Phase motion in the $Z^-(4430)$ amplitude in $B^0 \to \psi' \pi^- K^+$ decay

Ignacio Bediaga [a], Jussara M. de Miranda [a], Fernando Rodrigues [a,*], Marina Nielsen [b]

[a] *Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro, RJ, Brazil*
[b] *Instituto de Física, Universidade de São Paulo, C.P. 66318, 05314-970 São Paulo, SP, Brazil*

A B S T R A C T

In view of the proliferation in the number of new charmonium states, it is really important to have an experimental way to prove that an observed bump is, indeed, a real resonance. To do that, in this paper we present an alternative method to demonstrate the resonant behavior of a state. With this method, the phase variation of a generic complex amplitude can be directly revealed through interference in the Dalitz-plot region where it crosses a well established resonant state, used as a probe. We have tested the method for the $Z^-(4430)$ state by generating Monte Carlo samples for the $B^0 \to \psi(2S)\pi^- K^+$ decay channel. We have shown that the proposed method gives a clear oscillation behavior, related to the phase variation associated with a real resonant state, in the case where the $Z^-(4430)$ is considered as a regular resonance with a strong phase variation.

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Several experiments operating during the last decade, mainly BaBar at SLAC and Belle at KEK, CLEO-III and CLEO-c at CESR, CDF and DØ at Fermilab, BESIII at IHEP and LHCb and CMS at CERN, have vastly increased the available data on new charmonium-like states, called X, Y and Z states. Among these states, the charged ones are the most interesting, since they cannot be simple $c\bar{c}$ states. The $Z^+(4430)$, found by Belle Collaboration in 2007, was the first one observed [1–3]. Since the minimal quark content of this state is $c\bar{c}u\bar{d}$ this can only be achieved in a multiquark configuration. The BaBar Collaboration searched for the $Z^-(4430)$ signature in four decay modes and concluded that there is no significant evidence for a signal peak in any of these processes [4]. However, very recently the Belle and LHCb Collaborations have confirmed the $Z^+(4430)$ observation and have determined the preferred assignment of the quantum numbers to be $J^P = 1^+$ [3,5]. Curiously, the first evidence of this resonance in the $J/\psi\pi^+$ channel was reported only this year by Belle Collaboration [6].

The $Z^+(4430)$ observation motivated further studies of other $\bar{B}^0$ decays and, in 2008, Belle Collaboration reported the observation of other two resonance-like structures, called $Z_1^+(4050)$ and $Z_2^+(4250)$, in the exclusive process $\bar{B}^0 \to K^- \pi^+ \chi_{c1}$, in the $\pi^+ \chi_{c1}$ mass distribution [7]. Once again the BaBar Collaboration did not confirm these observations [8].

Following these observations, from March to October of 2013 four more charmonium charged states were reported. The first one was the $Z_c^+(3900)$, observed almost at the same time by BESIII [9] and Belle [10] Collaborations, in the $M(\pi^\pm J/\psi)$ mass spectrum of the $Y(4260) \to J/\psi\pi^+\pi^-$ decay channel. This structure was also confirmed by the authors of Ref. [11] using CLEO-c data.

Soon after the $Z_c^+(3900)$ observation, the BESIII related the observation of other three charges states: $Z_c^+(4025)$ [12], $Z_c^+(4020)$ [13] and $Z_c^+(3885)$ [14]. Up to now it is not clear if the states $Z_c^+(3900)$–$Z_c^+(3885)$ and the states $Z_c^+(4025)$–$Z_c^+(4020)$ are the same states seen in different decay channels, or if they are independent states.

Finally, in August 2014 the $Z_c^+(4200)$ was reported by Belle Collaboration in the $J/\psi\pi^+$ channel of the $\bar{B}^0$ decay, with a 6.2σ significance. As in the case of the $Z^+(4430)$, the preferred assignment of the quantum numbers is $J^P = 1^+$ [6]. We show these states in Table 1. For more details we refer the reader to the more comprehensive review articles [15–19].

In view of so many non-confirmed (NC) states in Table 1, it is really important to have an experimental way to prove that an observed excess of events is, indeed, a real resonance. In particular, bumps close to the threshold of a pair of particles should be treated with caution [21]. Sometimes they are identified as new particles, but they can also be a reflection of a resonance below threshold. As an example, in the case of the $Z_c^+(3885)$, it was shown in Ref. [21] that the signal reported in [14] could be also described by a $D\bar{D}^*$ resonance with a mass around 3875 MeV and

Table 1
The new charged states in the $c\bar{c}$ regions, ordered by mass. Masses m and widths Γ represent the weighted averages from the listed sources as in [19], or are taken from [20] when available. The citation given in red is for the first observation and the citation given in blue is for a non-confirmation. The Status column NC (needs confirmation) indicates that the state has been observed by only one, or was not confirmed by other, experiment. The Status is OK if at least two independent experiments saw the state.

State	m (MeV)	Γ (MeV)	$J^{P(C)}$	Process (mode)	Experiment	Year	Status
$Z_c^+(3885)$	3883.9 ± 4.5	25 ± 12	1^+	$Y(4260) \to \pi^-(D\bar{D}^{*+})$	BESIII [14]	2013	NC
$Z_c^+(3900)$	3896.7 ± 7.3	55 ± 35	1^+	$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BESIII [9], Belle [10], CLEO-c [11]	2013	OK
$Z_c^+(4020)$	4022.9 ± 2.8	7.9 ± 3.7	...	$e^+e^- \to \pi^-(\pi^+ h_c)$	BESIII [13]	2013	NC
$Z_c^+(4025)$	4026.9 ± 4.5	24.8 ± 9.5	$1^+, 2^+$	$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BESIII [12]	2013	NC
$Z_1^+(4050)$	4051^{+24}_{-43}	82^{+51}_{-55}	...	$B \to K(\pi^+ \chi_{c1}(1P))$	Belle [7], BaBar [8]	2008	NC
$Z_c^+(4200)$	4196^{+35}_{-30}	370^{+99}_{-110}	1^+	$B \to K(\pi^+ J/\psi)$	Belle [6]	2014	NC
$Z_2^+(4250)$	4248^{+185}_{-45}	177^{+321}_{-72}	...	$B \to K(\pi^+ \chi_{c1}(1P))$	Belle [7], BaBar [8]	2008	NC
$Z^+(4430)$	4458 ± 15	166^{+37}_{-32}	1^+	$B \to K^-(\pi^+ \psi(2S))$	Belle [1–3], BaBar [4], LHCb [5]	2007	OK
				$B \to K^-(\pi^+ J/\psi)$	Belle [6]		

width around 30 MeV. Also, in the case of the $Z_c^+(4025)$, it was shown in Ref. [22] that both, a resonance with $J^P = 1^+$ or a bound state with $J^P = 2^+$, are compatible with the data from Ref. [12]. Besides, it was also shown in Ref. [22] that the experimental data can also be explained with just a pure wave-D background. In the case of the $Z^+(4430)$, since its mass is close to the D^*D_1 threshold, it was suggested that it could be a $J^P = 1^+$ $D^*\bar{D}_1$ molecular state [23] or a cusp in the $D^*\bar{D}_1$ channel [24].

The first attempt to demonstrate the resonant behavior of the $Z^+(4430)$ state was done by LHCb in Ref. [5], where a fit was performed in which the Breit–Wigner amplitude was replaced by a combination of independent complex amplitudes at six equally spaced points in $m_{\psi(2S)\pi}$ range covering the $Z^+(4430)$ peak region. The resulting Argand diagram is consistent with a rapid phase transition at the peak of the amplitude, just as expected for a resonance. In Ref. [6] a similar method was applied to show the resonant behavior of the $Z^+(4200)$. The Breit–Wigner amplitude was replaced by a combination of constant amplitudes, with six bins in $m_{J/\psi\pi}$ range covering the $Z^+(4200)$ peak and two independent sets of constant amplitudes, to represent the two helicity amplitudes of the $Z_c^+(4200)$, H_0 and H_1. The results in the Argand diagram for H_1 clearly show a resonance-like change of the amplitude's absolute value and phase. However, they argue that because of the Argand diagram for the H_0 amplitudes has much larger relative errors, it was not possible to draw any conclusions from it. In any case, the Argand-plot approach, proposed by LHCb experiment, needs a high statistics sample to be able to give, in an undoubted way, the confirmation of the phase variation expected for a regular resonant state.

In this paper we propose a different method to demonstrate the resonant behavior of a state. It is a simple experimental method isobar-based Amplitude Difference (AD), that can be used to extract the phase motion of a complex amplitude in three-body heavy-meson decays [25]. With this method, the phase variation of a generic complex amplitude can be directly revealed through interference in the Dalitz-plot region where it crosses a well established resonant state, used as a probe. This method was successfully applied to Fermilab E791 data [26] to extract the well-known phase motion of the scalar amplitude $f_0(980)$ observed in $D_s^+ \to \pi^-\pi^+\pi^+$ decay. It was also successfully used to extract the phase motion, of a resonant scalar amplitude $\sigma(500)$ in $D^+ \to \pi^-\pi^+\pi^+$ decay [27], to confirm previous evidences of the existence of a light and broad scalar resonance presented by Fermilab experiment E791 [28].

In full Dalitz-plot analyses, each possible resonance amplitude is represented by a Breit–Wigner function multiplied by angular distributions associated with the spin of the resonance. The various contributions are combined in a coherent sum, with complex coefficients, that are extracted from fits to the data. The absolute

value of the coefficients is related to the relative fraction of each contribution and the phases take into account the final state interaction (FSI) between the resonance and the third particle.

Amplitude Difference method has a different approach. It concentrates in a particular region of the Dalitz plot, where the amplitude under study crosses a well-known resonance amplitude, called probe amplitude, represented by a Breit–Wigner. The phase variation of the complex amplitude can be directly revealed through the interference, in the Dalitz-plot region, where they cross each other.

There are two necessary conditions to extract the phase motion of a generic amplitude with the AD method:

- A crossing region between the amplitude under study and a probe resonance has to be dominated by these two contributions.
- The integrated amplitude of the probe resonance must be symmetric with respect to an effective mass squared (m_{eff}^2) that is the nominal mass of the resonance probe.

These two conditions are very well satisfied in many charmonium three-body B decays, where the phase space is large and the charmonium candidates are located in the central region of the Dalitz plot, possibly crossing with well established resonances. The probe resonances are, in general, placed at low $K\pi$, KK or $\pi\pi$ invariant mass. As a consequence, the charmonium amplitude candidates must cross basically all phase space, or at least the low $K\pi$ region, to be observed by the AD method. Therefore, if the amplitude under study is located only in a region of the Dalitz plot and does not cross the probe resonance, this method may not apply. One example could be a molecular state, as discussed in Ref. [29]. This will exclude the direct observation of the phase variation of molecular states with the AD method.

With the two above conditions, we can examine the $B^0 \to \psi\pi^-K^+$ decay (where ψ represents J/ψ or $\psi(2S)$), and write down an amplitude with two components: one representing the probe resonance, K^*, placed in the Dalitz variable s_{12} that is the square invariant mass of the pair π^- and K^+, through a Breit–Wigner and the angular distribution, and the other, representing the resonance under study, which we call generically by Z decaying in $J/\psi\pi$ or $\psi(2S)\pi$, placed in the Dalitz variable s_{13}, that is the square invariant mass of the pair π^- and J/ψ. This simple amplitude must be used only in the small part of the phase space where the interference between them occurs. Since the last one can have different dynamical origins, it can be written in a most generic way as:

$$A^Z(s_{13}) = \sin\delta(s_{13})e^{i\delta(s_{13})}. \tag{1}$$

This unitary equation is able to represent amplitudes with slow phase variation, as well as resonances with a large phase variation, of the order of 180°, around the nominal mass of the resonance, in the same way as the Argand plot used by LHCb [5]. The total amplitude for the $B^0 \to \psi \pi^- K^+$ decay, in the small part of the phase space where the interference between the resonances $K^* \to K^+ \pi^-$ and $Z^- \to \psi \pi^-$ occurs, can be written as [3]:

$$|\mathcal{A}(s_{12}, s_{13})|^2 = \sum_{\zeta=1,-1} |A(s_{12}, s_{13}, \zeta)|^2 ,$$

with $A(s_{12}, s_{13}, \zeta) = \sum_{\lambda=-1,0,1} A_{\lambda\zeta}^{K^*} + \sum_{\lambda'=-1,0,1} A_{\lambda'\zeta}^{Z} ,$ (2)

where ζ, λ and λ' are the helicities of the lepton pair, the K^* and Z respectively. We take the amplitudes of the decays $B^0 \to \psi(\to l^+l^-)K^*(\to K^+\pi^-)$ and $B^0 \to K^+ Z^-(\to \psi(\to l^+l^-)\pi^-)$ from [3]:

$$A_{\lambda\zeta}^{K^*} = H_\lambda^{K^*} A^{K^*}(s_{12}) d_{\lambda 0}^1(\theta_{K^*}) e^{i\lambda\varphi} d_{\lambda\zeta}^1(\theta_\psi) ,$$
$$A_{\lambda'\zeta}^{Z} = H_{\lambda'}^{Z} A^{Z}(s_{13}) d_{0\lambda'}^1(\theta_Z) e^{i\lambda'\tilde{\varphi}} d_{\lambda'\zeta}^1(\tilde{\theta}_\psi) e^{i\zeta\alpha} .$$ (3)

In Eqs. (3) H_λ^R is the helicity amplitude for the decay via the resonance R, $d_{mn}^J(\beta)$ are Wigner d functions, θ_R is the resonance helicity angle (the angle between π^- and K^+ or ψ momenta in the resonance rest frame), θ_ψ ($\tilde{\theta}_\psi$) is the ψ helicity angle (the angle between π^- and l^- momenta in the ψ rest frame), $\varphi(\tilde{\varphi})$ is the angle between the planes defined by the $(l^+\pi^-)$ and $(K^+\pi^-)$ momenta in the ψ rest frame and α is the angle between the planes defined by the $(l^+\pi^-)$ and (l^+K^*) momenta in the ψ rest frame. The amplitudes H_1^Z and H_{-1}^Z are related by parity conservation $H_\lambda^Z = H_{-\lambda}^Z$. Finally, $A^Z(s_{13})$ is given in Eq. (1) and $A^{K^*}(s_{12})$ is described by a Breit–Wigner:

$$A^{K^*}(s_{12}) = \frac{m_0\Gamma_0}{s_{12} - m_0^2 + im_0\Gamma_0} ,$$ (4)

where m_0 and Γ_0 are the mass and the width of the $K^*(892)$ respectively.

Let us define

$$a_\zeta^{K^*} e^{i\alpha_\zeta(K^*)} = H_{-1}^{K^*} d_{-10}^1(\theta_{K^*}) e^{-i\varphi} d_{-1\zeta}^1(\theta_\psi)$$
$$+ H_0^{K^*} d_{00}^1(\theta_{K^*}) d_{0\zeta}^1(\theta_\psi) + H_1^{K^*} d_{10}^1(\theta_{K^*}) e^{i\varphi} d_{1\zeta}^1(\theta_\psi),$$
$$a_\zeta^{Z} e^{i\alpha_\zeta(Z)} = H_1^Z \left(d_{0-1}^1(\theta_Z) e^{-i\tilde\varphi} d_{-1\zeta}^1(\tilde\theta_\psi) + d_{01}^1(\theta_Z) e^{i\tilde\varphi} d_{1\zeta}^1(\tilde\theta_\psi) \right)$$
$$+ H_0^Z d_{00}^1(\theta_Z) d_{0\zeta}^1(\tilde\theta_\psi) .$$ (5)

Using Eqs. (3) and (5), we can write

$$A(s_{12}, s_{13}, \zeta) = \sum_{\lambda=-1,0,1} A_{\lambda\zeta}^{K^*} + \sum_{\lambda'=-1,0,1} A_{\lambda'\zeta}^{Z}$$
$$= A^{K^*}(s_{12}) a_\zeta^{K^*} e^{i\alpha_\zeta(K^*)} + A^Z(s_{13}) e^{i\zeta\alpha} a_\zeta^Z e^{i\alpha_\zeta(Z)}. $$ (6)

Therefore,

$$|A(s_{12}, s_{13}, \zeta)|^2 = (a_\zeta^{K^*})^2 |A^{K^*}(s_{12})|^2 + (a_\zeta^Z)^2 \sin\delta(s_{13})$$
$$+ \frac{2a_\zeta^{K^*} a_\zeta^Z \sin\delta(s_{13}) m_0\Gamma_0}{(s_{12} - m_0^2)^2 + m_0^2\Gamma_0^2}$$
$$\times \left[(s_{12} - m_0^2) \cos(\delta(s_{13}) + \beta_\zeta) - m_0\Gamma_0 \sin(\delta(s_{13}) + \beta_\zeta) \right],$$ (7)

where we have used Eqs. (1) and (4) and we have defined $\beta_\zeta = \alpha_\zeta(K^*) - \alpha_\zeta(Z) - \zeta\alpha$.

The expression in Eq. (7) is very similar to the one obtained in [27], for two scalar resonances. For small Γ_0, $|A^{K^*}(s_{12})|^2$ can be considered as a symmetric function, therefore $|A^{K^*}(s_{12} = m_0^2 + \epsilon)|^2 - |A^{K^*}(s_{12} = m_0^2 - \epsilon)|^2 = 0$, where ϵ is small. Also, in the small part of the phase space where the interference between the K^* and the Z occurs, we can suppose that all the angles are almost constant. Consequently the difference of the amplitudes squared takes the simple form:

$$|A(m_0^2 + \epsilon, s_{13}, \zeta)|^2 - |A(m_0^2 - \epsilon, s_{13}, \zeta)|^2$$
$$= \frac{4a_\zeta^{K^*} a_\zeta^Z m_0\Gamma_0\epsilon \sin\delta(s_{13})}{\epsilon^2 + m_0^2\Gamma_0^2} \cos(\delta(s_{13}) + \beta_\zeta) .$$ (8)

We can rewrite Eq. (8) as:

$$\Delta|A_\zeta|^2 = |A(m_0^2 + \epsilon, s_{13}, \zeta)|^2 - |A(m_0^2 - \epsilon, s_{13}, \zeta)|^2$$
$$= \mathcal{C}_\zeta (\sin(2\delta(s_{13}) + \beta_\zeta) - \sin\beta_\zeta),$$ (9)

where $\mathcal{C}_\zeta = 2a_\zeta^{K^*} a_\zeta^Z \epsilon m_0\Gamma_0/(\epsilon^2 + m_0^2\Gamma_0^2)$.

Using Eqs. (2) and (9) we can write the difference of the amplitudes squared:

$$\Delta|\mathcal{A}|^2 = |\mathcal{A}(m_0^2 + \epsilon, s_{13})|^2 - |\mathcal{A}(m_0^2 - \epsilon, s_{13})|^2$$
$$= \sum_{\zeta=-1,1} \Delta|A_\zeta|^2$$
$$= \sum_{\zeta=-1,1} \mathcal{C}_\zeta (\sin(2\delta(s_{13}) + \beta_\zeta) - \sin\beta_\zeta) ,$$ (10)

where $|\mathcal{A}(m_0^2 - \epsilon, s_{13})|^2$ and $|\mathcal{A}(m_0^2 + \epsilon, s_{13})|^2$ are taken from data. The above equation can, finally, be rewritten as:

$$\Delta|\mathcal{A}|^2 = \sin(2\delta(s_{13})) \sum_{\zeta=-1,1} \mathcal{C}_\zeta \cos\beta_\zeta$$
$$+ (\cos(2\delta(s_{13})) - 1) \sum_{\zeta=-1,1} \mathcal{C}_\zeta \sin\beta_\zeta .$$ (11)

$\Delta|\mathcal{A}|^2$, in Eq. (11), directly reflects the behavior of $\delta(s_{13})$. A constant $\Delta|\mathcal{A}|^2$ would imply a constant $\delta(s_{13})$, and this would be the case of non-resonant contribution. In the same way, a slow phase motion will produce a slowly varying $\Delta|\mathcal{A}|^2$ and a full resonance phase motion produces a clear signature in $\Delta|\mathcal{A}|^2$ with the presence of zero, maximum and minimum values.

To clarify these possible behaviors of $\Delta|\mathcal{A}|^2$ and show the statistic feasibility of the AD method, we perform a simple Monte Carlo study. To do that we generated two Monte Carlo samples of $B^0 \to \psi(2S)\pi^- K^+$ decay channel, each one with a sample of 20,000 events with relative fractions 0.86 and 0.08 respectively for the $K^*/\psi(2S)$ and $Z^-(4430)/K$ contributions, similarly to the observed by the LHCb experiment with 3 fb^{-1} accumulated data [5]. In the first sample the $Z^-(4430)$ enters as a regular resonance with a strong phase variation, represented by a Breit–Wigner, while in the second, $Z^-(4430)$ is represented by a real bump amplitude with no strong phase associated. To simplify this study we use only one helicity of the lepton pair in Eq. (10) and assume that in the little phase space crossing region both amplitudes do not have a significant variation due to the angular distribution. In both cases we assume zero phase difference between these amplitudes. Finally we do not include background components in our simulation.

Fig. 1(a) shows the $\Delta|\mathcal{A}|^2$ distribution for the sample with a Breit–Wigner representing the $Z^-(4430)$ particle. One can see a clear oscillation behavior around the zero value of this function, with positive and negative regions, placed around the nominal mass value of this charmonium state. As it was discussed
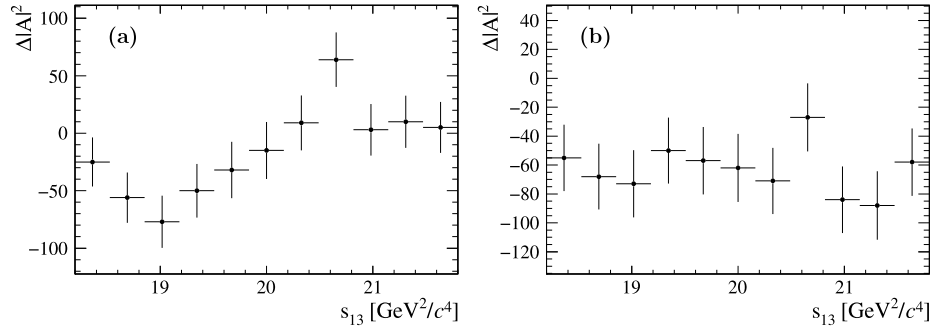
Fig. 1. $\Delta \mid \mathcal{A} \mid^2 = (\mid \mathcal{A}(m_0^2 + \epsilon, s_{13}) \mid^2 - \mid \mathcal{A}(m_0^2 - \epsilon, s_{13}) \mid^2)$, as a function of $s_{13} = m^2(\psi(2S)\pi)$, for a sample of 20,000 signal events, using K^* as a probe with $m_0 = 0.89594$ GeV$^2/c^4$ and $\epsilon = 0.06$ GeV$^2/c^4$. Two cases are illustrated: (a) considering both resonances represented as Breit–Wigner, and (b) representing the K^* resonance as a Breit–Wigner and a real bump around $s_{13} = 20$ GeV$^2/c^4$ as a non-resonant component.

in Ref. [25], this particular distribution is determined by the phase difference between the two amplitudes. Here we assume zero, but any other possible value to this phase difference produces the same signature: positive, negative and zero regions along the crossing region. The $\Delta \mid \mathcal{A} \mid^2$ distribution for the second Monte Carlo sample, with no phase variation associated with the $Z^-(4430)$ around the K^* mass region, is shown in Fig. 1(b). The behavior is clearly different from Fig. 1(a), with almost constant value for $\Delta \mid \mathcal{A} \mid^2$ distribution. The mean value of $\Delta \mid \mathcal{A} \mid^2$ is shifted from zero due to the constant behavior of $\delta(s_{13})$ phase, along the s_{13} variable in the phase space region considered.

In conclusion, we have discussed the new findings of several experiments operating during the last decade, with many indications of new charmonium states. We have identified the need to have a direct confirmation of these states through the study of the phase variation associated with a real resonant state. The first attempt to demonstrate such resonant behavior was done by the LHCb Collaboration [5], for the charged charmonium state $Z^-(4430)$ observed in the $B^0 \rightarrow \psi(2S)\pi^- K^+$ decay. In this paper we present an alternative method, called isobar-based Amplitude Difference (AD), already used in charm three-body decays [25–27], that can be used in cases where the amplitude under study crosses, in the Dalitz plot, a well established resonance. We have tested the method for the $Z^-(4430)$ state by generating two Monte Carlo samples of $B^0 \rightarrow \psi(2S)\pi^- K^+$ decay channel. The first where the $Z^-(4430)$ is considered as a regular resonance with a strong phase variation, represented by a Breit–Wigner, and the second where the $Z^-(4430)$ is represented by a real bump amplitude with no strong phase associated and, therefore, is not a real particle. Each one of these Monte Carlo simulations were generated with samples similar to the observed by the LHCb experiment [5]. We have shown that, in the first case, the AD method gives a clear oscillation behavior related to the phase variation associated with a real resonant state. For the second case no oscillation behavior was observed.

Acknowledgements

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