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A quasi-transversal Hopf bifurcations

Jorge A. Beloqui

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Jorge A. Beloqui

IME - USP

C.P. 20570

(01498) São Paulo (SP)

BRASIL

Bifurcations of one parameter families of vectorfields with simple recurrence are caused by, either

a) non-hyperbolicity of singularities,

b) non-hyperbolicity of periodic orbits,

c) non-transversal intersection of invariant manifolds.

The generic ones in a) are the saddle - node and the Andronov-Hopf bifurcation ({1},{2},{3},{4},{5}). Both are strongly stable. In the same way, quasi-transversality of the invariant manifolds of two hyperbolic elements is generic for c) ({6},{7},{8}).
Van Strien ({9}) gave a complete description of the conditions required for two generic one parameter families to be strongly equivalent. He did this in terms of moduli of stability ({10},{11},{12}, {13},{14},).

Here we shall see some results about one kind of two parameter families of vectorfields with simple recurrence in M^3. They shall be those whose central bifurcation is a vectorfield $X(0,0)$ which exhibits both a quasi-hyperbolic singularity (of the Hopf type) and a quasi transversal intersection of invariant manifolds.

More precisely,

Let $X_1(M^3)$ be the space of C^∞ vector fields endowed with the C^∞ Whitney topology and let $X_2(M)$ be the space of the mappings $X: I \times I \to X_1(M)$ with the usual C^∞ topology($I=[-1,1]$).
Suppose $X(\mu)$, is a 2-parameter family of vectorfields such that $X(0,0)$ is a field which exhibits:

a) a saddle type periodic orbit $\sigma_1(0)$ of period 1, hyperbolic, with associated eigenvalues $0 < |\beta_1| < 1 < |\beta_2|$, C^2 linearizable with associated transversal section Σ .

b) a quasi-hiperbolic singularity p(0) of saddle type such that $\dim W^u(p(0)) = 1$, associated to the eigenvalue $\alpha_3 > 0$ and $\dim W^s(p(0)) = 2$, associated to eigenvalues $\alpha_1 \pm i\alpha_2 (\alpha_1 = 0)$. Consider $X/W^s(p(0))$ as a "vague attractor".

c) a unique orbit γ in $W^u(p(0)) \cap W^s(\sigma_1(0))$ that is, a quasi-transversal intersection.

Call $E \subset \mathfrak{X}(M^3)$ the set of fields like X(0,0) and E' the set of families as the one described above.

A topological equivalence between two vectorfields X, $\tilde{X}$ on M is an homeomorphism $h: M \to M$ such that h sends orbits of X onto orbits of $\tilde{X}$, preserving time orientation. If addition h preserves time, that is $h X_t = \tilde{X}_t h$ holds, then h is called a conjugation. A vectorfields X is called structurally stable if it is equivalent to any nearby vectorfield.

A semilocal equivalence between X and $\tilde{X}$ and $\tilde{X} \in E$ shall be an equivalence defined from a neighorhood of $\bar{\gamma}$ onto a neighborhood of $\bar{\bar{\gamma}}$.

We say that a family X_μ at μ_0 is equivalent to $\tilde{X}_\mu$ at $\tilde{\mu}_0$, if there exists a reparametrisation (homeomorphism) $\lambda: (U \subset \mathbb{R}^2, \mu_0) \to (U, \tilde{\mu}_0)$ and a family of homeomorphisms $h_\mu: M \to M$ so that h_μ maps orbits of X_μ onto orbits of $\tilde{X}_{\lambda(\mu)}$ for μ near μ_0. If in addition the homeomorphisms h_μ depend continuously on μ, then we say that X_μ is continuously (or strongly) to equivalent to $\tilde{X}_\mu$ (at μ_0 and $\tilde{\mu}_0$). We say that X_μ has a <u>stable bifurcation</u> at μ_0, if every family $\tilde{X}_\mu$ sufficiently near X, has a bifurcation value $\tilde{\mu}_0$ such that X_μ and $\tilde{X}_\mu$ are equivalent (at μ_0 and $\tilde{\mu}_0$).

We also need semilocal versions of these definitions. We say that X_μ at μ_0 is semilocally equivalent to $\tilde{X}_\mu$ at $\tilde{\mu}_0$ if there is a reparametrisation $\lambda: (U, \mu_0) \to (U, \tilde{\mu}_0)$, a family of homeomorphisms $h_\mu : M \to M$ and a neighbourhood V of the closure of γ such that

(i) $h(\gamma) = \tilde{\gamma}$

(ii) $h_\mu \mid V$ maps orbits of $X_\mu \mid V$ onto orbits of $\tilde{X}_{\lambda(\mu)} | h^\mu(V)$ for μ near μ_0.

Let us state

Theorem A - Let X, X' be two vectorfields in E, ε - C^r near. Then they are semilocally topologically equivalent iff $\beta_2 = \tilde{\beta}_2$.

Sketch of the proof: a) Necessity of modulus.
By Takens [15], we can "linearize" the vectorfield, in a way that $X/W^s(p)$ is given by

$$\begin{cases} \dot{\rho} = b\rho^3 + o(\rho^5), \quad b < 0 \text{ (vague attractor)} \\ \dot{\theta} = \alpha_2 + o(\rho^2) \end{cases}$$

Let $\Lambda = \{x_3 = 1\}$ be a transversal section to the flow, contained in the lienarization domain of ρ.

Take $R=\{x_1=a_1, x_2=a_2, x_3>0\}$ (a line) and consider $F=\Lambda \cap \left(\bigcup_{t>0} X_t(R)\right)$. A sector F is a connected component of $F-W^s(\sigma_1)$.

Call e_m the maxima of the "upper" sectors.

Finally $c_1 \in W^u(\sigma_1) \cap \Sigma - \sigma_1$.

Define $N_m(F, \pi_2^{-1}(c_1), S_p) = \#\{S_j / S_j \cap X-n(\pi_2^{-1}(c_1)) \neq \emptyset, j \geq p\}$.

So we get $\quad e_{1+N_m} \leq c_1 \; \beta_2^{-m} \leq e_{N_m}$

and, in the same way

$$\tilde{e}_{1+N_m^{\cdot\cdot}} \leq \tilde{c}_1 \; \tilde{\beta}_2^{-m} \leq \tilde{e}_{N_m^{\cdot\cdot}}$$

where $\tilde{c}_1=h(c_1)$.

$$\text{Then} \quad \frac{e_{1+N_m}}{\tilde{e}_{N_m}} \leq \frac{c_1}{\tilde{c}_1} \quad \frac{\beta_2}{\tilde{\beta}_2}^{-m} \leq \frac{e_{N_m}}{\tilde{e}_{1+N_m}}$$

With a little calculation, we get

$$e_{N_m} \doteq A_{N_m} \; / \sqrt{1-2b\; t_{N_m}}$$

where t_{N_m} is the time needed in $W^c(p)$ to turn N_m times along an orbit, and $A_{N_m} \to A$.

So there exist two constants $\delta_1, \delta_2 > 0$ such that

$$\delta_1 \leq \frac{c_1}{\tilde{c}_1} \left(\frac{\beta_2}{\tilde{\beta}_2}\right)^{-m} \leq \delta_2$$

for all $m \in \mathbb{N}$. This means $\beta_2 = \tilde{\beta}_2$.

b) Sufficiency of the modulus.

It follows by an application of the same technique as in [14]. □

In this case we say that X has modulus of stability one, and that β_2 is a modulus for X. In some sense, we are parameterizing the equivalence classes in a neighborhood of X by the parameter β_2.

Let us mention which kind of vectorfields shall appear in a generic family of this type:

1) vectorfields in E

2) vectorfields in D, which are like those in E, except for the fact that the singularity p is hyperbolic.

3) vectorfields in C, which exhibit a singularity of the source type, hyperbolic and a periodic orbit σ_2 very near this singularity.

Let β_3 be its attractive eigenvalue.

This set C we subdivide into:

a) C_1, the set of fields such that $W^u(\sigma_2) \cap W^s(\sigma_1)$ consists only of one orbit, giving rise to a quasi transversal intersection.

b) C_2, the set of vectorfields such that $W^u(\sigma_2) \overline{\pitchfork} W^s(\sigma_1)$ along 2 orbits.

c) C_0, the set of vectorfields such that $W^u(\sigma_2) \cap W^s(\sigma_1) = \emptyset$.

4) Vectorfields in B, with a singularity p of saddle type, such that $W^u(p) \cap W^s(\sigma_1) = \emptyset$, which we subdivide into a) B_0 the set of fields where p is hyperbolic; b) B_1 the set of fields where p is quasi-hyperbolic.

Observe that $W^u(\sigma_2) \cap W^s(\sigma_1)$ consists alternatively of none, one or two orbits, and that there might be tangencies only for fields of C_1.

We can now state

Theorem B - Let $X(\mu) \varepsilon E'$ be a generic 2 - parameter family of vectorfields. Then

a) the modulus of stability for weak topological equivalence is one, namely β_2.

b) the modulus of stability for strong topological equivalence is infinite.

Sketch of the proof:

Take in Σ a C^1 family of C^2 linearizations. Choose a fundamental neighborhood N (that is, a neighborhood of a fundamental domain) and consider $v(\mu_1,\mu_2) = \pi_2 \, (N \cap W^{uu}(p(\mu_1,\mu_2))$, where π_2 is the trivial projection on $W^u(\sigma_1(\mu_1,\mu_2))$.

We say that $X\,(\mu_1,\mu_2) \varepsilon E'$ is generic if

$D\,(\alpha_1(\mu_1,\mu_2),\ v(\mu_1,\mu_2)\ (0,0)$ is a non singular matrix.

In this case, we can apply the Inverse Function Theorem and identify $\mu_1=\alpha_1$, $\mu_2=v$ (the "height" of $W^{uu}(p)$ over $W^s(\sigma_1)$).

Now we claim that for this type of family, the set of values (μ_1,μ_2) for which $W^u(\sigma_2(\mu_1,\mu_2) \cap W^s(\sigma_1(\mu_1,\mu_2)$ is non transversal is a differentiable curve $\mu_1=\mu_1(\mu_2)$, quadratically tangent to the μ_2 axis.

This is natural because for the Hopf bifurcation the radius σ_2 is approximatedly $A\sqrt{\mu_1}$. So if $W^u\,(\sigma_2)$ is tangent to $W^s(\sigma_1)$, its "center" $v(\mu_1,\mu_2)$ will have height $\mu_2 \simeq A\sqrt{\mu_1}$.

Take now another generic family $\tilde{X}_\mu$ in E'.

We recall that in (14) it is proven that the fields in D have modulus of stability 1, given by $\lambda_1 = \frac{\alpha_1}{\alpha_2} \ln \beta_2$. De Melo (11) , and Palis {10}, prove that a vectorfield exhibiting a quasi trans - versal connection between the invariant manifolds of periodic orbits has modulus 1, namely $\lambda_2 = \ln\beta_3/\ln\beta_2$.

Define the reparametrization p in order to get

$_1(\mu_1,0) = \tilde{\lambda}_1(p(\mu_1,0))$ for $\mu_1<0$ and $_2(\mu_1(\mu_2),\mu_2) = \lambda_2(p(\mu_1(\mu_2),\mu_2))$, for $\mu_1>0$.

b) Van Strien proves in {9} that a one parameter family in D' has 2 moduli of stability for strong equivalence:

$\lambda_{1,1}=\alpha_1/\alpha_2$; $\lambda_{1,2}=\ln\beta_2$, that is the modulus splits. For a familiy in C_1 the 2 moduli are $\lambda_{1,2} = \ln\beta_2$, $\lambda_{1,3}= \ln\beta_3$. Obviously for weak equivalence we have modulus 1.

For the sake of simplicity, change the parameters μ_1 and $\tilde{\mu}_1$ for $\frac{\mu_1}{\alpha_2}$ and $\frac{\tilde{\mu}_1}{\tilde{\alpha}_2}$ respectively. Observe that $\rho(\mu_1,0)=(\rho_1(\mu_1,0),0)$; hence $\rho_1(\frac{\mu_1}{\alpha_2},0)= \frac{\mu_1}{\alpha_2}$ if we have a strong equivalence.

Moreover, as $\frac{\mu_1}{\alpha_2} \to 0$ monotonously with μ_1, the existence of a strong equivalence between both families means that only $\tilde{X}(\frac{\mu_1}{\alpha_2},0)$ can be equivalent to $X(\frac{\mu_1}{\alpha_2},0)$ and in addition

$$\beta_2(\frac{\mu_1}{\alpha_2},0) = \tilde{\beta}_2(\frac{\mu_1}{\alpha_2},0)$$

for $\frac{\mu_1}{\alpha_2}$ in a left neighborbood of zero. This is an equality between two functions; more precisely, the functions $\beta_2(x,0)$ and $\tilde{\beta}_2(x,0)$ should define the same germ in a left neighborhood of zero.

In the same way we get two additional equalities of a similar type, by considering β_3, the attractive eigenvalue associated to σ_2, along $\mu_2^+(\mu_1)$ and $\mu_2^-(\mu_1)$ $(\mu_1>0)$, the two branches of the curve $\mu_1(\mu_2)$. Calling $\beta_3^+=\beta_3(\mu_1,\mu_2^+(\mu_1))$ and $\beta_3^-=\beta_3(\mu_1,\mu_2^-(\mu_1))$, we have equalities between germs of functions defined to the right of zero, given by β_3^- and β_3^+. $\square$

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