Nº 78

On a conjecture of Zassenhaus on torsion units in integral group rings II

César Polcino Milies, Jürgen Ritter and Sudarshan K. Sehgal

Janeiro de 1985

ON A CONJECTURE OF ZASSENHAUS ON TORSION UNITS IN INTEGRAL GROUP RINGS II*

César Polcino Milies, Jürgen Ritter and Sudarshan K. Sehgal

Introduction. Let $U_1\mathbb{Z}G$ be the group of units of augmentation one in the integral group ring $\mathbb{Z}G$ of a finite group G and let $TU_1\mathbb{Z}G$ be the subset of torsion units. For two elements α, β of the rational group algebra $\mathbb{Q}G$ (or of $\mathbb{Z}G$) we write $\alpha \sim \beta$ if there is a unit γ of $\mathbb{Q}G$ such that $\alpha = \gamma^{-1}\beta\gamma$. There is a well known conjecture of Zassenhaus:

$$\text{(ZC)} \qquad u \in TU_1\mathbb{Z}G \quad \Rightarrow \quad \exists\, g \in G \quad \text{such that} \quad u \sim g .$$

This conjecture has been confirmed in [1] and [3] for groups G which are nilpotent class 2 or are split metacyclic with $G = \langle a \rangle \rtimes \langle x \rangle$, $O(a)$ a prime power and $O(x)$ relatively prime to $O(a)$.

It is the purpose of this paper to extend this to some more metacyclic groups. We prove the

Theorem. Let G be the split metacyclic group, $G = \langle a \rangle \rtimes \langle x \rangle$, with $O(a) = n$, $O(x) = t$ and $(n,t) = 1$. Then for any unit $u \in TU_1\mathbb{Z}G$ there is a $g \in G$ such that $u \sim g$.

In the proof we need to use the main result of [3], namely that (ZC) holds if n is a prime power.

---

*This work has been supported by CNPQ of Brazil, DFG of W. Germany and NSERC of Canada.

§ 2. Some Lemmas

We recall Lemma 2 of [1].

Lemma 2.1. *Let G be a split extension $A \rtimes X$ where A is an abelian normal p-group and X any group. Let $u \in U_1\mathbb{Z}G$ be written as $u = vw$, $v \in U(1+\Delta(G,A))$, $w \in U\mathbb{Z}X$. If u has finite order not divisible by p, then $u \sim w$.*

Here, $\Delta(G,A)$ denotes the kernel of the natural map $\mathbb{Z}G \longrightarrow \mathbb{Z}(G/A)$.

Lemma 2.2. *Let $G = \langle a \rangle \rtimes \langle x \rangle$, $O(a) = n$, $O(x) = t$ with $(n,t) = 1$. If $u \in TU_1\mathbb{Z}G$ has prime power order, then $u \sim g$ for some $g \in G$.*

Proof. Let $O(u) = p^k$. If p divides $O(x)$, then we are done by repeated applications of the last lemma. Therefore, suppose

$$n = p^m n_1 \ , \ (p,n_1) = 1 \ , \ m > 0 \ .$$

Then $G = \langle a^{p^m} \rangle \rtimes G_1$, $G_1 = \langle a^{n_1} \rangle \rtimes \langle x \rangle$. It follows by (2.1) that $u \sim v$, $v \in U\mathbb{Z}G_1$. Thus the lemma is a consequence of the main result of [3].

Lemma 2.3.

Let $G = \langle a \rangle \rtimes \langle x \rangle$ with $a^x = a^j$, $O(a) = p^m$, $O(x) = t$, $(p,t) = 1$. Then for an element $a^i x^k$ of G either x^k is central or $(a^i x^k)^t = 1$.

Proof. Let $a^i = b$, $x^k = y$. Then

$$(by)^t = b^{1+j^k+\ldots+j^{k(t-1)}} \ .$$

Now, $(1-j^k)(1+j^k+\dots+j^{k(t-1)}) = (1-j^{kt}) \equiv 0 \pmod{p^m}$. Consequently, either $p \mid (1-j^k)$ in which case $j^{kp^{m-1}} \equiv 1 \pmod{p^m}$ implying that $j^k \equiv 1 \pmod{p^m}$ (because $(O(j),p) = 1$) and thus y to be central, or $p \nmid (1-j^k)$ and $1 + j^k + \dots + j^{k(t-1)} \equiv 0 \pmod{p^m}$ which forces $(by)^t = 1$.

__Lemma 2.4.__ __Let__ $G = \langle a \rangle \rtimes \langle x \rangle$, $O(a) = n$, $O(x) = t, (t,n) = 1$. __Suppose that__ $u \in TU_1\mathbb{Z}G$ __and that all primes dividing__ n __divide__ $O(u)$. __Then we can write__ $u = u_1u_2$, $u_1 \in 1 + \Delta(G,\langle a \rangle)$ __and__ u_2 __central.__

__Proof.__ If n is a prime power, then the result is true by the main result of [3] and the last lemma. Let $n = p^m n_1$, $m > 0$, $(p,n_1) = 1$, $n_1 > 1$. We use induction on the number of primes dividing n.

Notice that $u \equiv x^k \bmod \Delta(G,\langle a \rangle)$ for some k. Writing

$$G = \langle a^{p^m} \rangle \rtimes G_1 , \quad G_1 = \langle a^{n_1} \rangle \rtimes \langle x \rangle$$

we can express $u = v_1v_2$, $v_1 \in 1 + \Delta(G,\langle a^{p^m} \rangle)$, $v_2 \in \mathbb{Z}G_1$.

Going mod $\langle a^{p^m} \rangle$ and applying Corollary 1 of [1] we conclude that p divides the order of v_2. It follows that

$$v_2 = v_3x^k \quad \text{with} \quad v_3 \in \Delta(G_1,\langle a^{n_1} \rangle).$$

Thus $u = v_1v_3x^k$ with $(x^k,a^{n_1}) = 1$ and $v_1v_3 \in 1 + \Delta(G,\langle a \rangle)$. Repeating this argument for different prime divisors of n we come to the conclusion that $u = u_1x^k$, $u_1 \in \Delta(G,\langle a \rangle)$, x^k central, as desired.

3. Proof of Theorem - Some reductions.

We have a group $G = \langle a \rangle \rtimes \langle x \rangle$, $a^x = a^j$, $O(a) = n$, $O(x) = t$ with $(t,n) = 1$ and are given a unit $u \in TU_1\mathbb{Z}G$. We wish to show that there is a $g \in G$ such that $u \sim g$. To that effect we use induction on $|G|$ and thus assume that the result is true for all metacyclic groups G of this type of smaller order. We can restrict ourselves to the case that n is divisible by at least two different primes.

(3.1) We may assume that all prime divisors of n divide $O(u)$.

Proof. Suppose that p is a prime number with

$$n = p^m n_1\ , (p,n_1) = 1\ , \ m > 0 \ \text{ and } \ (O(u),p) = 1\ .$$

Write $G = \langle a^{n_1} \rangle \rtimes G_1$, $G_1 = \langle a^{p^m} \rangle \rtimes \langle x \rangle$.
Then it follows by (2.1) that $u \sim v$, $v \in U\mathbb{Z}G_1$, and so by induction $v \sim g$, $g \in G$.

(3.2) We may assume that no Sylow p-subgroup of $\langle a \rangle$ is central in G.

Proof. Suppose that $\langle a \rangle$ has a central Sylow p-subgroup $\langle a_1 \rangle$.
Write

$$\langle a \rangle = \langle a_1 \rangle \times A_1\ , \ G = \langle a_1 \rangle \rtimes G_1\ , \ G_1 = A_1 \rtimes \langle x \rangle\ .$$

Then $u = u_1u_2$, $u_1 \in 1 + \Delta(G, \langle a_1 \rangle)$, $u_2 \in \mathbb{Z}G_1$. Since a_1 is central in G it follows by ([4], p. 34) that $u_1 = a_1^r$, a central element of G.

Moreover, $u_2 \sim g$, $g \in G_1$ by induction. Thus $u \sim a_1^r g$ and the assertion follows.

(3.3) <u>We may assume that</u> $j \not\equiv 1 \pmod p$ <u>for any prime</u> p <u>dividing</u> n <u>and therefore,</u> $G' = \langle a \rangle$.

<u>Proof</u>. $j \equiv 1 \pmod p \Rightarrow j^{p^{m-1}} \equiv 1 \pmod{p^m} \Rightarrow j \equiv 1 \pmod{p^m}$, as the order of $j \bmod n$, being a divisor of t , is relatively prime to n . Therefore the Sylow p-subgroup is central and the assertion follows.

We shall prove as in ([1] and [3]) that for every absolutely irreducible representation T of G , $T(u) \sim T(g)$ for some $g \in G$. It is enough to do this for rationally inequivalent representations. This will complete the proof of the Theorem.

From [5, p. 62] we see that these representations up to rational equivalence are given by

$$T_{d,\mu}(a) = \begin{bmatrix} \zeta_d & & & \\ & \zeta_d^j & & \\ & & \ddots & \\ & & & \zeta_d^{j^{t_d - 1}} \end{bmatrix}_{t_d \times t_d} \qquad T_{d,\mu}(x) = \begin{bmatrix} 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & \cdot & 0 \\ & & & & & & \\ 0 & \cdot & \cdot & \cdot & 0 & 0 & 1 \\ \eta^\mu & 0 & \cdot & \cdot & \cdot & \cdot & 0 \end{bmatrix}_{t_d \times d}$$

where, d runs over the divisors of n , ζ is a fixed primitive n-th root of unity, $\zeta_d = \zeta^d$, t_d is the order of $j \bmod n/d$, and η is a primitive t/t_d - th root of unity; $\mu = 0,1,2,\ldots,t/t_d - 1$.

It therefore remains to prove the following propositions.

Proposition 3.4. $u \in T(1+\Delta(G,\langle a\rangle)) \Rightarrow T_{d,\mu}(u) \sim T_{d,\mu}(a^r)$ for some $r = r(d,\mu)$.

Notice that for all abelian representations T we have $T(u) = T(a^r) = 1$.

Proposition 3.5. We can choose an $r = r(d,\mu)$ which is independent of d and μ.

For proving Proposition 3.4 we may assume that all prime divisors of n divide $O(u)$ by (3.1) and moreover, by (3.3), that we are in the situation $G' = \langle a\rangle$ and $j \not\equiv 1 \bmod p$ for every prime divisor p of n.

4. Completion of Proof of Theorem

Lemma 4.1. $T_{1,\mu}$ is faithful on any cyclic group H of units of order dividing n.

Proof. Use induction on $|H|$ and (2.2), remembering that $T_{1,\mu}$ is faithful on $\langle a\rangle$.

Lemma 4.2. Let $u \in T(1+\Delta(G,\langle a\rangle))$. Suppose p is a prime dividing n. Then:

$$T_{1,\mu}(u^p) = 1 \Leftrightarrow T_{p,\mu}(u) = 1 .$$

The proof is the same as the one of (iii) on page 262 of [3] by writing $T_{1,\mu} = T_1$, $T_{p,\mu} = T_p$. Notice that (ii) which is used in that proof can be replaced by (4.1) above.

(4.3) Proof of (3.4).

Write $n = p_1^{m_1} p_2^{m_2} \ldots p_k^{m_k}$, a product of distinct primes. Then

$$O(u) = p_1^{\beta_1} p_2^{\beta_2} \ldots p_k^{\beta_k}, \quad 0 < \beta_i \leq m_i .$$

We express $u = u_1 \ldots u_k$, $O(u_i) = p_i^{\beta_i}$. Write $T = T_{1,\mu}$.
Then we know by (2.2) that $u_i \sim a^{\lambda_i}$ for some λ_i. Thus

$$T(u_i) \sim T(a^{\lambda_i}) = \begin{bmatrix} \zeta^{\lambda_i} & & & \\ & \zeta^{\lambda_i j} & & \\ & & \ddots & \\ & & & \zeta^{\lambda_i j^{\ell-1}} \end{bmatrix}, \quad \ell = O(j) \bmod n .$$

Now,

$$O(\zeta^{\lambda_i}) = O(\zeta^{\lambda_i j}) = O(a^{\lambda_i}) = O(u_i) = p_i^{\beta_i} .$$

This is because $(j,n) = 1$ and since T is faithful on $\langle a \rangle$. As the u_i's are powers of u, diagonalizing $T(u)$ gives a diagonal form for $T(u_i)$ as well; the eigenvalues of $T(u)$ are products

$$\zeta^{\lambda_i} \cdot \zeta^{\lambda_j j^{\mu}} \;\ldots\ldots .$$

Thus all eigenvalues of $T(u)$ have order $O(u)$. Let λ be an eigenvalue of $T(u)$. Then λ^j is also an eigenvalue of $T(u)$ as conjugating $T(u)$ by $T(x)$ means applying the autormorphism $\zeta \to \zeta^j$ to the entries of $T(u)$.

If $\lambda^{j^\nu} = \lambda$, then $j^\nu \equiv 1(O(u))$ and so $j^\nu \equiv 1(p_i^{\beta_i})$ for all i .
Because $O(j) \bmod n$ is relatively prime to n we conclude $j^\nu \equiv 1(p_i^{\alpha_i})$ for all i and therefore $j^\nu \equiv 1(\bmod n)$.

Thus we have

$$T(u) \sim \begin{bmatrix} \lambda & & & & & \\ & \lambda^{j} & & & & \\ & & \lambda^{j^2} & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & \cdot \ \lambda^{j^{\ell-1}} \end{bmatrix} \sim T(a^r) \text{ for some } r .$$

We wish to show that $T_{d,\mu}(u) \sim T_{d,\mu}(a^r)$ for a divisor d of n .
Clearly, $< a^{n/d} >$ is in the kernel of $T_{d,\mu}$. Write $\bar{G} = G/< a^{n/d} >$ and $\bar{T}_{d,\mu}$ for the corresponding representations of $\bar{G}$. We see that

$$\bar{T}_{1,\mu}(\bar{u}) = T_{d,\mu}(u) .$$

But from what we have already proved

$$\bar{T}_{1,\mu}(\bar{u}) \sim \bar{T}_{1,\mu}(\bar{a}^r) = T_{d,\mu}(a^r) .$$

This completes the proof of (3.4).

(4.4) Proof of (3.5).

(a) We shall first show that $r = r(d,\mu)$ is independent of d .
Write $T_{1,\mu} = T_1$, $T_{p,\mu} = T_p$ for a prime divisor p of n .

It is enough to prove:

$$T_1(u) \sim T_1(a^r) \ , \quad T_p(u) \sim T_p(a^t) \qquad \Rightarrow \qquad T_p(a^t) \sim T_p(a^r) \ .$$

We use induction on $O(u)$ and $|G|$. There are two cases: (i) $p^2 \nmid n$; (ii) $p^2 \mid n$. In the first case the dimension of T_p may be smaller than that of T_1. In the second case they are the same. We deal with the two cases separately.

<u>Case (i)</u>, $p \mid n$, $p^2 \nmid n$. Then we can assume that $p \mid O(u)$ as otherwise $u \sim$ a unit in a smaller group by the argument in (3.1). Thus

$$T_1(u^p) \sim T_1(a^{rp}) \ , \ T_p(u^p) \sim T_p(a^{tp}) \ \Rightarrow \ T_p(a^{rp}) \sim T_p(a^{tp}) \ ,$$

by induction.

Comparing eigenvalues we get

$$rp^2 \equiv tp^2 j^{\nu} (\mathrm{mod}\ n) \quad \text{for some} \quad \nu \ .$$

Therefore, $rp = tpj^{\nu} (\mathrm{mod}\ n/p)$ which implies $r \equiv tj^{\nu}(n/p)$ as $p^2 \nmid n$. It follows that $rp = tpj^{\nu} \bmod n$ and thus $T_p(a^r) \sim T_p(a^t)$ as desired.

<u>Case (ii)</u>, $p^2 \mid n$, dimension of T_1 = dimension of T_p. Let

$$u = \sum_{i=o}^{t-1} f_i(a) x^i$$

with integral polynomials $f_i(a)$ in a of degree $\leq n-1$. Then

$$T_{d,\mu}(u) = \begin{bmatrix} \sum_{\nu=o}^{\ell-1} f_{\nu s}(\zeta_d)\eta^{\mu\nu} & \sum_{\nu=o}^{\ell-1} f_{1+\nu s}(\zeta_d)\eta^{\mu\nu} & \cdots & \sum_{\nu=o}^{\ell-1} f_{s-1+\nu s}(\zeta_d)\eta^{\mu\nu} \\ \cdots\cdots & \sum_{\nu=o}^{\ell-1} f_{\nu s}(\zeta_d^j)\eta^{\mu\nu} & & \\ & & \ddots & \\ \cdots\cdots\cdots\cdots\cdots\cdots & & & \sum_{\nu=o}^{\ell-1} f_{\nu s}(\zeta_d^{j^{s-1}})\eta^{\mu\nu} \end{bmatrix}$$

where $s = t_d = O(j) \bmod n/d$, $\ell = t/s$, η is an ℓ th root of unity. By taking $d = 1$ and p , respectively, we obtain $T_1(u)$ and $T_p(u)$. We introduce, as in [1] and [3], the matrix

$$M(y) = \begin{bmatrix} \sum_{\nu=o}^{\ell-1} f_{\nu s}(y)\eta^{\mu\nu} & \sum_{\nu=o}^{\ell-1} f_{1+\nu s}(y)\eta^{\mu\nu} & \cdots & \sum_{\nu=o}^{\ell-1} f_{s-1+\nu s}(y)\eta^{\mu\nu} \\ & \sum_{\nu=o}^{\ell-1} f_{\nu s}(y^j)\eta^{\mu\nu} & & \\ \cdots\cdots & & & \\ \cdots\cdots\cdots\cdots\cdots\cdots & & & \sum_{\nu=o}^{\ell-1} f_{\nu s}(y^{j^{s-1}})\eta^{\mu\nu} \end{bmatrix}$$

where y is an indeterminate. This matrix has coefficients in $\mathbb{Z}[\eta][y]$. Write for the characteristic polynomials

$$Ch_{\mu(y)}(z) = \sum_{\mu=o}^{s-1} \psi_\mu(y) z^{s-\mu}$$

$$Ch_{T_1(u)}(z) = \prod_{\nu=o}^{s-1} (z-\zeta^{rj^\nu})$$

$$Ch_{T_p(u)}(z) = \prod_{\nu=o}^{s-1} (z-\zeta^{ptj^\nu}) .$$

Now we follow page 263 and 264 of [3] verbatim, only replacing q by s, and conclude $\zeta^{pt} = \zeta^{prj^\nu}$ for some ν. Hence $T_p(a^t) \sim T_p(a^r)$. (Observe that the n-th cyclotomic polynomial $\varphi_n(y)$ is also irreducible over the ring $\mathbb{Z}[\eta]$ of coefficients of the $\psi_\mu(y)$.)

(b) We have seen that $r(d,\mu) = r(\mu)$ is independent of d. It is enough to show that $T_{1,\mu}(a^{r(\mu)}) \sim T_{1,\mu}(a^{r(o)})$.

We are using induction on $|G|$ and $O(u)$. Consequently we may assume as in (3.1) that all primes dividing n divide $O(u)$. Thus for any proper divisor d of n

$$T_{1,\mu}(u^d) \sim T_{1,\mu}(a^{r(\mu)d}) , \; T_{1,o}(u^d) \sim T_{1,o}(a^{r(o)d})$$

implies that $T_{1,\mu}(a^{r(\mu)d}) \sim T_{1,\mu}(a^{r(o)d})$ (*) .

Let us consider the traces of the matrices $T_{1,\mu}(u)$:

$$\chi_{1,\mu}(u) = trT_{1,\mu}(u) = \sum_{\nu=o}^{\ell-1} tr_j f_{\nu s}(\zeta)\eta^{\mu\nu} = tr_j \zeta^{r(\mu)} = \chi_{1,\mu}(a^{r(\mu)}) ,$$

where for an element $\alpha \in \mathbb{Q}(\zeta)$ whe have denoted by $tr_j\alpha$ the trace of α with respect to the automorphism group generated by $\sigma_j : \zeta \longrightarrow \zeta^j$.

Adding up the equations over all μ we get

$$\begin{aligned}\sum_{\mu=o}^{\ell-1} tr_j \zeta^{r(\mu)} &= \sum_{\mu}\sum_{\nu} tr_j f_{\nu s}(\zeta)\eta^{\mu\nu} \\ &= \sum_{\nu} (tr_j f_{\nu s}(\zeta) \sum_{\mu} \eta^{\mu\nu}) \\ &= \ell \, tr_j f_o(\zeta)\end{aligned}$$

as for $\nu \neq 0$, $\sum_{\nu=o}^{\ell-1} \eta^{\mu\nu} = 0$.

Thus we have

$$\sum_{\mu=o}^{\ell-1} tr_j \zeta^{r(\mu)} \equiv 0 \pmod{\ell} .$$

Moreover, from (*) we have

$$tr_j \zeta^{r(\mu)d} = tr_j \zeta^{r(o)d} \quad \text{for all } \mu \text{ and all } d|n , d \neq 1 .$$

Recall that $t_1 = 0(j) \bmod n$. Let us write the class sum of $a^{r(\mu)}$ in G :

$$C_\mu = \sum_{\nu=o}^{t_1-1} a^{r(\mu)j^\nu} ,$$

and $\alpha = \sum_{\mu=o}^{\ell-1} C_\mu$. Then we have:

$$\alpha(\zeta) = \sum_{\mu=o}^{\ell-1} C_\mu(\zeta) = \sum_{\mu=o}^{\ell-1} \sum_{\nu=o}^{t_1-1} \zeta^{r(\mu)j^\nu} = \sum_{\mu=o}^{\ell-1} tr_j \zeta^{r(\mu)} \equiv 0 \pmod{\ell} ,$$

$$\alpha(\zeta^d) = \sum_{\mu=o}^{\ell-1} tr_j \zeta^{dr(\mu)} = \ell \, tr_j \zeta^{r(o)d} \equiv 0 \pmod{\ell} ,$$

$$\alpha(1) \equiv 0 \pmod{\ell} .$$

Thus in the embedding

$$\mathbb{Z}\langle a\rangle \longrightarrow \sum_{d|n} \mathbb{Z}[\zeta_d] = R$$

the image of α is $\equiv 0 \pmod{\ell}$. But we know by ([2], p. 379) that $O(a)R \subseteq \mathbb{Z}\langle a\rangle$. Since we have $(O(a),\ell) = 1$, it follows that $\alpha \equiv 0 \pmod{\ell}$. Therefore, $C_\mu = C_o$ for all μ .

We have proved that

$$tr_j \zeta^{r(\mu)} = tr_j \zeta^{r(o)} , \ \chi_{1,\mu}(u) = \chi_{1,\mu}(a^{r(o)}) .$$

Thus $\chi_{d,\mu}(u) = \chi_{d,\mu}(a^{r(o)})$ for all d,μ .

Repeating this for all powers u^k of u we have

$$\chi_{d,\mu}(u^k) = \chi_{d,\mu}(a^{r(o)k}) \quad \text{for all } d,\mu .$$

Hence $u \sim a^{r(o)}$ and the proof is complete.

REFERENCES

[1] C. Polcino Milies and S. K. Sehgal, Torsion units in integral group rings of metacyclic groups, J. Number Theory 19 (1984), 103-114.

[2] I. Reiner, Maximal orders, Academic Press, New York (1975).

[3] J. Ritter and S. K. Sehgal, On a conjecture of Zassenhaus on torsion units in integral group rings, Math. Ann. 264 (1983), 257 - 270.

[4] J. Ritter and S. K. Sehgal, Isomorphism of group rings, Arch. Math. 40 (1983), 32 - 39.

[5] J. P. Serre, Linear representations of finite groups, Graduate Texts in Mathematics, Vol. 42, Springer, New York (1977).

Universidade de São Paulo
Sao Paulo, Brasil,

Universität Augsburg
D-8900 Augsburg
West Germany

and

University of Alberta
Edmonton, Canada

TRABALHOS

DO

DEPARTAMENTO DE MATEMATICA

TITULOS PUBLICADOS

8001 - PLETCH, A. Local freeness of profinite groups. São Paulo, IME-USP, 1980. 10p.

8002 - PLETCH, A. Strong completeness in profinite groups. São Paulo, IME-USP, 1980. 8p.

8003 - CARNIELLI, W.A. & ALCANTARA, L.P. de. Transfinite induction on ordinal configurations. São Paulo, IME-USP, 1980. 22p.

8004 - JONES RODRIGUES, A.R. Integral representations of ciclic p-groups. São Paulo, IME-USP, 1980. 13p.

8005 - CORRADA, M. & ALCANTARA, L.P. de. Notes on many-sorted systems. São Paulo, IME-USP, 1980. /25/p.

8006 - POLCINO MILIES, F.C. & SEHGAL, S.K. FC-elements in a group ring. São Paulo, IME-USP, 1980. /10/p.

8007 - CHEN, C.C. On the Ricci condition and minimal surfaces with constantly curved Gauss map. São Paulo, IME-USP, 1980. 10p.

8008 - CHEN, C.C. Total curvature and topological structure of complete minimal surfaces. São Paulo, IME-USP, 1980. 21p.

8009 - CHEN, C.C. On the image of the generalized Gauss map of a complete minimal surface in R^4. São Paulo, IME-USP, 1980. 8p.

8110 - JONES RODRIGUES, A.R. Units of ZCp^n. São Paulo, IME-USP, 1981. 7p.

8111 - KOTAS, J. & COSTA, N.C.A. da. Problems of modal and discussive logics. São Paulo, IME-USP, 1981. 35p.

8112 - BRITO, F.B. & GONÇALVES, D.L. Algebras não associativas, sistemas diferenciais polinomiais homogeneos e classes características. São Paulo, IME-USP, 1981. 7p.

8113 - POLCINO MILIES, F.C. Group rings whose torsion units form a subgroup II. São Paulo, IME-USP, 1981. 1v. (não paginado)

8114 - CHEN, C.C. An elementary proof of Calabi's theorems on holomorphic curves. São Paulo, IME-USP, 1981. 5p.

8115 - COSTA, N.C.A. da & ALVES, E.H. Relations between paraconsistent logic and many-valued logic. São Paulo, IME-USP, 1981. 8p.

8116 - CASTILLA, M.S.A.C. On Przymusinski's theorem. São Paulo, IME-USP, 1981. 6p.

8117 - CHEN, C.C. & GOES, C.C. Degenerate minimal surfaces in R^4. São Paulo, IME-USP, 1981. 21p.

8118 - CASTILLA, M.S.A.C. Imagens inversas de algumas aplicações fechadas. São Paulo, IME-USP, 1981. 11p.

8119 - ARAGONA VALEJO, A.J. & EXEL FILHO, R. An infinite dimensional version of Hartogs' extension theorem. São Paulo, IME-USP, 1981. 9p.

8120 - GONÇALVES, J.Z. Groups rings with solvable unit groups. São Paulo, IME-USP, 1981. 15p.

8121 - CARNIELLI, W.A. & ALCANTARA, L.P. de. Paraconsistent algebras. São Paulo, IME-USP, 1981. 16p.

8122 - GONÇALVES, D.L. Nilpotent actions. São Paulo, IME-USP, 1981. 10p.

8123 - COELHO, S.P. Group rings with units of bounded exponent over the center. São Paulo, IME-USP, 1981. 25p.

8124 - PARMENTER, M.M. & POLCINO MILIES, F.C. A note on isomorphic group rings. São Paulo, IME-USP, 1981. 4p.

8125 - MERKLEN GOLDSCHMIDT, H.A. Hereditary algebras with maximum spectra are of finite type. São Paulo, IME-USP, 1981. 10p.

8126 - POLCINO MILIES, F.C. Units of group rings: a short survey São Paulo, IME-USP, 1981. 32p.

8127 - CHEN, C.C. & GACKSTATTER, F. Elliptic and hyperelliptic functions and complete minimal surfaces with handles. São Paulo, IME-USP, 1981. 14p.

8128 - POLCINO MILIES, F.C. A glance at the early history of group rings. São Paulo, IME-USP, 1981. 22p.

8129 - FERRER SANTOS, W.R. Reductive actions of algebraic groups on affine varieties. São Paulo, IME-USP, 1981. 52p.

8130 - COSTA, N.C.A. da. The philosophical import of paraconsistent logic. São Paulo, IME-USP, 1981. 26p.

8131 - GONÇALVES, D.L. Generalized classes of groups, spaces c-nilpotent and "the Hurewicz theorem". São Paulo, IME-USP, 1981. 30p.

8132 - COSTA, N.C.A. da & MORTENSEN, Chris. Notes on the theory of variable binding term operators. São Paulo, IME-USP, 1981. 18p.

8133 - MERKLEN GOLDSCHMIDT, H.A. Homogenes ℓ-hereditary algebras with maximum spectra. São Paulo, IME-USP, 1981. 32p.

8134 - PERESI, L.A. A note on semiprime generalized alternative algebras. São Paulo, IME-USP, 1981. 10p.

8135 - MIRAGLIA NETO, F. On the preservation of elementary equivalence and embedding by filtred powers and structures of stable continuous functions. São Paulo, IME-USP, 8136 - FIGUEIREDO, G.V.R. Catastrophe theory: some global theory a full proof. São Paulo, IME-USP, 1981. 91p.

8237 - COSTA, R.C.F. On the derivations of gametic algebras. São Paulo, IME-USP, 1982. 17p.

8238 - FIGUEIREDO, G.V.R. de. A shorter proof of the Thom-Zeeman global theorem for catastrophes of cod ≤ 5. São Paulo, IME-USP, 1982. 7p.

8239 - VELOSO, J.M.M. Lie equations and Lie algebras: the intransitive case. São Paulo, IME-USP, 1982. 97p.

8240 - GOES, C.C. Some results about minimal immersions having flat normal bundle. São Paulo, IME-USP, 1982. 37p.

8241 - FERRER SANTOS, W.R. Cohomology of comodules II. São Paulo, IME-USP, 1982. 15p.

8242 - SOUZA, V.H.G. Classification of closed sets and diffeos of one-dimensional manifolds. São Paulo, IME-USP, 1982. 15p.

8243 - GOES, C.C. The stability of minimal cones of codimension greater than one in R^n. São Paulo, IME-USP, 1982. 27p.

8244 - PERESI, L.A. On automorphisms of gametic algebras. São Paulo, IME-USP, 1982. 27p.

8245 - POLCINO MILIES, F.C. & SEHGAL, S.K. Torsion units in integral group rings of metacyclic groups. São Paulo, IME-USP, 1982. 18p.

8246 - GONÇALVES, J.Z. Free subgroups of units in group rings. São Paulo, IME-USP, 1982. 8p.

8247 - VELOSO, J.M.M. New classes of intransitive simple Lie pseudogroups. São Paulo, IME-USP, 1982. 8p.

8248 - CHEN, C.C. The generalized curvature ellipses and minimal surfaces. São Paulo, IME-USP, 1982. 10p.

8249 - COSTA, R.C.F. On the derivation algebra of zygotic algebras for polyploidy with multiple alleles. São Paulo, IME-USP, 1982. 24p.

8350 - GONÇALVES, J.Z. Free subgroups in the group of units of group rings over algebraic integers. São Paulo, IME-USP, 1983. 8p.

8351 - MANDEL, A. & GONÇALVES, J.Z. Free k-triples in linear groups. São Paulo, IME-USP, 1983. 7p.

8352 - BRITO, F.G.B. A remark on closed minimal hypersurfaces of S^4 with second fundamental form of constant length. São Paulo, IME-USP, 1983. 12p.

8353 - KIIHL, J.C.S. U-structures and sphere bundles. São Paulo, IME-USP, 1983. 8p.

8354 - COSTA, R.C.F. On genetic algebras with prescribed derivations. São Paulo, IME-USP, 1983. 23p.

8355 - SALVITTI, R. Integrabilidade das distribuições dadas por subalgebras de Lie de codimensão finita no gh(n,C). São Paulo, IME-USP, 1983. 4p.

8356 - MANDEL, A. & GONÇALVES, J.Z. Construction of open sets of free k-Tuples of matrices. São Paulo, IME-USP, 1983. 18p.

8357 - BRITO, F.G.B. A remark on minimal foliations of codimension two. São Paulo, IME-USP, 1983. 24p.

8358 - GONÇALVES, J.Z. Free groups in subnormal subgroups and the residual nilpotence of the group of units of group rings. São Paulo, IME-USP, 1983. 9p.

8359 - BELOQUI, J.A. Modulus of stability for vector fields on 3-manifolds. São Paulo, IME-USP, 1983. 40p.

8360 - GONÇALVES, J.Z. Some groups not subnormal in the group of units of its integral group ring. São Paulo, IME-USP, 1983. 8p.

8361 - GOES, C.C. & SIMÕES, P.A.Q. Imersoes minimas nos espaços hiperbolicos. São Paulo, IME-USP, 1983. 15p.

8462 - GIAMBRUNO, A.; MISSO, P. & POLCINO MILIES, F.C. Derivations with invertible values in rings with involution. São Paulo, IME-USP, 1984. 12p.

8463 - FERRER SANTOS, W.F. A note on affine quotients. São Paulo, IME-USP, 1984. 6p.

8464 - GONCALVES, J.Z. Free-subgroups and the residual nilpotence of the group of units of modular and p-adic group rings. São Paulo, IME-USP, 1984. 12p.

8465 - GONÇALVES, D.L. Fixed points of S^1-fibrations. São Paulo, IME-USP, 1984. 18p.

8466 - RODRIGUES, A.A.M. Contact and equivalence of submanifolds of homogenous spaces. São Paulo, IME-USP, 1984. 15p.

8467 - LOURENÇO, M.L. A projective limit representation of (DFC) spaces with the approximation property. São Paulo, IME-USP, 1984. 20p.

8468 - FORNARI, S. Total absolute curvature of surfaces with boundary. São Paulo, IME-USP, 1984. 25p.

8469 - BRITO, F.G.B. & WALCZAK, P.G. Totally geodesic foliations with integral normal bundles. São Paulo, IME-USP, 1984. 6p.

8470 - LANGEVIN, R. & POSSANI, C. Quase-folheacões e integrais de curvatura no plano. São Paulo, IME-USP, 1984. 26p.

8471 - OLIVEIRA, M.E.G.G. de Non-orientable minimal surfaces in R^N. São Paulo, IME-USP, 1984. 41p.

8472 - PERESI, L.A. On baric algebras with prescribed automorphisms. São Paulo, IME-USP, 1984. 42p.

8473 - MIRAGLIA NETO, F. & ROCHA FILHO, G.C. The measurability of Riemann integrable functions with values in Banach spaces and applications. São Paulo, IME-USP, 1984. 27p.

8474 - MERKLEN GOLDSCHMIDT, H.A. Artin algebras wich are equivalent to a hereditary algebra modulo preprojectives. São Paulo, IME-USP, 1984. 38p.

8475 - GOES, C.C. & SIMÕES, P.A.Q. The generalized Gauss map of minimal surfaces in H^3 and H^4. São Paulo, IME-USP, 1984. 16p.

8476 - GONÇALVES, J.Z. Normal and subnormal subgroups in the group of units of a group rings. São Paulo, IME-USP, 1984. 13p.

8477 - ARAGONA VALLEJO, A.J. On existence theorems for the $\bar{\partial}$ operator on generalized differential forms. São Paulo, IME-USP, 1985. 23p.

8478 - POLCINO MILIES, C.; RITTER, J. & SEHGAL, S.K. On a conjecture of Zassenhaus on torsion units in integral group rings II. São Paulo, IME-USP, 1985. 14p.