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by

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System's Performance Under Mixed Minimal and Imperfect Repair Maintenance Strategies

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Abstract

A technical item operates under a maintenance strategy that calls for preventive repair actions every T units of time and minimal repair actions whenever a failure occurs between the scheduled repairs. Preventive repair actions produce random age reductions of size not greater than T. Explicit mathematical expressions for typical reliability and maintainability characteristics are derived and important related optimization problems are presented.

Keywords: Maintenance strategies, minimal repair, imperfect repair, non-homogeneous Poisson processes.

1 Introduction

Since the pioneering paper of Barlow and Hunter (1960), the study of maintenance strategies for technical items has enjoyed a great deal of interest and space in the literature and models have been developed to accomodate different forms of modeling the item's degradation along time, as well as the frequency and effect of repair interventions. In particular, models for maintenance strategies involving both complete and minimal repair actions, are among the ones which received a great deal of attention in the late 1970's and 1980's. We refer to Barlow et al. (1965), Gertsbakh (1977), Block et al. (1990), (other references), as well as the references therein, for a partial account of these developments.

More recently, however, attention has shifted to maintenance strategies involving repair actions that can result in age reductions that can vary randomly between the extreme results of minimal and complete repair, that is, no reduction and full reduction (recall that while minimal repair actions merely put a failed item back in operation without affecting its current age and failure rate, complete repair actions completely recover the item's functional ability, restoring it to a "as good as new" condition). These repair actions, sometimes called "general" or "imperfect" repairs, have been treated by Gu (1993 and 1994). Similar actions appear also in Guo and Love (1994) and Dimitrov et al. (1999) who analyze the effect of proportional age reducing repairs in a reliability context.

In this report we consider the operation of a technical item under a maintenance strategy in which preventive maintenance check points (pmcp's) are scheduled after every T units of time. At each pmcp a repair action will instantly reduce the current age of the item by a random amount Z, $0 \leq Z \leq T$, that is, an imperfect repair action that cannot make the item any better that it was immediately after the previous preventive repair. Between consecutive pmcp's, failures of the item are instantly removed by minimal repair actions, that is, repair procedures that merely put the item back to work, without affecting its current age and failure rate. In this context we develop a stochastic model to describe the maintenance intervention times and establish explicit mathematical expressions for the system's reliability and maintainability characteristics of greatest interest. This will be done in section 2. The constructive approach used is easy to deal with and is sufficiently general to fit many interesting structure settings. In section 3 we give a brief description of some related optimization problems that are currently under investigation and will be the object of forthcomming articles.

2 Stochastic Model

A technical item of age $x_0 \geq 0$ starts operating at time 0 under the following conditions:

- preventive maintenance check points (pmcp's) are scheduled after every T units of time;
- at the n-th pmcp, $n \geq 1$, a repair action of cost $c_p(n)$ will be executed which will instantly and independently reduce the current age of the item by a random amount Z_n such that $P\{0 \leq Z_n \leq T\} = 1$;
- between pmcp's, a failure of the item at age u will be instantly removed by minimal repair action of cost $c_m(u)$.

We will refer to the maintenance procedure just described as a *mixture of minimal and imperfect preventive repairs* or, briefly, an *MMIPR*(x_0, T) strategy. The parameters x_0 and T indicate, respectively, the item's initial age and cycle length, that is, the time interval between consecutive pmcp's. With this provision, the cases in which a new item ($x_0 = 0$) or an old one ($x_0 > 0$), is put in operation at time 0, are treated simultaneously. The time interval ($(n-1)T, nT]$, $n \geq 1$, will be refered to as the n-th cycle.

We assume, throughout, that the item's original life distribution F is a continuous function such that $F(0) = 0$ and $F(x) < 1$ for all $x \geq 0$, and denote its corresponding survival distribution by $\bar{F} = 1 - F$.

We shall also assume that the independent non-negative random variables $Y_n = T - Z_n$, $n \geq 1$, have a common distribution function

$$G(y, T) = P\{Y_n \leq y\}, \quad 0 \leq y \leq T, \tag{1}$$

depending on the parameter T, such that

$$G(y, T) = G(\frac{y}{T}, 1), \quad 0 \leq y \leq T. \tag{2}$$

Recall that for an item with original life distribution F,

$$F(r|x) = 1 - \frac{\bar{F}(x+r)}{\bar{F}(x)}, \; r \geq 0. \tag{3}$$

describes both its residual life distribution at age x and its life distribution after being recovered by an age reducing repair action when failing at age y,

$y \geq x$, at any time. In the later case, $z = y - x$ is the age reduction achieved. Repair is called minimal if $z = 0$. Furthermore, if after start operating with age x at time t, all its failures are instantly removed by minimal repairs at times $t \leq T_1 \leq T_2 \leq \ldots$, the counting process $\{N(t) : n \geq 1\}$ corrresponding to the sequence $\{T_n - t : n \geq 1\}$ is a non-stationary Poisson process with mean function

$$M(s) = E[N(s)] = -log\bar{F}(s|x) = \Lambda(x+s) - \Lambda(x)\ , \quad s \geq 0, \tag{4}$$

where

$$\Lambda(x) = \int_0^x F(du)\bar{F}^{-1}(u) = -log\bar{F}(x)\ , \quad x \geq 0. \tag{5}$$

is the cumulative hazard function of F. If F is absolutely continuous with probability density function f, its failure rate function $\lambda = f\bar{F}^{-1}$ is also well defined and

$$\Lambda(x) = \int_0^x du\lambda(u)\ , \quad x \geq 0. \tag{6}$$

$M(s)$ gives us then the average number of minimal repair actions in the time interval $[t, t+s)$.

The relevant reliability and maintenance characteristics of a *mixture of minimal and imperfect preventive repairs* maintenance procedure are summarized in the following theorem.

Theorem 1 *Under an MMIPR*(x_0, T) *strategy:*

(i) The item's age at time t, $t \geq 0$, *is given by*

$$A_t = Y_0 + Y_1 + Y_2 + \ldots + Y_{n_t} + (t - n_t T)\ , \tag{7}$$

where $Y_0 = x_0$ *and* $n_t = [\frac{t}{T}]$ *denotes the integer part of* $\frac{t}{T}$. *Futhermore, the total age reduction up to time t,* $t \geq 0$, *provided by the maintenance strategy is given by*

$$D_t = Z_0 + Z_1 + Z_2 + \ldots + Z_{n_t}\ , \tag{8}$$

where $Z_0 = 0$.

(ii) If F is absolutely continuous,

$$r(t) = \lambda(A_t) = \lambda(x_0 + t - D_t) \qquad t \geq 0, \tag{9}$$

gives us the maintained item's conditional failure rate at time t, that is, the limit

$$\lim_{h \searrow 0} \frac{1}{h} P\{R_t \le h | A_t = a\} \,,$$

where R_t denotes the time to the first repair action after t.

(iii) If ν_n denotes the number of minimal repair actions in the $n-th$ cycle, then

$$\begin{aligned} E[\nu_n | A_{(n-1)T}] &= \int_0^T F(du | A_{(n-1)T}) \bar{F}^{-1}(u | A_{(n-1)T}) \\ &= \Lambda(A_{(n-1)T} + T) - \Lambda(A_{(n-1)T}). \end{aligned} \tag{10}$$

(iv) If $T_{n,1}$ denotes the first repair time in the $n-th$ cycle and $(n-1)T \le t < nT$, then

$$P\{T_{n,1} > t | A_{(n-1)T}\} =$$
$$= exp\{-[\Lambda(A_{(n-1)T} + t - (n-1)T) - \Lambda(A_{(n-1)T})]\}. \tag{11}$$

On the other hand, if $t \ge nT$, we have

$$P\{T_{n,1} > t | A_{(n-1)T}\} = 0. \tag{12}$$

(v) If $C_n(x_0, T)$ denotes the item's total maintenance cost in the $n-th$ cycle, then

$$E[C_n(x_0, T) | A_{(n-1)T}] = c_p(n) +$$
$$+ \int_0^T F(du | A_{(n-1)T}) \bar{F}^{-1}(u | A_{(n-1)T}) c_m(A_{(n-1)T} + u). \tag{13}$$

(vi) If $C(x_0, T, W)$ denotes the item's total maintenance cost over an arbitrary time interval $(0, W]$, then

$$E[C(x_0, T, W) | A_t : t \le W] = \sum_{i=0}^{n_W} C_i +$$
$$+ \int_0^{W - n_W T} F(du | A_{n_W T}) \bar{F}^{-1}(u | A_{n_W T}) c_m(A_{n_W T} + u) \,, \tag{14}$$

where $n_W = [\frac{W}{T}]$, $C_0 = 0$ and $C_i = E[C_i(x_0, T) | A_{(i-1)T}]$ if $i \ge 1$.

Proof: Since between pmcp's minimal repairs will keep item ageing as if no failures had occurred, and at the n-th such point an age reduction of Z_n time units is achieved, it is clear that

$$A_t = A_{(n-1)T} + t - (n-1)T, \quad \text{if} \quad (n-1)T \le t < nT, \tag{15}$$

and

$$\begin{aligned} A_{nT} &= A_{(n-1)T} + T - Z_n \\ &= A_{(n-1)T} + Y_n \\ &= Y_0 + \ldots + Y_n, \quad n \ge 1, \end{aligned} \tag{16}$$

provided that $A_0 = Y_0 = x_0$. Also, letting $Z_0 = 0$,

$$D_t = Z_0 + Z_1 \ldots + Z_{n-1}, \quad \text{if} \quad (n-1)T \le t < nT. \tag{17}$$

From (15),(16) and (17), (7) and (8) follow. Typical trajectories of the item's aging process $\{A_t : t \ge 0\}$, and of the age reduction process $\{D_t : t \ge 0\}$, are pictured in Figures 1 and 2.

To prove (ii), note that if $(n-1)T \le t < nT$ and $A_t = a$, the time, R_t, to the first repair action after t is such that

$$P\{R_t > h | A_t = a\} = 1_{[0,nT-t)}(h)\bar{F}(h|a), \qquad h \ge 0.$$

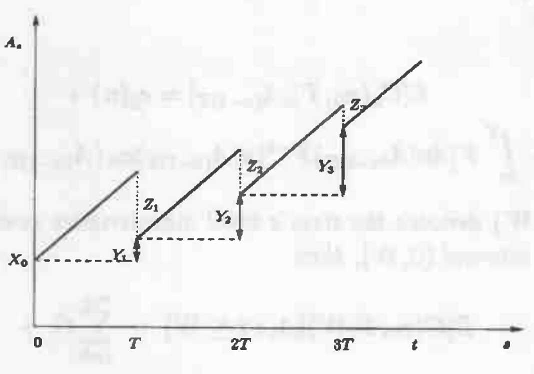

Fig. 1. Aging process of an item under a mixed minimal and imperfect preventive repairs strategy.

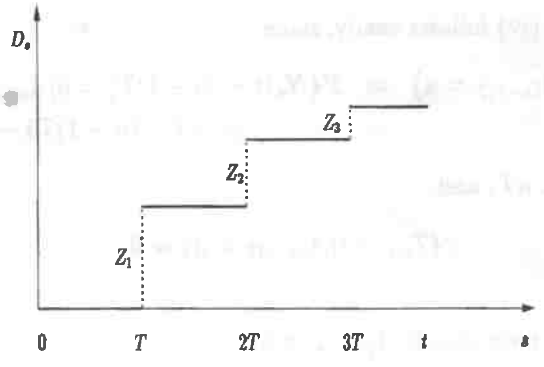

Fig. 2. Age reduction process of an item under a mixed minimal and imperfect preventive repairs strategy.

Consequently, as $h \searrow 0$,

$$\frac{1}{h}P\{R_t \le h | A_t = a\} = \frac{1}{h}\frac{F(a+h) - F(a)}{\bar{F}(a)} \to \lambda(a).$$

Equation (9) follows since $A_t + D_t = x_0 + t$ for all $t \ge 0$.

To prove (iii), recall that if $A_{(n-1)T} = a$ and

$$(n-1)T < T_{n,1} < T_{n,2} < \dots$$

denote the minimal repair action times in the n-th cycle,

$$N_n(s) = \sum_{i=1}^{\infty} 1_{(0,s]}(T_{n,i} - (n-1)T), \;\; 0 < s \le T,$$

behaves like a non-stationary Poisson process with mean function

$$M_n(s) = E[N_n(s)] = \int_0^s F(du|a)\bar{F}^{-1}(u|a) = \Lambda(a+s) - \Lambda(a).$$

Consequently, the conditional average number of minimal repair actions in the time interval $((n-1)T, nT]$ is given by $M_n(T)$, as stated in (10).

The result in (iv) follows easily, since

$$\begin{aligned} P\{T_{n,1} > t | A_{(n-1)T} = a\} &= P\{N_n(t-(n-1)T) = 0 | A_{(n-1)T} = a\} \\ &= exp\{-[\Lambda(a+t-(n-1)T) - \Lambda(a)]\} \end{aligned}$$

if $(n-1)T \le t < nT$, and

$$P\{T_{n,1} > t | A_{(n-1)T} = a\} = 0$$

if $t \ge nT$.

To prove (v), note that if $A_{(n-1)T} = a$,

$$\begin{aligned} C_n(x_0, T) &= c_p(n) + \sum_{i=1}^{\infty} 1_{(0,T]}(T_{n,i} - (n-1)T) \times c_m(a + T_{n,i} - (n-1)T) \\ &= c_p(n) + \int_0^T N_n(ds) c_m(a+s) . \end{aligned}$$

Equation (13) now follows by taking conditional expectation.

To prove (vi), note that if $(n-1)T < W \le nT$ and $A_{iT} = a_i$, for $i = 0, 1, \ldots, n-1$, with $a_0 = x_0$,

$$C(x_0, T, W) = \sum_{i=0}^{n-1} \xi_i + \int_0^{W-(n-1)T} N_n(ds) c_m(a_{n-1} + s) ,$$

with $\xi_0 = 0$ and

$$\xi_i = C_i(x_0, T) = c_p(i) + \int_0^T N_i(ds) c_m(a_{i-1} + s) ,$$

for $i = 1, \ldots, n-1$. Equation (14) now follows by taking conditional expectation since

$$E[C(x_0, T, W) | A_t : t \le W] = E[C(x_0, T, W) | A_0, A_T, \ldots, A_{(n-1)T}]$$

and

$$E[C_i(x_0, T) | A_0, A_T, \ldots, A_{(n-1)T}] = E[C_i(x_0, T) | A_{(i-1)T}] ,$$

for $i = 1, \ldots, n-1$.

□

Theorem 1 gives us constructive relationships between *model components* that specify important conditional sistem's reliability and maintainability characteristics. Unconditional averages of these characteristics can now be easily obtained by making use of the fact that $E[X] = E[E[X|Y]]$. The results are collected in the corollary below. As usual, the n–fold convolution of the cumulative distribution function G will be denoted by $G^{(n)}$.

Corollary 1 *Under an $MMIPR(x_0, T)$ strategy:*

(i) The item's expected age at time t and the expected total age reduction up to time t, $t \geq 0$, are given by

$$E[A_t] = x_0 + t - n_t T \int_0^1 daG(a,1) \tag{18}$$

and

$$E[D_t] = n_t T \int_0^1 daG(a,1) \; , \tag{19}$$

respectively, where $n_t = [\frac{t}{T}]$.

(ii) If F is absolutely continuous, the maintained item's unconditional failure rate, that is, the limit

$$\lim_{h \searrow 0} \frac{1}{h} P\{R_t \leq h\} \; ,$$

is given by

$$\rho(t) = \int_0^\infty G^{(n_t)}(da - x_0 - (t - n_t T), T)\lambda(a) \; , \tag{20}$$

where $G^{(0)} = 1_{[0,\infty)}$. This reduces to

$$\rho(t) = \lambda(x_0 + t) \tag{21}$$

if $0 \leq t < T$, and to

$$\rho(t) = \int_0^1 G(dx_1, 1) \ldots \int_0^1 G(dx_{n-1}, 1)\lambda(x_0 + t - T\sum_{i=1}^{n-1}(1 - x_i)) \tag{22}$$

if $(n-1)T \leq t < nT$, with $n > 1$.

(iii) The expected number of minimal repairs in the n-th cycle is given by

$$E[\nu_n] = \int_0^\infty G^{(n-1)}(da - x_0, T)[\Lambda(a+T) - \Lambda(a)] \; . \tag{23}$$

This reduces to

$$E[\nu_n] = \Lambda(x_0 + T) - \Lambda(x_0) \tag{24}$$

if $n = 1$, and to

$$E[\nu_n] = \int_0^1 G(dx_1, 1) \ldots \int_0^1 G(dx_{n-1}, 1) \times$$

$$\times [\Lambda(x_0 + T(1 + \sum_{i=1}^{n-1} x_i)) - \Lambda(x_0 + T \sum_{i=1}^{n-1} x_i)] \tag{25}$$

otherwise.

(iv) The expected time to first repair in the n-th cycle is given by

$$\mu_n = \int_0^T du \int_0^\infty G^{(n-1)}(da - x_0, T) exp\{-[\Lambda(a+u) - \Lambda(a)]\} \; . \tag{26}$$

This reduces to

$$\mu_1 = T \int_0^1 ds \; exp\{-[\Lambda(x_0 + sT) - \Lambda(x_0)]\} \tag{27}$$

if $n = 1$, and to

$$\mu_n = T \int_0^1 ds \int_0^1 G(dx_1, 1) \ldots \int_0^1 G(dx_{n-1}, 1) \times$$

$$\times exp\{-[\Lambda(x_0 + T(s + \sum_{i=1}^{n-1} x_i)) - \Lambda(x_0 + T \sum_{i=1}^{n-1} x_i)]\} \tag{28}$$

otherwise.

(v) If F is absolutely continuous, the item's expected total maintenance cost in the n-th cycle is given by

$$E[C_n(x_0, T)] = c_p(n) + \int_0^T du \int_0^\infty G^{(n-1)}(da - x_0, T) c_m(a+u) \lambda(a+u). \tag{29}$$

This reduces to

$$E[C_1(x_0,T)] = c_p(1) + T\int_0^1 ds\; c_m(x_0+sT)\lambda(x_0+sT) \tag{30}$$

if $n=1$, *and to*

$$E[C_n(x_0,T)] = c_p(n) + T\int_0^1 ds \int_0^1 G(dx_1,1)\ldots\int_0^1 G(dx_{n-1},1)\times$$

$$\times c_m(x_0+T(s+\sum_{i=1}^{n-1}x_i))\lambda(x_0+T(s+\sum_{i=1}^{n-1}x_i)) \tag{31}$$

otherwise.

(vi) If F is absolutely continuous, the item's expected total maintenance cost over an arbitrary time interval $(0,W]$ *is given by*

$$E[C(x_0,T,W)] = \sum_{i=0}^{n_W}\bar{C}_i +$$

$$+\int_0^{W-n_WT} du \int_0^\infty G^{(n_W)}(da-x_0,T)c_m(a+u)\lambda(a+u). \tag{32}$$

where $n_W=[\frac{W}{T}]$ *and* $\bar{C}_i = E[C_i]$, $i\geq 0$, *with* C_i *as defined in (v). Also as in (v), the second term in (32) reduces to*

$$\int_0^W du\; c_m(x_0+u)\lambda(x_0+u). \tag{33}$$

if $n_W=0$, *and to*

$$\int_0^{W-n_WT} du\int_0^1 G(dx_1,1)\ldots\int_0^1 G(dx_{n-1},1)\times$$

$$\times c_m(x_0+u+T\sum_{i=1}^{n_W}x_i)\lambda(x_0+u+T\sum_{i=1}^{n_W}x_i) \tag{34}$$

otherwise.

Proof: From (2) and (7),

$$\begin{aligned}
E[A_t] &= \int_0^\infty da\ P\{A_t > a\} \\
&= \int_0^\infty da\ \bar{G}^{(n_t)}(a-(t-n_tT)-x_0,T) \\
&= x_0+(t-n_tT)+n_t\int_0^T da\ \bar{G}(a,T) \\
&= x_0+(t-n_tT)+n_tT\int_0^1 da\ \bar{G}(a,1) \\
&= x_0+t-n_tT\int_0^T da\ G(a,1)\ ,
\end{aligned}$$

which proves (18). Equation (19) now follows from (18) and the fact that $A_t+D_t=x_0+t$ for all $t\geq 0$. This proves (i).

To prove (ii), observe that equations (20), (21) and (22) follow immediatelly from (2), (9) and (7) since

$$\begin{aligned}
P\{R_t\leq h\} &= \int_0^\infty P\{A_t\in da\}P\{R_t\leq h|A_t=a\} \\
&= \int_0^\infty G^{(n_t)}(da-x_0-(t-n_tT),T)P\{R_t\leq h|A_t=a\}\ ,
\end{aligned}$$

and $G^{(0)}=1_{[0,\infty)}$.

Similar arguments prove the other statements in the corollary. Equations (23), (24) and (25) follow immediatelly from (2), (10) and (7) since

$$\begin{aligned}
E[\nu_n] &= E[E[\nu_n|A_{(n-1)T}]] \\
&= \int_0^\infty P\{A_{(n-1)T}\in da\}[\Lambda(a+T)-\Lambda(a)] \\
&= \int_0^\infty G^{(n-1)}(da-x_0,T)[\Lambda(a+T)-\Lambda(a)]\ ,
\end{aligned}$$

which proves (iii).

Equations (26), (27) and (28) follow immediatelly from (2), (11), (12) and (7) since

$$\begin{aligned}
\mu_n &= \int_0^\infty duP\{T_{n,1}-(n-1)T>u\} \\
&= \int_0^\infty du\ E[P\{T_{n,1}>u+(n-1)T|A_{(n-1)T}\}] \\
&= \int_0^T du\int_0^\infty G^{(n-1)}(da-x_0,T)P\{T_{n,1}>u+(n-1)T|A_{(n-1)T}=a\}\ ,
\end{aligned}$$

which proves (iv).

Equations (29), (30) and (31) follow immediatelly from (2), (13), (7) and the basic fact that

$$\int_0^x F(du|a)\bar{F}^{-1}(u|a) = \int_0^x du\lambda(a+u),$$

if F is absolutely continuous, since

$$\begin{aligned} E[C_n(x_0,T)] &= \int_0^\infty G^{(n-1)}(da-x_0,T)E[C_n(x_0,T)|A_{(n-1)T}=a] \\ &= c_p(n) + \int_0^\infty G^{(n-1)}(da-x_0,T)\int_0^T du\; c_m(a+u)\lambda(a+u)\,, \end{aligned}$$

which proves (v).

Finally, the same argument used to prove (v) can be used to establish (32), (33) and (34) from (2), (14), (7) and the basic fact stated above, proving (vi). □

3 Related Optimization Problems

Several optimization problems of interest, related to the operation of an item under an $MMIPR(x_0,T)$, can be studied under the model and the results developed in the previous section. Some of them are currently under investigation and their results will be the object of future articles. We shall, however, briefly describe them for completion.

A. Optimization of the expected total maintenance cost in finite horizon. Under an $MMIPR(x_0,T)$ strategy, the question of which cycle length minimizes the item's expected total maintenance cost over an arbitrary but fixed finite time interval, is of great interest. Mathematically, the problem is that of finding the values T^* of T for which

$$E[C(x_0,T^*,W)] = \inf_{T>0} E[C(x_0,T,W)]\,,$$

where $E[C(x_0,T,W)]$ is given by (32) In certain settings, constraints on the total maintenance costs for each cycle are also imposed. In this case one is interested in solutions such that

$$E[C_n(x_0,T^*)] \leq C_{n0}, \quad n = 1,\, 2, \ldots\,,$$

where $\{C_{n0} : n \geq 1\}$ are given cycle cost limits.

□

B. Maximization of the mission time W. Under an $MMIPR(x_0,T)$ strategy, the equation

$$E[C(x_0,T,W)] = L$$

specifies the mission time W as an implicit function of the item's initial age, x_0, the cycle length, T, and the expected total maintenance cost, L. The problem of interest consists of finding the values T^* of T for which

$$W^* = W(x_0,T^*,L) = \sup_{T>0} W(x_0,T,L) .$$

□

C. The warranty cost optimization problem. In certain settings, the life of a product under warranty can also be viewed as a technical item under an $MMIPR(x_0,T)$ strategy, e.g. Blischke and Murthy (1996). In this context, the mission time W is interpreted as the product's warranty period, x_0 and T are given product characteristics, and the question of interest is which warranty period W will provide the best competitive position to the product in the marketplace. A precise mathematical formulation to this question can be given if we agree upon the use of the expected total warranty cost as a measure competitive position. With this provision the problem reduces to that of minimizing the expected total warranty cost. On behalf of the consumer, a constraint is usually imposed by requiring that the expected total age reduction at the end of the warranty period be no less than a given quantity. In summary, the problem of interest consists of finding the values W^* of W for which

$$E[C(x_0,T,W^*)] = \inf_{W>0} E[C(x_0,T,W)] ,$$

subject to the condition that

$$E[D_{W^*}] \geq D .$$

□

D. Optimization of the expected total maintenance cost in random horizon. Under an $MMIPR(x_0,T)$ strategy, Gu (1993, 1994) considered the problem of finding the cycle length that minimizes the expected total maintenance cost up to preventive maintenance time such that the average number of failures in the next cycle becomes large. Letting

$$K_n(x_0,T) = \sum_{j=1}^{n} C_j(x_0,T) , \quad n \geq 1 ,$$

the problem is that of finding the values T^* of T for which

$$K_N(x_0,T^*) = \inf_{T>0} K_N(x_0,T) ,$$

where

$$N = \inf\{n \geq 0 : E[\nu_{n+1}] \geq C\} .$$

A variety of interesting constrained optimization problems such this can be formulated by specifying different criteria. For instance, a different and appealing one would be stopping at a preventive maintenance time if the expected total maintenance cost in the next cycle becomes large, that is, defining N as

$$N = \inf\{n \geq 0 : E[C_{n+1}(x_0,T)] \geq C .$$

□

The solution to the optimization problems described above can also be of interest in special settings. Having general expressions for the maintenance characteristics of an item operating under an $MMIPR(x_0,T)$ strategy, one can explicitly write them down for any particular choice of the basic probability distributions and carry on the investigation under the apropriate conditions. One particular setting has been investigated by the authors in Dimitrov et al. (2000), who considered the problem of determining the cycle length that minimizes the expected total maintenance cost over an arbitrary but fixed finite time interval, for items whose original life distribution is Weibull with shape parameter $\alpha > 1$ and scale parameter $\mu > 0$, in the special case in which the age increments Y_n, $n \geq 1$, are proportional to the cycle length T, that is,

$$G(y,T) = \begin{cases} 0 & \text{if} \quad y < \delta T \\ 1 & \text{otherwise,} \end{cases}$$

for some fixed $\delta \in [0,1]$, and the cost functions $c_p(n)$ and $c_m(u)$ increase with n and u , respectively. In this case:

$$E[C(x_0,T,W)] = \frac{\alpha}{\mu^\alpha} \int_{x_0}^{x_0+W} c_m(s) s^{\alpha-1} ds$$

if $0 \leq W < T$,

$$E[C(x_0,T,W)] = \sum_{n=1}^{k} (c_p(n) + \frac{\alpha}{\mu^\alpha} \int_{x_0+(n-1)\delta T}^{x_0+(n-1)\delta T+T} c_m(s)s^{\alpha-1}ds)+$$

$$+\frac{\alpha}{\mu^\alpha} \int_{x_0+k\delta T}^{x_0+k\delta T+W-kT} c_m(s)s^{\alpha-1}du,$$

if $kT \leq W < (k+1)T$, $k \geq 1$, and, if $\alpha > 1$, it can be shown that there exists an integer $k \geq 0$ such that

$$\inf_{T>0} E[C(x_0,T,W)] = E[C(x_0, \frac{W}{k}, W)],$$

that is, the optimal cycle length is of the form $T^* = \frac{W}{k^*}$, where k^* is a non-negative integer such that $E[C(x_0, \frac{W}{k^*}, W)] = min\{E[C(x_0, \frac{W}{k}, W)] : k = 0, 1, \ldots\}$.

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