

Modeling of Single-Phase Flows in Porous Media and Passive Contaminant Transport

Henrique Casellato Vitorio Rodrigues da Costa, Nikolai V. Chemetov

Universidade de São Paulo, Brazil

A porous medium is a material characterized by its ability to store fluids and is found in a variety of contexts, such as sedimentary rocks for oil extraction, underground aquifers, biological tissues, and fuel cells. In general, computational simulations are essential tools for understanding chemical, physical, and flow processes, complementing field observations, laboratory experiments, and analytical models. This work aims to introduce concepts of Numerical Methods for Partial Differential Equations and their applications in modeling flow in porous media. Specifically, numerical simulation methods for elliptic and hyperbolic equations are investigated.

A Kruzhkov-Type Uniqueness Theorem for a System of Conservation Laws Describing Chemical Flooding

Yulia Petrova, Nikita Rastegaev

Universidade de São Paulo, Brazil

A system of two hyperbolic conservation laws describing a two-phase, three-component displacement (usually water and oil phases, as well as a component of a chemical agent dissolved in water) is considered. This system is neither truly nonlinear nor strictly hyperbolic, which limits the applicability of general results pertaining to strictly hyperbolic, truly nonlinear systems.

Solutions to some initial-boundary value problems (e.g., the Riemann problem or the problem of injecting a chemical agent slug) for this system have been previously investigated using the transition to Lagrangian coordinates, in which the equations are separated. The solutions are constructed using the method of characteristics, but the uniqueness of the resulting solutions has not been investigated. We used the proposed coordinate change to prove a Kruzhkov-type uniqueness theorem for solving the Cauchy problem under certain constraints on the initial data and the class of admissible weak solutions. A local small parameter criterion (vanishing viscosity) was used to determine the admissibility of discontinuities. The uniqueness result is applied to the analysis of the Riemann problem solutions for an S-shaped flow function changing monotonicity exactly once.

This talk is based on joint work with N. Rastegaev, <https://arxiv.org/abs/2512.07639>

An Extension of the Subadditive Ergodic Theorem

Jean Silva

Universidade Federal de Minas Gerais, Brazil

In this talk, it will be presented an extension of the Kingman's subadditive ergodic theorem. Here, we will consider subadditive quantities composed with stochastic deformations, in which the stationarity assumption is missed, and we will show that the same conclusions hold as in the classical case. The proof involves a rather simple reduction to the additive case, where Birkhoff's ergodic theorem can be applied.

Regular variation, maximal functions, and DiPerna-Lions flows

João Fernando Nariyoshi, Henrique Borrin

Universidade de São Paulo, Brazil

Using maximal functions with weights of regular variation, we investigate the regularity of flows generated by Sobolev velocity fields. This is an ongoing joint work with Henrique Borrin (FFCLRP-USP), and it is

supported in part by the São Paulo Research Foundation (FAPESP, Process 2024/21041-1) and by the Pró-Reitoria de Pesquisa e Inovação of the University of São Paulo (Programa de Apoio a Novos Docentes, Grant 22.1.09345.01.2).

Wave-Curve Construction of Riemann Solutions in Three-Phase Flow Models

Luis Fernando Lozano, Frederico Furtado, Aparecido de Souza, Dan Marchesin
Universidade Federal de Juiz de Fora, Brazil

We investigate the Riemann problem for a 2×2 system of conservation laws that exhibits non-strict hyperbolicity due to the coincidence of characteristic speeds at an interior state. This system arises in models featuring large viscosity contrasts and serves as a representative example of the non-strictly hyperbolic dynamics observed in several multiphase transport processes.

Our work provides a detailed construction and classification of Riemann solutions for left states constrained to one boundary of the state domain and right states spanning almost the entire admissible region. We apply the wave-curve method, utilizing the viscous-profile criterion for discontinuities, combined with numerical techniques, to determine the bifurcation loci that separate regions in the space of right states with distinct Riemann solution structures.

We also analyze the sensitivity of these solutions to perturbations in the initial data and present numerical evidence supporting their L^1_{loc} -stability and overall robustness. The results highlight how interior degeneracies influence wave interactions and solution organization in non-strictly hyperbolic systems.

Finally, we validate the semi-analytical constructions through fully numerical simulations, confirming the consistency and accuracy of the proposed classification across the considered parameter regimes.

Concentration Effects of H-Measures in Different Ergodic Settings

Luís Fernando Salvino, Jean Carlos da Silva
Universidade Federal de Minas Gerais, Brazil

The H-measure was introduced independently by L. Tartar, P. Gerárd and improved by E. Panov. It is a default measure which quantifies, in the phase (i.e. the physical space times the Fourier space of propagation directions), the lack of compactness of weakly converging sequences. In this paper, we address the task to reckon the H-measure generated by functions in known ergodic algebras, that is, in the almost periodic and in the weakly almost periodic algebras. We shall see that the H-measure generated by a function in these ergodic algebras has the same structure of one which is periodic: The direction of the Fourier variable ξ is a purely atomic measure. But this atomic behavior seems to be very restricted to the algebras of the "periodics families". Precisely, we shall compute the H-measure generated by an ergodic function which is beyond the weak almost periodic setting and its corresponding H-measure has a structure completely different: the direction of the Fourier variable ξ is a measure absolutely continuous with respect to Lebesgue measure.

Existence and large-time behavior of solutions to an incompressible inhomogeneous kinetic–fluid flocking system

Robson Moura, Gabriela Planas
Universidade Estadual de Campinas, Brazil

In this work, we prove the global existence of weak solutions and establish an a priori exponential time-asymptotic behavior for a kinetic–fluid flocking model. The kinetic component is the standard Cucker–Smale