}), \quad (4.20)$$

$$\varepsilon_{WW}^{(\prime)} = W^{(\prime)} - V^{(\prime)}, \quad (4.21)$$

and they are generated by the contact four-fermion operators in the rotated basis. Additionally, the correction to the  $W$  mass is

$$\frac{\Delta M_W}{\hat{M}_W} = \Delta g_W^O - \frac{1}{2} (V + 2V' - W - 2W'). \quad (4.22)$$

The relation between the parametrization in terms of oblique parameters and the USMEFT expressions in terms of coefficients of the operators in the rotated basis can be made explicit by computing the oblique parameters in the bosonic basis and then applying Eqs. (2.3) and (2.4). We list those expressions in Appendix B. As seen in Appendix B, within USMEFT up to dimension-eight operators, there are eight nonvanishing oblique parameters:  $\hat{S}$ ,  $\hat{T}$ ,  $\hat{U}$ ,  $W$ ,  $Y$ ,  $X$ ,  $W'$ , and  $Y'$ . Introducing the resulting expressions of these oblique parameters in terms of the coefficients in the rotated basis in Eqs. (4.15)–(4.29) one finds

$$\Delta g_1^O = -\frac{1}{4} [2\bar{\Delta}_{4F} + \bar{c}_{\Phi,1}] \frac{\hat{v}^2}{\Lambda^2} + \frac{\hat{e}^4}{8\hat{c}^4 \hat{s}^4} \left( \hat{s}^2 c_{\psi^4 D^2}^{(2)} + \hat{c}^2 c_{\psi^4 D^2}^{(3)} \right) \frac{\hat{v}^4}{\Lambda^4} = \Delta \bar{g}_1 + \Delta g'_1, \quad (4.23)$$

$$\Delta g_2^O = \frac{\hat{s}_2}{8\hat{c}_2} [\hat{s}_2 (2\bar{\Delta}_{4F} + \bar{c}_{\Phi,1}) + 4\bar{c}_{BW}] \frac{\hat{v}^2}{\Lambda^2} + \frac{\hat{e}^4}{8\hat{c}^2 \hat{s}^2} \left( c_{\psi^4 D^2}^{(3)} - c_{\psi^4 D^2}^{(2)} \right) \frac{\hat{v}^4}{\Lambda^4} = \Delta \bar{g}_2 + \Delta g'_2, \quad (4.24)$$

$$\Delta g_W^O = -\frac{1}{4\hat{c}_2} [2\hat{s}_2 \bar{c}_{BW} + 2\hat{c}_2^2 \bar{\Delta}_{4F} + \hat{c}_2^2 \bar{c}_{\Phi,1}] \frac{\hat{v}^2}{\Lambda^2} - \frac{1}{2} \bar{c}_{W^2 H^4}^{(3)} \frac{\hat{v}^4}{\Lambda^4} + \frac{\hat{e}^4}{8\hat{s}^4} c_{\psi^4 D^2}^{(3)} \frac{\hat{v}^4}{\Lambda^4} = \Delta \bar{g}_W + \Delta g'_W, \quad (4.25)$$

$$\varepsilon_{\gamma\gamma} = -\frac{\hat{e}^2}{2\hat{s}^2} (\hat{s}^2 \bar{c}_{2JW} + \hat{c}^2 \bar{c}_{2BW}) \frac{\hat{v}^2}{\Lambda^2} + \frac{\hat{c} \hat{e}^2}{4\hat{s}} c_{\psi^4 H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4} = \hat{c}^2 \bar{\mathcal{N}}_{\gamma\gamma}, \quad (4.26)$$

$$\varepsilon_{ZZ} = -\frac{\hat{e}^2}{2\hat{s}^2} (\hat{c}^2 \bar{c}_{2JW} + \hat{s}^2 \bar{c}_{2BW}) \frac{\hat{v}^2}{\Lambda^2} - \frac{\hat{c} \hat{e}^2}{4\hat{s}} c_{\psi^4 H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4} = \hat{c}^2 \bar{\mathcal{N}}_{ZZ}, \quad (4.27)$$

$$\varepsilon_{Z\gamma} = -\frac{\hat{e}^2 \hat{c}}{2\hat{s}} (\bar{c}_{2JW} - \bar{c}_{2BW}) \frac{\hat{v}^2}{\Lambda^2} + \hat{c}_2 \frac{\hat{e}^2}{8\hat{s}^2} c_{\psi^4 H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4} = \hat{c}^2 \bar{\mathcal{N}}_{\gamma Z}, \quad (4.28)$$

$$\varepsilon_{WW} = -\frac{\hat{e}^2}{2\hat{s}^2} \bar{c}_{2JW} \frac{\hat{v}^2}{\Lambda^2} = \bar{\mathcal{N}}_{WW}, \quad (4.29)$$

$$\varepsilon'_{\gamma\gamma} = -\frac{\hat{e}^4}{8\hat{s}^4} \left( \hat{s}^2 c_{\psi^2 D^2}^{(3)} + \hat{c}^2 c_{\psi^2 D^2}^{(2)} \right) \frac{\hat{v}^4}{\Lambda^4} = \hat{c}^4 \mathcal{N}'_{\gamma\gamma}, \quad (4.30)$$

$$\varepsilon'_{ZZ} = -\frac{\hat{e}^4}{8\hat{s}^4} \left( \hat{c}^2 c_{\psi^2 D^2}^{(3)} + \hat{s}^2 c_{\psi^2 D^2}^{(2)} \right) \frac{\hat{v}^4}{\Lambda^4} = \hat{c}^4 \mathcal{N}'_{ZZ} = -\hat{c}^4 \Delta g'_1, \quad (4.31)$$

$$\varepsilon'_{Z\gamma} = -\hat{c} \frac{\hat{e}^4}{8\hat{s}^3} \left( c_{\psi^2 D^2}^{(3)} - c_{\psi^2 D^2}^{(2)} \right) \frac{\hat{v}^4}{\Lambda^4} = \hat{c}^4 \mathcal{N}'_{\gamma Z} = -\hat{c}^4 \frac{1}{\hat{c} \hat{s}} \Delta g'_2, \quad (4.32)$$

$$\varepsilon'_{WW} = -\frac{\hat{e}^4}{8\hat{s}^4} c_{\psi^2 D^2}^{(3)} \frac{\hat{v}^4}{\Lambda^4} = \mathcal{N}'_{WW} = -\Delta g'_W. \quad (4.33)$$

TABLE III. Neutral- and charged-current Drell-Yan data considered in our analyses.

Channel	Distribution	No. of bins	Ranges	Dataset	Integrated luminosity
NC	$\frac{d^2\sigma}{dm_{\ell\ell}d y _{\ell\ell}}$	48	$116 \text{ GeV} \leq m_{\ell\ell} \leq 1.5 \text{ TeV}$ $0 \leq y_{\ell\ell} \leq 2.4$	ATLAS 8 TeV	$20.3 \text{ fb}^{-1}$ [2]
NC	$\frac{dN_{ev}}{dm_{e^+e^-}}$	20	$250 \text{ GeV} \leq m_{e^+e^-} \leq 5 \text{ TeV}$	ATLAS 13 TeV	$139 \text{ fb}^{-1}$ [7]
NC	$\frac{dN_{ev}}{dm_{\mu^+\mu^-}}$	20	$250 \text{ GeV} \leq m_{\mu^+\mu^-} \leq 5 \text{ TeV}$	ATLAS 13 TeV	$139 \text{ fb}^{-1}$ [7]
NC	$\frac{dN_{ev}}{dm_{e^+e^-}}$	20	$300 \text{ GeV} \leq m_{e^+e^-} \leq 6 \text{ TeV}$	CMS 13 TeV	$137 \text{ fb}^{-1}$ [8]
NC	$\frac{dN_{ev}}{dm_{\mu^+\mu^-}}$	20	$300 \text{ GeV} \leq m_{\mu^+\mu^-} \leq 7 \text{ TeV}$	CMS 13 TeV	$137 \text{ fb}^{-1}$ [8]
CC	$\frac{d\sigma}{dm_T}$	20	$200 \text{ GeV} \leq m_{T,\ell\nu} \leq 5 \text{ TeV}$	ATLAS 13 TeV	$140 \text{ fb}^{-1}$ [6]
CC	$\frac{dN}{dm_T}$	20	$440 \text{ GeV} \leq m_{T,e\nu} \leq 7 \text{ TeV}$	CMS 13 TeV	$138 \text{ fb}^{-1}$ [10]
CC	$\frac{dN}{dm_T}$	20	$600 \text{ GeV} \leq m_{T,\mu\nu} \leq 7 \text{ TeV}$	CMS 13 TeV	$138 \text{ fb}^{-1}$ [10]

The equalities above hold exactly when, in the most right-hand side, only the linear  $\mathcal{O}(\Lambda^{-2})$  from dimension-six operators and  $\mathcal{O}(\Lambda^{-4})$  from dimension-eight operators are included in the *overline coefficients*. In the same form the correction to the  $W$  mass obtained from Eq. (4.22) coincides with Eq. (3.19) when the dimension-six square terms are not included.

We finish by stressing that in the context of USMEFT, the parametrization of the universal effects in terms of oblique parameters obtained by linearly expanding the gauge boson self-energies does not provide a consistent series in  $(1/\Lambda)$  beyond  $(1/\Lambda^2)$ . The consistent expansion requires the inclusion of the terms quadratic in the Wilson coefficients as presented in Sec. III and, therefore, to go beyond the expansion of the amplitudes in terms of oblique parameters at linear order. Nevertheless, as expected, the amplitudes obtained in terms of oblique parameters match the full expressions for the terms linear in the operator coefficients and allows for a clean identification of the number of independent combinations of operator coefficients. Furthermore this analysis allows to compare the constraints derived on the oblique parameters when including all those generated at dimension-eight in USMEFT to those that had been derived in the literature including only the oblique parameters generated at dimension six.

## V. ANALYSIS FRAMEWORK

In this work, our goal is to study the constraints on the USMEFT Wilson coefficients imposed by neutral- and charged-current Drell-Yan processes in combination with EWPO.

Regarding the Drell-Yan processes, the larger LHC energy of the 13 TeV runs implies that these runs are more sensitive to the presence of anomalous couplings, as expected. Unfortunately only very recently the ATLAS Collaboration has presented a dedicated study of Drell-Yan CC process at 13 TeV [6]. No dedicated study of Drell-Yan NC process at this energy has been published with detailed enough information on the differential cross sections to allow

for analysis outside of the collaborations. Determination of the Drell-Yan NC cross sections has been presented only using 8 TeV data [2,3]. However, both ATLAS and CMS have searched for new resonances in the  $\ell^+\ell^-$  and  $\ell^\pm\nu_\ell$  channels with the full 13 TeV luminosity. These searches can be recast into bounds on the USMEFT by studying their data on the lepton pair invariant mass distribution and the transverse mass spectrum respectively. As such, we have included the high invariant mass part of those distributions in the analyses, and for convenience, we have also rebinned the data to guarantee a minimum number of events. We present in Table III a summary of the data included in our analyses as well as provide further details on our analyses in Appendix C.

As discussed in Sec. III a total of six dimension-six operators and eleven dimension-eight operators contributes to the amplitudes at order  $1/\Lambda^4$  of which we have identified eight combinations of Wilson coefficients entering linearly, the five *overline coefficients*  $\bar{\Delta}_{4F}$ ,  $\bar{c}_{BW}$ ,  $\bar{c}_{\Phi,1}$ ,  $\bar{c}_{W^2H^4}^3$ , and  $\bar{c}_{2JB}$  in Eqs. (3.9)–(3.12) and (3.21), together with the coefficients of the dimension-eight operators  $c_{\psi^4H^2}^{(7)}$ ,  $c_{\psi^4D^2}^{(2)}$ , and  $c_{\psi^4D^2}^{(3)}$ . In addition there are contributions purely quadratic in the four dimension-six coefficients  $\Delta_{4F}$ ,  $c_{BW}$ ,  $c_{\Phi,1}$ , and  $c_{2JB}$ .

The theoretical predictions needed for the analyses were obtained with MadGraph5\_aMC@NLO [48] at leading-order in QCD and QED, with the UFO files for the effective Lagrangian with the seventeen operators of the rotated basis generated with FeynRules [49,50] including also the  $\mathcal{O}(\Lambda^{-2})$  and  $\mathcal{O}(\Lambda^{-4})$  terms from the finite renormalization of the SM inputs. Parton shower and hadronization was performed using PYTHIA8 [54], and the fast detector simulation was carried out with DELPHES [55]. Jet analyses was done using FastJet [56]. Exclusively for the ATLAS NC data [7], the detector response was simulated using Rivet [57,58], with the analysis code provided by the experimental collaboration. For the ATLAS 8 TeV data [2], QCD NNLO corrections for the SM predictions were incorporated using MATRIX [59]. When required, we corrected

these predictions bin by bin by the SM correspondent  $k$ -factors for higher order QCD corrections.

In order to address the dependence of the results on the order of the expansion, we have performed the analyses at different orders. We label the different analyses as follows:

- (i)  $\mathcal{O}(\Lambda^{-4})$ : Including contributions from dimension-six and dimension-eight operators and keeping their effects in the observables up to quadratic order in the dimension-six operator coefficients and linear order in the dimension-eight operator coefficients. This is the main focus of our work. In addition we make the following analyses for comparison.
- (ii)  $\mathcal{O}(\Lambda^{-2})$ : Including only contributions from dimension-six operators and keeping their effects in the observables at linear order in the operator coefficients—that is, considering only the first two terms in Eq. (2.1).
- (iii) Dim-6 + (Dim-6)<sup>2</sup>: Including only contributions from dimension-six operators and keeping their effects in the observables up to quadratic order in the operator coefficients, i.e. not including the last term in Eq. (2.1).
- (iv) Dim-6 + Dim-8: Including contributions from dimension-six and dimension-eight operators and keeping their effects in the observable only at linear order in all operator coefficients. This means including neither the third nor fourth term of Eq. (2.1).

We present the results of this analysis also in terms of the generalized oblique parameters.

At  $\mathcal{O}(\Lambda^{-4})$  we find that even including the quadratic contributions, the analysis cannot break the degeneracies between the different Wilson coefficients entering in the *overline coefficients*. Thus we proceed by substituting  $\Delta_{4F}$ ,  $c_{BW}$ ,  $c_{\Phi,1}$ , and  $c_{2JB}$  with  $\tilde{\Delta}_{4F}$ ,  $\tilde{c}_{BW}$ ,  $\tilde{c}_{\Phi,1}$ , and  $\tilde{c}_{2JB}$  in the quadratic terms and in the process neglect terms of  $\mathcal{O}(\Lambda^{-6})$ . With this we define a  $\chi^2$  function which depends on the eight Wilson coefficient combinations,

$$\begin{aligned} \chi_{\text{DY,NC}}^2(\tilde{c}_{BW}, \tilde{c}_{\Phi,1}, \tilde{\Delta}_{4F}, \tilde{c}_{2JB}, c_{\psi^4 D^2}^{(2)}, c_{\psi^4 D^2}^{(3)}, c_{\psi^4 H^2}^{(7)}) \\ + \chi_{\text{DY,CC}}^2(\tilde{c}_{BW}, \tilde{c}_{\Phi,1}, \tilde{\Delta}_{4F}, \tilde{c}_{W^2 H^4}^{(3)}, c_{\psi^4 D^2}^{(3)}). \end{aligned} \quad (5.1)$$

With respect to the EWPO, we include 12Z-pole observables [60]:  $\Gamma_Z$ ,  $\sigma_h^0$ ,  $\mathcal{A}_\ell(\tau^{\text{pol}})$ ,  $R_\ell^0$ ,  $\mathcal{A}_\ell(\text{SLD})$ ,  $A_{\text{FB}}^{0,l}$ ,  $R_c^0$ ,  $R_b^0$ ,  $\mathcal{A}_c$ ,  $\mathcal{A}_b$ ,  $A_{\text{FB}}^{0,c}$ , and  $A_{\text{FB}}^{0,b}$  and two  $W$  pole observables  $M_W$  and  $\Gamma_W$  taken from [61]. Notice that the average leptonic  $W$  branching ratio is not included because it does not lead to any additional constraint on universal EFT's. We include in our analyses the correlations among these inputs, as given in Ref. [60], and the SM predictions and their uncertainties due to variations of the SM parameters were extracted from [62].

As discussed in Sec. III the  $Z$  and  $W$  pole observables to order  $\mathcal{O}(\Lambda^{-4})$  which are dominant can be expressed in

terms of four combinations  $\tilde{c}_{BW}$ ,  $\tilde{c}_{\Phi,1}$ ,  $\tilde{c}_{W^2 H^4}^{(3)}$ , and  $\tilde{\Delta}_{4F}$  which involve six of the eight combinations testable in Drell-Yan:  $\tilde{c}_{BW}$ ,  $\tilde{c}_{\Phi,1}$ ,  $\tilde{c}_{W^2 H^4}^{(3)}$ ,  $\tilde{\Delta}_{4F}$ ,  $c_{\psi^4 D^2}^{(2)}$ , and  $c_{\psi^4 D^2}^{(3)}$ . As with the Drell-Yan observables, the dimension-six square contributions to EWPO are not able to break the degeneracies, thus we can proceed by neglecting terms of  $\mathcal{O}(\Lambda^{-6})$  and we write the couplings relevant to the EWPO as Eqs. (3.4) and (3.5) where we substitute  $\Delta_{4F}$ ,  $c_{BW}$ , and  $c_{\Phi,1}$  with  $\tilde{\Delta}_{4F}$ ,  $\tilde{c}_{BW}$  and  $\tilde{c}_{\Phi,1}$  in  $\Delta g_i^\square$  and  $(\Delta M_W)^\square$ . With this, the EWPO chi-squared function is

$$\begin{aligned} \chi_{\text{EWPO}}^2(\tilde{c}_{BW}, \tilde{c}_{\Phi,1}, \tilde{\Delta}_{4F}, \tilde{c}_{W^2 H^4}^{(3)}, c_{\psi^4 D^2}^{(2)}, c_{\psi^4 D^2}^{(3)}) \\ \equiv \chi_{\text{EWPO}}^2(\tilde{c}_{BW}, \tilde{c}_{\Phi,1}, \tilde{c}_{W^2 H^4}^{(3)}, \tilde{\Delta}_{4F}), \end{aligned} \quad (5.2)$$

which can effectively only constrain the four *tilde coefficients* in Eq. (3.45).

## VI. RESULTS

We start by studying the complementarity and improvement on the sensitivity between the DY results and the EWPO. In order to do so we project the results of the analysis over the *tilde coefficients*. The result is shown in the left in Fig. 1 which contains the one- and two-dimensional projections of  $\Delta\chi^2$  functions for the  $\mathcal{O}(\Lambda^{-4})$  analyses of the EWPO and DY separately and their combination as a function of the four *tilde* parameters. The top row contains the one-dimensional marginalized projections of  $\Delta\chi^2$ 's as a function of the four combinations of Wilson coefficients. From the analysis we obtain the 95% CL allowed ranges for the coupling combinations taking part in the EWPO listed in Table IV.

The lower panels of Fig. 1 depict the two-dimensional  $1\sigma$  and  $2\sigma$  allowed regions for the different analyses. For comparison we show the equivalent results for the  $\mathcal{O}(\Lambda^{-2})$  analysis on the right which involves only three coefficients. First, comparing the result from the analyses of the EWPO (blue regions) in Fig. 1 we observe the wider allowed range of parameters  $\tilde{c}_{\Phi,1}$ ,  $\tilde{\Delta}_{4F}$  in the  $\mathcal{O}(\Lambda^{-4})$  analysis, and the well-known strong correlations among  $\tilde{c}_{\Phi,1}$ ,  $\tilde{\Delta}_{4F}$  and  $\tilde{c}_{W^2 H^4}^{(3)}$ . These correlations are due to the cancellation of their linear contributions to the  $Z$  couplings and  $W$  mass when

$$\tilde{c}_{\Phi,1} = -2\tilde{\Delta}_{4F} = -2\tilde{c}_{W^2 H^4}^{(3)} \frac{\hat{v}^2}{\Lambda^2}. \quad (6.1)$$

Along this direction the bounds on these three combinations dominantly come from  $\Gamma_W$  which is less precisely determined. This correlation weakens the limits on the Wilson coefficient combinations shown in Table IV by a factor of 2–3 with respect to the order  $\mathcal{O}(\Lambda^{-2})$  analysis.

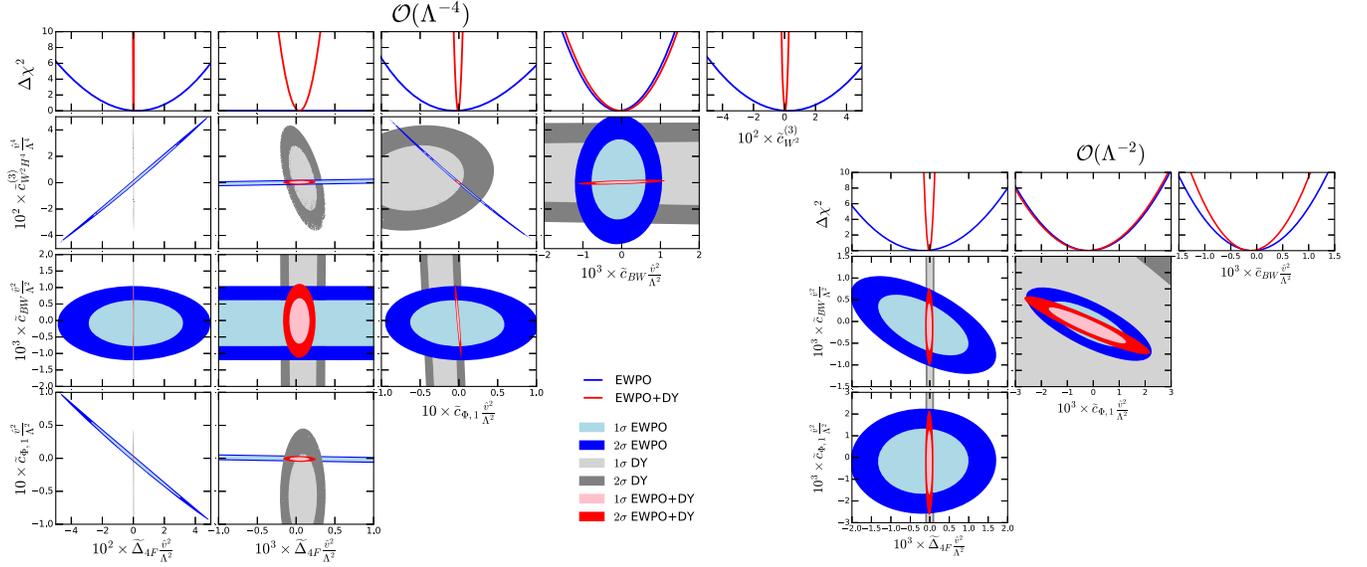


FIG. 1. One- and two-dimensional projections of  $\Delta\chi^2$  from the analyses of EWPO and Drell-Yan data performed to  $\mathcal{O}(\Lambda^{-4})$  on the left [ $\mathcal{O}(\Lambda^{-2})$  on the right] for the coefficients  $\tilde{c}_{BW}\hat{v}^2/\Lambda^2$ ,  $\tilde{c}_{\Phi,1}\hat{v}^2/\Lambda^2$ ,  $\tilde{\Delta}_{4F}\hat{v}^2/\Lambda^2$ , and  $\tilde{c}_{W^2H^4}^{(3)}\hat{v}^4/\Lambda^4$  ( $\tilde{c}_{BW}\hat{v}^2/\Lambda^2$ ,  $\tilde{c}_{\Phi,1}\hat{v}^2/\Lambda^2$ , and  $\tilde{\Delta}_{4F}\hat{v}^2/\Lambda^2$ ), as indicated in each panel after marginalizing over the 7/6 (2/1 at  $\mathcal{O}(\Lambda^{-2})$ ) undisplayed parameters for one- and two-dimensional projections respectively. Notice that the second column in the left figure is a magnification of the results on the first column for better visibility as well as the change of scale in the axis between the panels on the left and on the right figure.

In Fig. 1 we also see that the analysis of Drell-Yan data by itself provided a two orders of magnitude stronger constraint on the coefficient  $\tilde{\Delta}_{4F}$  which contains the four-fermion dimension-six operator coefficient  $c_{2JW}$ . This effect of *energy helping* accuracy discussed in Ref. [24] in the context of an  $\mathcal{O}(\Lambda^{-2})$  analysis arises from the contribution of the operator  $Q_{2JW}$  to the Drell-Yan four-fermion contact amplitudes; see Eqs. (3.27)–(3.30), or equivalently to the modified propagators (4.18)–(4.21) which dominate at higher invariant masses. Interestingly, in the  $\mathcal{O}(\Lambda^{-4})$  analysis the resulting effect in the combination of the DY with the EWPO is quantitatively more relevant because DY results contribute to breaking the very strong degeneracy present in the EWPO analysis. As a consequence, as seen in Fig. 1 and in Table IV the inclusion of DY results not only in a better determination of  $\tilde{\Delta}_{4F}$ , but also constraints on  $\tilde{c}_{\Phi,1}$  and  $\tilde{c}_{W^2H^4}^{(3)}$  of a factor  $\sim 20$  stronger.

The results of the full analyses in terms of the eight *overline coefficients* introduced in Sec. V are shown in Figs. 2 and 3 and summarized in Table V. First, from Fig. 2 we see that for none of the analyses presented do we find any favored deviation from the SM predictions, and the zero value for all of the parameters lies at  $\Delta\chi^2 < 1$ . With respect to the allowed ranges, comparing the results in the two left-most columns in Table V we see that for the analysis performed including only dimension-six operators, the constraints on the coefficients  $\tilde{c}_{BW}$ ,  $\tilde{c}_{\Phi,1}$ ,  $\tilde{c}_{2JW}$  (or equivalently  $\tilde{\Delta}_{4F}$ ), and  $\tilde{c}_{2JB}$  are robust under the inclusion of the dimension-six square contributions to the observables. Furthermore comparing these results with those of the analysis performed including the dimension-eight operators (see two right columns in Table V and red curves in Fig. 2) we learn that the bounds on  $\tilde{c}_{BW}$ ,  $\tilde{c}_{\Phi,1}$ ,  $\tilde{c}_{2JW}$ , ( $\tilde{\Delta}_{4F}$ ) become slightly weaker while the constraint on  $\tilde{c}_{2JB}$

TABLE IV. 95% CL allowed ranges for the effective couplings entering the EWPO analysis.

Coupling	95% CL allowed range			
	$\mathcal{O}(\Lambda^{-2})$		$\mathcal{O}(\Lambda^{-4})$	
	EWPO	EWPO + DY	EWPO	EWPO + DY
$\frac{\hat{v}^2}{\Lambda^2} \tilde{c}_{BW}$	$[-10, 8.4] \times 10^{-4}$	$[-8.5, 6.1] \times 10^{-4}$	$[-10, 8.4] \times 10^{-4}$	$[-9.3, 9.1] \times 10^{-4}$
$\frac{\hat{v}^2}{\Lambda^2} \tilde{c}_{\Phi,1}$	$[-2.1, 1.8] \times 10^{-3}$	$[-2.2, 1.7] \times 10^{-3}$	$[-8.0, 8.1] \times 10^{-2}$	$[-4.6, 2.7] \times 10^{-3}$
$\frac{\hat{v}^2}{\Lambda^2} \tilde{\Delta}_{4F}$	$[-1.7, 1.4] \times 10^{-3}$	$[-10, 7.6] \times 10^{-5}$	$[-3.9, 4.1] \times 10^{-2}$	$[-1.3, 2.1] \times 10^{-4}$
$\frac{\hat{v}^4}{\Lambda^4} \tilde{c}_{W^2H^4}^{(3)}$			$[-3.8, 4.3] \times 10^{-2}$	$[-1.2, 1.9] \times 10^{-3}$

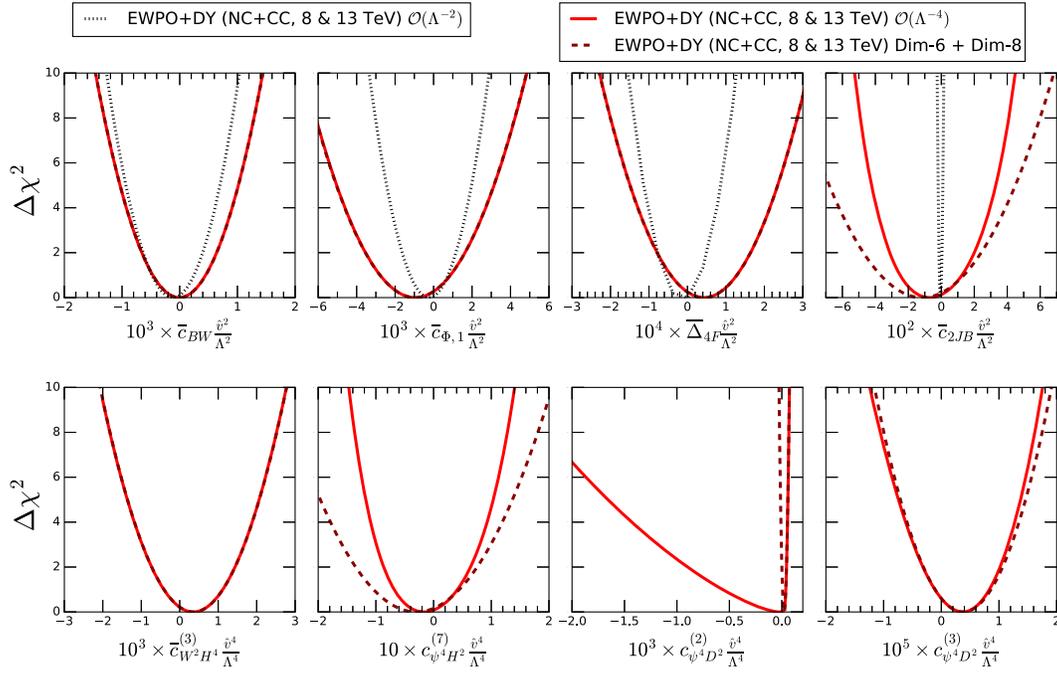


FIG. 2. One-dimensional projections of  $\Delta\chi^2$  after marginalization over all other coefficients for the different analyses performed as labeled in the figure.

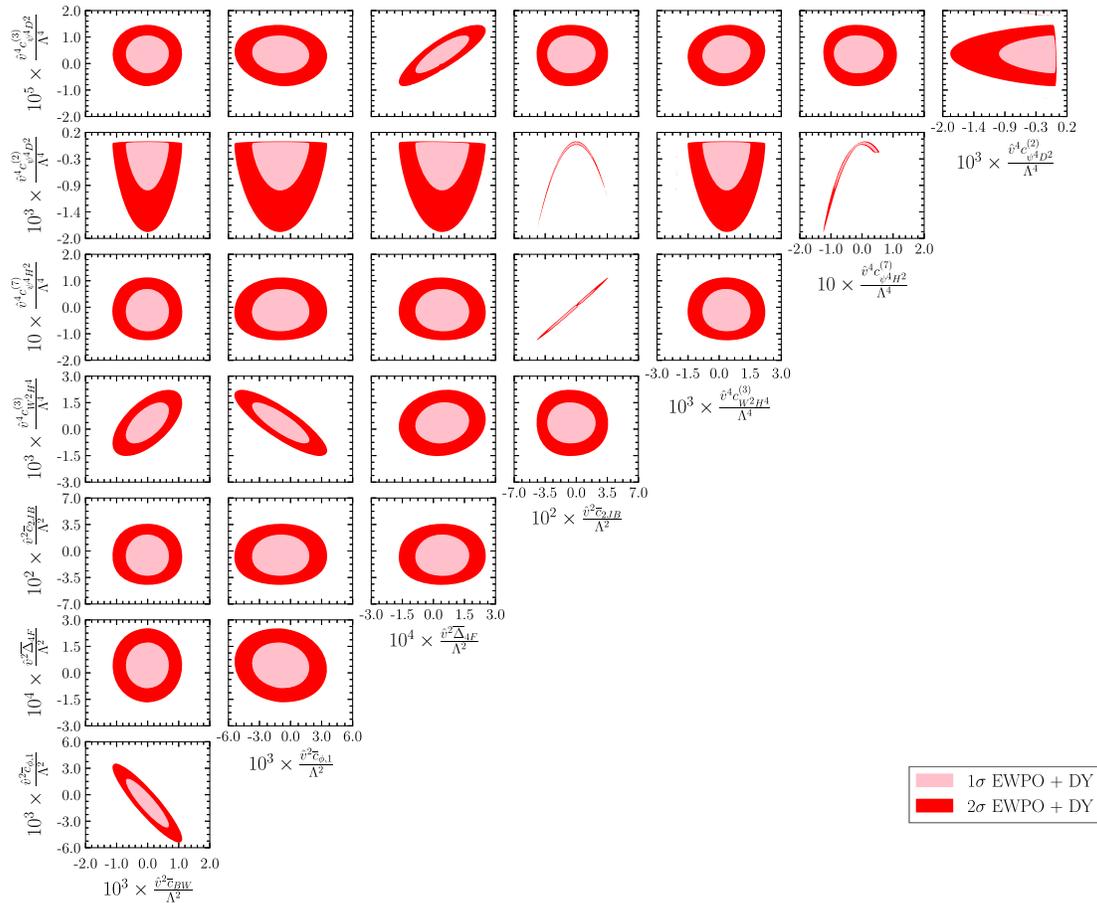


FIG. 3. Two-dimensional projection of the  $\Delta\chi^2$  from the analysis of EWPO+Drell-Yan data for the  $\mathcal{O}(\Lambda^{-4})$  analysis.

TABLE V. 95% CL allowed ranges for the Wilson coefficients from the combined analysis of EWPO and DY for the different analysis assumptions. In all cases the ranges are obtained after marginalization over all other coefficients entering the analysis. For convenience we list in the third and fourth lines the bounds on the contributions from the  $Q_{2JW}$  operator in terms of both  $\bar{c}_{2JW}$  and  $\bar{\Delta}_{4F} = -\frac{\hat{v}^2}{2\hat{c}}\bar{c}_{2JW}$ .

Coupling	95% CL allowed range EWPO + DY			
	$\mathcal{O}(\Lambda^{-2})$	Dim-6 + (Dim-6) <sup>2</sup>	$\mathcal{O}(\Lambda^{-4})$	Dim-6 + Dim-8
$\bar{c}_{BW} \frac{\hat{v}^2}{\Lambda^2}$	$[-8.5, 6.1] \times 10^{-4}$	$[-8.4, 6.1] \times 10^{-4}$	$[-9.3, 9.1] \times 10^{-4}$	$[-9.3, 9.1] \times 10^{-4}$
$\bar{c}_{\Phi,1} \frac{\hat{v}^2}{\Lambda^2}$	$[-2.2, 1.7] \times 10^{-3}$	$[-2.2, 1.7] \times 10^{-3}$	$[-4.6, 2.7] \times 10^{-3}$	$[-4.6, 2.7] \times 10^{-3}$
$\bar{\Delta}_{4F} \frac{\hat{v}^2}{\Lambda^2}$	$[-10, 7.6] \times 10^{-5}$	$[-10, 7.8] \times 10^{-5}$	$[-1.3, 2.1] \times 10^{-4}$	$[-1.3, 2.1] \times 10^{-4}$
$\bar{c}_{2JW} \frac{\hat{v}^2}{\Lambda^2}$	$[-3.6, 4.8] \times 10^{-4}$	$[-3.7, 5.0] \times 10^{-4}$	$[-10, 6.1] \times 10^{-4}$	$[-10, 6.1] \times 10^{-4}$
$\bar{c}_{2JB} \frac{\hat{v}^2}{\Lambda^2}$	$[-17, 7.9] \times 10^{-4}$	$[-21, 7.9] \times 10^{-4}$	$[-3.9, 2.9] \times 10^{-2}$	$[-6.2, 3.9] \times 10^{-2}$
$\bar{c}_{W^2H^4}^{(3)} \frac{\hat{v}^4}{\Lambda^4}$			$[-1.4, 2.0] \times 10^{-3}$	$[-1.4, 2.0] \times 10^{-3}$
$c_{\psi^4H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4}$			$[-1.1, 0.93] \times 10^{-1}$	$[-1.8, 1.1] \times 10^{-1}$
$c_{\psi^4D^2}^{(2)} \frac{\hat{v}^4}{\Lambda^4}$			$[-14, 0.54] \times 10^{-4}$	$[-9.0, 54] \times 10^{-6}$
$c_{\psi^4D^2}^{(3)} \frac{\hat{v}^4}{\Lambda^4}$			$[-6.1, 13] \times 10^{-6}$	$[-5.9, 13] \times 10^{-6}$

becomes much weaker. This is a consequence of cancellations between dimension-six and dimension-eight contributions which results into correlations between the allowed ranges.

The correlations in the determination of the coefficients is explicitly shown in Fig. 3 where we plot two-dimensional projections of the  $\chi^2$  function after marginalization over the other six coefficients. In the figure we see correlations between  $\bar{c}_{BW}$  and  $\bar{c}_{\Phi,1}$ , between  $\bar{c}_{\Phi,1}$  and  $\bar{c}_{W^2H^4}^3$  and between  $\bar{c}_{BW}$  and  $\bar{c}_{W^2H^4}^3$ . They are related to those already observed among the corresponding *tilde coefficients* in the  $\mathcal{O}(\Lambda^{-4})$  results on the left of Fig. 1. We also observe a moderate correlation between  $\bar{\Delta}_{4F}$  ( $\bar{c}_{2JW}$ ) and  $c_{\psi^4D^2}^{(3)}$ . We can trace its origin to the fact that linear combinations of these two coefficients enter in both the pole observables [see Eqs. (3.42)–(3.44)] and in the four-fermion contact DY amplitudes in Eqs. (3.23) and (3.22) [see also Eqs. (3.27)–(3.30) and (3.38)–(3.41)]. However the relative effects are different at the Z and W pole than in the DY amplitudes because of the momentum dependence of the  $c_{\psi^4D^2}^{(3)}$  contribution. Consequently the combination of EWPO and DY from both NC and CC processes can independently bound the two coefficients leaving only the correlation shown.

From Fig. 3 we also see that the most correlated bounds correspond to the coefficients  $\bar{c}_{2JB}$ ,  $c_{\psi^4H^2}^{(7)}$ , and  $c_{\psi^4D^2}^{(2)}$ . We first observe a very strong positive correlation between  $\bar{c}_{2JB}$  and  $c_{\psi^4H^2}^{(7)}$ . These two operators do not contribute to EWPO and only enter DY in the four-fermion NC contact amplitudes in Eq. (3.22) [see Eqs. (3.27)–(3.29)], of which  $\bar{\mathcal{N}}_{\gamma\gamma}$  is numerically larger. Consequently, the analysis provides the weakest bounds when  $\bar{\mathcal{N}}_{\gamma\gamma}$  cancels which occurs for

$$\bar{c}_{2JB} \frac{\hat{v}^2}{\Lambda^2} = \frac{\hat{s}}{2\hat{c}} c_{\psi^4H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4} \simeq 0.3 c_{\psi^4H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4}, \quad (6.2)$$

leading to the very strong correlation observed and the substantial weakening of the bounds on  $c_{2JB}$  when compared to the analysis performed at  $\mathcal{O}(\Lambda^{-2})$ . In addition we find very highly correlated nonelliptical allowed regions for  $\bar{c}_{2JB}$  and  $c_{\psi^4D^2}^{(2)}$  and also for  $c_{\psi^4H^2}^{(7)}$  and  $c_{\psi^4D^2}^{(2)}$ . We trace this behaviour to possible cancellations in the Drell-Yan NC distributions between the linear contribution from negative values of  $c_{\psi^4D^2}^{(2)}$  and the quadratic contribution from  $\bar{c}_{2JB}$  because both enter at  $\mathcal{O}(\Lambda^{-4})$ . We illustrate this behaviour in Fig. 4 where we show the predicted invariant mass distribution for DY NC cross section at 13 TeV for a set of values of  $\bar{c}_{2JB}$ ,  $c_{\psi^4H^2}^{(7)}$ , and  $c_{\psi^4D^2}^{(2)}$  within the  $2\sigma$  bounds from the  $\mathcal{O}(\Lambda^{-4})$  analysis. As seen in the figure the full  $\mathcal{O}(\Lambda^{-4})$  prediction (red line) is very similar to the SM in the range of invariant masses shown. Conversely once the dimension-six square contribution is not included (dotted blue line) the prediction departs substantially from the SM. Consequently, bounds on negative values of  $c_{\psi^4D^2}^{(2)}$  are much stronger if the dimension-six square contribution is not included as seen in the dashed lines in Fig. 2 and the last column in Table V. This degeneracy in the DY event rates can only be broken with larger statistics at the highest invariant masses.

Recent theoretical work on positivity bounds [16,63–65] can restrict the values of operator coefficients. Nevertheless, a more complete analysis taking into account positivity constraints in the context of USMEFT may modify our statistical results, but is beyond the scope of the current work.

We finish by presenting in Fig. 5 the results of an analysis performed in terms of the generalized oblique

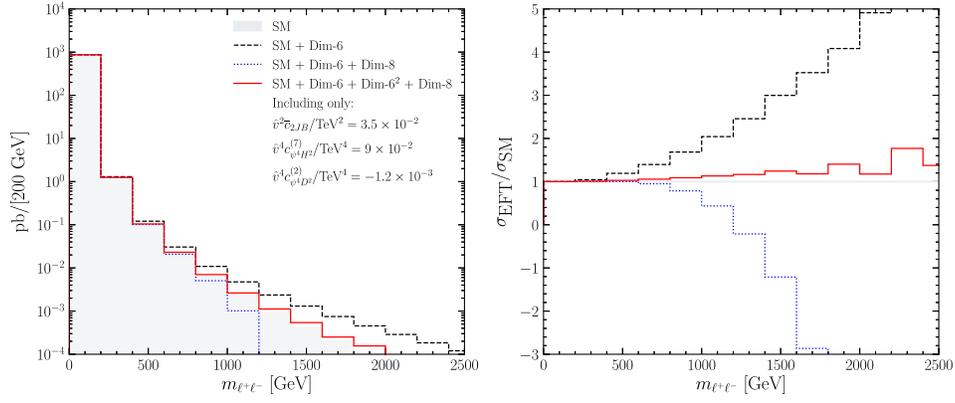


FIG. 4. Prediction of the DY NC invariant mass distribution for several values of the Wilson coefficients as labeled in the figure.

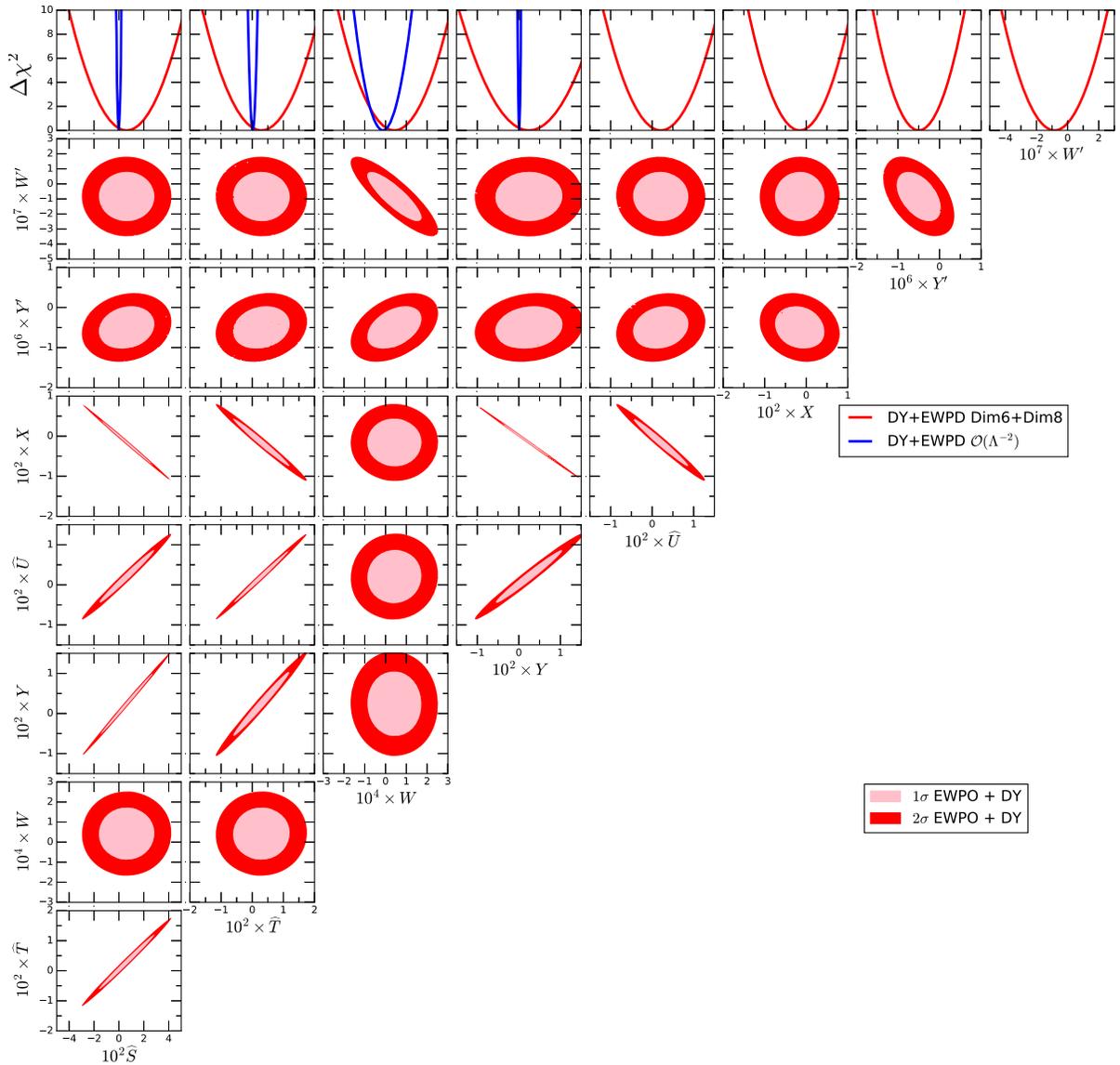


FIG. 5. One- and two-dimensional projection of the  $\Delta\chi^2$  from the analysis of EWPO and Drell-Yan data in terms of the generalized oblique parameters.

TABLE VI. 95% CL allowed ranges for the oblique parameters from the combined analysis of EWPO and DY for the different analysis assumptions. In all cases the ranges are obtained after marginalization over all other coefficients entering the analysis.

Parameter	95% CL allowed range EWPO + DY	
	$\mathcal{O}(\Lambda^{-2})$	Dim-6+Dim-8
$\hat{S}$	$[-1.5, 1.2] \times 10^{-3}$	$[-2.3, 3.6] \times 10^{-2}$
$\hat{T}$	$[-8.4, 11] \times 10^{-4}$	$[-9.0, 15] \times 10^{-3}$
$W$	$[-10, 7.6] \times 10^{-5}$	$[-1.3, 2.1] \times 10^{-4}$
$Y$	$[-1.7, 3.7] \times 10^{-4}$	$[-8.2, 13] \times 10^{-3}$
$\hat{U}$		$[-7.7, 12] \times 10^{-3}$
$X$		$[-9.4, 6.3] \times 10^{-3}$
$Y'$		$[-12, 2.0] \times 10^{-7}$
$W'$		$[-3.0, 1.3] \times 10^{-7}$

parameters introduced in Sec. IV. As discussed, in the framework of the operator expansion this corresponds to an analysis which neglects the dimension-six square effects. Therefore, by construction the  $\chi^2$  statistics are quadratic functions of all the oblique parameters and the two-dimensional projections, are elliptic regions. In this figure we see strong correlations among several of the allowed ranges of the oblique parameters. The correlations among  $\hat{S}$ ,  $\hat{T}$ , and  $\hat{U}$  dominantly stem from the same effect as the correlation among  $\bar{c}_{BW}$ ,  $\bar{c}_{\Phi,1}$ , and  $\bar{c}_{W^2H^4}^3$  observed in Fig. 3, or equivalently among  $\tilde{c}_{BW}$ ,  $\tilde{c}_{\Phi,1}$ , and  $\tilde{c}_{W^2H^4}^3$  in Fig. 1. The anticorrelation between  $W$  and  $W'$  arises from the same effects as  $\bar{\Delta}_{4F}$  ( $\bar{c}_{2JW}$ ) and  $c_{\psi^4D^2}^{(3)}$  discussed above. In addition we see a somewhat weak anticorrelation between  $W'$  and  $Y'$  stemming from their dominant contribution to the tail of the invariant mass distributions induced by  $e'_{YY}$ ,  $e'_{YZ}$ ,  $e'_{ZZ}$ , and  $e'_{WW}$ , (4.18)–(4.21). The correlations observed involving  $X$  and  $Y$  mostly result from the cancellations equivalent to that in Eq. (6.2); see Appendix B for the expressions of the oblique parameters in terms of the operator coefficients. For the sake of comparison we also show in the figure the one-dimensional projection of an analysis performed in terms of the oblique parameters which are generated by USMEFT dimension-six operators,  $\hat{S}$ ,  $\hat{T}$ ,  $W$ , and  $Y$ . From this figure we can see that as a consequence of the above correlations, the bounds on these four oblique parameters relax considerably when including the effects of  $\hat{U}$ ,  $X$ ,  $W'$ , and  $Y'$  as quantified in Table VI.

## VII. SUMMARY

We have studied Drell-Yan production in universal theories consistently including effects beyond those of USMEFT at dimension-six. We have focused on effects which are  $C$  and  $P$  conserving and found that eleven dimension-eight operators and six dimension-six operators contribute to our analyses. The chosen bases are listed in

Tables I and II. Working in the rotated basis in which operators with higher derivatives of the bosonic fields have been replaced by the equations of motion in favor of combinations of operators involving SM fermionic currents, we have identified the eight combinations of the 17 Wilson coefficients which are physically distinguishable by studying the invariant mass distribution of the lepton pairs produced:  $\bar{\Delta}_{4F}$ ,  $\bar{c}_{BW}$ ,  $\bar{c}_{\Phi,1}$ ,  $\bar{c}_{2JB}$ , and  $\bar{c}_{W^2H^4}^3$ , given in Eqs. (3.9)–(3.12) and (3.21), together with the coefficients of the dimension-eight operators  $c_{\psi^4H^2}^{(7)}$ ,  $c_{\psi^4D^2}^{(2)}$ , and  $c_{\psi^4D^2}^{(3)}$ . Of those eight, the four *tilde coefficients* in Eq. (3.45) contribute EWPO at the  $Z$  and  $W$  poles.

In Sec. IV we have introduced an extension of the parametrization of universal effects in terms of 11 oblique parameters obtained by linearly expanding the self-energies of the electroweak gauge bosons to  $\mathcal{O}(q^6)$ . Of those, eight are generated by the USMEFT at dimension-eight:  $\hat{S}$ ,  $\hat{T}$ ,  $W$ ,  $Y$ ,  $\hat{U}$ ,  $X$ , plus two additional which we label  $W'$  and  $Y'$ . The correspondence between these eight oblique parameters and the operator coefficients is given in Appendix B.

We have performed combined analyses to a variety of LHC dilepton data and the EWPO in order to quantify the constraints on the full parameter space and studied the dependence of the derived constraints with the order of the expansion considered. We first have quantified how the DY results can complement the constraints from EWPO on the four *tilde coefficients*. We found that in the  $\mathcal{O}(\Lambda^{-4})$  analysis the resulting effect of the combination of the DY with the EWPO is quantitatively more relevant than at  $\mathcal{O}(\Lambda^{-2})$  as, besides constraining the coefficient  $\bar{\Delta}_{4F}$  better from contact four-fermion DY amplitudes, DY results further contribute by breaking the very strong degeneracy present in the EWPO analysis when including the dimension-eight operators. As a consequence, as seen in Fig. 1 and in Table IV the inclusion of the DY results results not only in the better determination of  $\bar{\Delta}_{4F}$  but also in a factor  $\sim 20$  stronger constraint on  $\bar{c}_{\Phi,1}$ , and  $\bar{c}_{W^2H^4}^{(3)}$ .

The results on the full eight parameter space are shown in Figs. 2, 3, and Table V. They show that, when consistently including all effects to  $\mathcal{O}(\Lambda^{-4})$ , the combination of EWPO and DY provides robust constraints on the three coefficients with leading dimension-six contributions  $\bar{c}_{BW}$ ,  $\bar{c}_{\Phi,1}$ ,  $\bar{c}_{2JW}$  which are only weaker by at most a factor  $\sim 2$  with respect to the  $\mathcal{O}(\Lambda^{-2})$  bounds. Robust bounds are also obtained for the dimension-eight operator coefficients  $\bar{c}_{W^2H^2}^{(3)}$  and  $c_{\psi^4D^2}^{(3)}$  which are not affected by possible cancellations with dimension-six square contributions. Conversely, the bounds on the leading dimension-six coefficient  $\bar{c}_{2JB}$  is weakened by more than one order of magnitude with respect to the  $\mathcal{O}(\Lambda^{-2})$  limits due to cancellations with the  $\mathcal{O}(\Lambda^{-4})$  contributions from  $c_{\psi^4H^2}^{(7)}$  and  $c_{\psi^4D^2}^{(2)}$ . Consequently, the bounds on these two dimension-eight Wilson

coefficients, in particular  $c_{\psi^4 D^2}^{(2)}$  are the least robust. Finally we have quantified the constraints on the eight oblique parameters in Fig. 5 and Table VI.

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**DATA AVAILABILITY**

The data are not publicly available. The data are available from the authors upon reasonable request.

**APPENDIX A: UNIVERSAL BOSONIC OPERATOR BASIS AT DIMENSION EIGHT**

Murphy’s basis contains 89 bosonic operators in total. We list them in Table VII together with their transformation properties under  $C$  and  $P$ . In addition there are 86 bosonic operators present in universal models which contain higher derivatives and which can be rotated into fermionic operators by the SM EOM. We list them in Table VIII together with their transformation properties under  $C$  and  $P$ . For convenience we indicate the  $38 + 50$  operators that violate either  $C$  or  $P$  with a lighter shade.

TABLE VII. 89 purely bosonic operators present in Murphy’s basis.

1: $X^4, X^3 X^1$		$C$	$P$	$CP$	1: $X^2 X^1^2$		$C$	$P$	$CP$
$Q_{G^4}^{(1)}$	$(G_{\mu\nu}^A G^{A\mu\nu})(G_{\rho\sigma}^B G^{B\rho\sigma})$	✓	✓	✓	$Q_{G^2 W^2}^{(1)}$	$(W_{\mu\nu}^I W^{I\mu\nu})(G_{\rho\sigma}^A G^{A\rho\sigma})$	✓	✓	✓
$Q_{G^4}^{(2)}$	$(G_{\mu\nu}^A \tilde{G}^{A\mu\nu})(G_{\rho\sigma}^B \tilde{G}^{B\rho\sigma})$	✓	✓	✓	$Q_{G^2 W^2}^{(2)}$	$(W_{\mu\nu}^I \tilde{W}^{I\mu\nu})(G_{\rho\sigma}^A \tilde{G}^{A\rho\sigma})$	✓	✓	✓
$Q_{G^4}^{(3)}$	$(G_{\mu\nu}^A G^{B\mu\nu})(G_{\rho\sigma}^A G^{B\rho\sigma})$	✓	✓	✓	$Q_{G^2 W^2}^{(3)}$	$(W_{\mu\nu}^I G^{A\mu\nu})(W_{\rho\sigma}^J G^{A\rho\sigma})$	✓	✓	✓
$Q_{G^4}^{(4)}$	$(G_{\mu\nu}^A \tilde{G}^{B\mu\nu})(G_{\rho\sigma}^A \tilde{G}^{B\rho\sigma})$	✓	✓	✓	$Q_{G^2 W^2}^{(4)}$	$(W_{\mu\nu}^I \tilde{G}^{A\mu\nu})(W_{\rho\sigma}^J \tilde{G}^{A\rho\sigma})$	✓	✓	✓
$Q_{G^4}^{(5)}$	$(G_{\mu\nu}^A G^{A\mu\nu})(G_{\rho\sigma}^B \tilde{G}^{B\rho\sigma})$	✓	✗	✗	$Q_{G^2 W^2}^{(5)}$	$(W_{\mu\nu}^I \tilde{W}^{I\mu\nu})(G_{\rho\sigma}^A G^{A\rho\sigma})$	✓	✗	✗
$Q_{G^4}^{(6)}$	$(G_{\mu\nu}^A G^{B\mu\nu})(G_{\rho\sigma}^A \tilde{G}^{B\rho\sigma})$	✓	✗	✗	$Q_{G^2 W^2}^{(6)}$	$(W_{\mu\nu}^I W^{I\mu\nu})(G_{\rho\sigma}^A \tilde{G}^{A\rho\sigma})$	✓	✗	✗
$Q_{G^4}^{(7)}$	$d^{ABE} d^{CDE} (G_{\mu\nu}^A G^{B\mu\nu})(G_{\rho\sigma}^C G^{D\rho\sigma})$	✓	✓	✓	$Q_{G^2 W^2}^{(7)}$	$(W_{\mu\nu}^I G^{A\mu\nu})(W_{\rho\sigma}^J \tilde{G}^{A\rho\sigma})$	✓	✗	✗
$Q_{G^4}^{(8)}$	$d^{ABE} d^{CDE} (G_{\mu\nu}^A \tilde{G}^{B\mu\nu})(G_{\rho\sigma}^C \tilde{G}^{D\rho\sigma})$	✓	✓	✓	$Q_{G^2 B^2}^{(1)}$	$(B_{\mu\nu} B^{\mu\nu})(G_{\rho\sigma}^A G^{A\rho\sigma})$	✓	✓	✓
$Q_{G^4}^{(9)}$	$d^{ABE} d^{CDE} (G_{\mu\nu}^A G^{B\mu\nu})(G_{\rho\sigma}^C \tilde{G}^{D\rho\sigma})$	✓	✗	✗	$Q_{G^2 B^2}^{(2)}$	$(B_{\mu\nu} \tilde{B}^{\mu\nu})(G_{\rho\sigma}^A \tilde{G}^{A\rho\sigma})$	✓	✓	✓
$Q_{W^4}^{(1)}$	$(W_{\mu\nu}^I W^{I\mu\nu})(W_{\rho\sigma}^J W^{J\rho\sigma})$	✓	✓	✓	$Q_{G^2 B^2}^{(3)}$	$(B_{\mu\nu} G^{A\mu\nu})(B_{\rho\sigma} G^{A\rho\sigma})$	✓	✓	✓
$Q_{W^4}^{(2)}$	$(W_{\mu\nu}^I \tilde{W}^{I\mu\nu})(W_{\rho\sigma}^J \tilde{W}^{J\rho\sigma})$	✓	✓	✓	$Q_{G^2 B^2}^{(4)}$	$(B_{\mu\nu} \tilde{G}^{A\mu\nu})(B_{\rho\sigma} \tilde{G}^{A\rho\sigma})$	✓	✓	✓
$Q_{W^4}^{(3)}$	$(W_{\mu\nu}^I W^{J\mu\nu})(W_{\rho\sigma}^I W^{J\rho\sigma})$	✓	✓	✓	$Q_{G^2 B^2}^{(5)}$	$(B_{\mu\nu} \tilde{B}^{\mu\nu})(G_{\rho\sigma}^A G^{A\rho\sigma})$	✓	✗	✗
$Q_{W^4}^{(4)}$	$(W_{\mu\nu}^I \tilde{W}^{J\mu\nu})(W_{\rho\sigma}^I \tilde{W}^{J\rho\sigma})$	✓	✓	✓	$Q_{G^2 B^2}^{(6)}$	$(B_{\mu\nu} B^{\mu\nu})(G_{\rho\sigma}^A \tilde{G}^{A\rho\sigma})$	✓	✗	✗
$Q_{W^4}^{(5)}$	$(W_{\mu\nu}^I W^{I\mu\nu})(W_{\rho\sigma}^J \tilde{W}^{J\rho\sigma})$	✓	✗	✗	$Q_{G^2 B^2}^{(7)}$	$(B_{\mu\nu} G^{A\mu\nu})(B_{\rho\sigma} \tilde{G}^{A\rho\sigma})$	✓	✗	✗
$Q_{W^4}^{(6)}$	$(W_{\mu\nu}^I W^{J\mu\nu})(W_{\rho\sigma}^I \tilde{W}^{J\rho\sigma})$	✓	✗	✗	$Q_{W^2 B^2}^{(1)}$	$(B_{\mu\nu} B^{\mu\nu})(W_{\rho\sigma}^I W^{I\rho\sigma})$	✓	✓	✓
$Q_{B^4}^{(1)}$	$(B_{\mu\nu} B^{\mu\nu})(B_{\rho\sigma} B^{\rho\sigma})$	✓	✓	✓	$Q_{W^2 B^2}^{(2)}$	$(B_{\mu\nu} \tilde{B}^{\mu\nu})(W_{\rho\sigma}^I \tilde{W}^{I\rho\sigma})$	✓	✓	✓
$Q_{B^4}^{(2)}$	$(B_{\mu\nu} \tilde{B}^{\mu\nu})(B_{\rho\sigma} \tilde{B}^{\rho\sigma})$	✓	✓	✓	$Q_{W^2 B^2}^{(3)}$	$(B_{\mu\nu} W^{I\mu\nu})(B_{\rho\sigma} W^{I\rho\sigma})$	✓	✓	✓
$Q_{B^4}^{(3)}$	$(B_{\mu\nu} B^{\mu\nu})(B_{\rho\sigma} \tilde{B}^{\rho\sigma})$	✓	✗	✗	$Q_{W^2 B^2}^{(4)}$	$(B_{\mu\nu} \tilde{W}^{I\mu\nu})(B_{\rho\sigma} \tilde{W}^{I\rho\sigma})$	✓	✓	✓
$Q_{G^3 B}^{(1)}$	$d^{ABC} (B_{\mu\nu} G^{A\mu\nu})(G_{\rho\sigma}^B G^{C\rho\sigma})$	✓	✓	✓	$Q_{W^2 B^2}^{(5)}$	$(B_{\mu\nu} \tilde{B}^{\mu\nu})(W_{\rho\sigma}^I W^{I\rho\sigma})$	✓	✗	✗
$Q_{G^3 B}^{(2)}$	$d^{ABC} (B_{\mu\nu} \tilde{G}^{A\mu\nu})(G_{\rho\sigma}^B \tilde{G}^{C\rho\sigma})$	✓	✓	✓	$Q_{W^2 B^2}^{(6)}$	$(B_{\mu\nu} B^{\mu\nu})(W_{\rho\sigma}^I \tilde{W}^{I\rho\sigma})$	✓	✗	✗

(Table continued)

TABLE VII. (Continued)

1: $X^4, X^3 X'$		$C$	$P$	$CP$	1: $X^2 X'^2$		$C$	$P$	$CP$
$Q_{G^3 B}^{(3)}$	$d^{ABC}(B_{\mu\nu}\tilde{G}^{A\mu\nu})(G_{\rho\sigma}^B G^{\rho\sigma})$	✓	✗	✗	$Q_{W^2 B^2}^{(7)}$	$(B_{\mu\nu}W^{1\mu\nu})(B_{\rho\sigma}\tilde{W}^{1\rho\sigma})$	✓	✗	✗
$Q_{G^3 B}^{(4)}$	$d^{ABC}(B_{\mu\nu}G^{A\mu\nu})(G_{\rho\sigma}^B \tilde{G}^{\rho\sigma})$	✓	✗	✗					
2: $H^8$					4: $H^4 D^4$				
$Q_{H^8}$	$(H^\dagger H)^4$	✓	✓	✓	$Q_{H^4}^{(1)}$	$(D_\mu H^\dagger D_\nu H)(D^\nu H^\dagger D^\mu H)$	✓	✓	✓
3: $H^6 D^2$					$Q_{H^4}^{(2)}$	$(D_\mu H^\dagger D_\nu H)(D^\mu H^\dagger D^\nu H)$	✓	✓	✓
$Q_{H^6}^{(1)}$	$(H^\dagger H)^2(D_\mu H^\dagger D^\mu H)$	✓	✓	✓	$Q_{H^4}^{(3)}$	$(D^\mu H^\dagger D_\mu H)(D^\nu H^\dagger D_\nu H)$	✓	✓	✓
$Q_{H^6}^{(2)}$	$(H^\dagger H)(H^\dagger \tau^I H)(D_\mu H^\dagger \tau^I D^\mu H)$	✓	✓	✓					
					5: $X^3 H^2$				
$Q_{G^3 H^2}^{(1)}$	$f^{ABC}(H^\dagger H)G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	✓	✓	✓	$Q_{W^3 H^2}^{(2)}$	$\epsilon^{IJK}(H^\dagger H)W_\mu^{I\nu}W_\nu^{J\rho}\tilde{W}_\rho^{K\mu}$	✓	✗	✗
$Q_{G^3 H^2}^{(2)}$	$f^{ABC}(H^\dagger H)G_\mu^{A\nu}G_\nu^{B\rho}\tilde{G}_\rho^{C\mu}$	✓	✗	✗	$Q_{W^2 BH^2}^{(1)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)B_\mu^J W_\nu^{K\rho}W_\rho^{K\mu}$	✓	✓	✓
$Q_{W^3 H^2}^{(1)}$	$\epsilon^{IJK}(H^\dagger H)W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	✓	✓	✓	$Q_{W^2 BH^2}^{(2)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)(\tilde{B}^{\mu\nu}W_{\nu\rho}^J W_\mu^{K\rho} + B^{\mu\nu}W_{\nu\rho}^J \tilde{W}_\mu^{K\rho})$	✓	✗	✗
					6: $X^2 H^4$				
$Q_{G^2 H^4}^{(1)}$	$(H^\dagger H)^2 G_\mu^A G^{A\mu\nu}$	✓	✓	✓	$Q_{W^2 H^4}^{(4)}$	$(H^\dagger \tau^I H)(H^\dagger \tau^I H)\tilde{W}_{\mu\nu}^I W^{J\mu\nu}$	✓	✗	✗
$Q_{G^2 H^4}^{(2)}$	$(H^\dagger H)^2 \tilde{G}_\mu^A G^{A\mu\nu}$	✓	✗	✗	$Q_{B^2 H^4}^{(1)}$	$(H^\dagger H)^2 B_{\mu\nu} B^{\mu\nu}$	✓	✓	✓
$Q_{W^2 H^4}^{(1)}$	$(H^\dagger H)^2 W_\mu^I W^{I\mu\nu}$	✓	✓	✓	$Q_{B^2 H^4}^{(2)}$	$(H^\dagger H)^2 \tilde{B}_{\mu\nu} B^{\mu\nu}$	✓	✗	✗
$Q_{W^2 H^4}^{(2)}$	$(H^\dagger H)^2 \tilde{W}_\mu^I W^{I\mu\nu}$	✓	✗	✗	$Q_{W^2 H^4}^{(3)}$	$(H^\dagger \tau^I H)(H^\dagger \tau^I H)W_\mu^I W^{J\mu\nu}$	✓	✓	✓
$Q_{WBH^4}^{(1)}$	$(H^\dagger H)(H^\dagger \tau^I H)W_\mu^I B^{\mu\nu}$	✓	✓	✓	$Q_{WBH^4}^{(2)}$	$(H^\dagger H)(H^\dagger \tau^I H)\tilde{W}_\mu^I B^{\mu\nu}$	✓	✗	✗
					7: $X^2 H^2 D^2$				
$Q_{G^2 H^2 D^2}^{(1)}$	$(D^\mu H^\dagger D^\nu H)G_{\mu\rho}^A G_\nu^{A\rho}$	✓	✓	✓	$Q_{B^2 H^2 D^2}^{(1)}$	$(D^\mu H^\dagger D^\nu H)B_{\mu\rho} B_\nu^\rho$	✓	✓	✓
$Q_{G^2 H^2 D^2}^{(2)}$	$(D^\mu H^\dagger D_\mu H)G_{\nu\rho}^A G^{\nu\rho}$	✓	✓	✓	$Q_{B^2 H^2 D^2}^{(2)}$	$(D^\mu H^\dagger D_\mu H)B_{\nu\rho} B^{\nu\rho}$	✓	✓	✓
$Q_{G^2 H^2 D^2}^{(3)}$	$(D^\mu H^\dagger D_\mu H)G_{\nu\rho}^A \tilde{G}^{A\nu\rho}$	✓	✗	✗	$Q_{B^2 H^2 D^2}^{(3)}$	$(D^\mu H^\dagger D_\mu H)B_{\nu\rho} \tilde{B}^{\nu\rho}$	✓	✗	✗
$Q_{W^2 H^2 D^2}^{(1)}$	$(D^\mu H^\dagger D^\nu H)W_{\mu\rho}^I W_\nu^{I\rho}$	✓	✓	✓	$Q_{WBH^2 D^2}^{(1)}$	$(D^\mu H^\dagger \tau^I D_\mu H)B_{\nu\rho} W^{I\nu\rho}$	✓	✓	✓
$Q_{W^2 H^2 D^2}^{(2)}$	$(D^\mu H^\dagger D_\mu H)W_{\nu\rho}^I W^{I\nu\rho}$	✓	✓	✓	$Q_{WBH^2 D^2}^{(2)}$	$(D^\mu H^\dagger \tau^I D_\mu H)B_{\nu\rho} \tilde{W}^{I\nu\rho}$	✓	✗	✗
$Q_{W^2 H^2 D^2}^{(3)}$	$(D^\mu H^\dagger D_\mu H)W_{\nu\rho}^I \tilde{W}^{I\nu\rho}$	✓	✗	✗	$Q_{WBH^2 D^2}^{(3)}$	$i(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho} W_\nu^{I\rho} - B_{\nu\rho} W_\mu^{I\rho})$	✗	✓	✗
$Q_{W^2 H^2 D^2}^{(4)}$	$i\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H)W_{\mu\rho}^J W_\nu^{K\rho}$	✓	✓	✓	$Q_{WBH^2 D^2}^{(4)}$	$(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho} W_\nu^{I\rho} + B_{\nu\rho} W_\mu^{I\rho})$	✓	✓	✓
$Q_{W^2 H^2 D^2}^{(5)}$	$\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H)(W_{\mu\rho}^J \tilde{W}_\nu^{K\rho} - \tilde{W}_{\mu\rho}^J W_\nu^{K\rho})$	✓	✗	✗	$Q_{WBH^2 D^2}^{(5)}$	$i(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho} \tilde{W}_\nu^{I\rho} - B_{\nu\rho} \tilde{W}_\mu^{I\rho})$	✗	✗	✓
$Q_{W^2 H^2 D^2}^{(6)}$	$i\epsilon^{IJK}(D^\mu H^\dagger \tau^I D^\nu H)(W_{\mu\rho}^J \tilde{W}_\nu^{K\rho} + \tilde{W}_{\mu\rho}^J W_\nu^{K\rho})$	✓	✗	✗	$Q_{WBH^2 D^2}^{(6)}$	$(D^\mu H^\dagger \tau^I D^\nu H)(B_{\mu\rho} \tilde{W}_\nu^{I\rho} + B_{\nu\rho} \tilde{W}_\mu^{I\rho})$	✓	✗	✗
					8: $X H^4 D^2$				
$Q_{WH^4 D^2}^{(1)}$	$(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H)W_{\mu\nu}^I$	✓	✓	✓	$Q_{WH^4 D^2}^{(4)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^I D^\nu H)\tilde{W}_{\mu\nu}^K$	✗	✗	✓
$Q_{WH^4 D^2}^{(2)}$	$(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H)\tilde{W}_{\mu\nu}^I$	✓	✗	✗	$Q_{BH^4 D^2}^{(1)}$	$(H^\dagger H)(D^\mu H^\dagger D^\nu H)B_{\mu\nu}$	✓	✓	✓
$Q_{WH^4 D^2}^{(3)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^I D^\nu H)W_{\mu\nu}^K$	✗	✓	✗	$Q_{BH^4 D^2}^{(2)}$	$(H^\dagger H)(D^\mu H^\dagger D^\nu H)\tilde{B}_{\mu\nu}$	✓	✗	✗

TABLE VIII. Additional 86 bosonic operators present in universal theories.

$H^6 D^2$		$C P CP$	$H^6 D^2$		$C P CP$
$R_{H^6 D^2}^{(1)}$	$(D^2 H^\dagger H + H^\dagger D^2 H)(H^\dagger H)(H^\dagger H)$	✓ ✓ ✓	$R_{H^6 D^2}^{(2)}$	$i(H^\dagger D^2 H - D^2 H^\dagger H)(H^\dagger H)(H^\dagger H)$	✗ ✓ ✗
$H^4 D^4$					
$R_{H^4 D^4}^{(1)}$	$(D^2 H^\dagger \tau^\dagger H + H^\dagger \tau^\dagger D^2 H)(D^\mu H^\dagger \tau^\dagger D_\mu H)$	✓ ✓ ✓	$R_{H^4 D^4}^{(6)}$	$I(H^\dagger D^2 H - D^2 H^\dagger H)(D^\mu H^\dagger D_\mu H)$	✗ ✓ ✗
$R_{H^4 D^4}^{(2)}$	$(D^2 H^\dagger D_\mu H)(H^\dagger D^\mu H) + (D_\mu H^\dagger D^2 H)(D^\mu H^\dagger H)$	✓ ✓ ✓	$R_{H^4 D^4}^{(7)}$	$(D^2 H^\dagger D^2 H)(H^\dagger H)$	✓ ✓ ✓
$R_{H^4 D^4}^{(3)}$	$i((D_\mu H^\dagger D^2 H)(D^\mu H^\dagger H) - (D^2 H^\dagger D_\mu H)(H^\dagger D^\mu H))$	✗ ✓ ✗	$R_{H^4 D^4}^{(8)}$	$(D^2 H^\dagger H)(D^2 H^\dagger H) + (H^\dagger D^2 H)(H^\dagger D^2 H)$	✓ ✓ ✓
$R_{H^4 D^4}^{(4)}$	$i(H^\dagger \tau^\dagger D^2 H - D^2 H^\dagger \tau^\dagger H)(D_\mu H^\dagger \tau^\dagger D^\mu H)$	✗ ✓ ✗	$R_{H^4 D^4}^{(9)}$	$(D^2 H^\dagger H)(H^\dagger D^2 H)$	✓ ✓ ✓
$R_{H^4 D^4}^{(5)}$	$(D^2 H^\dagger H + H^\dagger D^2 H)(D_\mu H^\dagger D^\mu H)$	✓ ✓ ✓	$R_{H^4 D^4}^{(10)}$	$i((H^\dagger D^2 H)(H^\dagger D^2 H) - (D^2 H^\dagger H)(D^2 H^\dagger H))$	✗ ✓ ✗
$H^2 D^6$			$X^2 D^4$		
$R_{H^2 D^6}^{(1)}$	$(D^\mu D^2 H^\dagger D^\mu D^2 H)$	✓ ✓ ✓	$R_{B^2 D^4}^{(1)}$	$D^\rho D^\alpha B_{\alpha\mu} D_\rho D^\beta B_\beta^\mu$	✓ ✓ ✓
			$R_{W^2 D^4}^{(1)}$	$D^\rho D^\alpha W_{\alpha\mu}^I D_\rho D^\beta W_\beta^{I,\mu}$	✓ ✓ ✓
			$R_{G^2 D^4}^{(1)}$	$D^\alpha D^\mu G_{\mu\nu}^A D_\alpha D^\rho G_\rho^{A,\nu}$	✓ ✓ ✓
$X^3 D^2, X^2 X' D^2$					
$R_{W^3 D^2}^{(1)}$	$W_{\mu\nu}^I (D_\alpha W^{J,\alpha\mu})(D_\beta W^{K,\beta\nu}) \epsilon^{IJK}$	✓ ✓ ✓	$R_{G^3 D^2}^{(1)}$	$G_{\mu\nu}^A (D_\alpha G^{B,\alpha\mu})(D_\beta G^{C,\beta\nu}) f^{ABC}$	✓ ✓ ✓
$R_{W^3 D^2}^{(2)}$	$\tilde{W}_{\mu\nu}^I (D_\alpha W^{J,\alpha\mu})(D_\beta W^{K,\beta\nu}) \epsilon^{IJK}$	✓ ✗ ✗	$R_{G^3 D^2}^{(2)}$	$\tilde{G}_{\mu\nu}^A (D_\alpha G^{B,\alpha\mu})(D_\beta G^{C,\beta\nu}) f^{ABC}$	✓ ✗ ✗
$R_{W^3 D^2}^{(3)}$	$W_{\mu\nu}^I W_{\rho}^{J,\nu} (D^\mu D_\alpha W^{K,\alpha\rho}) \epsilon^{IJK}$	✓ ✓ ✓	$R_{G^3 D^2}^{(3)}$	$G_{\mu\nu}^A G_\rho^{B,\nu} (D^\mu D_\alpha G^{C,\alpha\rho}) f^{ABC}$	✓ ✓ ✓
$R_{W^3 D^2}^{(4)}$	$W_{\mu\nu}^I \tilde{W}_{\rho}^{J,\nu} (D^\mu D_\alpha W^{K,\alpha\rho} - D^\rho D_\alpha W^{K,\alpha\mu}) \epsilon^{IJK}$	✓ ✗ ✗	$R_{G^3 D^2}^{(4)}$	$G_{\mu\nu}^A \tilde{G}_\rho^{B,\nu} (D^\mu D_\alpha G^{C,\alpha\rho} - D^\rho D_\alpha G^{C,\alpha\mu}) f^{ABC}$	✓ ✗ ✗
$R_{BW^2 D^2}^{(1)}$	$(D^\mu B_{\mu\nu}) W^{I,\nu\rho} (D^\alpha W_{\rho\alpha}^I)$	✗ ✓ ✗	$R_{BG^2 D^2}^{(1)}$	$G_{\mu\nu}^A (D_\alpha G^{A,\alpha\mu})(D_\beta B^{\beta\nu})$	✗ ✓ ✗
$R_{BW^2 D^2}^{(2)}$	$(D^\mu B_{\mu\nu}) \tilde{W}^{I,\nu\rho} (D^\alpha W_{\rho\alpha}^I)$	✗ ✗ ✓	$R_{BG^2 D^2}^{(2)}$	$\tilde{G}_{\mu\nu}^A (D_\alpha G^{A,\alpha\mu})(D_\beta B^{\beta\nu})$	✗ ✗ ✓
$R_{BW^2 D^2}^{(3)}$	$B_{\mu\nu} W_{\rho}^{I,\nu} (D^\mu D_\alpha W^{I,\alpha\rho} - D^\rho D_\alpha W^{I,\alpha\mu})$	✗ ✓ ✗	$R_{BG^2 D^2}^{(3)}$	$B_{\mu\nu} G_\rho^{A,\nu} (D^\mu D_\alpha G^{A,\alpha\rho} - D^\rho D_\alpha G^{A,\alpha\mu})$	✗ ✓ ✗
$R_{BW^2 D^2}^{(4)}$	$B_{\mu\nu} \tilde{W}_{\rho}^{I,\nu} (D^\mu D_\alpha W^{I,\alpha\rho} - D^\rho D_\alpha W^{I,\alpha\mu})$	✗ ✗ ✓	$R_{BG^2 D^2}^{(4)}$	$B_{\mu\nu} \tilde{G}_\rho^{A,\nu} (D^\mu D_\alpha G^{A,\alpha\rho} - D^\rho D_\alpha G^{A,\alpha\mu})$	✗ ✗ ✓
$X^2 H^2 D^2$					
$R_{B^2 H^2 D^2}^{(1)}$	$B_{\mu\nu} B^{\mu\nu} (D^2 H^\dagger H + H^\dagger D^2 H)$	✓ ✓ ✓	$R_{G^2 H^2 D^2}^{(1)}$	$G_{\mu\nu}^A G^{A\mu\nu} (D^2 H^\dagger H + H^\dagger D^2 H)$	✓ ✓ ✓
$R_{B^2 H^2 D^2}^{(2)}$	$i B_{\mu\nu} B^{\mu\nu} (H^\dagger D^2 H - D^2 H^\dagger H)$	✗ ✓ ✗	$R_{G^2 H^2 D^2}^{(2)}$	$i G_{\mu\nu}^A G^{A\mu\nu} (H^\dagger D^2 H - D^2 H^\dagger H)$	✗ ✓ ✗
$R_{B^2 H^2 D^2}^{(3)}$	$B_{\mu\nu} \tilde{B}^{\mu\nu} (D^2 H^\dagger H + H^\dagger D^2 H)$	✓ ✗ ✗	$R_{G^2 H^2 D^2}^{(3)}$	$G_{\mu\nu}^A \tilde{G}^{A\mu\nu} (D^2 H^\dagger H + H^\dagger D^2 H)$	✓ ✗ ✗
$R_{B^2 H^2 D^2}^{(4)}$	$i B_{\mu\nu} \tilde{B}^{\mu\nu} (H^\dagger D^2 H - D^2 H^\dagger H)$	✗ ✗ ✓	$R_{G^2 H^2 D^2}^{(4)}$	$i G_{\mu\nu}^A \tilde{G}^{A\mu\nu} (H^\dagger D^2 H - D^2 H^\dagger H)$	✗ ✗ ✓
$R_{B^2 H^2 D^2}^{(5)}$	$(D^\mu B_{\mu\nu}) B^{\alpha\nu} D_\alpha (H^\dagger H)$	✓ ✓ ✓	$R_{G^2 H^2 D^2}^{(5)}$	$(D^\mu G_{\mu\nu}^A) G^{A\alpha\nu} D_\alpha (H^\dagger H)$	✓ ✓ ✓
$R_{B^2 H^2 D^2}^{(6)}$	$i (D^\mu B_{\mu\nu}) B^{\alpha\nu} (H^\dagger \overleftrightarrow{D}_\alpha H)$	✗ ✓ ✗	$R_{G^2 H^2 D^2}^{(6)}$	$i (D^\mu G_{\mu\nu}^A) G^{A\alpha\nu} (H^\dagger \overleftrightarrow{D}_\alpha H)$	✗ ✓ ✗
$R_{B^2 H^2 D^2}^{(7)}$	$(D^\mu B_{\mu\nu}) \tilde{B}^{\alpha\nu} D_\alpha (H^\dagger H)$	✓ ✗ ✗	$R_{G^2 H^2 D^2}^{(7)}$	$(D^\mu G_{\mu\nu}^A) \tilde{G}^{A\alpha\nu} D_\alpha (H^\dagger H)$	✓ ✗ ✗
$R_{B^2 H^2 D^2}^{(8)}$	$i (D^\mu B_{\mu\nu}) \tilde{B}^{\alpha\nu} (H^\dagger \overleftrightarrow{D}_\alpha H)$	✗ ✗ ✓	$R_{G^2 H^2 D^2}^{(8)}$	$i (D^\mu G_{\mu\nu}^A) \tilde{G}^{A\alpha\nu} (H^\dagger \overleftrightarrow{D}_\alpha H)$	✗ ✗ ✓
$R_{B^2 H^2 D^2}^{(9)}$	$(D^\mu B_{\mu\alpha})(D_\nu B^{\nu\alpha})(H^\dagger H)$	✓ ✓ ✓	$R_{G^2 H^2 D^2}^{(9)}$	$(D^\mu G_{\mu\alpha}^A)(D_\nu G^{A\nu\alpha})(H^\dagger H)$	✓ ✓ ✓
$R_{W^2 H^2 D^2}^{(1)}$	$W_{\mu\nu}^I W^{I,\mu\nu} (D^2 H^\dagger H + H^\dagger D^2 H)$	✓ ✓ ✓	$R_{BWH^2 D^2}^{(1)}$	$B_{\mu\nu} W^{I,\mu\nu} (H^\dagger \tau^\dagger D^2 H + H^\dagger \tau^\dagger D^2 H)$	✓ ✓ ✓
$R_{W^2 H^2 D^2}^{(2)}$	$i W_{\mu\nu}^I W^{I,\mu\nu} (H^\dagger D^2 H - D^2 H^\dagger H)$	✗ ✓ ✗	$R_{BWH^2 D^2}^{(2)}$	$i B_{\mu\nu} W^{I,\mu\nu} (D^2 H^\dagger \tau^\dagger H - H^\dagger \tau^\dagger D^2 H)$	✗ ✓ ✗
$R_{W^2 H^2 D^2}^{(3)}$	$W_{\mu\nu}^I \tilde{W}^{I,\mu\nu} (D^2 H^\dagger H + H^\dagger D^2 H)$	✓ ✗ ✗	$R_{BWH^2 D^2}^{(3)}$	$B_{\mu\nu} \tilde{W}^{I,\mu\nu} (H^\dagger \tau^\dagger D^2 H + D^2 H^\dagger \tau^\dagger H)$	✓ ✗ ✗
$R_{W^2 H^2 D^2}^{(4)}$	$i W_{\mu\nu}^I \tilde{W}^{I,\mu\nu} (H^\dagger D^2 H - D^2 H^\dagger H)$	✗ ✗ ✓	$R_{BWH^2 D^2}^{(4)}$	$i B_{\mu\nu} \tilde{W}^{I,\mu\nu} (D^2 H^\dagger \tau^\dagger H - H^\dagger \tau^\dagger D^2 H)$	✗ ✗ ✓
$R_{W^2 H^2 D^2}^{(5)}$	$(D^\mu W_{\mu\nu}^I) W^{I,\alpha\nu} D_\alpha (H^\dagger H)$	✓ ✓ ✓	$R_{BWH^2 D^2}^{(5)}$	$(D^\mu B_{\mu\alpha}) W^{I,\alpha\nu} D_\nu (H^\dagger \tau^\dagger H)$	✓ ✓ ✓
$R_{W^2 H^2 D^2}^{(6)}$	$i (D^\mu W_{\mu\nu}^I) W^{I,\alpha\nu} (H^\dagger \overleftrightarrow{D}_\alpha H)$	✗ ✓ ✗	$R_{BWH^2 D^2}^{(6)}$	$i (D^\mu B_{\mu\alpha}) W^{I,\alpha\nu} (H^\dagger \overleftrightarrow{D}_\nu H)$	✗ ✓ ✗
$R_{W^2 H^2 D^2}^{(7)}$	$(D^\mu W_{\mu\nu}^I) \tilde{W}^{I,\alpha\nu} D_\alpha (H^\dagger H)$	✓ ✗ ✗	$R_{BWH^2 D^2}^{(7)}$	$(D^\mu B_{\mu\alpha}) \tilde{W}^{I,\alpha\nu} D_\nu (H^\dagger \tau^\dagger H)$	✓ ✗ ✗
$R_{W^2 H^2 D^2}^{(8)}$	$i (D^\mu W_{\mu\nu}^I) \tilde{W}^{I,\alpha\nu} (H^\dagger \overleftrightarrow{D}_\alpha H)$	✗ ✗ ✓	$R_{BWH^2 D^2}^{(8)}$	$i (D^\mu B_{\mu\alpha}) \tilde{W}^{I,\alpha\nu} (H^\dagger \overleftrightarrow{D}_\nu H)$	✗ ✗ ✓

(Table continued)

TABLE VIII. (Continued)

$H^6 D^2$		$C P CP$	$H^6 D^2$		$C P CP$
$R_{W^2 H^2 D^2}^{(9)}$	$(D^\mu W_{\mu\alpha}^I)(D_\nu W^{I,\nu\alpha})(H^\dagger H)$	✓ ✓ ✓	$R_{BWH^2 D^2}^{(9)}$	$(D^\mu W_{\mu\nu}^I)B^{\nu\alpha}D_\alpha(H^\dagger \tau^I H)$	✓ ✓ ✓
$R_{W^2 H^2 D^2}^{\prime(10)}$	$\epsilon^{IJK}(D^\mu W_{\mu\nu}^I)W^{J,\rho\nu}D_\rho(H^\dagger \tau^K H)$	✗ ✓ ✗	$R_{BWH^2 D^2}^{\prime(10)}$	$i(D^\mu W_{\mu\nu}^I)B^{\nu\alpha}(H^\dagger \overset{\leftrightarrow}{D}_\alpha^I H)$	✗ ✓ ✗
$R_{W^2 H^2 D^2}^{(11)}$	$i\epsilon^{IJK}(D^\mu W_{\mu\nu}^I)W^{J,\rho\nu}(H^\dagger \overset{\leftrightarrow}{D}_\rho^K H)$	✓ ✓ ✓	$R_{BWH^2 D^2}^{(11)}$	$(D^\mu W_{\mu\nu}^I)\tilde{B}^{\nu\alpha}D_\alpha(H^\dagger \tau^I H)$	✓ ✗ ✗
$R_{W^2 H^2 D^2}^{\prime(12)}$	$\epsilon^{IJK}(D^\mu W_{\mu\nu}^I)\tilde{W}^{J,\rho\nu}D_\rho(H^\dagger \tau^K H)$	✗ ✗ ✓	$R_{BWH^2 D^2}^{\prime(12)}$	$i(D^\mu W_{\mu\nu}^I)\tilde{B}^{\nu\alpha}(H^\dagger \overset{\leftrightarrow}{D}_\alpha^I H)$	✗ ✗ ✓
$R_{W^2 H^2 D^2}^{(13)}$	$i\epsilon^{IJK}(D^\mu W_{\mu\nu}^I)\tilde{W}^{J,\rho\nu}(H^\dagger \overset{\leftrightarrow}{D}_\rho^K H)$	✓ ✗ ✗	$R_{BWH^2 D^2}^{(13)}$	$(D^\mu B_{\mu\alpha})(D_\nu W^{I,\nu\alpha})(H^\dagger \tau^I H)$	✓ ✓ ✓
$XH^2 D^4$			$XH^4 D^2$		
$R_{BH^2 D^4}^{(1)}$	$(D^\mu H^\dagger D^2 H + D^2 H^\dagger D^\mu H)(D^\nu B_{\mu\nu})$	✗ ✓ ✗	$R_{BH^4 D^2}^{(1)}$	$i(D_\alpha B^{\alpha\mu})(H^\dagger \overset{\leftrightarrow}{D}_\mu H)(H^\dagger H)$	✓ ✓ ✓
$R_{BH^2 D^4}^{(2)}$	$i(D^\mu H^\dagger D^2 H - D^2 H^\dagger D^\mu H)(D^\nu B_{\mu\nu})$	✓ ✓ ✓	$R_{WH^4 D^2}^{(1)}$	$i(D^\mu W_{\mu\nu}^I)(H^\dagger \overset{\leftrightarrow}{D}^{I\nu} H)(H^\dagger H)$	✓ ✓ ✓
$R_{BH^2 D^4}^{(3)}$	$i(D_\alpha D^\nu B_{\mu\nu})(D^\mu H^\dagger D^\alpha H - D^\alpha H^\dagger D^\mu H)$	✓ ✓ ✓	$R_{WH^4 D^2}^{(2)}$	$\epsilon^{IJK}(H^\dagger \tau^I H)D^\nu(H^\dagger \tau^J H)(D^\mu W_{\mu\nu}^K)$	✓ ✓ ✓
$R_{WH^2 D^4}^{(1)}$	$(D^\mu H^\dagger \tau^I D^2 H + D^2 H^\dagger \tau^I D^\mu H)(D^\nu W_{\mu\nu}^I)$	✗ ✓ ✗	$R_{WH^4 D^2}^{(3)}$	$i\epsilon^{IJK}(H^\dagger \tau^I H)(H^\dagger \overset{\leftrightarrow}{D}^{J\nu} H)(D^\mu W_{\mu\nu}^K)$	✗ ✓ ✗
$R_{WH^2 D^4}^{(2)}$	$(D^\mu H^\dagger \tau^I D^2 H + D^2 H^\dagger \tau^I D^\mu H)(D^\nu W_{\mu\nu}^I)$	✓ ✓ ✓			
$R_{WH^2 D^4}^{(3)}$	$i(D_\alpha D^\nu W_{\mu\nu}^I)(D^\mu H^\dagger \tau^I D^\alpha H - D^\alpha H^\dagger \tau^I D^\mu H)$	✓ ✓ ✓			

## APPENDIX B: OBLIQUE PARAMETERS FROM DIMENSION-SIX AND DIMENSION-EIGHT OPERATORS

At linear order in the coefficients of dimension-six and dimension-eight operators in SMEFT eight nonzero oblique parameters are generated. We first compute them explicitly in terms of the coefficients of the operators in the bosonic basis. Subsequently we express the results in terms of the coefficients in the rotated basis by applying the relations in Eqs. (2.3) and (2.4):

$$\begin{aligned}
\hat{S} &= \frac{\hat{c}}{\hat{s}} \frac{\hat{v}^2}{\Lambda^2} \left[ b_{BW} + \frac{\hat{v}^2}{\Lambda^2} \left( b_{WBH^4}^{(1)} - \frac{g'}{4} r_{WH^4 D^2}^{(1)} - \frac{g}{4} r_{BH^4 D^2}^{(1)} \right) \right] \\
&= \left[ \frac{\hat{c}}{\hat{s}} \bar{c}_{BW} - \frac{\hat{v}^2}{2\hat{s}^2} (\bar{c}_{2JW} + \bar{c}_{2JB}) \right] \frac{\hat{v}^2}{\Lambda^2} - \left[ \frac{\hat{v}^2}{8\hat{c}\hat{s}^3} c_{\psi^4 H^2}^{(7)} + \frac{\hat{v}^4}{8\hat{s}^4 \hat{c}^2} (\hat{s}^2 c_{\psi^4 D^2}^{(2)} + \hat{c}^2 c_{\psi^4 D^2}^{(3)}) \right] \frac{\hat{v}^4}{\Lambda^4}, \\
\hat{T} &= -\frac{\hat{v}^2}{2\Lambda^2} \left( b_{\Phi,1} + \frac{\hat{v}^2}{\Lambda^2} b_{H^6}^{(2)} \right) = -\frac{1}{2} \left[ \bar{c}_{\Phi,1} + \frac{\hat{v}^2}{\hat{c}^2} \bar{c}_{2JB} \right] \frac{\hat{v}^2}{\Lambda^2} - \frac{\hat{v}^2}{2\hat{s}^2} c_{\psi^4 H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4}, \\
W &= \frac{g^2 \hat{v}^2}{4\Lambda^2} \left[ r_{2W} - \frac{\hat{v}^2}{\Lambda^2} r_{W^2 H^2 D^2}^{(9)} \right] = -\frac{\hat{v}^2}{2\hat{s}^2} \bar{c}_{2JW} \frac{\hat{v}^2}{\Lambda^2} = \bar{\Delta}_{4F} \frac{\hat{v}^2}{\Lambda^2}, \\
Y &= \frac{g^2 \hat{v}^2}{4\Lambda^2} \left[ r_{2B} - \frac{\hat{v}^2}{\Lambda^2} r_{B^2 H^2 D^2}^{(9)} \right] = -\frac{\hat{v}^2}{2\hat{s}^2} \bar{c}_{2JB} \frac{\hat{v}^2}{\Lambda^2}, \\
\hat{U} &= \frac{\hat{v}^4}{\Lambda^4} \left[ b_{W^2 H^4}^{(3)} + \frac{g}{2} r_{WH^4 D^2}^{\prime(2)} \right] = \left[ \bar{c}_{W^2 H^4}^{(3)} - \frac{\hat{v}^2}{2\hat{s}^2} c_{\psi^4 H^2}^{(7)} - \frac{\hat{v}^4}{2\hat{s}^2} c_{\psi^4 D^2}^{(2)} \right] \frac{\hat{v}^4}{\Lambda^2}, \\
X &= \frac{g^2 \hat{v}^4}{8\Lambda^4} r_{BWH^2 D^2}^{(13)} = \frac{\hat{v}^2}{8\hat{s}^2} c_{\psi^4 H^2}^{(7)} \frac{\hat{v}^4}{\Lambda^4}, \\
W' &= -\frac{g^4 \hat{v}^4}{8\Lambda^4} r_{W^2 D^4}^{(1)} = -\frac{\hat{v}^4}{8\hat{s}^4} c_{\psi^4 D^2}^{(3)} \frac{\hat{v}^4}{\Lambda^4}, \\
Y' &= -\frac{g^4 \hat{v}^4}{8\Lambda^4} r_{B^2 D^4}^{(1)} = -\frac{\hat{v}^4}{8\hat{s}^4} c_{\psi^4 D^2}^{(2)} \frac{\hat{v}^4}{\Lambda^4}, \\
X' &= V = V' = 0.
\end{aligned} \tag{B1}$$

### APPENDIX C: ADDITIONAL INFORMATION ON THE DRELL-YAN DISTRIBUTIONS

We considered the cross-section measurements for the NC channel at 8 TeV [2] and for the CC channel at 13 TeV [6], both provided by the ATLAS Collaboration. The HepData record for the NC data is available at [66], while the auxiliary material containing the experimental information for the CC data can be found at [67]. In both cases, we constructed the covariance matrices for the measurements using the experimental information provided on the pages mentioned above, and used them to build the following  $\chi^2$  test statistic for each of the channels as

$$\chi^2 = \sum_{i,j} (\mathcal{O}_i^{\text{dat}} - \mathcal{O}_i^{\text{th}}) V_{ij}^{-1} (\mathcal{O}_j^{\text{dat}} - \mathcal{O}_j^{\text{th}}), \quad (\text{C1})$$

where  $\mathcal{O}_i^{\text{dat}}$  and  $\mathcal{O}_i^{\text{th}}$  denote the measured cross section and the theoretical prediction for bin  $i$ , respectively, and  $V$  denotes the covariance matrix. For each bin  $i$ , the predictions  $\mathcal{O}_i^{\text{th}}$  include the SM signal simulation, which is

provided by the ATLAS Collaboration, along with the new physics contributions parametrized by the dimension-six and dimension-eight Wilson coefficients.

Moreover, we also reinterpret the experimental searches for heavy new particles in the NC and CC channels. In these cases, we rebinned the distributions to ensure at least ten events per bin. Our binning choices are summarized in Table IX, along with references to the corresponding HepData records for the original distributions. For each of these channels, the limits on the Wilson coefficients were extracted by means of a  $\chi^2$ -test statistic,

$$\chi^2 = \sum_i \frac{(N_i^{\text{dat}} - N_i^{\text{th}} - N_i^b)^2}{N_i^{\text{dat}} + (\delta N_i^b)^2}, \quad (\text{C2})$$

where  $N_i^{\text{dat}}$ ,  $N_i^b$ , and  $N_i^{\text{th}}$  denote the observed number of events, the background prediction, and the EFT contribution for bin  $i$ , respectively. The values of the background predictions  $N_i^b$  and their corresponding uncertainties  $\delta N_i^b$  were extracted from the HepData records listed in Table IX.

TABLE IX. Additional information on the binning and distributions of the Drell-Yan data from experimental searches for heavy particles considered in our analysis.

Channel	Distribution	Binning (GeV)	Data set	Integrated luminosity	HepData record	reference
NC	$\frac{dN_{\text{ev}}}{dm_{e^+e^-}}$	249.38, 497.07, 536.66, 579.41, 625.56, 675.39, 729.18, 787.27, 849.97, 917.68, 990.77, 1069.7, 1154.9, 1246.9, 1346.2, 1453.4, 1630.5, 1760.4, 2052.0, 2391.9, 4588.4	ATLAS 13 TeV	139 fb <sup>-1</sup> [7]	[68]	
NC	$\frac{dN_{\text{ev}}}{dm_{\mu^+\mu^-}}$	269.24, 460.40, 497.07, 536.66, 579.41, 625.56, 675.39, 729.18, 787.27, 849.97, 917.68, 990.77, 1069.7, 1154.9, 1246.9, 1346.2, 1453.4, 1569.2, 1760.4, 1974.8, 4588.4	ATLAS 13 TeV	139 fb <sup>-1</sup> [7]	[69]	
NC	$\frac{dN_{\text{ev}}}{dm_{e^+e^-}}$	300, 420, 460, 500, 540, 580, 630, 690, 750, 810, 870, 950, 1050, 1150, 1250, 1370, 1490, 1680, 1890, 2210, 6070	CMS 13 TeV	139 fb <sup>-1</sup> [8]	[70]	
NC	$\frac{dN_{\text{ev}}}{dm_{\mu^+\mu^-}}$	266.25, 429.49, 465.12, 503.71, 545.49, 590.74, 639.75, 692.82, 750.29, 812.54, 879.94, 952.94, 1032.0, 1117.6, 1210.3, 1310.7, 1419.4, 1537.2, 1952.4, 2289.7, 7000.0	CMS 13 TeV	139 fb <sup>-1</sup> [8]	[71]	
CC	$\frac{dN_{\text{ev}}}{dm_{\tau,\nu}}$	440, 520, 600, 680, 760, 840, 920, 1000, 1080, 1160, 1240, 1320, 1400, 1480, 1560, 1640, 1760, 1880, 2040, 2400, 7000	CMS 13 TeV	138 fb <sup>-1</sup> [10]	[72]	
CC	$\frac{dN_{\text{ev}}}{dm_{\tau,\mu\nu}}$	610, 708, 806, 904, 1002, 1100, 1198, 1296, 1394, 1492, 1590, 1688, 1982, 2276, 7000	CMS 13 TeV	138 fb <sup>-1</sup> [10]	[73]	

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