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A TWO-PLAYER GAME ON GRAPH FACTORS

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A Two-Player Game on Graph Factors

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Abstract

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1 Introduction

Motivated by a complexity issue raised in the analysis of a distributed database query evaluation problem[2], we introduce and analyze a two-player game on bipartite graphs.

In Section 1.1 we introduce the concepts of *factor*, *subfactor* and *left-factor* of a bipartite graph. In Section 1.2 we introduce a two-player game on left-factors of bipartite graphs. In Section 2 we state and prove some basic facts on the structure of left-factors of a bipartite graph. Finally, in Section 3 we introduce a strategy for one of the players of the game and prove the main result, Theorem 6, on how the game evolves when this strategy is followed.

We make use of standard notation. Let $G = (V(G), E(G))$ is a graph and $v \in V(G)$. We denote the *neighbourhood of v in G* by $\Gamma_G(v)$ and the *degree of v in G* by $\deg_G(v)$. For a set $X \subseteq V(G)$, $G[X]$ denotes the *subgraph of G induced by X* and $\Gamma_G(X)$ denotes the neighbourhood X. For a set $X \subseteq E(G)$, $G[X]$ denotes the *subgraph of G induced by X*, $G\{X\} = (V(G), X)$ denotes the *spanning subgraph of G induced by X*.

We denote by $\partial_G(v)$ the *star of v in G*, that is, the set of edges of G incident to v.

1.1 Factors and Left-Factors of a Bipartite Graph

Let G be a graph and let $f\colon V(G) \to \mathbf{N}$ be a function. We say that a set $F \subseteq E(G)$ is an *f-subfactor* of G if, for every $v \in V(G)$, the degree of v in $G[F]$ is upper bounded by $f(v)$, that is,

$$\deg_{G\{F\}}(v) \leq f(v).$$

If equality holds for each $v \in V(G)$, we say that F is an *f-factor* of G.

In the case in which G is a bipartite graph with vertex classes $L(G)$ and $R(G)$, and $f\colon L(G) \to \mathrm{N}$, we say that a set $F \subseteq E(G)$ is a *left-f-subfactor* of G if, for every $l \in L(G)$ and every $r \in R(G)$,

$$\begin{aligned} \deg_{G\{F\}}(l) &\leq f(l) \text{ and} \\ \deg_{G\{F\}}(r) &\leq 1. \end{aligned}$$

Again, if equality holds for each $v \in L(G)$, we say that F is a *left-f-factor* of G.

For brevity, we fix once and for all the following notation and terminology: B stands for a bipartite graph and we refer to the vertex classes $L(B)$ and $R(B)$ of B as the *left* and *right* classes. Generic elements of each class are denoted by l and r. Also, f stands for a function $f\colon L(B) \to \mathrm{N}$.

Given a set $X \subseteq L(B)$, we denote the restriction of f to X by f_X. As usual, we let $f(X) = \sum_{x \in X} f(x)$. The restriction of f to $L(B) - X$ is denoted $\overline{f_X}$. The subgraph of B induced by $X \cup \Gamma_B(X)$ is denoted B_X and its complement, $B - V(B_X)$ is denoted $\overline{B_X}$, that is

$$B_X = B[X \cup \Gamma_B(X)], \tag{1}$$

$$\overline{B_X} = B - V(B_X) = B - (X \cup \Gamma_B(X)). \tag{2}$$

For every $l \in L(B)$ with $f(l) > 0$, we define the function $(f - l)\colon L(B) \to \mathbb{N}$ by

$$(f-l)(v) = \begin{cases} f(v) - 1, & \text{if } v = l; \\ f(v), & \text{otherwise.} \end{cases}$$

1.2 A Game on Left-Factors of a Bipartite Graph

We now describe the game $\mathcal{G}(B, f)$ between two players, the hider and the seeker. In this game. B and f are given to both players and the hider chooses a left-f-subfactor F of B which she keeps hidden from the seeker.

A *move of the game* is a pair $(e, a) \in E(B) \times \{\text{yes}, \text{no}\}$, which is interpreted as the seeker querying "*does* $e \in F$*?*" and the hider answering yes or no, accordingly. The game finishes when the seeker has determined F.

One should conceive $\mathcal{G}(B, f)$ as a competitive game in which the hider and the seeker are mutual adversaries, the second doing his best to determine the chosen left-f-subfactor as soon as possible while the former evades the questions as best as she can so as to delay the end of the game as much as possible.

Note that, since F does not have to be disclosed until the very end, the hider does not have to make up her mind about which F to pick at the beginning; she may answer the queries as the game evolves, just making sure that her answers are consistent with *some* choice of F.

We are interested in a notion related to the *elusiveness of a graph property* as presented in [1]. Indeed, our main result (Theorem 6) will be that it is enough for the hider to follow a very natural and simple strategy (see Section 3) in order to force the seeker to query *every* edge incident to some of the vertices in $L(B)$, regardless of the strategy followed by the seeker.

2 Some Results on Left-Factors

We start by stating a well known result (see [1], Chapter 2) on graph factors in a form convenient for us.

Theorem 1. *The graph B has a left-f-factor if and only if for every $X \subseteq L(B)$, we have*

$$f(X) \leq |\Gamma_B(X)|. \tag{3}$$

Corollary 2. *Let $e = \{l, r\} \in E(B)$ be such that B has a left-f-factor and $B - e$ has no left-f-factor. Then, e belongs to every left-f-factor of B and there is a set $l \in A \subseteq L(B)$ such that*

$$f(A) = |\Gamma_B(A)|.$$

Proof. If $B - e$ has no left-f-factor, from Theorem 1 we know that there is some $A \subseteq L(B - e)$ for which $f(A) > |\Gamma_{B-e}(A)|$. On the other hand, as B has a left-f-factor, we have $f(A) \leq |\Gamma_B(A)|$. As $|\Gamma_{B-e}(A)| \leq |\Gamma_B(A)| + 1$, we conclude $f(A) = |\Gamma_B(A)|$ and $l \in A$. □

The following is stated without proof, since it is immediate from the definitions.

Lemma 3. *Let $F \subseteq E(B)$ and $e = \{l, r\} \in F$. The set F is a left-f-factor of B. if and only if $F - \{e\}$ is an left-$(f - l)$-factor of $B - r$.*

The following result gives information about the structure of a left-f-factor in the case in which equality holds in (3) for a given set A. Figure 1 shows an example of a graph B and a set A as in Lemma 4. In this example, we have $f(l) = 2$ for every $l \in L(B)$. The edges of B_A and $\overline{B_A}$ are shown in thick lines.

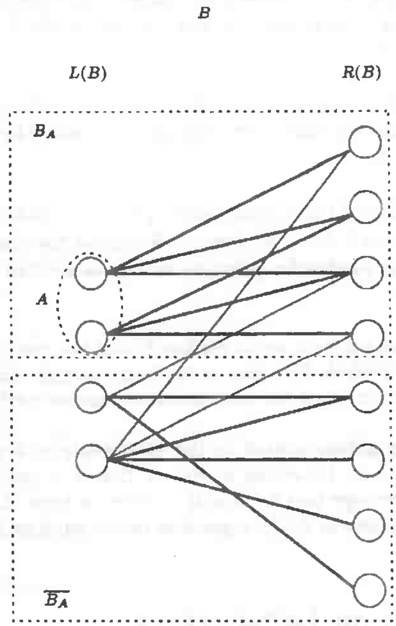

Figure 1: The decomposition in Lemma 4

Lemma 4. *Let $A \subseteq L(B)$ be such that $f(A) = |\Gamma_B(A)|$. A set $F \subseteq E(B)$ is a left-f-factor of B if and only if $F = F_A \cup \overline{F_A}$, where $F_A = F \cap E(B_A)$ is a left-f_A-factor of B_A and $\overline{F_A} = F \cap E(\overline{B_A})$ is a left-$\overline{f_A}$-factor of $\overline{B_A}$.*

Proof. It is immediate from the definitions that if F_A is a left-f_A-factor of B_A and $\overline{F_A}$ is a left-$\overline{f_A}$-factor of $\overline{B_A}$, then $F = F_A \cup \overline{F_A}$ is a left-f-factor of B.

To prove the converse, it is enough to show that every edge in F is either in $E(B_A)$ or in $E(\overline{B_A})$.

Let $e = \{l, r\} \in F - E(B_A)$. It is immediate from the definition of B_A that $l \in L(\overline{B_A}) = L(B) - A$. If $e \notin E(\overline{B_A})$ then $r \notin R(\overline{B_A}) = R(B) - \Gamma_B(A)$, that is, $r \in \Gamma_B(A)$. This is not possible, because $e \in F$ and $|\Gamma_B(A)| = f(A)$. □

3 A Strategy for the hider

A *realization of the game* $\mathcal{G}(B,f)$ is a sequence $\mathcal{R} = ((e_1,a_1),\dots,(e_m,a_m))$ of moves of the game from beginning to end. We denote by $Q(\mathcal{R},k)$ the set of edges queried by the seeker up to (and including) move k, and by $N(\mathcal{R},k)$ the subset of these edges for which the hider answers no. Also, we denote by $B(\mathcal{R},k)$, the graph obatained from B by deleting $N(\mathcal{R},k)$. More formally,

$$Q(\mathcal{R},k) = \{e_i : 1 \le i \le k\}, \tag{4}$$
$$N(\mathcal{R},k) = \{e_i : 1 \le i \le k,\ a_i = \text{no}\}, \tag{5}$$
$$B(\mathcal{R},k) = B - N(\mathcal{R},k). \tag{6}$$

For convenience, we also let

$$\begin{aligned} Q(\mathcal{R}) &= Q(\mathcal{R},m), \\ N(\mathcal{R}) &= N(\mathcal{R},m), \\ B(\mathcal{R}) &= B(\mathcal{R},m). \end{aligned}$$

In the case when B has a left-f-factor, we define *Strategy* $\mathcal{S}$ *for the* hider as the strategy in which she always answers no to any query, unless she has no choice but answering yes in order to have a left-f-factor at the end of the game. More precisely, Strategy $\mathcal{S}$ is the strategy in which. for every $k > 0$, with the sequence $\mathcal{R} = ((e_1,a_1),\dots,(e_{k-1},a_{k-1}))$ of moves from the begining of the game, we have

$$a_k = \text{yes if and only if } B(\mathcal{R},k-1) - e_k \text{ has no left-}f\text{-factor.}$$

The following remarks underline some immediate facts about the realizations of the game in which the hider follows Strategy $\mathcal{S}$. These facts will be important in the proof of our main result.

Remark 5. *In any realization* $\mathcal{R} = ((e_1,a_1),\dots,(e_m,a_m))$ *of the game* $\mathcal{G}(B,f)$ *in which the* hider *follows Strategy* $\mathcal{S}$*:*

1. *The set of edges of* B *to which the* hider *answers* yes*, namely,* $F = Q(\mathcal{R}) - N(\mathcal{R})$*, is a left-*$f$*-factor of* B*. Moreover,* F *is the only left-*f*-factor of* $B(\mathcal{R})$*.*
2. *The* hider *answers* yes *in exactly* $f(L(B))$ *moves of the game, that is,* $|Q(\mathcal{R}) - N(\mathcal{R})| = f(L(B))$*.*
3. *If* $f(L(B)) > 0$*, the game ends precisely at the last move in which the* hider *answers* yes*, that is,* $a_m =$ yes*.*

Our goal is to prove that in any realization of this game, if the hider follows Strategy $\mathcal{S}$, then the seeker must query every edge incident to some vertex in $L(B)$.

Theorem 6. *Let* $\mathcal{R}$ *be a realization of the game* $\mathcal{G}(B,f)$ *in which* B *has a left-*f*-factor and* $f(L(B)) > 0$*. If the* hider *follows strategy* $\mathcal{S}$ *in* $\mathcal{R}$*, then there is a vertex* $l \in L(B)$ *for which*

$$\partial_B(l) \subseteq Q(\mathcal{R}).$$

Proof. The proof is by induction on $f(L(B))$.

Suppose $f(L(B)) = 1$, and let $\mathcal{R} = ((e_1, a_1), \ldots, (e_m, a_m))$ be a realization of $\mathcal{G}(B, f)$. From Remark 5.3 we have $a_m =$ yes. Let l be the endpoint of e_m in $L(B)$, it is clear that if there was an edge $e \in \partial_{B(\mathcal{R},m)}(l) - \{e_m\}$, then $\{e\}$ would be a left-f-factor of $B(\mathcal{R}, m-1) - e_m$, which contradicts strategy $\mathcal{S}$. As a consequence,

$$\partial_B(l) \subseteq Q(\mathcal{R}, m) = Q(\mathcal{R}),$$

as required.

Let us now consider the case in which $f(L(B)) = t > 1$ under the assumption that the statement holds whenever $f(L(B)) < t$.

As before, let $\mathcal{R} = ((e_1, a_1), \ldots, (e_m, a_m))$ and let k be the first move in which the hider answers yes, that is,

$$k = \min\{1 \le i \le m \colon a_i = \text{yes}\}.$$

For every $1 \le i \le m$, let $e_i = \{l_i, r_i\}$, with $l_i \in L(B)$ and $r_i \in R(B)$.

According to the definition of Strategy $\mathcal{S}$, the graph $B(\mathcal{R}, k-1) - e_k$ has no left-f-factor, and then, by Corollary 2, we know that there is some $A \subseteq L(B(\mathcal{R}, k-1)) = L(B(\mathcal{R}, k))$ for which $f(A) = |\Gamma_{B(\mathcal{R},k)}(A)|$.

Let $F = Q(\mathcal{R}) - N(\mathcal{R})$ be the left-f-factor of B determined by the seeker in $\mathcal{R}$ (see Remark 5.1). From Corollary 2 we have $e_k \in F$ and $l_k \in A$.

Suppose that $f(A) = |\Gamma_{B(\mathcal{R},k)}(A)| = 1$. Then we have $\Gamma_{B(\mathcal{R},k)}(A) = \{r_k\}$. It is clear, then, that

$$\partial_B(l_k) \subseteq \{e_k\} \cup N(\mathcal{R}, k) \subseteq Q(\mathcal{R}, k) \subseteq Q(\mathcal{R}),$$

as required.

Let us now suppose $f(A) = |\Gamma_{B(\mathcal{R},k)}(A)| > 1$, let $\mathcal{R}_k$ be the subsequence obtained from $\mathcal{R}$ by deleting the moves (e_i, a_i) in which either $i \le k$ or $e_i \notin E(B_A)$ and let

$$B_k = B_A - r_k, \tag{7}$$
$$f_k = f_A - l_k. \tag{8}$$

Claim 7. *The sequence $\mathcal{R}_k$ is a realization of the game $\mathcal{G}(B_k, f_k)$ in which the hider follows Strategy $\mathcal{S}$.*

Proof. First we need to verify that $\mathcal{G}(B_k, f_k)$ is a well defined game, that Strategy $\mathcal{S}$ is eligible for the hider in this game, and that $\mathcal{R}_k$ is a realization of it.

That $\mathcal{G}(B_k, f_k)$ is a well defined game follows directly from the definitions of B_k and f_k in (7) and (8). Also, that $\mathcal{R}_k$ is a realization of this game, follows from the definition of $\mathcal{R}_k$. Finally, the fact that Strategy $\mathcal{S}$ is eligible for the hider in this game follows, first, from the hypothesis, in that $f(L(B)) > 1$ implies $f_k(L(B_k)) > 0$ and, second, from the fact that B_k has a left-f_k-factor $\mathcal{G}(B_k, f_k)$. This is so because, from Lemma 4, we have that $F \cap E(B_A)$ is a left-f_A-factor of B_A and hence, from Lemma 3, we have that $F - e_k$ is a left-f_k-factor of $B_k = B_A - r_k$.

Let $\mathcal{R}_k = ((e'_1, a'_1), \ldots, (e'_s, a'_s)) = ((e_{i_1}, a_{i_1}), \ldots, (e_{i_s}, a_{i_s}))$ and let $1 \le j \le s$. From the definition of $\mathcal{R}_k$ we have

$$a'_j = \text{yes if and only if } a_{i_j} = \text{yes}. \tag{9}$$

From the definition of Strategy $\mathcal{S}$ we have

$$a_{i_j} = \text{yes if and only if } B(\mathcal{R}, i_j - 1) - e_{i_j} \text{ has no left-}f\text{-factor}. \tag{10}$$

For convenience, let $B' = B(\mathcal{R}, i_j - 1) - e_{i_j}$ so that (10) can be rewriten as

$$a_{i_j} = \text{yes if and only if } B' \text{ has no left-}f_A\text{-factor}. \tag{11}$$

It is a consequence of Lemma 4 that

$$B' \text{ has no left-}f_A\text{-factor if and only if } B'_A \text{ has no left-}f_A\text{-factor}. \tag{12}$$

From the definitions in (1) and (5), we have

$$\begin{aligned} B'_A = B'[A \cup \Gamma_{B'}(A)] = (B(\mathcal{R}, i_j - 1) - e_{i_j})[A \cup \Gamma_{B'}(A)] &= B_A - N(\mathcal{R}, i_j - 1) - e_{i_j} \\ = B_A - (N(\mathcal{R}, i_j - 1) \cap E(B_A)) - e_{i_j} &= B_A - N(\mathcal{R}_k, j - 1) - e_{i_j}. \end{aligned}$$

On the other hand

$$\begin{aligned} B_k(\mathcal{R}_k, j-1) - e_{i_j} = (B_A - r_k)(\mathcal{R}_k, j-1) - e_{i_j} &= B_A(\mathcal{R}_k, j-1) - r_k - e_{i_j} \\ &= B_A - N(\mathcal{R}_k, j-1) - r_k - e_{i_j} = B'_A - r_k. \end{aligned} \tag{13}$$

From Lemma 3 we have

$$B'_A - r_k \text{ has no left-}(f_A - l_k)\text{-factor if and only if } B'_A \text{ has no left-}f_A\text{-factor}.$$

which, by (8) and (13), is the same as

$$B_k(\mathcal{R}_k, j-1) - e_{i_j} \text{ has no left-}f_k\text{-factor if and only if } B'_A \text{ has no left-}f_A\text{-factor}. \tag{14}$$

In short, from (9), (11), (12) and (14)

$$a'_j = \text{yes if and only if } B_k(\mathcal{R}_k, j-1) - e'_j \text{ has no left-}f_k\text{-factor},$$

which is the same as saying that $\mathcal{R}_k$ is a realization of $\mathcal{G}(B_k, f_k)$ in which the hider follows strategy $\mathcal{S}$. □

Returning to the main argument, we have $0 < f_k(L(B_k)) = f(A) - 1 < f(L(B))$. and so, we can apply the induction hypothesis to the realization $\mathcal{R}_k$ of $\mathcal{G}(B_k, f_k)$, and conclude that. for some $l \in L(B_k)$ we have

$$\partial_{B_k}(l) \subseteq Q(\mathcal{R}_k).$$

As the definition of B_k implies that $\partial_B(l) \subseteq \partial_{B_k}(l) \cup Q(\mathcal{R}, k)$, we have

$$\partial_B(l) \subseteq \partial_{B_k}(l) \cup Q(\mathcal{R}, k) \subseteq Q(\mathcal{R}),$$

as required. □

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