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A parallelism for Conformal Sub-Riemannian Geometry

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1 Introduction

Conformal geometry is a classic topic in differential geometry. A modern introduction to it by means of second order frames is in [K]. The essencial feature is the existence of a Cartan connection on an appropriate bundle. Recently, attention has been given to sub-Riemannian geometry [S], and the natural question of the corresponding subconformal geometry is naturally posed.

Sub-Riemannian geometry deals with a metric which is defined only on a distribution of a given manifold. A class of conformally related sub-Riemannian structures defines a subconformal structure. We will restrict our attention to the case of contact distributions.

The treatment of the equivalence problem for subconformal geometry of codimension 1 presented in this work follows the approach given, in the case of CR-structures, by Chern in [CM], following the steps of Cartan's work [C] in dimension 3. In this work we consider the line bundle, which we call E, of all conformal metrics to a fixed sub-Riemannian one over a manifold, and over this line bundle an appropriate coframe bundle Y which solves the equivalence problem. The solution is understood to be a parallelism which gives a complete set of subconformal invariants. In dimension 2, there exists a bijection between conformal structures on a surface and complex structures. The same result holds in dimension 3 if we substitute the conformal structure by subconformal structure, and complex structure by CR-structure. We also examine the relation between those structures in higher dimensions and show that the parallelism obtained in [CM] is a special case of the parallelism obtained for conformal sub-Riemannian structures.

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2 CR-structures and Sub-Conformal Structures

Let D be a distribution on a manifold M. We will consider the following structures

Definition 2.1 *1) (M,D,J) is a CR-structure if $J : D \to D$ satisfies $J^2 = -I$.*
2) (M,D,g) is a Sub-Riemannian structure if g is a metric on D.
3) (M,D,$\bar{g}$) is a Conformal Sub-Riemannian structure if $\bar{g}$ is a conformal class of sub-riemannian metrics.

Let D be a distribution on a manifold M and $\pi : TM \to TM/D$ the quotient map.

Definition 2.2 *The Levi form $\alpha : D \times D \to TM/D$ is the antissymetric form defined as $\alpha(X,Y) = -\pi([X,Y])$.*

In the following, we suppose that D is of codimension 1. In this case, fixing a base v of TM/D defines the Levi form α_v as a real valued form. Let θ_v be the contact form of this distribution such that $\theta_v(\pi^{-1}v) = 1$, then the Levi form is given by

$$d\theta_v(X,Y) = \alpha_v(X,Y)$$

Proposition 2.1 *1) If (M,D,J) is a CR-structure, for each vector v of TM/D, there exists a basis, compatible with J, such that*

$$[\alpha_v] = \begin{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

2) If (M,D,g) is a subriemannian manifold, for each vector v of TM/D, there exists an orthonormal basis such that

$$[\alpha_v] = \begin{pmatrix} \begin{pmatrix} 0 & \lambda_1 \\ -\lambda_1 & 0 \end{pmatrix} & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & \begin{pmatrix} 0 & \lambda_n \\ -\lambda_n & 0 \end{pmatrix} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

3) If $(M,D,\bar{g})$ is a conformal sub-riemannian manifold, for each vector v of TM/D, there exists an orthogonal basis such that the matrix $[\alpha_v]$ is as in 2), with a normalizing condition, for instance $\lambda_1 = 1$.

We will consider non-degenerate Levi forms in this work. In this case we can choose the normalizing condition in 3) above to be $det[\alpha_v] = 1$. Also, in 2) above, there exists only one v in TM/D such that $det[\alpha_v] = 1$, in the non-degenerate case.

Definition 2.3 *1)A non-degenerate CR-structure is strongly-pseudoconvex if the 2×2 blocks in the normal form of the Levi form are equal.*

2)A non-degenerate sub-riemannian structure is strongly-pseudoconvex if the 2×2 blocks in the normal form of the Levi form are equal, that is $\lambda_1 = \cdots = \lambda_n = 1$ for the unique v above. We call a sub-conformal structure strongly-pseudoconvex if it is strongly-pseudoconvex for one of the sub-riemannian metrics of the sub-conformal class.

We will establish next the equivalence between CR-structures satisfying certain conditions and certain Sub-Conformal structures. Before stating the proposition, we need the following definitions.

Definition 2.4 *1) A CR-structure is quasi-integrable if the Levi-form satisfies the condition $\alpha(X,Y) = \alpha(JX,JY)$.*

2) A CR-structure is integrable if it is quasi-integrable and we have $J([JX,JY] + [X,Y]) = [JX,Y] + [X,JY]$.

Proposition 2.2 *The following structures are equivalent:*

1) strongly pseudoconvex quasi-integrable CR-structures

2) strongly pseudoconvex sub-conformal structures

Proof. Consider a strongly pseudoconvex quasi-integrable CR structure. We will define a conformal class of sub-riemannian metrics by $g_v(X,Y) = \alpha_v(JX,Y)$ for $X, Y \in D$. A different choice of v will define a conformally related metric. Conversely, given a metric in the conformal class of a sub-conformal structure, define the J-operator to be the matrix of the normal Levi form α_v.

□

If we had allowed sub-Lorentzian metrics to appear in this work, the proposition could be generalized to establish the equivalence between quasi-integrable CR-structures of type (p,q) and subconformal structures of type (2p,2q).

3 The Bundles E and Y

Let (M, D, $\bar{g}$) be a nondegenerate subconformal structure. We let E' to be the line-bundle of all subriemannian metrics in the conformal class $\bar{g}$. Given a subriemannian metric, there exists a canonical contact form θ such that

$$d\theta = h_{ij}\theta^i \wedge \theta^j + h_i\theta^i \wedge \theta$$

where θ^i is a dual basis of an orthonormal basis of D, and $det(h_{ij}) = 1$. We could consider the line bundle E of all contact forms associated to the subriemannian metrics in this sense. It is clear then, that there exists a fiber bundle isomorphism between those two bundles. To explicit this isomorphism, consider a trivialization of E', that is, a choice of a subriemannian metric g and the corresponding trivialization of E, that is, the contact form θ. Then the bundle map is defined as $\lambda g \rightarrow \lambda\theta$. We will identify E with E' in the following considerations.

Over the bundle E we will construct a bundle Y of forms. The construction is very similar to the construction of the corresponding bundle in the case of CR-structures [CM].

We begin by defining the tautological form ω. Given a point e in E, consider a coframe θ^i as above. On e we consider the pull-back θ^i and all forms defined by

$$\omega^i = \sqrt{\lambda}a^i_j\theta^j + v^i\omega \quad \text{where} \quad (a^i_j) \in O(2n)$$

finally we define the form ϕ, by imposing the equation

$$d\omega = \omega \wedge \phi + h_{ij}\omega^i \wedge \omega^j \tag{1}$$

Observe that each choice of ω^i fixes a matrix h_{ij} and ϕ is then any form in the familly

$$\phi = -\frac{d\lambda}{\lambda} + 2h_{ij}v^j\omega^i + s\omega$$

The bundle of all forms ω, ω^i, ϕ is denoted by Y. Unfortunately it is not a principal bundle, and we will obtain a parallelism which doesn't have all the niceties of connections over principal bundles.

We could reduce further the bundle Y, but we loose in the other hand in the unity of treatment allowed by the general case.

This can be done fixing an antissymetric matrix g_{ij}, and considering the familly of forms ω, ω^i, ϕ satisfying the equation

$$d\omega = \omega \wedge \phi + g_{ij}\omega^i \wedge \omega^j$$

It is important to note that (g_{ij}) is not arbitrary, and cannot in general be chosen constant (although it can be chosen to be constant on the fibers of the line bundle). The best choice is the normal form defined above with the conditon $det(g_{ij}) = 1$, which fixes the scalar. We call Y^h this bundle, which is well defined when the normal form has the same number of distinct nonvanishing elements varying smoothly with the same multiplicity. The group G of matrices of the form

$$\begin{pmatrix} 1 & 0 & 0 \\ v^i & u^i_j & 0 \\ s & -2g_{kl}v^k u^l_j & 1 \end{pmatrix}$$

where $g_{ij}u^i_k u^j_l = g_{kl}$, and $u^i_j \in O(2n)$ acts on Y^h. The above conditions implies $u^i_j \in U(d_1) \times \cdots \times U(d_k)$, where $d_1, \cdots, d_k$ are the multiplicities of the normal form. In case g_{ij} are constant, then Y^h is a G-structure on E.

In the case of CR-structures we also form the line bundle E of contact forms and denote also by ω the tautological form. Y will be the G-structure of all coframes satisfying the equation

$$d\omega = ig_{\alpha\bar{\beta}}\omega^\alpha \wedge \omega^{\bar{\beta}} + \omega \wedge \phi$$

Here $g_{\alpha\bar{\beta}}$ is a fixed hermitian matrix. The natural choice is $g_{\alpha\bar{\beta}} = \delta_{\alpha\beta}$. The group G_1 of this G-structure is the group of matrices of the form

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ v^\alpha & u^\alpha_\beta & 0 & 0 \\ v^{\bar{\alpha}} & 0 & u^{\bar{\alpha}}_{\bar{\beta}} & 0 \\ s & ig_{\rho\bar{\sigma}}u^\rho_\beta v^{\bar{\sigma}} & -ig_{\rho\bar{\sigma}}u^{\bar{\sigma}}_{\bar{\beta}}v^\rho & 1 \end{pmatrix}$$

where $g_{\alpha\bar{\beta}}u^\alpha_\rho u^{\bar{\beta}}_{\bar{\sigma}} = g_{\rho\bar{\sigma}}$.

4 A Parallelism

To start defining the parallelism, we define forms ω^i_j, ϕ^i such that the following equation is satisfied

$$d\omega^i = -\frac{1}{2}\phi \wedge \omega^i - \omega^i_j \wedge \omega^j - \phi^i \wedge \omega \tag{2}$$

It is our goal to impose conditions on those forms so that they are intrinsically and uniquelly defined.

Let $\omega^i_j, \bar{\omega}^i_j$ and $\phi^i, \bar{\phi}^i$ forms satisfying the equation 2. Then

$$(\omega^i_j - \bar{\omega}^i_j) \wedge \omega^j + (\phi^i - \bar{\phi}^i) \wedge \omega = 0$$

Using Cartan's lemma we get

$$\omega^i_j - \bar{\omega}^i_j = c^i_{jk}\omega^k + c^i_j\omega$$

$$\phi^i - \bar{\phi}^i = c^i_j\omega^j + c^i\omega$$

with $c^i_{jk} = c^i_{kj}$. It is clear now that we can impose first that $\omega^i_j = -\omega^j_i$ and solve for c^i_{jk}. Therefore

$$\omega^i_j - \bar{\omega}^i_j = c^i_j\omega \tag{3}$$

$$\phi^i - \bar{\phi}^i = c^i_j\omega^j + c^i\omega$$

with $c^i_j = -c^j_i$.

Differentiating equation 1 we obtain

$$(dh_{ij} - h_{kj}\omega^k_i + h_{ki}\omega^k_j)\omega^i \wedge \omega^j - (d\phi - 2h_{ij}\phi^i \wedge \omega^j) \wedge \omega = 0 \tag{4}$$

Lemma 4.1 *Let A be a 2-form and B_{ij} be 1-forms with $B_{ij} = -B_{ij}$ and $A\omega + B_{ij}\omega^i \wedge \omega^j = 0$ then $A = -b_{ij}\omega^i \wedge \omega^j + \psi \wedge \omega$ and $B_{ij} = b_{ijk}\omega^k + b_{ij}\omega$ where $b_{ij} = -b_{ji}$, $b_{ijk} + b_{kij} + b_{jki} = 0$ and $b_{ijk} = -b_{jik}$*

Applying the lemma to equation 4 we obtain

$$d\phi - 2h_{ij}\phi^i \wedge \omega^j + \psi \wedge \omega = b_{ij}\omega^i \wedge \omega^j \tag{5}$$

$$dh_{ij} - h_{kj}\omega^k_i + h_{ki}\omega^k_j = b_{ijk}\omega^k + b_{ij}\omega \tag{6}$$

If we use 3 in 6 above we obtain

$$(h_{jk}c^k_i + h_{ki}c^k_j)\omega = (b_{ijk} - \bar{b}_{ijk})\omega^k + (b_{ij} - \bar{b}_{ij})\omega$$

Then

$$b_{ijk} = \bar{b}_{ijk}$$

$$h_{jk}c^k_i - h_{ik}c^k_j = b_{ij} - \bar{b}_{ij} \tag{7}$$

To continue further, consider in $gl(2n, \mathbf{R})$ the scalar product

$$< A, B >= Tr(AB^T)$$

and the linear map $ad_H(A) = HA - AH = [H, A]$ where $H = (h_{ij})$. Let $g = o(2n) \cap ker(ad_H)$ and $g' = sim(2n) \cap ker(ad_H)$, where sim(2n) is the set of symmetric matrices of dimension 2n. Then $g^\perp = o(2n) \cap Im(ad_H)$, and $g'^\perp = sim(2n) \cap Im(ad_H)$. We state the property we need in the following lemma

Lemma 4.2 *$ad_H : g^{\perp}(g'^{\perp}) \to g^{\perp}(g'^{\perp})$ is an isomorphism*

Observe now that equation 7 can be writtewn in the form

$$-[H, C] = B - \bar{B}$$

where $C = (c^i_j), B = (b_{ij})$.

Let $\bar{B} = \bar{B}^1 + \bar{B}^2$ where $\bar{B}^1 \in g$ and $\bar{B}^2 \in g^{\perp}$

Using the lemma we can solve the equation $[H, C] = \bar{B}^2$.

We proved that there exists antissimetric forms ω^i_j such that

$$B \in g \tag{8}$$

We still have the following ambiguity

$$\omega^i_j - \bar{\omega^i_j} = c^i_j \omega$$

where $(c^i_j) \in g$, that is,

$$h_{ik}c^k_j - c^i_k h_{kj} = 0 \tag{9}$$

If we differentiate the equation 2 and using 5, 2 and 1, we obtain

$$(d\phi^i - \frac{1}{2}\phi \wedge \phi^i - \phi^j \wedge \omega^i_j + \frac{1}{2}\psi \wedge \omega^i) \wedge \omega +$$
$$(d\omega^i_j + \omega^i_l \wedge \omega^l_j + h_{lj}\omega^i \wedge \phi^l - \frac{1}{2}b_{lj}\omega^l \wedge \omega^i - h_{lj}\phi^i \wedge \omega^l) \wedge \omega^j = 0 \tag{10}$$

We define

$$\Phi^i_j = d\omega^i_j + \omega^i_l \wedge \omega^l_j + h_{lj}\omega^i \wedge \phi^l - h_{li}\omega^j \wedge \phi^l - \frac{1}{2}b_{lj}\omega^l \wedge \omega^i +$$
$$\frac{1}{2}b_{li}\omega^l \wedge \omega^j - h_{lj}\phi^i \wedge \omega^l + h_{li}\phi^j \wedge \omega^l + h_{ij}\omega^k \wedge \phi^k$$

Then we have that

$$\Phi^i_j + \Phi^j_i = 0$$

$$\Phi^i_j \wedge \omega^j \equiv 0 \qquad \mathrm{mod}\,\omega$$

From those two properties follows easily the following lemma

Lemma 4.3

$$\Phi^i_j \equiv S^i_{jkl}\omega^k \wedge \omega^l \qquad mod\ \omega$$

where $S^i_{jkl} = -S^j_{ikl} = -S^i_{jlk}$ and $S^i_{jkl} + S^i_{ljk} + S^i_{klj} = 0$.

Consider two sets of forms ω^i_j, ϕ^i and $\tilde{\omega}^i_j, \tilde{\phi}^i$, then

$$\Phi^i_j - \tilde{\Phi}^i_j \equiv (c^i_j h_{kl} - h_{rj}c^r_k\delta^i_l + h_{ri}c^r_k\delta^j_l - h_{lj}c^i_k + h_{li}c^j_k + h_{ij}c^k_l)\omega^k \wedge \omega^l \quad mod\ \omega$$

Then

$$S^i_{jkl} - \tilde{S}^i_{jkl} = c^i_j h_{kl} - \frac{1}{2}h_{rj}c^r_k\delta^i_l + \frac{1}{2}h_{rj}c^r_l\delta^i_k + \frac{1}{2}h_{ri}c^r_k\delta^j_l - \frac{1}{2}h_{ri}c^r_l\delta^j_k - \frac{1}{2}h_{lj}c^i_k + \frac{1}{2}h_{kj}c^i_l + \frac{1}{2}h_{li}c^j_k - \frac{1}{2}h_{ki}c^j_l + h_{ij}c^k_l$$

Let $S_{jl} = \sum S^i_{jil}$ and $S = \sum S_{ii}$.

Lemma 4.4 *There are unique forms ω^i_j satisfying 2, 3, 9 and such that $(S_{jl}) \in g'^{\perp}$*

Proof: Using 9 and the formula above, we get

$$S_{jl} - \tilde{S}_{jl} = -(n+2)c^j_i h_{il} - \frac{1}{2}c^r_i h_{ir}\delta^j_l$$

and

$$S - \tilde{S} = -(2n+2)c^r_i h_{ir}$$

then

$$S_{jl} - \tilde{S}_{jl} = -(n+2)c^j_i h_{il} + \frac{S - \tilde{S}}{4(n+1)}\delta^j_l$$

By lemma 1, we can write $\tilde{S}_{jl} = \tilde{S}^1_{jl} + \tilde{S}^2_{jl}$ where $\tilde{S}^1_{jl} \in g'$ and $\tilde{S}^2_{jl} \in g'^{\perp}$.

As $ad_H(I) = 0$, we have that $< (S_{jl})^2, I >= 0$, that is $S^2 = 0$, or $S = S^1 = 0$. We then write the equation above as the following two equations

$$-\tilde{S}^1_{jl} + \frac{\tilde{S}^1}{4(n+1)}\delta^j_l = -(n+2)c^j_i h_{il}$$

$$(S_{jl})^2 = (\tilde{S}_{jl})^2$$

and this determines uniquely $(c^i_j) \in g$. □

From equation 10 we get the following

$$\Phi^i_j = S^i_{jkl}\omega^k \wedge \omega^l + \lambda^i_j \wedge \omega \qquad (11)$$

where $\lambda^i_j = -\lambda^j_i$ and

$$d\phi^i = \frac{1}{2}\phi \wedge \phi^i + \phi^j \wedge \omega^i_j - \frac{1}{2}\psi \wedge \omega^i + \lambda^i_j \wedge \omega^j + \nu^i \wedge \omega \qquad (12)$$

where ν^i are 1-forms.

Differentiating equation 5, and using equations 1, 2, 6 and 12, we find

$$(d\psi + \psi \wedge \phi + 2h_{ij}\nu^i \wedge \omega^j - 2h_{ij}\phi^i \wedge \phi^j - 4b_{ij}\phi^i \wedge \omega^j) \wedge \omega +$$
$$(-db_{ij} + 2b_{lji}\phi^l - 2h_{lj}\lambda^l_i + b_{ij}\phi + b_{lj}\omega^l_i + b_{il}\omega^l_j) \wedge \omega^i \wedge \omega^j = 0$$

Using lemma 1, we obtain the following expressions

$$d\psi + \psi \wedge \phi - 2h_{ij}\phi^i \wedge \phi^j - 4b_{ij}\phi^i \wedge \omega^j + 2h_{ij}\nu^i \wedge \omega^j = \rho \wedge \omega - P_{ij}\omega^i \wedge \omega^j \tag{13}$$

$$-db_{ij} + b_{ijl}\phi^l - h_{lj}\lambda^l_i + h_{li}\lambda^l_j + b_{ij}\phi + b_{lj}\omega^l_i - b_{li}\omega^l_j = P_{ij}\omega + P_{ijk}\omega^k \tag{14}$$

where ρ is a 1-form, $P_{ij} = -P_{ji}$, $P_{ijk} = -P_{jik}$, $P_{ijk} + P_{kij} + P_{jki} = 0$.

In order to determine the forms ϕ^i, we need to have some information about the forms λ^i_j. To obtain this information we differentiate equation 11 and use 1, 2, 6, 12, 14. After a tedious computation, we colect the term which contains $\omega^i \wedge \omega^j$. By lemma 1, its coeficient is 0 mod ω, ω^i, as all other terms contain ω.

$$dS^i_{jkl} - S^i_{jkl}\phi - S^i_{jrl}\omega^r_k - S^i_{jkr}\omega^r_l - S^i_{rkl}\omega^r_j + S^r_{jkl}\omega^i_r - \lambda^i_j h_{kl} - h_{ij}\lambda^k_l + \tfrac{1}{4}(-h_{rj}\lambda^r_l\delta^i_k + h_{rj}\lambda^r_k\delta^i_l + h_{ri}\lambda^r_l\delta^j_k - h_{ri}\lambda^r_k\delta^j_l - h_{rl}\lambda^r_j\delta^i_k + h_{rl}\lambda^r_i\delta^j_k - h_{rk}\lambda^r_i\delta^j_l + h_{rk}\lambda^r_j\delta^i_l) + \tfrac{1}{2}(h_{lj}\lambda^i_k - h_{kj}\lambda^i_l - h_{li}\lambda^j_k + h_{ki}\lambda^j_l) + [\tfrac{1}{4}(b_{rjl} + b_{rlj})\delta^i_k - \tfrac{1}{4}(b_{ril} + b_{rli})\delta^j_k - \tfrac{1}{4}(b_{rjk} + b_{rkj})\delta^i_l + \tfrac{1}{4}(b_{rik} + b_{rki})\delta^j_l + \tfrac{1}{2}b_{lki}\delta^j_r + \tfrac{1}{2}b_{ijl}\delta^k_r + \tfrac{1}{2}b_{klj}\delta^i_r - \tfrac{1}{2}b_{ijk}\delta^l_r]\phi^r \equiv 0 \qquad mod\ \omega, \omega^i$$

Leting i = k, and summing over i we get,

$$dS_{jl} - S_{jl}\phi - S_{jr}\omega^r_l - S_{rl}\omega^r_j - \tfrac{(n+2)}{2}(-\lambda^j_r h_{rl} + h_{rj}\lambda^r_l) - \tfrac{1}{2}h_{ri}\lambda^r_i\delta^j_l + [\tfrac{n}{2}(b_{rjl} + b_{rlj}) + \tfrac{1}{2}(b_{rii}\delta^j_l + b_{lii}\delta^j_r + b_{jii}\delta^l_r)]\phi^r \equiv 0 \qquad mod\ \omega, \omega^i$$

Leting j = l and summing, we obtain

$$2(n+1)h_{rl}\lambda^r_l \equiv (2n+1)b_{rll}\phi^r \qquad mod\ \omega, \omega^i$$

Substituting again in the formula above

$$dS_{jl} - S_{jl}\phi - S_{jr}\omega^r_l - S_{rl}\omega^r_j - \tfrac{(n+2)}{2}(-\lambda^j_r h_{rl} + h_{rj}\lambda^r_l) + [\tfrac{n}{2}(b_{rjl} + b_{rlj}) + \tfrac{1}{2}(\tfrac{1}{2(n+1)}b_{rii}\delta^j_l + b_{lii}\delta^j_r + b_{jii}\delta^l_r)]\phi^r \equiv 0 \qquad mod\ \omega, \omega^i$$

We will apply the following lemma to obtain λ^i_j. It is important to observe first that from equation 14 we get that

$$h_{lr}\lambda^r_j - \lambda^l_r h_{rj} \equiv 0\ mod\ \omega, \omega^i, \phi^i \tag{15}$$

Lemma 4.5 *Let S, A be functions with values in $Im(ad_H)$ and $Ker(ad_H)$ respectively. Then*

$$dA + [\omega, A] \in Ker(ad_H) \qquad mod\ \omega, \omega^i$$

$$dS + [\omega, S] \in Im(ad_H) \qquad mod\ \omega, \omega^i$$

where $\omega = (\omega^i_j)$.

Proof: Differentiating $[A, H] = 0$, and using the formula 6 in the form $dH + [\omega, H] \equiv 0 \quad mod\ \omega, \omega^i$, we get $[dA + [\omega, A], H] \equiv 0 \quad mod\ \omega, \omega^i$. This proves the first part of the lemma. To prove the second part, we use the formula $< [X, Y], Z > = < Y, [X^T, Z] >$ to the differentiation of $< S, A > = 0$. □

Using lemma 4.5 and the above formula for dS_{jl} we get

$$\lambda^i_j = V^i_{jk}\omega^k + W^i_{jk}\phi^k$$

with $V^i_{jk} = -V^j_{ik}, W^i_{jk} = -W^j_{ik}$. The essencial point to define the parallelism is the fact that λ^i_j does not contain terms in ψ.

Lemma 4.6 *There exist uniquely defined* ϕ^i *such that* $\sum V^i_{ji} = 0$.

Proof: We write equation 12 as

$$\Phi^i = d\phi^i - \frac{1}{2}\phi \wedge \phi^i - \phi^j \wedge \omega^i_j + \frac{1}{2}\psi \wedge \omega^i = \frac{1}{2}(V^i_{jk} - V^i_{kj})\omega^k \wedge \omega^j + W^i_{jk}\phi^k \wedge \omega^j + \nu^i \wedge \omega \quad (16)$$

We have the following ambiguity on the choice of ϕ^i and ψ

$$\phi^i - \overline{\phi}^i = c^i\omega$$

$$\psi - \overline{\psi} = -2h_{ij}c^i\omega^j + G\omega$$

Taking the difference of equation 16 with two choices for the forms ϕ^i and ψ, and taking the coeficient of the term $\omega^k \wedge \omega^l$ gives us

$$c^i h_{kl} - \frac{1}{2}\delta^i_l h_{rk}c^r + \frac{1}{2}\delta^i_k h_{rl}c^r = \frac{1}{2}V^i_{lk} - \frac{1}{2}\overline{V}^i_{lk} - \frac{1}{2}V^i_{kl} + \frac{1}{2}\overline{V}^i_{kl}$$

Summing for i = k we have

$$(n + \frac{1}{2})h_{il}c^i = \frac{1}{2}V^i_{li} - \frac{1}{2}\overline{V}^i_{li}$$

It follows that the condition $V^i_{li} = 0$ determines c^i by the formula

$$(n + \frac{1}{2})h_{il}c^i = -\frac{1}{2}\overline{V}^i_{li}$$

□

To complete defining the parallelism it remains to determine uniquely ψ. Although this is not strictly necessary, we first obtain more information on ν^i for future use. We differentiate equation 16, and get the following equations

$$dW^i_{lr} - W^s_{sr}\omega^s_l + \omega^i_s W^s_{lr} - W^i_{ls}\omega^s_r - \frac{1}{2}W^i_{lr}\phi \equiv X^i_{lrk}\omega^k + Z^i_{lrs}\phi^s \quad mod\ \omega \tag{17}$$

where $X^i_{lrk} = -X^l_{irk}$ and $Z^i_{lrs} = -Z^l_{irs} = Z^i_{lsr}$, and collecting mod ω, ω^i, the coefficient of the term which contains $\omega^k \wedge \omega^l$ and making i = k and summing we obtain

$$-(2n+1)h_{jl}\nu^j - \frac{1}{2}W^i_{li}\psi + (3nb_{rl} - 2S_{rl} + W^i_{ls}W^s_{ir} - X^i_{lri})\phi^r \equiv 0 \quad mod\ \omega, \omega^i$$

We conclude that

$$\nu^i = P^i_j\omega^j + R^i_j\phi^j + U^i\psi$$

It follows that

$$\begin{aligned} d\phi^i &= \frac{1}{2}\phi \wedge \phi^i + \phi^j \wedge \omega^i_j - \frac{1}{2}\psi \wedge \omega^i + \frac{1}{2}(V^i_{jk} - V^i_{kj})\omega^k \wedge \omega^j + W^i_{jk}\phi^k \wedge \omega^j \\ &+ P^i_j\omega^j \wedge \omega + R^i_j\phi^j \wedge \omega + U^i\psi \wedge \omega \end{aligned} \tag{18}$$

and

$$d\psi + \psi\wedge\phi - 2h_{ij}\phi^i\wedge\phi^j + (2h_{ij}R^i_k + 4b_{jk})\phi^k\wedge\omega^j + 2h_{ij}U^j\psi\wedge\omega^j = \rho\wedge\omega + Q_{ij}\omega^i\wedge\omega^j \tag{19}$$

where $Q_{ij} = h_{ki}P^k_j - h_{kj}P^k_i - P_{ij}$.

Lemma 4.7 *There exists a unique form ψ defined by $\sum P^i_i = 0$*

Proof : We take the difference of equation 16 with two different forms ψ and $\overline{\psi}$. The ambiguity is $\psi - \overline{\psi} = G\omega$. The term in $\omega^j \wedge \omega$ is

$$-\frac{1}{2}G\delta^i_j = P^i_j - \overline{P}^i_j$$

It is clear then that the condition $P^i_i = 0$ determines ψ. □

5 Reduction to the Complex Structure on D

We will first establish some algebraic lemmas. The matrix $H = (h_{ij})$ will be considered in $u(n)$, that is, $h_{\alpha\beta} = h_{\alpha+n\,\beta+n}$ and $h_{\alpha+n\,\beta} = -h_{\alpha\,\beta+n}$, where from now on, greek letters range from 1 to n. Then $[H, J] = 0$, where $J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}$.

Lemma 5.1 *If $A \in o(2n)$ and $[A, H] = 0$ with $det(H) \neq 0$, then $A \in u(n)$.*

Lemma 5.2 $[u(n), u(n)^\perp] \subset u(n)^\perp$ *and* $[u(n)^\perp, u(n)^\perp] \subset u(n)$

Lemma 5.3 $u(n)^\perp = \{A \in o(2n) \ : \ AJ + JA = 0\}$, *that is,* $a^{\alpha+n}_{\beta+n} = -a^\alpha_\beta$, $a^{\alpha+n}_\beta = a^\alpha_{\beta+n}$.

Lemma 5.4 *If $A \in o(2n)$ and $A = A^u + A^\perp$ with $A^u \in u(n)$ and $A^\perp \in u(n)^\perp$, then*

$$a^{u\alpha}_{\ \ \beta} = \frac{a^\alpha_\beta + a^{\alpha+n}_{\beta+n}}{2} = a^{u\alpha+n}_{\ \ \beta+n}, \ a^{u\alpha+n}_{\ \ \beta} = \frac{a^{\alpha+n}_\beta - a^\alpha_{\beta+n}}{2} = -a^{u\alpha}_{\ \ \beta+n}$$

$$a^{\perp\alpha}_{\ \ \beta} = \frac{a^\alpha_\beta - a^{\alpha+n}_{\beta+n}}{2} = -a^{\perp\alpha+n}_{\ \ \beta+n}, \ a^{\perp\alpha+n}_{\ \ \beta} = \frac{a^{\alpha+n}_\beta + a^\alpha_{\beta+n}}{2} = a^{\perp\alpha}_{\ \ \beta+n}$$

Puting $U^\alpha_\beta = a^{u\alpha}_{\ \ \beta} + ia^{u\alpha+n}_{\ \ \beta}$, $V^\alpha_{\bar\beta} = a^{\perp\alpha}_{\ \ \beta} + ia^{\perp\alpha+n}_{\ \ \beta}$ *then* $\overline{U^\alpha_\beta} = -U^\beta_\alpha$ *and* $V^\alpha_{\bar\beta} = -V^\beta_{\bar\alpha}$

Lemma 5.5 *If $B \in sim(2n)$, with $B = B'^u + B'^\perp$, $B'^u \in u(n)'$, $B'^\perp \in u(n)'$, $B'^\perp \in u(n)'^\perp$, where $u(n)' = ker\ ad_H \cap sim(2n)$, then*

$$b'^{u\alpha}_{\ \ \beta} = \frac{b^\alpha_\beta + b^{\alpha+n}_{\beta+n}}{2} = b'^{u\alpha+n}_{\ \ \beta+n}, \ b'^{u\alpha+n}_{\ \ \beta} = \frac{b^{\alpha+n}_\beta - b^\alpha_{\beta+n}}{2} = -b'^{u\alpha}_{\ \ \beta+n}$$

Lemma 5.6 *If $(S^i_{jkl}) \in g$, for any fixed k,l, then $(S_{jl}) \in g'$.*

We will consider the reduction of the bundle Y to the bundle of coframes where $(h_{ij}) \in u(n)$. Decompose $(\omega^i_j) = (\omega^{1\,i}_{\ j}) + (\tau^{1\,i}_{\ j})$, where $\omega^1 = (\omega^{1\,i}_{\ j}) \in u(n)$ and $\tau^1 = (\tau^{1\,i}_{\ j}) \in u(n)^\perp$. From equation 6 we obtain

$$dH = [H, \tau^1] + B^u_k\omega^k + B^\perp_k\omega^k + B\omega + [H, \omega^1]$$

where $B_k = (b_{ijk}) = B^u_k + B^\perp_k$. Using lemmas 5.1 and 5.2, we have

$$dH = B^u_k\omega^k + B\omega + [H, \omega^1] \tag{20}$$

and

$$[H, \tau^1] = -B^\perp_k\omega^k \tag{21}$$

It follows from the equation above that we may write $\tau^{1\,i}_{\ j} = \tau^{1\,i}_{\ jk}\omega^k$.

Equation 2 is written as

$$d\omega^i = -\frac{1}{2}\phi \wedge \omega^i - \omega^{1\,i}_{\ j} \wedge \omega^j - \tau^{1\,i}_{\ j} \wedge \omega^j - \phi^i \wedge \omega \tag{22}$$

Introduce the complex functions

$$ig_{\alpha\overline{\beta}} = h_{\alpha\overline{\beta}} = h_{\alpha\beta} + ih_{\alpha\,\beta+n}$$

and complex forms

$$\zeta^{\alpha} = \omega^{\alpha} + i\omega^{\alpha+n}$$
$$\eta^{1\alpha}_{\ \beta} = \omega^{1\alpha}_{\ \beta} + i\omega^{1\alpha+n}_{\ \beta}$$
$$\varphi^{\alpha} = \phi^{\alpha} + i\phi^{\alpha+n}$$
$$\gamma^{1\alpha}_{\ \overline{\beta}} = \tau^{1\alpha}_{\ \beta} + i\tau^{1\alpha+n}_{\ \beta}$$

In complex form, equation 1 is written as

$$d\omega = \omega\wedge\phi + h_{\alpha\overline{\beta}}\zeta^{\alpha}\wedge\zeta^{\overline{\beta}} \tag{23}$$

and 22 as

$$d\zeta^{\alpha} = -\frac{1}{2}\phi\wedge\zeta^{\alpha} - \eta^{1\alpha}_{\ \beta}\wedge\zeta^{\beta} - \gamma^{1\alpha}_{\ \overline{\beta}}\wedge\zeta^{\overline{\beta}} - \varphi^{\alpha}\wedge\omega \tag{24}$$

where $\zeta^{\overline{\beta}} = \overline{\zeta^{\beta}}$.

We have also that

$$\gamma^{1\alpha}_{\ \overline{\beta}} = \gamma^{1\alpha}_{\ \overline{\beta}\mu}\zeta^{\mu} + \gamma^{1\alpha}_{\ \overline{\beta}\overline{\mu}}\zeta^{\overline{\mu}}$$

where

$$\gamma^{1\alpha}_{\ \overline{\beta}\mu} = \frac{1}{2}(\tau^{1\alpha}_{\ \beta\mu} + \tau^{1\alpha+n}_{\ \beta\mu+n}) + \frac{i}{2}(\tau^{1\alpha+n}_{\ \beta\mu} - \tau^{1\alpha}_{\ \beta\mu+n})$$
$$\gamma^{1\alpha}_{\ \overline{\beta}\overline{\mu}} = \frac{1}{2}(\tau^{1\alpha}_{\ \beta\mu} - \tau^{1\alpha+n}_{\ \beta\mu+n}) + \frac{i}{2}(\tau^{1\alpha+n}_{\ \beta\mu} + \tau^{1\alpha}_{\ \beta\mu+n})$$

Equation 5 in complex form becomes

$$d\phi - h_{\alpha\overline{\beta}}\varphi^{\alpha}\wedge\zeta^{\overline{\beta}} + h_{\beta\overline{\alpha}}\varphi^{\overline{\alpha}}\wedge\zeta^{\beta} + \psi\wedge\omega = b_{\alpha\overline{\beta}}\zeta^{\alpha}\wedge\zeta^{\overline{\beta}} \tag{25}$$

where $\varphi^{\overline{\alpha}} = \overline{\varphi^{\alpha}}$ and $b_{\alpha\overline{\beta}} = b_{\alpha\beta} + ib_{\alpha\beta+n}$. We get analogously

$$dh_{\alpha\overline{\beta}} = b_{\alpha\overline{\beta}}\omega + b_{\alpha\overline{\beta}\mu}\zeta^{\mu} + b_{\alpha\overline{\beta}\overline{\mu}}\zeta^{\overline{\mu}} + h_{\alpha\overline{\mu}}\eta^{1\overline{\mu}}_{\ \overline{\beta}} + \eta^{1\mu}_{\ \beta}h_{\mu\overline{\beta}} \tag{26}$$

where

$$b_{\alpha\overline{\beta}\mu} = \frac{1}{2}(b^{u}_{\alpha\beta\mu} + b^{u}_{\alpha\beta+n\mu+n}) + \frac{i}{2}(b^{u}_{\alpha\beta+n\mu} - b^{u}_{\alpha\beta\mu+n})$$
$$b_{\alpha\overline{\beta}\overline{\mu}} = \frac{1}{2}(b^{u}_{\alpha\beta\mu} - b^{u}_{\alpha\beta+n\mu+n}) + \frac{i}{2}(b^{u}_{\alpha\beta+n\mu} + b^{u}_{\alpha\beta\mu+n})$$

In the following we will write equation 11 in complex form. By lemma 5.2, writing $\Phi^{i}_{j} = \Phi^{ui}_{j} + \Phi^{\perp i}_{j}$, we have

$$\Phi^{ui}_{j} = d\omega^{1i}_{\ j} + \omega^{1i}_{l}\wedge\omega^{1l}_{j} + \tau^{1i}_{l}\wedge\tau^{1l}_{j} + T^{ui}_{jkl}\omega^{k}\wedge\phi^{l} - B^{ui}_{jkl}\omega^{k}\wedge\omega^{l}$$

and

$$\Phi^{\perp i}_{\ j} = d\tau^{1i}_j + \tau^{1i}_l \wedge \omega^{1l}_j + \omega^{1i}_l \wedge \tau^{1l}_j + T^{\perp i}_{jkl}\omega^k \wedge \phi^l - B^{\perp i}_{jkl}\omega^k \wedge \omega^l$$

where we introduced

$$T^i_{jkl} = h_{lj}\delta^i_k - h_{li}\delta^j_k + h_{kj}\delta^i_l - h_{ki}\delta^j_l + h_{ij}\delta^k_l$$

$$B^i_{jkl} = \frac{1}{4}(b_{lj}\delta^i_k - b_{li}\delta^j_k - b_{kj}\delta^i_l + b_{ki}\delta^j_l)$$

and where $T^{\perp i}_{jkl}, T^{ui}_{jkl}, B^{\perp i}_{jkl}, B^{ui}_{jkl}$ are obtained using lemma 5.4. Analogously, we obtain

$$\Phi^{ui}_{\ j} = V^{ui}_{jk}\omega^k \wedge \omega + W^{ui}_{jk}\phi^k \wedge \omega + S^{ui}_{jkl}\omega^k \wedge \omega^l \quad (27)$$

and

$$\Phi^{\perp i}_{\ j} = V^{\perp i}_{jk}\omega^k \wedge \omega + W^{\perp i}_{jk}\phi^k \wedge \omega + S^{\perp i}_{jkl}\omega^k \wedge \omega^l \quad (28)$$

which are decompositions of equation 11.

Writing $\tilde{\Phi}^{u\alpha}_{\beta} = \Phi^{u\alpha}_{\beta} + i\Phi^{u\alpha+n}_{\beta}$ and $\tilde{\Phi}^{\perp\alpha}_{\beta} = \Phi^{\perp\alpha}_{\beta} + i\Phi^{\perp\alpha+n}_{\beta}$, and using lemma 5.4 we have

$$\begin{aligned}\tilde{\Phi}^{u\alpha}_{\beta} &= d\eta^{1\alpha}_{\beta} + \eta^{1\alpha}_{\iota} \wedge \eta^{1\iota}_{\beta} + \gamma^{1\alpha}_{\bar\iota} \wedge \gamma^{1\bar\iota}_{\beta} + \frac{1}{2}(h_{\beta\bar\iota}\delta^{\alpha}_{\mu} + h_{\mu\bar\alpha}\delta^{\beta}_{\iota} + h_{\beta\bar\alpha}\delta^{\mu}_{\iota})\varphi^{\bar\iota} \wedge \zeta^{\mu}\\ &+\frac{1}{2}(h_{\iota\bar\alpha}\delta^{\beta}_{\mu} + h_{\beta\bar\mu}\delta^{\alpha}_{\iota} + h_{\beta\bar\alpha}\delta^{\mu}_{\iota})\varphi^{\iota} \wedge \zeta^{\bar\mu}\\ &+\frac{1}{8}(b_{\beta\bar\iota}\delta^{\alpha}_{\mu} - b_{\mu\bar\alpha}\delta^{\beta}_{\iota})\zeta^{\mu} \wedge \zeta^{\bar\iota} + \frac{1}{8}(b_{\iota\bar\alpha}\delta^{\beta}_{\mu} - b_{\beta\bar\mu}\delta^{\alpha}_{\iota})\zeta^{\bar\mu} \wedge \zeta^{\iota}\end{aligned} \quad (29)$$

and

$$\begin{aligned}\tilde{\Phi}^{\perp\alpha}_{\beta} &= d\gamma^{1\alpha}_{\bar\beta} + \eta^{1\alpha}_{\iota} \wedge \gamma^{1\iota}_{\bar\beta} + \gamma^{1\alpha}_{\bar\iota} \wedge \eta^{1\bar\iota}_{\bar\beta} + (h_{\iota\bar\beta}\delta^{\alpha}_{\mu} - h_{\iota\bar\alpha}\delta^{\beta}_{\mu} +\\ &h_{\mu\bar\beta}\delta^{\alpha}_{\iota} - h_{\mu\bar\alpha}\delta^{\beta}_{\iota})\zeta^{\mu} \wedge \varphi^{\iota} + \frac{1}{4}(b_{\iota\bar\alpha}\delta^{\beta}_{\mu} - b_{\iota\bar\beta}\delta^{\alpha}_{\mu} - b_{\mu\bar\alpha}\delta^{\beta}_{\iota} + b_{\mu\bar\beta}\delta^{\alpha}_{\iota})\zeta^{\mu} \wedge \zeta^{\iota}\end{aligned} \quad (30)$$

where $\eta^{1\bar\iota}_{\bar\beta} = \overline{\eta^{1\iota}_{\beta}}$ and $\gamma^{1\bar\iota}_{\beta} = \overline{\gamma^{1\iota}_{\bar\beta}}$

Using equation 27 we get

$$\begin{aligned}\tilde{\Phi}^{u\alpha}_{\beta} &= \tilde{S}^{u\alpha}_{\beta\mu\iota}\zeta^{\mu} \wedge \zeta^{\iota} + \tilde{S}^{u\alpha}_{\beta\mu\bar\iota}\zeta^{\mu} \wedge \zeta^{\bar\iota} + \tilde{S}^{u\alpha}_{\beta\bar\mu\iota}\zeta^{\bar\mu} \wedge \zeta^{\iota} + \tilde{S}^{u\alpha}_{\beta\bar\mu\bar\iota}\zeta^{\bar\mu} \wedge \zeta^{\bar\iota}\\ &+\tilde{V}^{u\alpha}_{\beta\mu}\zeta^{\mu} \wedge \omega + \tilde{V}^{u\alpha}_{\beta\bar\mu}\zeta^{\bar\mu} \wedge \omega + \tilde{W}^{u\alpha}_{\beta\mu}\varphi^{\mu} \wedge \omega + \tilde{W}^{u\alpha}_{\beta\bar\mu}\varphi^{\bar\mu} \wedge \omega\end{aligned} \quad (31)$$

where the coeficients are easilly computed using the coeficients of equation 27.

Analogously we have

$$\begin{aligned}\tilde{\Phi}^{\perp\alpha}_{\beta} &= \tilde{S}^{\perp\alpha}_{\beta\mu\iota}\zeta^{\mu} \wedge \zeta^{\iota} + \tilde{S}^{\perp\alpha}_{\beta\mu\bar\iota}\zeta^{\mu} \wedge \zeta^{\bar\iota} + \tilde{S}^{\perp\alpha}_{\beta\bar\mu\iota}\zeta^{\bar\mu} \wedge \zeta^{\iota} + \tilde{S}^{\perp\alpha}_{\beta\bar\mu\bar\iota}\zeta^{\bar\mu} \wedge \zeta^{\bar\iota} +\\ &\tilde{V}^{\perp\alpha}_{\beta\mu}\zeta^{\mu} \wedge \omega + \tilde{V}^{\perp\alpha}_{\beta\bar\mu}\zeta^{\bar\mu} \wedge \omega + \tilde{W}^{\perp\alpha}_{\beta\mu}\varphi^{\mu} \wedge \omega + \tilde{W}^{\perp\alpha}_{\beta\bar\mu}\varphi^{\bar\mu} \wedge \omega\end{aligned} \quad (32)$$

We have $\tilde{S}^{uα}_{\mu\beta\bar\iota} = -\tilde{S}^{uα}_{\mu\bar\iota\beta} = \overline{\tilde{S}^{u\mu}_{\alpha\iota\bar\beta}}$ and $\overline{\tilde{S}^{uα}_{\beta\bar\mu\bar\iota}} = -\tilde{S}^{u\beta}_{\alpha\mu\iota}$ and lemma 4.3 gives the following relations

$$\tilde{S}^{uα}_{\beta\mu\iota} + \tilde{S}^{uα}_{\mu\iota\beta} + \tilde{S}^{uα}_{\iota\beta\mu} = 0$$

$$\tilde{S}^{uα}_{\beta\mu\bar\iota} + \tilde{S}^{uα}_{\mu\bar\iota\beta} + \tilde{S}^{\perp α}_{\bar\iota\beta\mu} = 0$$

$$\tilde{S}^{uα}_{\beta\bar\mu\bar\iota} + \tilde{S}^{\perp α}_{\bar\mu\bar\iota\beta} + \tilde{S}^{\perp α}_{\bar\iota\beta\bar\mu} = 0$$

$$\tilde{S}^{\perp α}_{\overline{\beta\mu\iota}} + \tilde{S}^{\perp α}_{\overline{\mu\iota\beta}} + \tilde{S}^{\perp α}_{\overline{\iota\beta\mu}} = 0$$

The condition in lemma 4.4 is written as $(S'^{m}_{jl}) \in g'^{\perp} \cap u(n)'$ where (S'^{m}_{jl}) is the component of $(S_{jl}) \in sim(2n)$ in $u(n)'$. A simple computation shows that in complex form this condition becomes

$$(S^{uα}_{\beta\alpha\bar\mu} + \overline{\tilde{S}^{\perp α}_{\beta\alpha\mu}}) \in g'^{\perp} \cap u(n)' \tag{33}$$

Equation 16 is written

$$\begin{aligned} d\phi^i &= \frac{1}{2}\phi \wedge \phi^i + \phi^j \wedge \omega^{1i}_{\ j} + \phi^j \wedge \tau^{1i}_{\ j} - \frac{1}{2}\psi \wedge \omega^i \\ &\quad + (\frac{1}{2}(V^i_{jk} - V^i_{kj})\omega^k + W^i_{jk}\phi^k) \wedge \omega^j + \nu^i \wedge \omega \end{aligned} \tag{34}$$

In complex form, we obtain

$$\begin{aligned} \tilde{\Phi}^α &= d\varphi^α - \frac{1}{2}\phi \wedge \varphi^α - \zeta^\mu \wedge \eta^{1α}_{\ \mu} - \zeta^{\bar\mu} \wedge \gamma^{1α}_{\ \bar\mu} + \frac{1}{2}\psi \wedge \zeta^α = \frac{1}{2}(\tilde{V}^{uα}_{\mu\iota} - \tilde{V}^{uα}_{\iota\mu})\zeta^\mu \wedge \zeta^\iota \\ &+ \frac{1}{2}(\tilde{V}^{uα}_{\mu\bar\iota} - \tilde{V}^{\perp α}_{\bar\iota\mu})\zeta^\mu \wedge \zeta^{\bar\iota} + \frac{1}{2}(\tilde{V}^{\perp α}_{\bar\mu\iota} - \tilde{V}^{uα}_{\iota\bar\mu})\zeta^{\bar\mu} \wedge \zeta^\iota + \frac{1}{2}(\tilde{V}^{\perp α}_{\bar\mu\bar\iota} - \tilde{V}^{\perp α}_{\bar\iota\bar\mu})\zeta^{\bar\mu} \wedge \zeta^{\bar\iota} + \\ &\tilde{W}^{uα}_{\mu\iota}\varphi^\mu \wedge \zeta^\iota + \tilde{W}^{uα}_{\mu\bar\iota}\varphi^\mu \wedge \zeta^{\bar\iota} + \tilde{W}^{\perp α}_{\bar\mu\iota}\varphi^{\bar\mu} \wedge \zeta^\iota + \tilde{W}^{\perp α}_{\bar\mu\bar\iota}\varphi^{\bar\mu} \wedge \zeta^{\bar\iota} + \tilde\nu^α \wedge \omega \end{aligned} \tag{35}$$

where $\tilde\nu^α = \nu^α + i\nu^{α+n}$. Then

$$\tilde\nu^α = \tilde{P}^α_\mu \zeta^\mu + \tilde{P}^α_{\bar\mu}\zeta^{\bar\mu} + \tilde{R}^α_\mu \varphi^\mu + \tilde{R}^α_{\bar\mu}\varphi^{\bar\mu} + \tilde{U}^α \psi \tag{36}$$

where $\tilde{U}^α = U^α + iU^{α+n}$, $\tilde{P}^α_\mu = \frac{1}{2}(P^α_\mu + P^{α+n}_{\mu+n}) + \frac{i}{2}(P^{α+n}_\mu - P^α_{\mu+n})$ and $\tilde{P}^α_{\bar\mu} = \frac{1}{2}(P^α_\mu - P^{α+n}_{\mu+n}) + \frac{i}{2}(P^{α+n}_\mu + P^α_{\mu+n})$, and analogous formulas for $\tilde{R}^α_\mu$ and $\tilde{R}^α_{\bar\mu}$.

The condition on lemma 4.6 is

$$\tilde{V}^{uα}_{\beta α} = -\overline{\tilde{V}^{\perp α}_{\beta α}} \tag{37}$$

Analogously, equation 13 in complex form is

$$\begin{aligned} &d\psi + \psi \wedge \phi - 2h_{α\bar\beta}\varphi^α \wedge \varphi^{\bar\beta} - 2b_{α\bar\beta}\varphi^α \wedge \zeta^{\bar\beta} + 2b_{\beta\bar α}\varphi^{\bar\beta} \wedge \zeta^α + h_{α\bar\beta}\tilde\nu^α \wedge \zeta^{\bar\beta} - \\ &h_{\beta\bar α}\tilde\nu^{\bar α} \wedge \zeta^\beta = \rho \wedge \omega - \tilde{P}_{α\beta}\zeta^α \wedge \zeta^\beta - \tilde{P}_{\bar α\beta}\zeta^{\bar α} \wedge \zeta^\beta - \tilde{P}_{α\bar\beta}\zeta^α \wedge \zeta^{\bar\beta} - \tilde{P}_{\bar α\bar\beta}\zeta^{\bar α} \wedge \zeta^{\bar\beta} \end{aligned} \tag{38}$$

where $\tilde{\nu}^{\bar{\alpha}} = \overline{\tilde{\nu}^{\alpha}}$ and

$$\tilde{P}_{\alpha\beta} = \frac{1}{2}(P^{\perp}_{\alpha\beta} - iP^{\perp}_{\alpha+n\beta}) = \overline{\tilde{P}_{\bar{\alpha}\bar{\beta}}}$$

$$\tilde{P}_{\alpha\bar{\beta}} = \frac{1}{2}(P^{u}_{\alpha\beta} + iP^{u}_{\alpha\beta+n}) = \overline{\tilde{P}_{\bar{\alpha}\beta}}$$

The condition on lemma 4.7 is written as

$$Re(\tilde{P}^{\alpha}_{\alpha}) = 0 \tag{39}$$

In the following proposition of this section, we obtain the construction of the parallelism of Chern- Moser[CM] for CR-structures as a particular case of the conformal sub-Riemannian parallelism.

A CR-structure corresponds to having $H = J$ and $\tau^{1}{}^{i}_{j} = 0$, or $h_{\alpha\bar{\beta}} = i\delta_{\alpha\beta}$ and $\gamma^{1}{}^{\alpha}_{\bar{\beta}} = 0$.

Proposition 5.1 *The parallelism for CR-structures obtained in [CM] is a special case of the parallelism obtained above for a conformal sub-Riemannian structure*

Proof: Observe that in this case $g = u(n)$. Using equations 20 and 21 we get .

$$B^{u}_{k} = B^{\perp}_{k} = B = 0 \tag{40}$$

so $b_{\alpha\bar{\beta}} = b_{\alpha\bar{\beta}\mu} = b_{\alpha\bar{\beta}\bar{\mu}} = 0$.

The equation 23 corresponds to equation 4.10 of [CM]. The equation 24 with $\gamma^{1}{}^{\alpha}_{\bar{\beta}} = 0$ corresponds to 4.16 of [CM], where

$$\varphi^{\alpha}_{\beta} = \eta^{1}{}^{\alpha}_{\beta} + \frac{1}{2}\delta^{\alpha}_{\beta}\phi$$

The equation 4.21 of [CM] is precisely $\overline{\eta^{1}{}^{\alpha}_{\beta}} + \eta^{1}{}^{\beta}_{\alpha} = 0$ which follows from our definition. The equation 25 with $b_{\alpha\bar{\beta}} = 0$ corresponds to 4.26 of [CM]. It follows from 28 that

$$S^{\perp i}_{jkl} = V^{\perp i}_{jk} = 0 \tag{41}$$

so $S^{i}_{jkl} = S^{ui}_{jkl}$, $V^{i}_{jk} = V^{ui}_{jk}$. It follows then, that

$$\tilde{S}^{\alpha}_{\beta\mu\iota} = \tilde{S}^{\alpha}_{\mu\beta\iota} \text{ and } \tilde{S}^{\alpha}_{\beta\overline{\mu\iota}} = 0$$

and

$$\tilde{S}^{\alpha}_{\beta\mu\iota} = 0$$

It follows from 40 that the coeficient of ϕ^{r} in the equation $dS_{jl} - \cdots \equiv 0 \;\; mod\ \omega,\ \omega^{i}$ vanishes, so that

$$W^{i}_{jk} = 0 \tag{42}$$

implying $\tilde{W}^{\nu\alpha}_{\beta\mu} = \tilde{W}^{\nu\alpha}_{\beta\bar{\mu}} = 0$. Equation 29 reduces to

$$d\eta^{1\alpha}_{\beta} + \eta^{1\alpha}_{\iota} \wedge \eta^{1\iota}_{\beta} + i\varphi^{\bar{\beta}} \wedge \zeta^{\alpha} + i\varphi^{\alpha} \wedge \zeta^{\bar{\beta}} + \frac{i}{2}\delta^{\alpha}_{\beta}(\varphi^{\bar{\iota}} \wedge \zeta^{\iota} + \varphi^{\iota} \wedge \zeta^{\bar{\iota}}) =$$
$$2\tilde{S}^{\alpha}_{\beta\mu\bar{\iota}}\zeta^{\mu} \wedge \zeta^{\bar{\iota}} + \tilde{V}^{\alpha}_{\beta\mu}\zeta^{\mu} \wedge \omega + \tilde{V}^{\alpha}_{\beta\bar{\mu}}\zeta^{\bar{\mu}} \wedge \omega \tag{43}$$

Using 25 we obtain equation 4.53 of [CM]. From 41 we get $\tilde{S}^{\perp\alpha}_{\beta\alpha\mu} = 0$. The condition 33 is then written as

$$\tilde{S}^{\alpha}_{\beta\alpha\bar{\mu}} = 0 \tag{44}$$

which is condition 4.37 of [CM]]. Using 42 we have $X^{i}_{lrk} = 0$, and this implies

$$R^{i}_{j} = U^{i} = 0$$

Using 41 , 42 in 35 we obtain 4.54 of [CM] and from the equation above in 36 we obtain

$$\tilde{\nu}^{\alpha} = \tilde{P}^{\alpha}_{\mu}\zeta^{\mu} + \tilde{P}^{\alpha}_{\bar{\mu}}\zeta^{\bar{\mu}} \tag{45}$$

which is 4.61 of [CM]. From 37 and 41 follows that

$$\tilde{V}^{\alpha}_{\beta\alpha} = 0 \tag{46}$$

which is 4.58 of [CM]. Equation 38 corresponds to equations 4.59 and 4.64 of [CM] and equation 39 corresponds to 4.70 of [CM], and this completes the proof of the proposition.

6 Final Reduction

The matrix $(g_{\alpha\bar{\beta}})$ being antissymetric, it could be diagonalized. The eigenvalues are invariants of the subconformal structure and functions over the manifold. We suppose that we get precisely r eigenvalues ρ_i with constant multiplicities d_i. Let $\zeta^1, \cdots, \zeta^n$ a complex coframe satisfying

$$g_{\alpha\bar{\beta}} = \delta^{\alpha}_{\beta}\lambda_{\beta} \tag{47}$$

and

$$\lambda_{d_1+\cdots+d_{k-1}+1} = \cdots = \lambda_{d_1+\cdots+d_k} = \rho_k \tag{48}$$

for $1 \le k \le r$. Consider a second coframe $\zeta'^1, \cdots, \zeta'^n$ where 47 is valid. Writing $\zeta'^{\alpha} = a^{\alpha}_{\beta}\zeta^{\beta}$, it is easy to see that $(a^{\alpha}_{\beta}) \in U(d_1) \times \cdots \times U(d_r)$. This allows us to reduce further the structure. We denote $G = U(d_1) \times \cdots \times U(d_r)$, and g its Lie algebra which is $g = ker(ad_H) \cap o(2n)$. The scalar product in $o(2n)$, restricted to $u(n)$, in complex form is

$$< A, B >= 2ReTr(A\overline{B}^T)$$

For $A \in u(n)$, we write $A = (A_{jk})_{r\times r}$, where each A_{jk} is a matrix $d_j \times d_k$, with $\overline{A}^T_{jk} = -A_{kj}$. In particular $H_{jk} = 0$ if $j \neq k$ and $H_{jj} = i\rho_j I_{d_j}$.

Lemma 6.1 *$A \in g$ if and only if $A_{jk} = 0$ for $j \neq k$, and $A_{jj} \in u(d_j)$. $B \in g^\perp \cap u(n)$ if and only if $B_{jj} = 0$, where $1 \leq j, k \leq r$.*

Proof: The first assertion is clear. To prove the second one, we use the formula

$$< A, B >= -2ReTr(A_{jj}B_{jj})$$

□

Lemma 6.2 $[g, g^\perp] \subset g^\perp$

We decompose η^1 as

$$\eta^1 = \eta + \gamma^2$$

where η takes values in g and γ^2 in $g^\perp \cap u(n)$.

We will use indices $\alpha_j, \beta_j, \cdots$ with the following range $d_1 + \cdots + d_{j-1} + 1 \leq \alpha_j, \beta_j \leq d_1 + \cdots + d_j$. Then

$$\eta^{\alpha_j}_{\beta_j} = \eta^{1\alpha_j}_{\beta_j} \qquad \eta^{\alpha_j}_{\beta_k} = 0$$

$$\gamma^{2\alpha_j}_{\beta_j} = 0 \qquad \gamma^{2\alpha_j}_{\beta_k} = \eta^{1\alpha_j}_{\beta_k}$$

where $j \neq k$.

Equation 26 splits into the following equations

$$\gamma^{2\beta_j}_{\alpha_k} = \frac{i}{\rho_j - \rho_k}(b_{\alpha_k\overline{\beta}_j\mu}\zeta^\mu + b_{\alpha_k\overline{\beta}_j\overline{\mu}}\zeta^{\overline{\mu}}) \tag{49}$$

$$i\delta^{\alpha_j}_{\beta_j} d\rho_j = b_{\alpha_j\beta_j}\omega + b_{\alpha_j\beta_j\mu}\zeta^\mu + b_{\alpha_j\beta_j\overline{\mu}}\zeta^{\overline{\mu}} \tag{50}$$

Writing $d\rho_j = \rho_{j0}\omega + \rho_{j\mu}\zeta^\mu + \rho_{j\overline{\mu}}\zeta^{\overline{\mu}}$ then

$$b_{\alpha_j\overline{\beta}_j} = i\delta^{\alpha_j}_{\beta_j}\rho_{j0}$$

$$b_{\alpha_j\beta_j\mu} = i\delta^{\alpha_j}_{\beta_j}\nu_{j\mu}$$

$$b_{\alpha_j\beta_j\overline{\mu}} = i\delta^{\alpha_j}_{\beta_j}\nu_{j\overline{\mu}}$$

Using those expressions we can write equation 23

$$d\omega = \omega \wedge \phi + i\rho_j\zeta^{\alpha_j} \wedge \zeta^{\overline{\alpha}_j} \tag{51}$$

Equation 24 as

$$d\zeta^{\alpha_j} = -\frac{1}{2}\phi \wedge \zeta^{\alpha_j} - \eta^{\alpha_j}_{\beta_j} \wedge \zeta^{\beta_j} - \gamma^{2\alpha_j}_{\beta_k} \wedge \zeta^{\beta_k} - \gamma^{1\alpha_j}_{\overline{\beta}} \wedge \zeta^{\overline{\beta}} - \varphi^{\alpha_j} \wedge \omega \tag{52}$$

and equation 25 is written as

$$d\phi - i\rho_j\varphi^{\alpha_j}\wedge\zeta^{\overline{\alpha_j}} + i\rho_j\varphi^{\overline{\alpha_j}}\wedge\zeta^{\alpha_j} + \psi\wedge\omega = i\rho_{j0}\zeta^{\alpha_j}\wedge\zeta^{\overline{\alpha_j}} \tag{53}$$

Equation 29 splits into

$$\begin{aligned}\tilde{\Phi}^{u\alpha_j}_{\beta_j} &= d\eta^{\alpha_j}_{\beta_j} + \eta^{\alpha_j}_{\iota_j}\wedge\eta^{\iota_j}_{\beta_j} + \gamma^{2\alpha_j}_{\iota_k}\wedge\gamma^{2\iota_k}_{\beta_j} + \gamma^{1\alpha_j}_{\bar{\iota}}\wedge\gamma^{1\bar{\iota}}_{\beta_j}\\ &+i\rho_j(\varphi^{\overline{\beta_j}}\wedge\zeta^{\alpha_j} + \varphi^{\alpha_j}\wedge\zeta^{\overline{\beta_j}}) + \frac{i}{2}\delta^{\alpha_j}_{\beta_j}\rho_j(\varphi^{\bar{\mu}}\wedge\zeta^{\mu} + \varphi^{\mu}\wedge\zeta^{\bar{\mu}})\end{aligned} \tag{54}$$

and

$$\begin{aligned}\tilde{\Phi}^{u\alpha_j}_{\beta_k} &= d\gamma^{2\alpha_j}_{\beta_k} + \eta^{\alpha_j}_{\iota_j}\wedge\gamma^{2\iota_j}_{\beta_k} + \gamma^{2\alpha_j}_{\iota_k}\wedge\eta^{\iota_k}_{\beta_k} + \gamma^{2\alpha_j}_{\iota_l}\wedge\gamma^{2\iota_l}_{\beta_k} + \gamma^{1\alpha_j}_{\bar{\iota}}\wedge\gamma^{1\bar{\iota}}_{\beta_k}\\ &+\frac{i}{2}(\rho_k+\rho_j)(\varphi^{\overline{\beta_k}}\wedge\zeta^{\alpha_j} + \varphi^{\alpha_j}\wedge\zeta^{\overline{\beta_k}}) + \frac{i}{4}(\rho_{k0}-\rho_{j0})\zeta^{\alpha_j}\wedge\zeta^{\overline{\beta_k}}\end{aligned} \tag{55}$$

Equation 31 is valid with the corresponding indices of 54 and 55. Analogously equation 30 is written as

$$\begin{aligned}\tilde{\Phi}^{\perp\alpha_j}_{\overline{\beta}_k} &= d\gamma^{1\alpha_j}_{\overline{\beta}_k} + \eta^{\alpha_j}_{\iota_j}\wedge\gamma^{1\iota_j}_{\overline{\beta}_k} + \gamma^{1\alpha_j}_{\bar{\iota}_k}\wedge\eta^{\bar{\iota}_k}_{\overline{\beta}_k} + \gamma^{2\alpha_j}_{\iota_l}\wedge\gamma^{1\iota_l}_{\overline{\beta}_k} + \gamma^{1\alpha_j}_{\bar{\iota}_l}\wedge\gamma^{2\bar{\iota}_l}_{\overline{\beta}_k}\\ &i(\rho_k-\rho_j)(\zeta^{\alpha_j}\wedge\varphi^{\beta_k} + \zeta^{\beta_k}\wedge\varphi^{\alpha_j}) - \frac{i}{2}(\rho_{j0}+\rho_{k0})\zeta^{\alpha_j}\wedge\zeta^{\beta_k}\end{aligned} \tag{56}$$

In this equation k could be equal to j. Equation 32 is valid with corresponding indices. It follows from 33 that

$$\tilde{S}^{u\alpha}_{\beta_j\alpha\mu_j} + \tilde{S}^{\perp\alpha}_{\overline{\beta}_j\alpha\mu_j} = 0 \tag{57}$$

because this matrix is in $g'\cap g'^{\perp}$. The first member of equation 34 becomes

$$\tilde{\Phi}^{\alpha_j} = d\varphi^{\alpha_j} - \frac{1}{2}\phi\wedge\varphi^{\alpha_j} - \zeta^{\beta_j}\wedge\eta^{\alpha_j}_{\beta_j} - \zeta^{\beta_k}\wedge\gamma^{2\alpha_j}_{\beta_k} - \zeta^{\bar{\mu}}\wedge\gamma^{1\alpha_j}_{\bar{\mu}} + \frac{1}{2}\psi\wedge\zeta^{\alpha'} \tag{58}$$

Equation 37 continues to be valid and 38 becomes

$$\begin{aligned}&d\psi + \psi\wedge\phi - 2i\rho_j\varphi^{\alpha_j}\wedge\varphi^{\bar{\alpha}_j} - 2i\rho_{j0}(\varphi^{\alpha_j}\wedge\zeta^{\bar{\alpha}_j} - \varphi^{\bar{\alpha}_j}\wedge\zeta^{\alpha_j}) +\\ &i\rho_j(\tilde{\nu}^{\alpha_j}\wedge\zeta^{\bar{\alpha}_j} - \tilde{\nu}^{\bar{\alpha}_j}\wedge\zeta^{\alpha_j}) = \rho\wedge\omega - \tilde{P}_{\alpha\beta}\zeta^{\alpha}\wedge\zeta^{\beta} -\\ &\tilde{P}_{\bar{\alpha}\beta}\zeta^{\bar{\alpha}\wedge\zeta^{\beta}} - \tilde{P}_{\alpha\overline{\beta}}\zeta^{\alpha}\wedge\zeta^{\overline{\beta}} - \tilde{P}_{\bar{\alpha}\overline{\beta}}\zeta^{\bar{\alpha}}\wedge\zeta^{\overline{\beta}}\end{aligned} \tag{59}$$

Finally, equation 39 continues valid.

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