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Non-embeddable CR-structures and dilations

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1 Introduction

CR-manifolds arise naturally as boundaries of complex spaces. The class of strictly pseudoconvex CR-manifolds arise more specifically as boundaries of Stein spaces. The question of whether an abstract CR-manifold is the boundary of a complex manifold is still the object of research, and is best understood in the strictly pseudoconvex case.

In [BM] Boutet de Monvel showed that any compact, orientable n-dimensional , $n \geq 5$, strictly pseudoconvex CR-manifold is embeddable in some C^N. On the other hand 3-dimensional compact orientable CR-manifolds are not necessarily embeddable, the first example is attributed to Andreotti in [R].

Nirenberg [N] constructed, for the first time, examples of locally non-embeddable of 3-dimensional strictly pseudoconvex CR-manifolds. The situation was further clarified by Jacobowitz and Treves [JT1,2], to show that, in fact, non-embeddable CR-structures are dense in the space of CR-structures over a 3-dimensional manifold. See [F] for a different approach based on pasting of structures. In the higher dimensional case, local embeddability occurs always in dimensions greater than 5 [K],[A], but the 5-dimensional case is not settled.

In this paper we construct global examples of open non-embeddable strictly pseudoconvex CR-manifolds in any dimensions and use dilations to get examples in small open sets. In dimension 3 we obtain, in a limit, local non-embeddable examples, but in higher dimensions the limit structure loses continuity at one point.

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2 Global examples

We start collecting the following definitions. Let M be a real manifold of dimension 2n+1.

Definition 1 *A CR-structure over M is given by a subbundle $\Delta^{1,0}$ of the complexified tangent bundle* $T_{\mathbb{C}}M$ *such that*

i)$\Delta^{1,0} \cap \overline{\Delta^{1,0}} = 0$

ii)$\Delta^{1,0}$ *is involutive, that is, if L and L' are local sections of* $\Delta^{1,0}$*, then so is [L,L'].*

Observe that if M is embedded in $\mathbb{C}^{n+1}$ then naturally $\Delta^{1,0} = T_{\mathbb{C}}M \cap T^{1,0}(\mathbb{C}^{n+1})$.

If M is three dimensional, a CR-structure is given simply by a complex vector field with linearly independent real and imaginary parts at every point.tex

Consider a manifold M with CR-structure $\Delta^{1,0}$.

Definition 2 *A CR-function on M is a function f such that Lf=0 for any local section L of* $\Delta^{1,0}$.

Definition 3 *A CR-structure* $\Delta^{1,0}$ *on M is embeddable if there exists an embedding* $F : M \to \mathbb{C}^N$ *for some N, where* $F_*(\Delta^{1,0}) \subset T^{1,0}(\mathbb{C}^N)$.

A CR-structure on M is locally embeddable at $p \in M$, *if there exists a neighborhood of p which is embeddable.*

In [F] global examples of non-embeddable CR-structures are constructed over the 3-dimensional sphere S^3. The examples can be defined on sufficiently large open subset of S^3.

As a specific example, consider SU(2) acting on $\mathbb{C}^2$ as a matrix group. If $G \subset SU(2)$ is a finite group, it is a classical result that $G\backslash\mathbb{C}^2$ can be given a structure of an analytic space with one isolated singularity.

For instance, if

$$G = G_k = \left\{ \begin{pmatrix} g & 0 \\ 0 & \bar{g} \end{pmatrix} \in SU(2) \mid g^k = 1 \right\}$$

then $A_{k-1} = G_k\backslash\mathbb{C}^2$ is isomorphic to the hypersurface defined by the equation $x_1x_2 - x_3^k = 0$ We have then the diagram

$$\begin{array}{ccc} S^3 & \to & C^2 \\ \downarrow & & \downarrow \\ G_k \backslash S^3 & \to & A_{k-1} \end{array}$$

Let $A^c_{k-1} = \{ (x_1, x_2, x_3) \in C^3 \mid x_1x_2 - x_3{}^k = c \}$ be a smoothing of the singularity. Then $M_c = A^c_{k-1} \cap S^5$ is diffeomorphic to $G_k \backslash S^3$ for small c. As M_c is an hypersurface in A^c_{k-1}, it has an induced CR-structure. Let the CR-structure on S^3 be given by the pull-back by a covering map $\pi_c : S^3 \to M_c$.

Theorem 1 *Consider* S^3 *with the CR-structure given by* $\pi_c^{-1}\{\Delta^{1,0}(M_c)\}$. *Then CR-functions on* S^3 *are pullbacks of CR-functions on* M_c.

Rossi's example is contained in this list. Especifically we have: Let (z_1, z_2) be coordinates on C^2 and $S^3 \subset C^2$ given by $| z_1 |^2 + | z_2 |^2 = R$. Consider the smoothing $V_\epsilon = \{(x, y, z) \in C^3 \mid xy - z^2 = \epsilon\}$ and the map $\pi_\epsilon : S^3 \to V_\epsilon$ given by

$$x = z_1^2 + \frac{\epsilon}{R^2}\bar{z}_2{}^2$$

$$y = z_2^2 + \frac{\epsilon}{R^2}\bar{z}_1{}^2$$

$$z = z_1z_2 - \frac{\epsilon}{R^2}\bar{z}_1\bar{z}_2$$

It is a simple matter to verify that the vector field on S_3

$$L = z_2\frac{\partial}{\partial \bar{z}_1} - z_1\frac{\partial}{\partial \bar{z}_2}$$

defines the standard CR-structure induced by the standard imbedding in C^2.

The CR-structure induced by the immersion π_ϵ is given by

$$Z = L + \frac{\epsilon}{R^2}\bar{L}$$

In this CR-structure CR-functions are symmetric, that is, $f(z_1, z_2) = f(-z_1, -z_2)$. We will use the following generalization of this example:

Theorem 2 **([F])** *Even if Z is only defined on a neighborhood of the equator* $\mathrm{Re} z_1 = 0$, *CR-functions are also symmetric on a smaller neighborhood of the equator.*

To get global higher dimensional examples, we will consider the (2n-1)-dimensional sphere $S^{2n-1} \subset C^n$, $S^{2n-1} = \{ (z_1, ..., z_n) \in C^n \mid |z_1|^2 + |z_2|^2 + ... + |z_n|^2 = 1 \}$.

Our construction will be on a neighborhood of the $z_1 z_2$ – equator. Let $E_\delta = \{ (z_1, ..., z_n) \in S^{2n-1} \mid |z_3|^2 + ... + |z_n|^2 < \delta \}$ The standard structure is generated by the fields:

$$L_{ij} = z_i \frac{\partial}{\partial \bar{z}_j} - z_j \frac{\partial}{\partial \bar{z}_i}$$

For instance on the $z_1 z_2$ – equator we have the vector field

$$L_{21} = z_2 \frac{\partial}{\partial \bar{z}_1} - z_1 \frac{\partial}{\partial \bar{z}_2}$$

To construct the non-embeddable example, define the action $Z_2 \times C^n \rightarrow C^n$

$$(z_1, ..., z_n) \rightarrow (-z_1, ..., -z_n)$$

As in the two dimensional case we obtain

$$Z_2 \backslash C_n \simeq V_0 = \{ (x, y, z, w_3, ..., w_n) \mid xy - z^2 = 0 \}$$

And if we define the smoothing

$$V_\epsilon = \{ (x, y, z, w_3, ..., w_n) \mid xy - z^2 = \epsilon \}$$

and the map $\pi_\epsilon : E_\delta \rightarrow V_\epsilon$ given by

$$x = z_1^2 + \frac{\epsilon}{R^2} \bar{z}_2^{\,2}$$

$$y = z_2^2 + \frac{\epsilon}{R^2} \bar{z}_1^{\,2}$$

$$z = z_1 z_2 - \frac{\epsilon}{R^2} \bar{z}_1 \bar{z}_2$$

$$w_3 = z_3$$

$$\vdots$$

$$w_n = z_n$$

where $R = 1 - (|z_3|^2 + ... + |z_n|^2)$.

Using theorem 1 we obviously get the following:

Theorem 3 *The CR-structure on E_δ induced by π_ϵ is not embeddable.*

The CR-structure is generated by the vector fields:

$$Z_{ij} = z_i \frac{\partial}{\partial \bar{z}_j} - z_j \frac{\partial}{\partial \bar{z}_i}$$

if i and j are different from 1 and 2 and

$$Z_i = z_i(\bar{z}_1 \frac{\partial}{\partial \bar{z}_1} + \bar{z}_2 \frac{\partial}{\partial \bar{z}_2}) - R \frac{\partial}{\partial \bar{z}_i}$$

and

$$Z_{21} = L + \frac{\epsilon}{R^2} \overline{L} \quad \text{where} \quad L = L_{21}$$

Observe that, as in the case of the 3-dimensional sphere, we need this structure defined only in a sufficiently large open subset of E_δ. Also the deformation of the CR-structure is reflected only in the field Z_{21}.

3 Dilations and local examples

We will work out the formulas for the three dimensional case. The higher dimensional case is similar. The simplest way to describe a dilation is using the quadric instead of the sphere. We relate them using the Cayley transform.

$$Q = \{ (x,z) \in \mathbf{C}^2 \mid \mathrm{Im} z = |x|^2 \}$$

$$S^3 = \{ (z_1, z_2) \in \mathbf{C}^2 \mid |z_1|^2 + |z_2|^2 = 1 \}$$

we define

$$C^{-1} : Q \to S^3 \text{ as } z_1 = \frac{z - i}{z + i} \quad z_2 = \frac{2x}{z + i}$$

The inverse transform is

$$C : S^3 \to Q \quad \text{as} \quad z = -i \frac{1 + z_1}{z_1 - 1} \quad x = -i \frac{z_2}{z_1 - 1}$$

The standard vector field over Q is given by

$$C_*(z_2 \frac{\partial}{\partial \bar{z}_1} - z_1 \frac{\partial}{\partial \bar{z}_2}) = \frac{(\bar{z} - i)^2}{z + i} [\frac{1}{2} \frac{\partial}{\partial \bar{x}} - ix \frac{\partial}{\partial \bar{z}}]$$

The non-embeddable structure constructed in the previous section is therefore the vector field on the quadric given by

$$C_*(L_{21} + \frac{\epsilon}{R^2}\overline{L}_{21}) = \frac{(\bar{z}-i)^2}{z+i}[\frac{1}{2}\frac{\partial}{\partial \bar{x}} - ix\frac{\partial}{\partial \bar{z}}] + \frac{\epsilon}{R^2}\frac{(z+i)^2}{\bar{z}-i}[\frac{1}{2}\frac{\partial}{\partial x} + i\bar{x}\frac{\partial}{\partial z}]$$

As in the case of the sphere, this structure is not embeddable even if it is defined on a small neighborhood of the image of the equator $(\mathrm{Im} z_1)^2 + |z_2|^2 = 1$, that is, $|x|^4 + (\mathrm{Re} z)^2 = 1$.

We will construct now a local example by bringing this non-embeddable structure arbitrarily close to the origin using dilations.

A dilation in the quadric is the map $T_t(x,z) = (tx, t^2 z)$ where $t \in \mathbf{R}^+$. If we suppose that t is a function of (x,z) then we obtain after some computation $T_*^{-1}(Z_\epsilon) = L + \epsilon\overline{L}$ where

$$L = \frac{(t^2\bar{z}-i)^2}{t(t^2 z+i)}\left[\frac{1}{2}\frac{\partial}{\partial \bar{x}} - ix\frac{\partial}{\partial \bar{z}} - \frac{\frac{1}{2}\frac{\partial t}{\partial \bar{x}} - ix\frac{\partial t}{\partial \bar{z}}}{t + \bar{x}\frac{\partial t}{\partial \bar{x}} + x\frac{\partial t}{\partial x} + 2\bar{z}\frac{\partial t}{\partial \bar{z}} + 2z\frac{\partial t}{\partial z}}\left(\bar{x}\frac{\partial}{\partial \bar{x}} + x\frac{\partial}{\partial x} + 2\bar{z}\frac{\partial}{\partial \bar{z}} + 2z\frac{\partial}{\partial z}\right)\right]$$

If t is constant we obtain the vector field

$$L = \frac{(t^2\bar{z}-i)^2}{t(t^2 z+i)}\left[\frac{1}{2}\frac{\partial}{\partial \bar{x}} - ix\frac{\partial}{\partial \bar{z}}\right]$$

Therefore the CR-structure defined by $L + \epsilon\overline{L}$ is the same as the one defined by the vector field

$$Z = \frac{1}{2}\frac{\partial}{\partial \bar{x}} - ix\frac{\partial}{\partial \bar{z}} + \epsilon\frac{(t^2 z+i)^3}{(t^2\bar{z}-i)^3}[\frac{1}{2}\frac{\partial}{\partial x} + i\bar{x}\frac{\partial}{\partial z}]$$

Let t(x,z) be a function which is constantly equal to $t_n = n$ on the annulus $A_n = \{(x,z) \in Q \mid \frac{1}{(n+\delta)^4} < |x|^4 + (\mathrm{Re} z)^2 < \frac{1}{(n-\delta)^4}\}$. By using the dilation T_{t_n} we obtain a CR-structure which is not embeddable on this annulus. Observe that it is a tubular neighborhood of the image of the equator $|x|^4 + (\mathrm{Re} z)^2 = 1$ by the inverse dilation $T_{t_n}^{-1}$. We find now a function $\epsilon(x,z)$, constant on those annuli, which makes the field Z_ϵ^t differentiable at the origin. For instance, we can choose the function

$$\epsilon\frac{(t^2 z+i)^3}{(t^2\bar{z}-i)^3} = \sum \epsilon_n(x,z)\frac{(t_n^2 z+i)^3}{(t_n^2\bar{z}-i)^3}$$

where $\epsilon_n(x,z)$ has compact support on

$$\frac{1}{(n+\delta')^4} < |x|^4 + (\mathrm{Re} z)^2 < \frac{1}{(n-\delta')^4}$$

and is constant on the annuli A_n with $0 < \delta < \delta'$ and such that the C^∞ – norm decreases fast enough. We obtained the following

Theorem 4 *Z^t_ϵ is not locally embeddable at the origin $(x,z)=(0,0)$.*

Observations:

1)Z^t_ϵ can be made arbitrarily close to the standard field Z by making ϵ arbitrarily small.

2)We didn't use the formula for the pull-back with variable $t(x,z)$. If we had used it, we would lose continuity of the structure at the origin. In the higher dimensional case we are obliged to use that formula, to garantee the integrability of the CR-structure, and therefore we get a CR-structure on the neighborhood of the origin which is not continuous at the origin.

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