Book of Abstracts

SEE2023 - Symposium on Evolution Equations

Universidade Federal de Santa Catarina

Florianópolis - 2023

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Lectures

Schedule

Monday - July 24

13:40 - 14:20	Registration
	Chair: Marcelo Rempel Ebert
14:20 - 15:05	Michael Reissig - Critical exponent versus critical regularity in semilinear evolution models
15:05 - 15:50	Alexandre Arias Junior - 3-evolution semilinear equations in projective Gevrey classes
15:50 - 16:20	**Coffee Break**
16:20 - 17:05	Alessia Ascanelli - Semilinear stochastic pdes of Schrödinger type with variable coefficients

Tuesday - July 25

	Chair: Michael Reissig
9:45 - 10:30	Marco Cappiello - Gevrey well-posedness for p-evolution equations
10:30 - 10:50	**Coffee Break**
10:50 - 11:35	Halit Sevki Aslan - On the asymptotic behavior of the energy for evolution models with oscillating time-dependent damping
11:35 - 12:20	Arthur Cavalcante Cunha - Smoothing and finite-dimensionality of uniform attractors in Banach spaces
12:20 - 14:00	**Lunch**
	Chair: Ruy Coimbra Charão
14:00 - 14:45	Marcello D'Abbicco - Asymptotics for two-terms fractional diffusion problems
14:45 - 15:30	Massimo Gobbino - Monotonicity results for generalized solutions to a class of forward- backward parabolic equations
15:30 - 15:50	**Coffee Break**
15:50 - 16:35	Tiago Henrique Picon - Sobolev solvability of elliptic homogenous linear equations on Borel measures
16:35 - 17:20	Radouan Daher - Old and New Results on Fourier Analysis

Wednesday - July 26

	Chair: Marcello D'Abbicco
9:00 - 9:45	Jacson Simsen - Autonomous evolution inclusions with large diffusion and variable exponents
9:45 - 10:30	Marina Ghisi - Almost global existence for Kirchhoff equations
10:30 - 10:50	**Coffee Break**
10:50 - 11:35	Jorge Manuel Silva Marques - Critical exponent of Fujita type for semilinear damped wave equations in Friedmann-Lemaître-Robertson-Walker spacetime
11:35 - 12:20	Wanderley Nunes do Nascimento - A shift of a Fujita type exponent for a class of semilinear evolution equations with time-dependent damping
12:20 - 14:00	**Lunch**
14:30	Visit to Lagoa da Conceição
20:00	Conference Dinner

Thursday - July 27

	Chair: Marco Cappiello
9:00 - 9:45	Benjamin Aina Peter - Reactive Flow of Unsteady Eyring-Powell Nanofluid Fluid with Thermocapillary Convec tive Boundary Conditions, Variable Thermal Conductivity and Radiant Heat through a Porous
9:45 - 10:30	Marcelo Rempel Ebert - Asymptotic behaviour of solutions for a strongly damped wave equation
10:30 - 10:50	**Coffee Break**
10:50 - 11:35	Alexandre N. Oliveira Sousa - Topological structural stability for nonautonomous random dynamical systems and applications in a SIR model
11:35 - 12:20	Yuta Wakasugi - Asymptotic expansion of solutions to the wave equation with space-dependent damping

3-evolution semilinear equations in projective Gevrey classes

Alexandre Arias Junior

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We consider the quasilinear Cauchy problem

$$\begin{cases} P(t,x,u(t,x),D_t,D_x)u(t,x) = f(t,x),\ (t,x) \in [0,T] \times \mathbb{R}, \\ u(0,x) = g(x),\ x \in \mathbb{R}, \end{cases} \tag{1}$$

where

$$P(t,x,u,D_t,D_x) = D_t + a_3(t)D_x^3 + a_2(t,x,u)D_x^2 + a_1(t,x,u)D_x + a_0(t,x,u),$$

$a_j(t,x,w)$, $0 \leq j \leq 2$, are continuous functions of time t, projective Gevrey regular with respect to the space variable x and holomorphic in the complex parameter w. The coefficient $a_3(t)$ is assumed to be a continuous function which never vanishes.

In this talk we shall discuss how to apply the Nash-Moser inversion theorem in order to obtain local in time well-posedness in projective Gevrey classes for the Cauchy problem (1).

This is a joint work with:

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Semilinear stochastic pdes of Schrödinger type with variable coefficients

Alessia Ascanelli

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In this talk we discuss the Cauchy problem for Schrödinger type stochastic partial differential equations with uniformly bounded x-depending coefficients:

$$\begin{cases} P(x,\partial_t,\partial_x)u(t,x) = \gamma(t,x,u(t,x)) + \sigma(t,x,u(t,x))\dot{\Xi}(t,x), & (t,x) \in [0,T] \times \mathbb{R}^d, \\ u(0,x) = u_0(x), \quad x \in \mathbb{R}^d, \end{cases} \tag{1}$$

$$\begin{aligned} P(x,\partial_t,\partial_x) &= i\partial_t + \frac{1}{2}\sum_{j,\ell=1}^{d} \partial_{x_j}\left(a_{j\ell}(x)\partial_{x_\ell}\right) + m_1(x,-i\partial_x) + m_0(x,-i\partial_x) \\ &= i\partial_t + a(x,D_x) + a_1(x,D_x) + m_1(x,D_x) + m_0(x,D_x), \end{aligned}$$

$D_x = -i\partial_x$, where:

- $a(x,\xi) := -\frac{1}{2}\sum_{j,\ell=1}^{d} a_{j\ell}(x)\xi_j\xi_\ell$, $a_{j\ell} = a_{\ell j}$, $j,\ell = 1,\ldots,d$, is the Hamiltonian of the equation,

  $a_1(x,\xi) := \frac{i}{2}\sum_{j,\ell=1}^{d} \partial_{x_j} a_{j\ell}(x)\xi_\ell$, $m_1(x,\xi)$ comes from a magnetic field and $m_0(x,\xi)$ is a potential term;
- γ and σ, are real-valued functions representing, respectively, drift and diffusion;
- Ξ is an $\mathcal{S}'(\mathbb{R}^d)$-valued Gaussian process, white in time and coloured in space, with correlation measure Γ and spectral measure $\mathfrak{M}$;
- u is an unknown stochastic process, called solution of the Cauchy problem (1).

We give conditions on the coefficients, on the drift and diffusion terms, on the Cauchy data, and on the spectral measure associated with the noise, such that the Cauchy problem admits a unique function-valued mild solution in the sense of Da Prato and Zabczyc.

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On the asymptotic behavior of the energy for evolution models with oscillating time-dependent damping

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In the present work, we consider the following Cauchy problem for a linear σ-evolution equation with a time-dependent damping term $b(t)u_t$:

$$\begin{cases} u_{tt} + (-\Delta)^\sigma u + b(t)u_t = 0, & (t,x) \in [0,\infty) \times \mathbb{R}^n, \\ u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), & x \in \mathbb{R}^n, \end{cases} \tag{1}$$

where $\sigma > 0$ and b is a continuous and positive function. The term $b(t)u_t$ represents a damping, a term whose action may dissipate the energy

$$E(t) = \frac{1}{2}\|u_t(t,\cdot)\|^2_{L^2} + \frac{1}{2}\|(-\Delta)^{\frac{\sigma}{2}}u(t,\cdot)\|^2_{L^2}. \tag{2}$$

Our purpose is to study the energy behavior of solutions to the Cauchy problem (1) depending on the properties of time-dependent coefficient $b(t) = \mu(t) + \omega(t)$, where $\mu = \mu(t)$ is a shape function and $\omega = \omega(t)$ is an oscillating function. To achieve this we use the energy method in the phase space and method of the zones similarly as considered in the diagonalization procedure. This method allows us to have less regularity on b without further control on the oscillations. Our goal is to understand if oscillations on the time-dependent coefficient $b(t)$ satisfying $\limsup_{t\to\infty} tb(t) < \infty$ have any influence in the energy behavior of solutions to (1).

1. Models with the behavior of energy solutions determined at high frequencies

Hypothesis 1 *We assume that the function b is positive, continuous and can be defined by $b(t) = \mu(t) + \omega(t)$, where $\mu = \mu(t)$ is a shape function and $\omega = \omega(t)$ is an oscillating function. These functions satisfy the following conditions:*

(A1) $\omega \in L^1([0,\infty))$,

(A2) $\limsup_{t\to\infty} tb(t) < 2$,

(A3) $b(t) \approx \mu(t)$ *for* $t \geq t_0$, $t_0 > 0$ *is sufficiently large,*

(A4) $\mu(t) > 0$, $\mu'(t) < 0$, $-\mu'(t) = O(\mu(t)^2)$ *and* $2\mu'(t) + \mu(t)^2$ *is a strictly increasing function for* $t \geq t_0$.

Example 1 *As a simple example we may take*

$$\omega(t) = \frac{\sin\big(t/(\ln(e+t))^\alpha\big)}{(1+t)(\ln(e+t))^\gamma} \quad \text{with} \quad \alpha \in \mathbb{R}, \quad \gamma > 1 \quad \text{and} \quad \mu(t) = \frac{\beta}{1+t} \quad \text{with} \quad \beta \in (0,2).$$

Theorem 1 *Let $u_0 \in H^\sigma$, $u_1 \in L^2$ and let us assume Hypothesis 1. Then, the energy solution to the Cauchy problem (1) satisfies the estimate*

$$E(t) \leq C \exp\Big(-\int_0^t \mu(s)ds\Big)\big(E(0) + \|u_0\|^2_{L^2}\big),$$

where $E(t)$ is defined in (2).

Example 2 *We consider*

$$b(t)=\frac{\beta\big(\ln(e+t)\big)^{\gamma}}{1+t}+\omega(t),\quad \gamma\in[-1,0],$$

with $\beta\in(0,2)$ if $\gamma=0$ and $\omega(t)$ satisfies Hypothesis 1. Then, Theorem 1 implies the following estimates:

$$E(t)\leq C\big(E(0)+\|u_0\|^2_{L^2}\big)\begin{cases}(1+t)^{-\beta} & \text{if } \gamma=0,\\ \exp\Big(-\frac{\beta}{\gamma+1}\big(\big(\ln(e+t)\big)^{\gamma+1}\Big) & \text{if } \gamma\in(-1,0),\\ \big(\ln(e+t)\big)^{-\beta} & \text{if } \gamma=-1.\end{cases}$$

2. A model with the behavior of energy solutions determined at low frequencies

In this section we are interested in damping terms $b(t)u_t$ satisfying $\liminf_{t\to\infty} tb(t)>2$ and $\limsup_{t\to\infty} tb(t)<\infty$. For this reason, in the following we restrict ourselves to the case that $b(t)$ is a perturbation of the scale invariant case with $\beta>2$.

Hypothesis 2 *We assume that the function b is positive, continuous and can be written as $b(t)=\mu(t)+\omega(t)$, where $\mu(t)$ will determine the shape of the coefficient, while $\omega(t)$ contains oscillations. These two functions satisfy the following conditions:*

(B1) $\mu(t)=\dfrac{\beta}{1+t},\ \ \beta>2,$

(B2) *there exists $\varepsilon>0$ such that $\beta-3\varepsilon>2$ and $|\omega(t)|\leq\varepsilon(1+\varepsilon)^{-1}\mu(t)$.*

Theorem 2 *Let $u_0\in H^\sigma$, $u_1\in L^2$ and let us assume Hypothesis 2. Then, the energy solution to the Cauchy problem (1) satisfies the estimate*

$$E(t)\leq C(1+t)^{-2}\big(E(0)+\|u_0\|^2_{L^2}\big),$$

where $E(t)$ is defined in (2).

Example 3 *Let us consider*

$$b(t)=\underbrace{\frac{\beta}{1+t}}_{=:\mu(t)}+\underbrace{\frac{\varepsilon(1+\varepsilon)^{-1}\beta\sin t}{1+t}}_{=:\omega(t)},$$

where $\beta>2+3\varepsilon$, with $\varepsilon>0$ sufficiently small. From Theorem 2 we conclude the same decay rate $(1+t)^{-2}$ for the energy of solutions to (1) for a class of effective damping $b(t)u_t$ that does not satisfy the condition $|b'(t)|\leq C(1+t)^{-1}b(t)$ in order to apply the diagonalization procedure.

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Gevrey well-posedness for p-evolution equations

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We study the Cauchy problem

$$\begin{cases} P(t,x,D_t,D_x)u(t,x) = f(t,x) \\ u(0,x) = g(x) \end{cases}, \qquad (t,x) \in [0,T] \times \mathbb{R},$$

for p-evolution operators of the form

$$P(t,x,D_t,D_x) = D_t + a_p(t)D_x^p + \sum_{j=1}^{p-1} a_j(t,x)D_x^j, \qquad (t,x) \in [0,T] \times \mathbb{R},$$

where $a_p \in C([0,T],\mathbb{R})$ and $a_j \in C([0,T],C^\infty(\mathbb{R};\mathbb{C})), j = 0,\dots,p-1$, in the Gevrey functional setting. When the coefficients $a_j(t,x), j = 0,\dots,p-1$, of the lower order terms are complex-valued, it is possible to obtain well-posedness results in Gevrey spaces under suitable decay assumptions on a_j for $|x| \to \infty$. In the first part of the talk, we present a well-posedness result for 3-evolution equations obtained in [1]. In the second part we discuss the sharpness of the decay conditions assumed and the generalization of these conditions to the case of p-evolution equations for an arbitrary positive integer p.

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Smoothing and finite-dimensionality of uniform attractors in Banach spaces

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The aim of this talk is to find an upper bound for the fractal dimension of uniform attractors in Banach spaces. The main technique we employ is essentially based on a compact embedding of some auxiliary Banach space into the phase space and a corresponding smoothing effect between these spaces. Our bounds on the fractal dimension of uniform attractors are given in terms of the dimension of the symbol space and the Kolmogorov entropy number of the embedding. In addition, a dynamical analysis on the symbol space is also given, showing that the finite-dimensionality of the hull of a time-dependent function is fully determined by the tails of the function, which allows us to consider more general non-autonomous terms than quasi-periodic functions. As applications, we show that the uniform attractor of a reaction-diffusion equation is finite-dimensional in L^2 and in L^p, with $p > 2$.

Now we give definitions on this topic and summarize our main results on the *dimensionality of uniform attractors*.

Let (Ξ, d_Ξ) be a complete metric space and let $\{\theta_s\}_{s\in\mathbb{R}}$ be a group of continuous operators acting on Ξ, i.e., $\theta_0\sigma = \sigma$ and $\theta_t(\theta_s\sigma) = \theta_{t+s}\sigma$ for all $\sigma \in \Xi$, $t, s \in \mathbb{R}$, and for each $s \in \mathbb{R}$, $\theta_s : \Xi \to \Xi$ is a continuous map in Ξ. Let $\Sigma \subseteq \Xi$ be a *compact* subset of Ξ which is invariant under $\{\theta_s\}_{s\in\mathbb{R}}$, that is, $\theta_s\Sigma = \Sigma$ for all $s \in \mathbb{R}$.

Considering a collection of evolution processes $\{U_\sigma(t, s)\}_{\sigma\in\Sigma}$ indexed by $\sigma \in \Sigma$, i.e., for each $\sigma \in \Sigma$ the two-parameter family $\{U_\sigma(t, s) : t \geq s\}$ in a Banach space X satisfies $U(s, s) = Id_X$ and $U(t, \tau)U(\tau, s) = U(t, s)$ for all $t, \tau, s \in \mathbb{R}$ with $t \geq \tau \geq s$, it will be called a *system* if the *translation-identity* is satisfied:

$$U_{\theta_h\sigma}(t, s) = U_\sigma(t + h, s + h), \quad \forall \sigma \in \Sigma,\ t \geqslant s,\ h \in \mathbb{R}.$$

In this case, the parameter σ is called the *symbol* of the process $\{U_\sigma(t, s)\}$ and the set Σ the *symbol space* of the system $\{U_\sigma(t, s)\}_{\sigma\in\Sigma}$. The uniform attractor for such systems of processes is defined as follows.

Definition. A compact set $\mathcal{A}_\Sigma \subset X$ is said to be the *uniform (w.r.t. $\sigma \in \Sigma$) attractor* of a system $\{U_\sigma(t, s)\}_{\sigma\in\Sigma}$ if

(i) $\mathcal{A}_\Sigma$ is *uniformly attracting*, i.e., for any bounded set $E \subset X$ it holds

$$\sup_{\sigma\in\Sigma} \operatorname{dist}_X\big(U_\sigma(t, 0, E), \mathcal{A}_\Sigma\big) \longrightarrow 0, \quad \text{as } t \to \infty,$$

where for non-empty sets $A, B \subset X$ we denote the *Hausdorff semi-distance* $\operatorname{dist}_X(A, B) := \sup\limits_{a\in A} \inf\limits_{b\in B} \|a - b\|_X$.

(ii) (Minimality) If $\mathcal{A}'_\Sigma$ is a closed set in X uniformly attracting, then $\mathcal{A}_\Sigma \subset \mathcal{A}'_\Sigma$.

Definition. Let A be a non-empty precompact subset of X. The *(upper) fractal dimension* of A in X is defined as

$$\dim_F(A; X) := \limsup_{\varepsilon\to 0^+} \frac{\ln N_X[A; \varepsilon]}{-\ln \varepsilon},$$

where $N_X[A; \varepsilon]$ denotes the minimum number of open ε-balls in X that are necessary to cover A.

Now we give a criterion for the uniform attractor $\mathcal{A}_\Sigma$ of $\{U_\sigma(t, s)\}_{\sigma\in\Sigma}$ to be finite-dimensional. The main idea is an (X, Y)-smoothing property combined with assumptions on the dimension of the symbol space. Let

$\mathcal{B} \subset X$ be a closed bounded uniformly absorbing set of $\{U_\sigma(t,s)\}_{\sigma\in\Sigma}$ (it has necessarily to satisfy $\mathcal{A}_\Sigma \subseteq \mathcal{B}$) and suppose that:

(H_1) There is an auxiliary Banach space Y *compactly embedded* in the Banach space X.

(H_2) The symbol space Σ has finite fractal dimension in space Ξ, i.e., $\dim_F(\Sigma;\Xi) < \infty$.

(H_3) The system $\{U_\sigma(t,s)\}_{\sigma\in\Sigma}$ is $(\Sigma \times X, X)$-*continuous.*

(H_4) $\{U_\sigma(t,s)\}_{\sigma\in\Sigma}$ is *uniformly* (X,Y)-*smoothing* on the absorbing set $\mathcal{B}$, i.e., for any $t > 0$ there exists a $\kappa(t) > 0$ such that

$$\sup_{\sigma\in\Sigma} \|U_\sigma(t,0)u - U_\sigma(t,0)v\|_Y \leqslant \kappa(t)\|u-v\|_X, \quad \forall u,v \in \mathcal{B}.$$

(H_5) $\{U_\sigma(t,s)\}_{\sigma\in\Sigma}$ is (Σ, X)-*Lipschitz* on the absorbing set $\mathcal{B}$, satisfying

$$\|U_{\sigma_1}(t,0)u - U_{\sigma_2}(t,0)u\|_X \leqslant L(t) d_\Xi(\sigma_1,\sigma_2), \quad \forall t \geq 1,\ \sigma_1,\sigma_2 \in \Sigma,\ u \in \mathcal{B},$$

where $1 \leq L(t) \leq c_1 e^{\beta t}$ for some positive constants $c_1, \beta > 0$ for $t \geq 1$.

Our main result guarantees that under these hypotheses $\mathcal{A}_\Sigma$ has finite fractal dimension in X. More precisely:

Theorem. Let $\{U_\sigma(t,s)\}_{\sigma\in\Sigma}$ be a system in X with uniform attractor $\mathcal{A}_\Sigma$. If conditions (H_1) - (H_5) hold, then the uniform attractor $\mathcal{A}_\Sigma$ has finite fractal dimension in X with

$$\dim_F\big(\mathcal{A}_\Sigma; X\big) \leqslant \ln N_X\big[B_Y(0,1); 1/(2e\kappa)\big] + (\beta+1)\dim_F\big(\Sigma;\Xi\big),$$

for some $\kappa = \kappa(T_\mathcal{B})$ depending on $\mathcal{B}$ with $T_\mathcal{B} \geq 1$ an absorption time after which $\mathcal{B}$ uniformly absorbs itself.

About the symbol space we also provide the following result in order to construct finite-dimensional examples.

Theorem. Let $\mathcal{M}$ be a complete metric space and $g_+,\ g_- \in \Xi = \mathcal{C}(\mathbb{R};\mathcal{M})$ with finite-dimensional hulls $\mathcal{H}(g_+)$ and $\mathcal{H}(g_-)$ in Ξ, respectively, where $\mathcal{H}(\xi) = \overline{\{\xi(\cdot+s) : s \in \mathbb{R}\}}^\Xi$ for $\xi \in \Xi$. Suppose that $g \in \Xi$ is a function such that

(G1) g is Lipschitz continuous from $\mathbb{R}$ to $\mathcal{M}$;

(G2) g converges forwards to g_+ and backwards to g_- eventually exponentially.

Then the hull $\mathcal{H}(g)$ of g is compact and finite-dimensional in Ξ with

$$\dim_F\left(\mathcal{H}(g);\Xi\right) \leq \max\Big\{1,\ \dim_F\left(\mathcal{H}(g_+);\Xi\right),\ \dim_F\left(\mathcal{H}(g_-);\Xi\right)\Big\}.$$

As an application of our theoretical results we investigate the reaction-diffusion equation

$$\begin{aligned} &v_t + \lambda v - \Delta v = f(v) + \sigma(x,t),\\ &v(x,t)|_{t=\tau} = v_\tau(x), \quad v(x,t)|_{\partial\mathcal{O}} = 0, \quad x \in \mathcal{O},\ t \geq \tau, \end{aligned}$$

where $\mathcal{O} \subset \mathbb{R}^N$, $N \in \mathbb{N}$, is a bounded smooth domain and $\lambda > 0$ is a constant. The nonlinear term $f(\cdot) \in \mathcal{C}^1(\mathbb{R};\mathbb{R})$ is assumed to satisfy the following standard conditions

$$\begin{array}{rclcrcl} f(s)s & \leq & -\alpha_1|s|^p + \beta_1, & \qquad & |f'(s)| & \leq & \kappa_2|s|^{p-2} + l_2,\\ |f(s)| & \leq & \alpha_2|s|^{p-1} + \alpha_2, & & f'(s) & \leq & -\kappa_1|s|^{p-2} + l_1, \end{array}$$

where $p \geq 2$ and all the coefficients are positive. The non-autonomous symbol σ is in a symbol space Σ constructed as the hull $\mathcal{H}(g)$ of a given non-autonomous function $g \in \Xi := \mathcal{C}\big(\mathbb{R}; L^2(\mathcal{O})\big)$, i.e., for $\theta_r g(\cdot) := g(\cdot + r)$ we have

$$\Sigma = \mathcal{H}(g) := \overline{\{\theta_r g : r \in \mathbb{R}\}}.$$

We suppose that Σ has finite fractal dimension (see the previous Theorem). For such problem we prove that it has a uniform attractor which is finite-dimensional in L^2 and in L^p, with $p > 2$.

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Old and New Results on Fourier Analysis

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On this talk, we give some old and new results on Fourier analysis. In particular, the absolute convergence of trigonometric Fourier series is studied. Furthermore, the Szasz and Stechkin theorems are established in this framework.

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Asymptotics for two two-terms fractional diffusion problems

Marcello D'Abbicco

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We consider the Cauchy-type problems associated to the time fractional differential equation

$$\begin{cases} \partial_t u + \partial_t^\beta u - \Delta u = g(t,x), & t>0,\ x\in\mathbb{R}^n \\ u(0,x) = u_0(x), \end{cases}$$

with $\beta \in (0,1)$, where the fractional derivative ∂_t^β is in Caputo sense, and to the space-time fractional differential equation

$$\begin{cases} \partial_t u + \partial_t^\beta (-\Delta)^{1-\beta} u - \Delta u = g(t,x), & t>0,\ x\in\mathbb{R}^n \\ u(0,x) = u_0(x), \end{cases}$$

where $(-\Delta)^{1-\beta}$ is the fractional Laplace operator of order $1-\beta$.

For the first model (see [1]), we provide sufficient conditions on the perturbation g which guarantees that the solution remains in $\mathcal{C}([0,\infty), H^s)$, and we exploit a dissipative-smoothing effect which allows to describe the asymptotic profile of the solution in low space dimension.

For the second model (see [2]), we provide sufficient conditions on the perturbation g which guarantees that the solution satisfies the same long-time decay estimates of the case $g=0$, assuming initial datum in $H^{s,m}$ for some $s>0$ and $m\in(1,\infty)$.

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Asymptotic behaviour of solutions for a strongly damped wave equation

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We derive long time $L^p - L^q$ decay estimates, in the full range $1 \leq p \leq q \leq \infty$, for time-dependent multipliers in which an interplay between an oscillatory component and a diffusive component with different scaling appears. We estimate $\|m(t,\cdot)\|_{M_p^q}$ as $t \to \infty$ (see [3]) for multipliers of type

$$m(t,\xi) = e^{-t|\xi|^\theta} \operatorname{sinc}(t\omega(\xi)),$$

where $\operatorname{sinc}\rho = \rho^{-1}\sin\rho$ is the cardinal sin function and $\omega(\xi) \sim |\xi|^\sigma$ as $\xi \to 0$, under the additional assumption that at law frequencies the scaling of the diffusive component is worse, i.e., $\theta > \sigma \geq 1$. These multipliers are related to the fundamental solutions to the Cauchy problem for the Viscoelastic Damped Plate equation [1]

$$u_{tt} + \Delta^2 u + \Delta^2 u_t = 0, \quad t \geq 0,\ x \in \mathbf{R}^n,$$

and to the Viscoelastic Damped Wave equation ([2] and [4])

$$u_{tt} - \Delta u - \Delta u_t = 0, \quad t \geq 0,\ x \in \mathbf{R}^n.$$

As it is done in the case of the free damped evolution equation $u_{tt} + (-\Delta)^\sigma u = 0$, we have to split our analysis for $\sigma > 1$ and $\sigma = 1$.

This talk is a joint work with

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Almost global existence for Kirchhoff equations

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Let H be a real Hilbert space, and let A be a positive self-adjoint operator on H with dense domain $D(A)$. We consider the abstract evolution equation

$$u''(t) + m\left(|A^{1/2}u(t)|^2\right)Au(t) = 0, \tag{1}$$

where $m : [0, +\infty) \to \mathbb{R}$ is a nonlinearity that we always assume to be of class C^1 and to satisfy the strict hyperbolicity assumption

$$m(\sigma) \geq \nu_0 > 0 \qquad \forall \sigma \geq 0. \tag{2}$$

Equation (1) is an abstract version of the hyperbolic partial differential equation introduced by G. Kirchhoff as a model for the small transversal vibrations of elastic strings or membranes.

Existence of local/global solutions to equation (1) with initial data

$$u(0) = u_0 \qquad\qquad u'(0) = u_1, \tag{3}$$

has been deeply investigated in the literature. Global-in-time solutions to problem (1)–(3) are known to exist in many different special cases, involving either special initial data (such as analytic data, quasi-analytic data, lacunary data), or special nonlinearities, or dispersive equations and small data, or spectral gap operators. For the moment, there are no examples of local-in-time solutions that blow-up in some sense in finite time and this remains the main notorious open question for Kirchhoff equations.

Let us now consider equation (1) with small initial data

$$u(0) = \epsilon u_0, \qquad\qquad u'(0) = \epsilon u_1,$$

where $(u_0, u_1) \in D(A^{3/4}) \times D(A^{1/4})$ and ϵ is a positive real number. In this case it turns out that the solution is defined at least in some interval $[0, T_\epsilon]$, where $T_\epsilon \to +\infty$ as $\epsilon \to 0^+$. Results of this type are usually referred to as "almost global existence". The simpler result of almost global existence is a by-product of the classical local existence result of A. Arosio and S. Panizzi that implies that, if

$$|u_1|^2 + |A^{1/4}u_1|^2 + |A^{1/2}u_0|^2 + |A^{3/4}u_0|^2 \leq \epsilon^2, \tag{4}$$

then the solution to problem (1)–(3) is defined at least on an interval $[0, T_\epsilon]$ with

$$T_\epsilon \geq \frac{c_0}{\epsilon^2}, \tag{5}$$

where c_0 is a suitable constant that depends on the behavior of m in a neighborhood of the origin. More refined results have been proved in the last years. In this direction R. Manfrin and later P. Baldi and E. Haus obtained an estimate of the form

$$T_\epsilon \geq \frac{c_1}{\epsilon^4}. \tag{6}$$

The question that we asked ourselves was the following.

> Assume that $\{(u_{0\epsilon}, u_{1\epsilon})\}$ is a family of initial data that converges in some sense to some limiting initial datum (u_0, u_1), and assume that equation (1) admits a global-in-time solution with initial datum (u_0, u_1). Can we conclude that the life span T_ϵ of solutions with initial data $(u_{0\epsilon}, u_{1\epsilon})$ tends to $+\infty$ as $\epsilon \to 0^+$?

We give a positive answer to this question.

- From the qualitative point of view, we prove more generally that the life span of solutions (which is either a positive real number or $+\infty$) is lower semicontinuous with respect to convergence of initial data in $D(A^{3/4}) \times D(A^{1/4})$, and that solutions depend continuously on initial data in the longest possible time interval. In particular, if for some reason the limiting initial datum (u_0, u_1) originates a global solution, then the life span of solutions with approximating initial data tends to $+\infty$.
- From the quantitative point of view, we provide estimates from below for T_ϵ when (u_0, u_1) belongs to three special classes of initial data for which global existence is known, namely data that are finite linear combinations of eigenvectors of A, analytic data, and quasi-analytic data.

The quantitative estimates are rather weak, for example in the classical quasi-analytic case they involve three nested logarithms, but we point out that in this result we have no smallness assumptions on $(u_{0\epsilon}, u_{1\epsilon})$.

From the technical point of view, our estimates from below for T_ϵ are based on some estimates from above for the growth in time of the energy of the solution with the limiting initial condition (u_0, u_1). These growth estimates are a by-product of the classical global existence results in analytic and quasi-analytic classes, but probably they had not been stated or proved explicitly before.

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Monotonicity results for generalized solutions to a class of forward-backward parabolic equations

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The one dimensional Perona-Malik equation is the partial differential equation

$$u_t = \left(\frac{u_x}{1+u_x^2}\right)_x = \frac{1-u_x^2}{(1+u_x^2)^2}\, u_{xx} \tag{1}$$

with homogeneous Neumann boundary conditions in some interval, and an initial datum at $t = 0$.

It is the one-dimensional version of the celebrated equation introduced in the 90s (see [3]) as a tool for image denoising. The main feature is that it is a *forward parabolic* equation in the subcritical regime where $|u_x| < 1$, and a *backward parabolic* equation in the supercritical regime where $|u_x| > 1$. Despite the analytical ill-posedness, numerical simulations exhibit more stability than expected. A satisfactory theory that explains this unexpected stability, usually called "the Perona-Malik paradox", is still out of reach. By "satisfactory theory" we mean a notion of solution that exists for a reasonable class of initial data, depends in a reasonable way on the initial condition, and to which solutions of approximating models converge.

A natural approach (see [2]) consists in defining generalized solutions as all possible limits of solutions to the semi-discrete approximation in which derivatives with respect to the space variable are replaced by difference quotients.

In this talk we present the two main results of [1]. The first one is a pathological example in which the initial data converge strictly as bounded variation functions, but strict convergence is not preserved for all positive times, and in particular many basic quantities, such as the supremum or the total variation, do not pass to the limit. Nevertheless, the second result shows that all generalized solutions satisfy some of the properties of classical smooth solutions, namely the maximum principle and the monotonicity of the total variation.

The verification of the counterexample relies on a comparison result with suitable sub/supersolutions. The monotonicity results are proved for a more general class of evolution curves, which contains weak solutions to many parabolic equations (and even forward-backward equations) in divergence form.

This talk is based on a joint work with

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Critical exponent of Fujita type for semilinear damped wave equations in Friedmann-Lemaître-Robertson-Walker spacetime

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We study the Cauchy problem for the semilinear massless damped wave equation:

$$u_{tt}(t,x) - (1+t)^{-2\ell}\Delta u(t,x) + \frac{\beta}{1+t}u_t(t,x) = |u|^p$$

with $0 < \ell < 1$, $\beta > 0$ and $p > 1$, in FLRW spacetime. The solution u^{lin} of the corresponding linear Cauchy problem will be represented in the explicit form using Fourier multipliers operators with multipliers expressed in terms of special functions. Our main goal is to prove global (in time) existence of solutions in the case of decelerating expansion universe with small initial data belongs to $L^1(\mathbf{R}^n) \cap H^{k-1}(\mathbf{R}^n)$, $k \geq 1$. We are focused in finding the critical exponent of Fujita type: $p_c(n,\ell) = 1 + \frac{2}{n(1-\ell)}$. It means that, if $1 < p \leq p_c(n,\ell)$ there exist small data for which u blow-up in finite time while if $p > p_c(n,\ell)$ the global solution has the same long time behavior of u^{lin}. We derive optimal $L^p - L^q$ estimates of u^{lin} and then we use Duhamel's principle to prove our results. This talk is a joint work with

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A shift of a Fujita type exponent for a class of semilinear evolution equations with time-dependent damping

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The goal of this presentation is to find a sharp exponent p_c, such that for $p > p_c$ there exists global existence (in time) of small data solution for a class of semilinear σ-evolution equations with effective scale-invariant time-dependent damping

$$u_{tt} + (-\Delta)^\sigma u + \frac{\mu}{1+t} u_t = |u|^p, \;\; u(0,x) = 0, \;\; u_t(x,0) = u_1(x), \quad t \geq 0, \quad x \in \mathbb{R}^n. \tag{1}$$

We suppose that $\mu, p, \sigma > 1$. To achieve this goal we use a fixed point argument in a special operator defined on a suitable function space. Under the assumption of small initial data $u_1 \in L^m(\mathbb{R}^n) \cap L^2(\mathbb{R}^n), m = 1, 2$, we find the critical exponent

$$p_c = \bar{p}(\gamma_m) = 1 + \frac{2}{\gamma_m}, \;\; \gamma_m = \frac{n}{m\sigma}.$$

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Reactive Flow of Unsteady Eyring-Powell Nanofluid Fluid with Thermocapillary Convective Boundary Conditions, Variable Thermal Conductivity and Radiant Heat through a Porous Medium

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The development of new products, reliability, and the efficient functioning of the system all depend on finding solutions to heat transfer issues in fluid flow in a variety of technical and industrial processes. As a result, effective thermal management is crucial for preventing overheating and achieving functional success. The development of nanofluids as a result of nanotechnology has significantly enhanced engineering heat transfer processes and cooling technologies. In this study, thermal conductivity is studied for an unsteady Eyring-Powell hyperbolic fluid with saturated porous media, thermocapillary convective boundary conditions, and variable thermal conductivity. The thermal conductivity and fluid viscosity are temperature-dependent properties. The Maple 18 Software package, the Spectral element approach, the C++ programming language, and the Garlekin weighted residue technique are used to generate and solve the differential equations governing the fluid flow numerically. Skin friction coefficient and the local Nusselt number rise when the values of the Marangon convective parameter, Biot number, stretching parameter, and Eyring-Powell material parameters increase. The fluctuations of the physical factors on the distributions of fluid temperature and velocity are shown graphically and quantitatively discussed in depth. Keywords: Porous media, Nanofluid, Thermal conductivity, Radiant energy, Reactive flows, Petroleum, Thermodynamic states and processes.

This is a joint work with:

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Sobolev solvability of elliptic homogenous linear equations on Borel measures

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In this talk, we present new results on Lebesgue solvability of the equation $A^*(D)f = \mu$, where $A(D)$ is an elliptic homogeneous differential operator and μ is a Borel measure. We emphasize the solvability for $p = \infty$ by duality method based on L^1-type estimates on measures for a special class of operators.

Joint work with:

- Victor Biliatto (UFSCar).

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Critical exponent versus critical regularity in semilinear evolution models

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In the last decades we have a lot of papers devoted to finding of critical exponents in semilinear evolution models with source nonlinearity of power type. The critical exponent is a threshold for blow-up in the subcritical case and global (in time) existence of small data Sobolev solutions in the supercritical case. Some of the exponents are well-known as, for example, the Strauss exponent or the Fujita exponent. Firstly, we will explain recent results for σ-evolution models with friction and visco-elastic type damping as well. Different methods for proving blow-up or global (in time) existence are sketched.
Then we explain that the question for critical exponent is a bit not precise. Instead one should pose the question for critical regularity. We present two recent results for semilinear wave models. Some open questions for de Sitter models of cosmology complete the talk.

Joint work with:

- Chen Wenhui, Marcelo Rempel Ebert, Giovanni Girardi, Abdelatif Kainane Mezadek and Mohamed Kainane Mezadek.

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Autonomous evolution inclusions with large diffusion and variable exponents

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In this talk I will present an overview about results on continuity of solutions with respect to initial conditions and parameters and as well upper semicontinuity of global attractors for autonomous evolution inclusions with variable exponents and large diffusion.

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Topological structural stability for nonautonomous random dynamical systems and applications in a SIR model

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In this work we study the continuity and topological structural stability of attractors for nonautonomous random differential equations obtained by small bounded random perturbations of autonomous semilinear problems. First, we study the existence and permanence of unstable sets of hyperbolic solutions. Then, we use this to establish the lower semicontinuity of nonautonomous random attractors and to show that the gradient structure persists under nonautonomous random perturbations. We apply the abstract results in an autonomous SIR model under nonautonomous random perturbations.

The main abstract results are in [1], which is a joint work with:

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And the applications in the SIR model is a work in progress with

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Asymptotic expansion of solutions to the wave equation with space-dependent damping

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Let $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ be an exterior domain with smooth boundary or $\Omega = \mathbb{R}^N$ $(N = 1)$. We consider the initial-boundary value problem

$$\begin{cases} \partial_t^2 u - \Delta u + a(x)\partial_t u = 0, & x \in \Omega, t > 0, \\ u(x,t) = 0, & x \in \partial\Omega, t > 0, \\ u(x,0) = u_0(x),\ \partial_t u(x,0) = u_1(x), & x \in \Omega. \end{cases} \tag{1}$$

Here, $u = u(x,t)$ is a real-valued unknown. The damping coefficient $a(x)$ is a positive and smooth function defined on $\mathbb{R}^N$ having bounded derivatives. Moreover, we assume that there are constants $a_0 > 0$ and $\alpha \in [0,1)$ such that

$$\lim_{|x|\to\infty} |x|^\alpha a(x) = a_0. \tag{2}$$

The aim is to obtain the asymptotic behavior of solutions as $t \to \infty$. In particular, we prove the higher-order asymptotic expansion of the solution in terms of the corresponding heat equation.

To state the result, we prepare the following. Let V_0 be the solution of

$$\begin{cases} a(x)\partial_t V_0 - \Delta V_0 = 0, & x \in \Omega, t > 0, \\ V_0(x,t) = 0, & x \in \partial\Omega, t > 0, \\ V_0(x,0) = u_0(x) + a(x)^{-1}u_1(x), & x \in \Omega. \end{cases} \tag{3}$$

and let $V_1, \dots, V_n$ be successively defined as the solutions of

$$\begin{cases} a(x)\partial_t V_j - \Delta V_j = -\partial_t V_{j-1}, & x \in \Omega, t > 0, \\ V_j(x,t) = 0, & x \in \partial\Omega, t > 0, \\ V_j(x,0) = -(-a(x))^{-j-1}u_1(x), & x \in \Omega. \end{cases} \tag{4}$$

Also, we define, for $s \in \mathbb{N} \cup \{0\}, m \in \mathbb{R}$,

$$H^{s,m}(\Omega) = \{f : \Omega \to \mathbb{R}; \|f\|_{H^{s,m}} = \sum_{|\gamma|\leq s} \|\langle x\rangle^m \partial_x^\gamma f\|_{L^2} < \infty\},$$
$$H_0^{s,m}(\Omega) = \overline{C_0^\infty(\Omega)}^{\|\cdot\|_{H^{s,m}}},$$

where $\langle x\rangle = \sqrt{1+|x|^2}$. The main result reads as follows:

Theorem. *Let $n \in \mathbb{N}\cup\{0\}$. Assume $n+1 < \frac{N-\alpha}{2\alpha}$. For any $\lambda \in (\frac{2\alpha}{2-\alpha}(n+1), \frac{N-\alpha}{2-\alpha})$, there exist $s = s(n) \in \mathbb{N}$ and $m = m(n,\alpha,\lambda) > 0$ such that if*

$$u_0 \in H^{s+1,m}(\Omega) \cap H_0^{s,m}(\Omega), \quad u_1 \in H_0^{s,m}(\Omega), \tag{5}$$

then the solution u to (1) *satisfies*

$$\left\| u(t) - \sum_{j=0}^{n} \partial_t^j V_j(t) \right\|_{L^2} \leq C(1+t)^{-\frac{\lambda}{2} - \frac{(2n+1)(1-\alpha)}{2-\alpha} + \frac{\alpha}{2(2-\alpha)}} \tag{6}$$

for $t > 0$.

The conclusion of the theorem implies that the solution u can be asymptotically expanded as

$$u = V_0 + \partial_t V_1 + \partial_t^2 V_2 + \cdots + \partial_t^n V_n + \text{“remainder term”}$$

in terms of the solutions of the parabolic problems (3) and (4).

This talk is based on a joint work with

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