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Finite field-energy of a point charge in QED

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Abstract

We consider a simple nonlinear (quartic in the fields) gauge-invariant modification of classical electrodynamics, to show that it possesses a regularizing ability sufficient to make the field energy of a point charge finite. The model is exactly solved in the class of static central-symmetric electric fields. Collation with quantum electrodynamics (QED) results in the total field energy of a point elementary charge about twice the electron mass. The proof of the finiteness of the field energy is extended to include any polynomial selfinteraction, thereby the one that stems from the truncated expansion of the Euler–Heisenberg local Lagrangian in QED in powers of the field strength.

Keywords: nonlinear electrodynamics, point charge energy, soliton

1. Introduction

One may say that the triumph of the renormalization theory is not in that it has solved all the problems associated with divergencies, but in that it succeeded in solving most of them, while completely isolating others, so that their solution became less topical and might have been postponed to an undetermined future. Among these is the problem of the field origin of masses. In quantum electrodynamics (QED) the values of the electron mass and charge are treated as mutually independent external empirical parameters fixed to be finite by infinite renormalization. On the other hand, if the field energy of the electron had not diverged, its mass might have been, at least partially, related to its charge. We say 'partially', because one cannot expect that all the electron mass may be of pure electromagnetic origin, since the electron is subject to other interactions. The divergence of the integral for the electrostatic energy of the Coulomb field produced by a charge forces to attribute a certain finite 'classical radius' to the electron in order to equalise the field energy with the electron rest mass.

The problem of the finite field energy is readily solved within the nonlinear self-interaction model of Born and Infeld [1] and other models [2], where the maximum electric field is limited from above. Therefore, the electric field in the close vicinity of a point charge remains finite, so does its field energy. The Lagrangians in these models contain singularities at the maximum values of the electric field, just necessary to provide the finiteness of the field energy. But it is more important that these models are not associated with the only successful theory of electromagnetic interactions—QED, neither with its electro-weak extension.

In the present paper we consider the nonlinear electromagnetic self-interaction caused by the quantum effects that distinguish QED from the linear Maxwell electrodynamics. In QED the self-interaction of electromagnetic field originates from the fact that a photon creates a virtual electron–positron pair that interacts with the electromagnetic field before it annihilates to the photon again. This is how the photon senses the electromagnetic field of its own. Formally, the nonlinearity is, in the simplest manner, taken into account by the known action of Euler–Heisenberg. If expanded in powers of the field this action supplies the modified Maxwell equations with nonlinear terms. This is the vacuum realization of equations of nonlinear optics.

In section 2 we consider the lowest, quadratic, term of this expansion of the Euler–Heisenberg action in powers of the electromagnetic field invariant $\mathfrak{F}(x) = \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \frac{1}{2}\left(B^2 - E^2\right)$ as an independent model. (The other field invariant $\mathfrak{G}(x) = (\mathbf{B}\cdot\mathbf{E})$ is kept equal to zero, because it is not involved in the electrostatic problem under consideration.) We find that in this model the modified Coulomb field of a point charge can be written explicitly by solving a cubic equation and that, although the electric field turns into infinity in the point, where the charge is located, the space integral for the field energy concentrated in the modified Coulomb field converges. This is a new mechanism providing the finiteness of the energy a point charge, different from what was known within the Born–Infeld-like models.

In section 3 we prove the same finiteness of the field energy for the nonlinear action taken as any finite-power polynomial of $\mathfrak{F}$. This means that QED truncated at any finite-power term of the Taylor expansion of the Heisenberg–Euler action also provides this property.

In section 4 we come back to the simplest model of section 2, which admits an explicit solution, and define the self-coupling parameter in it as corresponding to the first term of the expansion of the Heisenberg–Euler–Lagrangian. In this way the field mass of the point charge e equal to that of electron becomes a function of e with the numerical value comparable in the order of magnitude with the mass of the electron.

In Conclusion we discuss the unsolved problems risen by the present approach.

2. The quartic model

We define the Lagrangian of a minimally nonlinear electrodynamics as

$$L(x) = -\mathfrak{F}(x) + \frac{\gamma}{2}(\mathfrak{F}(x))^2, \tag{1}$$

where $-\mathfrak{F}(x)$ makes the Lagrangian density of the standard linear Maxwell electrodynamics, while the second term in (1) is the quartic in the field-strength addition to it[4]. The field-strength tensor is related to the four-vector potential $A^\mu(x)$ as $F^{\mu\nu} = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$, where $\partial^\mu = \frac{\partial}{\partial x_\mu}$. Hence, the first pair of the Maxwell equations, $\nabla \times \mathbf{B} = 0$, $\nabla \times \mathbf{E} + \partial^0 \mathbf{B} = 0$, with the electric and magnetic field strengths $E_i = F^{i0}$ and $B_i = \epsilon_{ijk} F_{jk}$, remains standard. The self-coupling constant γ is presumably small enough. It has dimension of inverse fourth power of mass.

The causality and unitarity principles applied to the local effective action of any nonlinear electrodynamics result in some requirements [3], of which the first one is $\gamma > 0$. Then, the other requirements (in the present case, where the second field invariant $\mathfrak{G} = \mathbf{E} \cdot \mathbf{B}$ is not involved) reduce to $\frac{\mathrm{d}L}{\mathrm{d}\mathfrak{F}} \leqslant 0$, and $\frac{\mathrm{d}L}{\mathrm{d}\mathfrak{F}} + \frac{\mathrm{d}^2 L}{\mathrm{d}\mathfrak{F}^2} \leqslant 0$. These are satisfied up to an infinite field strength, provided that $\mathfrak{F} < 0$. This is a certain advantage as compared to the Born–Infeld model [1], that becomes inconsistent at too large electric field value. We shall be dealing with this case of electric field taken alone in a special Lorentz frame, $2\mathfrak{F}(x) = -E^2$, in the present paper.

[4] Greek indices span the four-dimensional Minkowski space–time taking the values 0, 1, 2, 3, while the Roman indices are 1, 2, 3. The metric tensor is $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, +1, +1)$, and bold symbols are reserved for three-dimensional Euclidean vectors. The Heaviside–Lorentz system of units is used throughout.

We shall see that the model (1) possesses an attractive property of providing finiteness to the field energy of a point charge, like the Born–Infeld model [1] and unlike the linear theory of electromagnetism. If we might think of it as of a bare theory for possibly subsequent quantizing we are to point a way of fixing the coupling constant γ. The quartic term $\mathfrak{F}^2$, added to the Lagrangian, would participate within the slow- and long-wave approximation in many processes, where (real or virtual) light-by-light scattering is involved, like, for instance: the photon (Delbrück) scattering and splitting off the electric field of a nucleus and off a laser field [4], the photon splitting off a magnetic field, formation of an anisotropic medium for the photon propagation in an external field, modification of the Coulomb field in a magnetic field [5], nonlinear renormalization of electric and magnetic moments *in vacio* [6] [5]. Out of those nonlinear phenomena only the ones associated with atoms have been up to now observed [7]. However, the experimental precision in the latter cases is not enough to fix the constant γ in (1). On the other hand, the same quartic term contributes to anomalous magnetic moments of electron and muon, measured and calculated in QED with high precision, and to atomic spectra. Recently a new process depending on the light-by-light scattering was proposed to be measured in heavy atom collision in LHC [8]. But even in these experimentally valuable cases a direct separation of the contribution of γ would be difficult and, after all, unnecessary. It will suffice to refer to QED as to a theory perfectly responsible for the whole body of existing data, relating to electromagnetism, except, perhaps, the discrepancy with the muon magnetic moment. Correspondingly, the scale of our coupling constant γ will be matched to QED, to be more precise, it will be taken over from the local Euler–Heisenberg effective action functional [9] that contains the light-by-light scattering amplitude of soft long-wave photons in its first term of the Taylor expansion in powers of the field invariant[6].

2.1. Field equations

The least action principle applied to the functional $S[A] = \int L(x)\mathrm{d}^4 x$ provides equations of motion $\frac{\delta S}{\delta A_\nu(x)} = 0$, which are the second pair of the Maxwell equations that are nonlinear:

$$\partial_\mu \left[(1 - \gamma \mathfrak{F}(x)) F^{\mu\nu} \right] = 0. \tag{2}$$

We are interested in purely electrostatic spherically symmetric solution produced by a point-like static charge e, placed in the origin. Then everywhere, except the point $\mathbf{x} = 0$ this equation is reduced to

$$\nabla \left[\left(1 + \frac{\gamma}{2} E^2 \right) \mathbf{E} \right] = 0. \tag{3}$$

Bearing in mind that at large $r = |\mathbf{x}|$ the standard Coulomb

[5] Certainly, another quartic addition, $\mathfrak{G}^2$, would also contribute to these processes to the same order, but it is not important for our present purposes.

[6] See [10], where the authors are basing their estimates of the free parameter in the Born–Infeld–Lagrangian on comparison with QED.

field of the point charge e

$$\mathbf{E}(\mathbf{x}) = \mathbf{E}^{\mathrm{lin}}(\mathbf{x}) = E^{\mathrm{lin}}(r)\frac{\mathbf{x}}{r} = \frac{e}{4\pi r^2}\frac{\mathbf{x}}{r} \qquad (4)$$

should be implied as the boundary condition, we rewrite (3), up to a curl, as

$$\left(1 + \frac{\gamma}{2}E^2(r)\right)\mathbf{E}(\mathbf{x}) = \mathbf{E}^{\mathrm{lin}}(\mathbf{x}). \qquad (5)$$

In understanding that the coordinate $\mathbf{x}$ is the only vector in the central-symmetric problem we may write $\mathbf{E}(\mathbf{x}) = E(r)\frac{\mathbf{x}}{r}$. Then the mentioned curl must be discarded, because it cannot be formed with $\mathbf{x}$ being the only vector, and the first Maxwell equation $\nabla \times \mathbf{E} = 0$ is trivially satisfied.

2.2. An exact solution to the quartic action

Now equation (5) becomes the cubic equation for $E(r)$ (see the procedure in [6])

$$\left(1 + \frac{\gamma}{2}E^2(r)\right)E(r) = \frac{e}{4\pi r^2}, \qquad (6)$$

whose only real solution is given by the Cardano formula

$$E(r) = \sqrt[3]{\sqrt{\left(\frac{E_{\mathrm{lin}}(r)}{\gamma}\right)^2 + \left(\frac{2}{3\gamma}\right)^3} + \frac{E_{\mathrm{lin}}(r)}{\gamma}} - \sqrt[3]{\sqrt{\left(\frac{E_{\mathrm{lin}}(r)}{\gamma}\right)^2 + \left(\frac{2}{3\gamma}\right)^3} - \frac{E_{\mathrm{lin}}(r)}{\gamma}}. \qquad (7)$$

For large distances, $r \to \infty$, solution (7) behaves as the Coulomb field $E(r) \sim E^{\mathrm{lin}}(r) = \frac{e}{4\pi r^2}$, which corresponds to neglect of the nonlinear quadratic term inside the bracket in (6), because it is much less than unity in this limit. The next-to-leading term of expansion of solution (7) at $r \to \infty$ is $-\frac{\gamma}{2}\left(E_{\mathrm{lin}}(r)\right)^3$. This result is readily obtainable by solving equation (6) perturbatively [9] [7] and would lead to the enhancing of the Coulomb singularity from r^{-2} to r^{-6}, if continued blindly to short distances, $r \to 0$, which is not allowed, of course. As a matter of fact the asymptote of solution (7) at $r \to 0$ is

$$E(r) \sim \left(\frac{2E_{\mathrm{lin}}(r)}{\gamma}\right)^{\frac{1}{3}} = \left(\frac{e}{2\pi\gamma}\right)^{\frac{1}{3}}\left(\frac{1}{r}\right)^{\frac{2}{3}}. \qquad (8)$$

This might also be immediately obtained from (6) if we neglected the unity in its left-hand side in favor of the quadratic term, much larger than unity in this limit. The behavior of the electrostatic field (8), produced by the point charge e via the nonlinear field equations (2), is essentially less singular in the vicinity of the charge than the standard Coulomb field $E^{\mathrm{lin}}(r) = \frac{e}{4\pi r^2}$. We shall see below that this suppression of the singularity is enough to provide convergence of the integrals giving the energy of the field configuration (7). Note that, contrary to the customary situation [1] in the Born–Infeld model, the singularity in our case is not totally removed, but only suppressed to a sufficient extent.

[7] In QED, with γ identified as equation (13) below, this nonlinear large-distance correction to the Coulomb field is due to Wichmann and Kroll [11].

Before proceeding with the field energy, we would like to make a remark concerning the effect the present results may have on the known phenomenon [12] of spontaneous electron–positron pair creation by an overcharged nucleus and also on the phenomenon of strong absorption of electrons by such nucleus occurring when the latter is treated as a (non-gravitational) black hole [13]. These phenomena are owing to the singularity of $-1/r^2$, to which the singularity of the Coulomb potential $-1/r$ gives rise after the set of the Dirac equations is reduced to one second-order differential equation. This singularity causes the so-called fall-down on the center, when the nucleus is point-like and its charge exceeds the value $Z = 137e$.

With our singularity of the potential weaker than $1/r$, the fall-down takes place at no charge value, hence the electron level does not sink into the Dirac see infinitely deep. However, it may reach its surface at a certain charge, larger than 137 to give rise to the above phenomena. Note that if we borrow from QED the value (13) for γ, our electric field (7) starts deviating from the Coulomb law at the distances

$$\left(\frac{Ze}{4\pi}\right)^{\frac{1}{2}}\left(\frac{3}{2}\right)^{\frac{3}{4}}\gamma^{\frac{1}{4}} = (Ze)^{\frac{1}{2}}\left(\frac{3}{40}\right)^{\frac{1}{4}}\left(\frac{1}{137\pi}\right)^{\frac{1}{2}}m^{-1}$$
$$= \frac{Z^{\frac{1}{2}}}{m}\frac{1}{137^{\frac{3}{4}}}\left(\frac{3}{10\pi}\right)^{\frac{1}{4}} = Z^{\frac{1}{2}}0.014\, m^{-1}$$

from the point charge Ze. If we just cut-off the potential at this distance, similarly to what Zel'dovich and Popov [14] did at the real size of the nucleus, we get the critical charge somewhat larger than their $Z = 172$. The genuine critical charge is expected to be smaller than this, because the potential in the cut-off region is not zero, but it is attractive. A more thorough study of solutions of the Dirac equation with the potential found by us is needed to determine or better estimate the value of the critical charge.

2.3. Finiteness of the field energy of a point charge

The Noether energy–momentum tensor for the Lagrange density (1) is

$$T^{\rho\nu} = (1 - \gamma\mathfrak{F}(x))F^{\mu\nu}\partial^\rho A_\mu - \eta^{\rho\nu}L(x). \qquad (9)$$

By subtracting the full derivative $\partial_\mu\left[\left(1 - \gamma\mathfrak{F}(x)F^{\mu\nu}\right)A^\rho\right]$, equal to $\left[\left(1 - \gamma\mathfrak{F}(x)F^{\mu\nu}\right)\partial_\mu A^\rho\right]$ due to the field equations (2), the gauge-invariant and symmetric under the transposition $\rho \leftrightarrows \nu$ energy–momentum tensor

$$\Theta^{\rho\nu} = (1 - \gamma\mathfrak{F}(x))F^{\mu\nu}F_\mu{}^\rho - \eta^{\rho\nu}L(x)$$

is obtained. When there is only spherically symmetric electric field, the energy density is

$$\Theta^{00} = \left(1 + \frac{\gamma E^2}{2}\right)E^2 - \frac{E^2}{2}\left(1 + \frac{\gamma E^2}{4}\right) = \frac{E^2}{2} + \frac{3\gamma E^4}{8}. \qquad (10)$$

By multiplying (6) by E we obtain the relation $\frac{\gamma}{2}E^4(r)=$

$E^{\text{lin}}(r)\;\; E(r) - E^2(r)$. Taking it into account the energy density becomes

$$\Theta^{00} = -\frac{E^2(r)}{4} + \frac{3}{4}E^{\text{lin}}(r)E(r).$$

Therefore, in order to determine the full electrostatic energy $\int \Theta^{00}\, \mathrm{d}^3x$ stored in solution (7) we have to calculate two integrals. The first one is

$$\int E^2(r)\mathrm{d}^3x = |e|^{\frac{3}{2}}\left(\frac{3}{2\gamma(4\pi)^2}\right)^{\frac{1}{4}}\frac{3}{2}I_1,$$

where

$$I_1 = \int_0^\infty y^{\frac{2}{3}}\left(\sqrt[3]{\sqrt{1+y^4}+1} - \sqrt[3]{\sqrt{1+y^4}-1}\right)^2\mathrm{d}y = 0.885.$$

The second one is

$$\int E^{\text{lin}}(r)E(r)\mathrm{d}^3x = e\int_0^\infty E(r)\mathrm{d}r = |e|^{\frac{3}{2}}\left(\frac{3}{2\gamma(4\pi)^2}\right)^{\frac{1}{4}}I_2,$$

where

$$I_2 = \int_0^\infty y^{-\frac{2}{3}}\left(\sqrt[3]{\sqrt{1+y^4}+1} - \sqrt[3]{\sqrt{1+y^4}-1}\right)\mathrm{d}y = 3.984.$$

Finally the energy is

$$\int \Theta^{00}\mathrm{d}^3x = |e|^{\frac{3}{2}}\left(\frac{3}{2\gamma(4\pi)^2}\right)^{\frac{1}{4}}\frac{1}{4}\left(3I_2 - \frac{3}{2}I_1\right) = 2.65\;|e|^{\frac{3}{2}}\left(\frac{3}{2\gamma(4\pi)^2}\right)^{\frac{1}{4}} < \infty. \quad (11)$$

3. Polynomial model

It is straightforward to extend the above statement about the finiteness of the field energy of a point charge to any nonlinear electrodynamics, with the effective Lagrange density $\mathfrak{L}(\mathfrak{F})$—in place of the quartic function $\frac{\gamma}{2}(\mathfrak{F}(x))^2$ used in (1)—being any function of $\mathfrak{F}$ that grows as a finite power of its argument, say $\mathfrak{F}^{n+1}$, $n \geqslant 1$, when $\mathfrak{F} \to \infty$. Neglecting again the unity in the equation to appear in place of (6) we obtain in place of (8) that the electric field's singularity near the origin is

$$E(r) \sim \left(\frac{e2^n n!}{4\pi\mathfrak{L}^{(n+1)}r^2}\right)^{\frac{1}{2n+1}}(-1)^{n+1}, \quad (12)$$

where $\mathfrak{L}^{(n+1)}$ is the $(n+1)$-st derivative of $\mathfrak{L}$ taken at $\mathfrak{F} = 0$. (Note that, given the sign of the charge, the leading electric field may have different signs depending on whether n is even or odd.) On the other hand, the leading-in-the origin contribution to the field energy density calculated as Noether's Θ^{00} will now, instead of E^4 in (10), be proportional to E^{2n+2}. In spite of this higher power, the integral for the field energy $\int \Theta^{00}\mathrm{d}^3x$ with the substitution of (12) converges at the lower limit as $\int_0 r^{-\frac{2}{2n+1}}\,\mathrm{d}\,r$, i.e., even faster than that of (10). As for convergence at large distances, it is ever provided by the standard Coulomb long-range behavior of any nonlinear solution with the long-range boundary condition $E(r) \sim E^{\text{lin}}(r)$, when all nonlinearity in the equation of motion should be disregarded.

The remark of the previous paragraph results in the claim that in QED, if one truncates (like in [6]) the Taylor series expansion of its nonlinearity at any given power of the field invariant $\mathfrak{F}$, the solution of the corresponding nonlinear Maxwell equations for electrostatic field of a point charge is a finite-energy field configuration. It is meant that the effective action of QED defined as the generating functional of the one-particle-irreducible vertex functions, or the Legendre transform of the generating functional of the photon Green functions [15], is taken in the local, or infrared, approximation [3, 6, 16]. It may be thought of as the Euler–Heisenberg action calculated with the accuracy of any number of loops.

4. Towards field-mass of electron in QED

To estimate the result (11) we may substitute the value of the coupling constant γ in (1) taken equal to the coefficient by the corresponding quartic term in the expansion of the one-loop Euler–Heisenberg–Lagrangian density $\mathfrak{L}^{\text{EH}}(\mathfrak{F}, \mathfrak{G})$, i.e. (note that $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ in the Heaviside–Lorentz system used here) [9]

$$\gamma = \left.\frac{\mathrm{d}^2\mathfrak{L}^{\text{EH}}(\mathfrak{F}, 0)}{\mathrm{d}\mathfrak{F}^2}\right|_{\mathfrak{F}=0} = \frac{e^4}{45\pi^2 m^4}, \quad (13)$$

where m is the electron mass, and e is the electron charge in understanding that the point charge previously denoted by the same letter now also is that of the electron. With this substitution the static field energy (11) of the electron considered as a purely electric point-like monopole gives the result

$$\int \Theta^{00}\mathrm{d}^3x = 7.15\alpha^{\frac{1}{4}}m = 2.09\;m, \quad (14)$$

about twice as large as the electron mass.

Equation (14) carries us back to the old idea, put forward by Abraham–Lorentz [17], and most advanced in Born–Infeld electrodynamics [1], of an electron being a particle-like finite-energy field configuration, in contemporary terms, the soliton, whose energy would be of a completely field nature. This idea got its extreme appearance in a rather successful attempt by Infeld [18], called a 'historical curiosity' in [10], to determine the fine-structure constant by equating the free dimensional parameter, inherent in the Born–Infeld model and fixed by the requirement, that the electron mass be of purely field origin, with its value matching QED. Nowadays there is little basis to believe that the electron mass may be due to its electromagnetic field alone, because electron is involved in other interactions, too. Nevertheless, it remains right that the combination of the quantum field theory, as long as it is able to produce such nonlinearity, which would make the field energy of electron finite, with the postulate of the field origin of its mass is apt of fixing the coupling constant. In this respect the present finding that the finite mass may be produced without going beyond the most reliable theory of electromagnetism, the QED, should be interesting.

Are there prospects for making the value (14) closer to m? Higher powers of nonlinearity converge faster and faster as the power grows, and produce some corrected values to replace (14). Although these terms depend on Feynman diagrams with six, eight and more even-number prongs, the corrected values of (14) will differ by more than powers of the fine structure constant: note that equations (11) with (13) is of the order of $\sqrt{|e|}$, so we do not face a perturbative series. On the other hand, the realistic electron, besides being an electric monopole, is also a point-like magnetic dipole, so the associated magnetic field energy should be expected to contribute to the total field mass [19]. A more challenging problem is to take the both mutually interacting fields together, to which end at least the term proportional to $\mathfrak{G}(x)^2$ is to be added to the Lagrangian (1).

5. Conclusion

We have considered classical models of nonlinear electrodynamics with polynomial self-interaction of the electromagnetic field, which may be viewed upon as created by the power expansion of the local effective action of QED truncated at any fixed power of the field invariant. Within the one-loop approximation in QED this action is that of Heisenberg–Euler. The point charge in every of these models is a soliton in the sense that its field makes a finite-energy configuration. This configuration—the nonlinearly modified Coulomb field—has been found analytically in the simplest case, where the nonlinearity reduces to a monomial of the fourth power of the field. The classical radius of this charge is zero and its field tends to infinity when approaching the charge slowly enough to keep the field energy integral converging to what may be referred to as the soliton rest mass. By equating the self-coupling coefficient of that simplest model to the corresponding coefficient in the field expansion of the Heisenberg–Euler action we find that this mass is of the order of magnitude of the mass of the electron.

We must stress that the essence of our work is not in finding nonlinear corrections to the Coulomb field of a point charge. This is the case only when sufficiently large distances from the charge are concerned, where the nonlinearity is small. On the contrary, our goal is to define an extension of the bulk of electromagnetic data from the larger distances, where these are well established experimentally and are perfectly covered by the Maxwell theory of electromagnetism, to small distances, different from the extension of the same bulk of data defined by the Maxwell equations. Naturally, our modified Coulomb law drastically differs from the usual one at short distances, and this difference is beyond the scope of perturbation.

The above results, certainly, do not finally solve the problem of divergence of the self-energy of the electron, because the treatment of high field strength inherent in it requires involvement of hight-field asymptotic behaviour of the effective action of QED, for which the power series expansion is only an asymptotic series [9], with the Schwinger effect of spontaneous pair creation by strong electric field consequently lost. There are other troubles concerning an applicability of the local approximation for the effective action (i.e. its independence of space- and time-derivatives of the field arguments) that may be destroyed in the vicinity of the charge. The problem of finiteness of the magnetic energy of the electron due to its magnetic moment also remains. We hope to continue studying these and other related matters in future.

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