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EXACT EXPRESSIONS FOR THE POSTERIOR MODE OF A FINITE POPULATION SIZE: CAPTURE-RECAPTURE SEQUENTIAL SAMPLING.

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SUMMARY

Using data obtained by a multiple capture-recapture experiment that depends on the catchability of the individuals in a closed population, an exact analytical expression for the mode of the posterior distribution of the population size is derived. The prior structure considered is noninformative. It is shown that there are cases where the posterior mode is always finite, a result that was conjectured by Leite et al (1986). Condition for the unicity of the posterior mode is also considered.

1. Introduction

The objective of this paper is to derive a closed analytical expression for the posterior mode of the population size, N, of a closed population, when a capture-recapture model that depends on the "catchability" of the individuals is considered. It is shown that there are situations where the posterior mode is always finite, a result that was conjectured by Leite et al (1986), when deriving exact expressions for the ML estimate of N. But unfortunately there are situations where the ML estimate is infinite, what also may happen with the posterior mode, under a particular class of improper priors.

As in Castledine (1981), the capture probabilities aré assumed to be the same for all animals, but with the possibility of changing over time. A review of the literature is given in Seber (1985).

In Section 2, the basic notation is presented and the posterior distribution of N is derived. Section 3 presents the main results of the paper. The behavior of the posterior mode is illustrated for various practical situations.

2. Basic notation and the posterior distribution of N.

As in Castledine (1981) let N denote the unknown population size, s ($s \geq 2$) the number of samples taken, p_i, $1 \leq i \leq s$, the unknown probability of each animal to be captured in the i-th sample, X_i the unmarked number of animals in the i-th sample, Y_i the number of marked animals in the i-th sample ($Y_1 = 0$) and M_i the number of marked animals in the population just before the i-th sample ($M_1 = 0$). Note that $M_{i+1} = M_i + X_i = \sum_{j=1}^{i} X_j$. To complete the notation, let $\underset{\sim}{p} = (p_1, p_2, \ldots, p_s)$ and

$$\mathcal{D} = \{(x_i, y_i),\ i = 1,2,\ldots,s,\ y_1 = 0\},$$

the observed data. X_i and Y_i, $i = 1,2,\ldots,s$ are assumed to be independent. Therefore, under the assumptions made above,

$$X_i | p_i \sim \mathcal{B}(N - M_i, p_i, x_i), \quad i = 1,2,\ldots,s$$

$$Y_i | p_i \sim \mathcal{B}(M_i, p_i, y_i), \quad i = 2,3,\ldots,s,$$

from where it follows that the likelihood function of N and $\underset{\sim}{p}$ is

$$L(N, \underset{\sim}{p} | \mathcal{D}) = \prod_{i=1}^{s} \binom{N-M_i}{x_i} \binom{M_i}{y_i} p_i^{n_i} (1-p_i)^{N-n_i}$$

$$\propto \binom{N}{r} \prod_{i=1}^{s} p_i^{n_i} (1-p_i)^{N-n_i},$$

where $n_i = x_i+y_i$ and $r = \sum_{i=1}^{s} x_i$, the total number of different animals captured.

Considering a prior distribution for $(N,\underset{\sim}{p})$ which density is such that

$$(1) \qquad \pi(N,\underset{\sim}{p}) = \pi(N)\pi(\underset{\sim}{p}) \quad ,$$

where $\pi(N)$ and $\pi(\underset{\sim}{p})$ are noninformative distributions, it follows that the joint posterior density of $(N,\underset{\sim}{p})$ is such that

$$(2) \qquad \pi(N,\underset{\sim}{p}|\mathcal{D}) \propto L(N,\underset{\sim}{p}|\mathcal{D}) .$$

By integrating out $\underset{\sim}{p}$ in (2), it follows that

$$(3) \qquad \pi(N|\mathcal{D}) \propto \frac{\binom{N}{r}}{[(N+1)!]^{s}} \prod_{i=1}^{s} (N-n_i)! \quad ,$$

where $N \geq r$ and $\max\{n_1,n_2,\ldots,n_s\} \leq r \leq \sum_{j=1}^{s} n_j$.

If the improper prior

$$(4) \qquad \pi(N,\underset{\sim}{p}) = \pi(N)\pi(\underset{\sim}{p}) \propto \prod_{i=1}^{s} p_i^{-1}$$

is taken for N and $\underset{\sim}{p}$, then, it follows that the posterior probability function of N is given by

$$(5) \qquad \pi(N|\mathcal{D}) \propto \frac{\binom{N}{r}}{[N!]^{s}} \prod_{i=1}^{s} (N-n_i)! \quad .$$

The behavior of the mode of the posterior distribution (5) is exactly the same as the behavior of the ML estimate considered in Leite et al. (1986).

3. Analytical expression for the Mode of (3).

For the observed data $\mathcal{D}$, the posterior mode of N is a point $\hat{N} \in \{n \in \mathbb{N}^*;\ n \geq r\}$ ($\mathbb{N}^* = \{1,2,\ldots\}$) such that it maximizes the posterior probability function of N. After simple algebraic manipulations, (3) may be written as

$$\pi(N|\mathcal{D}) \propto \frac{N!}{(N-r)!\prod_{j=1}^{s}\binom{N+1}{n_j+1}} . \tag{6}$$

The following result introduces the mode of (6) for the extreme case where $r = \max\{n_1, n_2, \ldots, n_s\}$.

Lemma 1. If $r = \max\{n_1, n_2, \ldots, n_s\}$, then $\hat{N} = r$ is the unique mode of (3).

Proof. After simple algebraic manipulations, (6) may be written as

$$\pi(N|\mathcal{D}) \propto \frac{\binom{N+1}{r+1}}{(N+1)\prod_{j=1}^{s}\binom{N+1}{n_j+1}} , \quad N \geq r . \tag{7}$$

Supposing, without any loss of generality, that $r = n_s$, it follows from (7) that

$$\pi(N|\mathcal{D}) \propto \frac{1}{(N+1)\prod_{j=1}^{s-1}\binom{N+1}{n_j+1}} \quad , N \geq r \; ,$$

from where the result follows.

Considering the case where $r > \max\{n_1, n_2, \ldots, n_s\}$, let

$$K(N) = \frac{N!}{(N-r)!\prod_{j=1}^{s}\binom{N+1}{n_j+1}} \quad , N \geq r \; .$$

It follows from (6) that $\hat{N}$ is the point which maximizes $K(N)$ and, for all $N \geq r$,

$$\frac{K(N+1)}{K(N)} = \left(1 - \frac{r}{N+1}\right)^{-1}\left(1 + \frac{1}{N+1}\right)^{-s}\prod_{j=1}^{s}\left(1 - \frac{n_j}{N+1}\right) \; .$$

Define the function

$$g(x) = (1-rx)^{-1}(1+x)^{-s}\prod_{j=1}^{s}(1-n_j x) \quad , \; x \in [0, \tfrac{1}{r}) \; .$$

For all $N \geq r$,

$$g\left(\frac{1}{N+1}\right) = \frac{K(N+1)}{K(N)} \; .$$

In the theorem that follows, the behavior of the function g is studied.

Theorem 1. If $r > \max\{n_1, n_2, \ldots, n_s\}$, then the equation $g(x) = 1$ has only one non null root x_o in the interval $[0, \frac{1}{r})$. For $x \in (0, x_o)$, $g(x) < 1$ and, for $x \in (x_o, \frac{1}{r})$, $g(x) > 1$.

Proof. Consider the functions $g_1(x) = \prod_{j=1}^{s} (1 - n_j x)$ and $g_2(x) = (1 - rx)(1+x)^s$, for all real x. The first and second derivatives of g_1 are such that

$$g_1'(x) = \sum_{i=1}^{s} (-n_i) \prod_{\substack{j=1 \\ j \neq i}}^{s} (1 - n_j x) < 0$$

and

$$g_1''(x) = \sum_{i=1}^{s} \sum_{\substack{j=1 \\ j \neq i}}^{s} n_i n_j \prod_{\substack{k=1 \\ k \neq i,j}}^{s} (1 - n_k x) > 0 ,$$

for all x in the interval $(0, \frac{1}{r})$.

Therefore, g_1 is a decreasing convex continuous function in the interval $[0, \frac{1}{r}]$. The function g_2 is continuous and its first and second derivatives are

$$g_2'(x) = [-r(s+1)x + s - r](1+x)^{s-1}$$

and

$$g_2''(x) = [-rs(s+1)x + (s-1)(s-r) - r(s+1)](1+x)^{s-2} ,$$

respectively, for all real x. Then,

(a) if $r \geq s$, $g_2'(x) < 0$ and $g_2''(x) < 0$, for all $x \in (0, \frac{1}{r})$, that is, g_2 is a decreasing and concave function in the interval $[0,\frac{1}{r}]$. At the origin, $g_1(0) = g_2(0) = 1$, and at the point $\frac{1}{r}$, $g_1(\frac{1}{r}) > 0$ and $g_2(\frac{1}{r}) = 0$. The derivative of g_2-g_1 evaluated at the origin is

$$g_2'(0) - g_1'(0) = (\sum_{i=1}^{s} n_i - r) + s > 0 .$$

Therefore, there is a positive real number δ, such that $g_2'(x)-g_1'(x) > 0$, for all x in the interval $[0,\delta)$. It follows, from the Mean Value Theorem, that $g_2(x) > g_1(x)$, for all x in the interval $(0,\delta)$. Consequently, there is a unique point $x_o \in (0,\frac{1}{r})$, such that

$$g_1(x_o) = g_2(x_o) ,$$

$$g_1(x) < g_2(x) , \quad \text{for all } x \in (0,x_o), \text{ and}$$

$$g_1(x) > g_2(x) , \quad \text{for all } x \in (x_o,\tfrac{1}{r}).$$

Since g is the restriction of $\frac{g_1}{g_2}$ to the interval $[0,\frac{1}{r})$, the result follows.

(b) If $r < s$, $g_2'(x) > 0$ for all $x \in (0,\frac{s-r}{r(s+1)})$; $g_2'(x) < 0$ and $g_2''(x) < 0$ for all $x \in (\frac{s-r}{r(s+1)},\frac{1}{r})$, implying that g_2 is increasing in the interval $[0,\frac{s-r}{r(s+1)}]$ and decreasing and concave in $[\frac{s-r}{r(s+1)},\frac{1}{r}]$. So, there is a unique point $x_o \in (\frac{s-r}{r(s+1)},\frac{1}{r})$ such that

$$g_1(x_o) = g_2(x_o) \; ,$$

$$g_1(x) < g_2(x) \quad , \text{ for all } \; x \in (0,x_o), \text{ and}$$

$$g_1(x) > g_2(x) \quad , \text{ for all } \; x \in (x_o,\tfrac{1}{r}).$$

Since $g = \frac{g_1}{g_2}$ in the interval $[0,\frac{1}{r}]$, the proof is completed.

Figure 1 below illustrates the behavior of the functions g_1 and g_2 in the interval $[0,\frac{1}{r}]$, as described in Theorem 1, in a general situation.

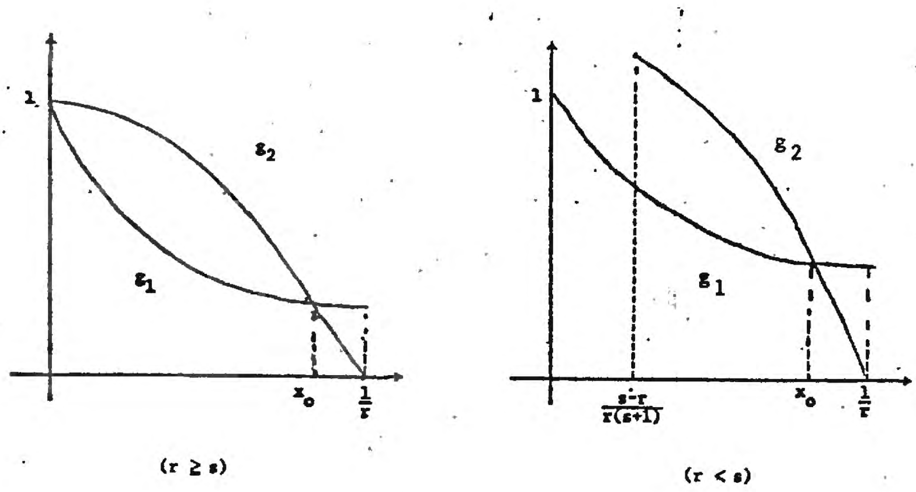

Figure 1: Functions g_1 and g_2 in a general situation.

The main result of the paper is stated next. Let $m = \max\{n_1,n_2,\ldots,n_s\}$ and

$$n_r = \min\{n \in \mathbb{N}^*; \prod_{j=1}^{s} (r+n-n_j) < \frac{n}{r+n}(r+n+1)^s\}$$

Theorem 2. For all $s \geq 2$, a mode of $\pi(N|\mathcal{D})$, $\hat{N}$, exists and is given by

$$\hat{N} = \begin{cases} m & \text{if } r = m \\ r+n_r-1 & \text{if } r > m. \end{cases}$$

The mode $\hat{N}$ is unique, unless

$$\prod_{j=1}^{s}(r+n_r-1-n_j) = \frac{n_r-1}{r+n_r-1}(r+n_r)^s \quad ,$$

in which case the only two modes are $r+n_r-1$ and $r+n_r-2$.

Proof. From Lema 1, $\hat{N} = r$ if $r = m$. If $m < r$, it follows from Theorem 1 that exists $n_o \in \mathbb{N}^*$, $n_o > 1$, such that, for $n \in \mathbb{N}^*$

$$g\left(\frac{1}{r+n}\right) \begin{cases} < 1 & \text{if } n \geq n_o \\ \geq 1 & \text{if } n = n_o-1 \\ > 1 & \text{if } n < n_o-1 \ . \end{cases}$$

Therefore , $n_o = \min\{n \in \mathbb{N}^* \ ; \ g(\frac{1}{r+n}) < 1\} =$

$$= \min\{n \in \mathbb{N}^* \ ; \ \prod_{j=1}^{s}(r+n-n_j) < \frac{1}{r+n}(r+n+1)^s\};$$

that is, $n_o = n_r$ and so,

(i) if $\prod_{j=1}^{s}(r+n_r-1-n_j) > \frac{n_r-1}{r+n_r-1}(r+n_r)^s$, then

$$g\left(\frac{1}{r+n}\right) \begin{cases} < 1 & \text{for } n \in \mathbb{N}^*,\ n \geq n_r \\ > 1 & \text{for } n \in \mathbb{N}^*,\ n \leq n_r-1. \end{cases}$$

From $g\left(\frac{1}{r+n}\right) = \frac{K(r+n)}{K(r+n-1)}$, $n \in \mathbb{N}^*$, it follows that

$$K(r+n_r-1) > K(r+n_r-2) > \ldots > K(r)$$

and

$$K(r+n_r-1) > K(r+n_r) > K(r+n_r+1) > \ldots \quad .$$

So, $\hat{N} = r+n_r-1$ is the only mode of $\pi(N|\mathcal{D})$, given by (3).

(ii) If $\prod_{j=1}^{s}(r+n_r-1-n_j) = \frac{n_r-1}{r+n_r-1}(r+n_r)^s$, then

$$g\left(\frac{1}{r+n}\right) \begin{cases} < 1 & \text{for } n \in \mathbb{N}^*,\ n \geq n_r \\ > 1 & \text{for } n \in \mathbb{N}^*,\ n < n_r-1 \end{cases}$$

and

$$g\left(\frac{1}{r+n_r-1}\right) = 1 .$$

Then, from the fact that $g\left(\frac{1}{r+n}\right) = \frac{K(r+n)}{K(r+n-1)}$, $n \in \mathbb{N}^*$, it follows that

$$K(r+n_r-1) = K(r+n_r-2) > K(r+n_r-3) > \ldots > K(r)$$

and

$$K(r+n_r-1) > K(r+n_r) > K(r+n_r+1) > \ldots .$$

So, $\hat{N} = r+n_r-1$ and $\hat{N}-1 = r+n_r-2$ are the two modes of $\pi(N|\mathcal{D})$ given by (3). The next result is a direct consequence of Theorem 2.

<u>Corollary</u>. If $r > m$, then $\hat{N} = r$ is the unique mode of (3), if and only if,

$$\prod_{j=1}^{s}(r+1-n_j) < \frac{1}{r+1}(r+2)^s .$$

From the posterior probability function (5), it follows that if an improper prior distribution is taken for $\underline{p}$, then the posterior mode may be infinite, as it happens with the ML estimate. On the other hand, if a proper prior is taken for $\underline{p}$, it follows from Theorem 2 that the posterior mode of N is always finite. Therefore, the conjecture made by Leite et al (1986) is right, as long as a proper prior distribution is taken for $\underline{p}$.

As a final remark, we note that for the simple one by one case, there exists a unique mode, whose expression follows from Theorem 2. The proof is similar to that of Theorem 7 in Leite et al (1986).

Table 1 bellow illustrates the behavior of the posterior mode given in Theorem 2, for some numerical examples.

For the sunfish data in Castledine (1981), it follows from Theorem 2 that $n_r = 160$ and then $\hat{N} = 295$.

Table 1: Examples of Posterior Modes of (3)

$n_1, n_2, \ldots, n_s$	r	ML estimate (mode of (5)	Posterior mode of (3) (Theorem 2)
(40,60)	62	63	63
	80	119 & 120	116
	100	∞	1299
(1,5,8)	10	12	11
	11	16	13
	12	25	17
	14	∞	30
(40,60,80)	90	92	92
	120	152	150
	140	239 & 240	232
	179	10381	2715
	180	∞	3628
(3,3,4,4,5)	6	6	6
	7	7	7
	10	11	10
	17	67	29
	18	139	35
	19	∞	44
(15,20,25,30,50)	60	61	61
	80	95	93
	98	149 & 150	143
	120	347	298
	139	7449	1336
	140	∞	1609

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