

Teorema. *Seja $g \in A$ com número de rotação de tipo limitado por $N > 0$. Então $DR(g)$ é um operador linear compacto com exatamente um autovalor expansivo e o restante do espectro inteiramente contido no disco $D(0; r)$, onde $0 < r < 1$ depende somente de N .*

A prova deste teorema utiliza certas técnicas holomorfas de D. Sullivan e resultados bastante recentes de C. McMullen, *Renormalization and 3-manifolds which fiber over the circle*, 1995. Este resultado tem importantes conseqüências, dentre as quais citamos apenas a seguinte.

Corolário. *Se $f, g \in C^r(r \geq 3)$ possuem o mesmo número de rotação α irracional de tipo limitado, então existe $0 < \beta < 1$ dependendo apenas de α tal que f e g são $C^{1+\beta}$ -conjugados.*

Continuamos investindo nossos esforços no sentido de estender estes resultados para qualquer número de rotação irracional, e não meramente aqueles de tipo limitado. — (18 de abril de 1995).

ATTRACTORS FOR PARABOLIC PROBLEMS WITH NONLINEAR BOUNDARY CONDITIONS IN FRACTIONAL POWER SPACES

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Let Ω be a bounded smooth domain of $\mathbb{R}^n$. In this work we consider reaction diffusion equations with dispersion of the form

$$\begin{cases} u_t = \text{Div}(a \nabla u) - \sum_{j=1}^n B_j(x) \frac{\partial u}{\partial x_j} - \\ \quad - \lambda u + f(u), \quad \text{in } \Omega, \\ \frac{\partial u}{\partial n_a} = g(u), \quad \text{on } \partial\Omega. \end{cases} \quad (1)$$

where $a \in C^1(\bar{\Omega})$, $a(x) > m_0 > 0$, $x \in \Omega$, $\lambda > 0$,

$$\frac{\partial u}{\partial n_a} = \langle a \nabla u, \vec{n} \rangle,$$

$\vec{n}$ is the outward normal and B_j is continuous in $\bar{\Omega}$, $j = 1, \dots, n$. Let $f: \mathbb{R} \rightarrow \mathbb{R}$, $g: \mathbb{R} \rightarrow \mathbb{R}$ be smooth functions, such that there exists ξ^0 such that, there are constants $c^0 = c^0(\xi^0)$ and $d^0 = d^0(\xi^0)$ with $\frac{f(s)}{s} < c^0$ and $\frac{g(s)}{s} < d^0$, for all $\xi^0 < |s|$ (the dissipative condition). Moreover,

given the eigenvalue problem

$$\begin{cases} -\text{Div}(a \nabla v) + \sum_{j=1}^n B_j(x) \frac{\partial v}{\partial x_j} + \\ \quad + \lambda v - c_0 v = \mu v, \quad \text{in } \Omega, \\ \frac{\partial v}{\partial n_a} = d_0 v \quad \text{on } \partial\Omega \end{cases} \quad (2)$$

we will assume that c^0 and d^0 are such that the principal eigenvalue (one with smaller real part) of the problem (2) is positive.

Observe that our dissipation condition on the equation allow either c_0 or d_0 to be positive. In other words, we allow either f or g to be a source of heat.

It has been shown by C. V. Pao [Asymptotic Behavior and Nonexistence of Global Solutions for a Class of Nonlinear Boundary Value Problems of Parabolic Type, *Journal of Math. Anal. and Appl.*, **65** (1978), 616-637] that if f is a source of heat and if $g = 0$ then we have blow up in finite time. Our aim here is to control the increase of heat by means of a dissipative flux through the boundary. To accomplish this goal we need to introduce some kind of "competition" between f and g . The condition on c_0 and d_0 is a precise formulation of this "competition" between f and g .

Our goal is to prove that this competition between f and g is sufficient to assure the existence of global attractors. To accomplish this we will work in $L^p(\Omega)$, for a suitable choice of $1 < p < \infty$, instead of $L^2(\Omega)$, and then follow the general approach developed by H. Amann [Semigroups and Nonlinear Evolution Equations, *Linear Algebra and its Appl.*, **84** (1986), 3-32] and [Parabolic Evolution Equations and Nonlinear Boundary Conditions, *Journal of Diff. Eq.*, **72** (1988), 201-269]. The main difference is that, in our approach, instead of working in the Sobolev spaces $W^{k,p}(\Omega)$, we work in the fractional power spaces associated to the operator defined by the linear part of (1) with homogeneous boundary conditions. These fractional power spaces turn out to be the so called "Lebesgue Spaces" $H_p^s(\Omega)$ (see H. Triebel [Interpolation Theory, Function Spaces, Differential Operators, North-Holland, Amsterdam, 1978], for a general discussion about these spaces and their relation with differential operators and interpolation theory). In this way we are able to use the well developed theory of sectorial operators as described, for example, in D. Henry [Geometric Theory of Semilinear Parabolic Equations, *Lectures Notes in Mathematics*, **840**, Springer-Verlag, 1981]. This ap-

proach leads, in our opinion, to a considerable simplification of Amann's arguments.

We then consider an abstract problem, whose solutions are classical solutions of (1) at any positive time. Within this abstract framework we are then able to prove existence of solution and finally establish existence of a global attractor, using comparison results. — (18 de abril de 1995).

SMOOTH TRIANGULAR MAPS OF THE SQUARE WITH CLOSED SET OF PERIODIC POINTS

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For $I = [0, 1]$, let $F: I^2 \rightarrow I^2$ be a continuous map of the square I^2 such that $F(x, y) = (f(x), g_x(y))$, where g_x is a family of maps of the interval depending continuously on x . As in (Kolyada, 1992), the map F will be said to be a *triangular map of the square* and f will be called the *base of F* .

Some time ago, Sharkovsky suggested that many of the regularity properties in the dynamics of maps of the interval might also hold for triangular maps of the square. This suggestion has been extensively investigated by Kolyada, 1992, partly in collaboration with Sharkovsky. While certain important properties carry over, others do not.

In this paper we consider the property that, if the set $\text{Per}(F)$ of the periodic points is closed, then $\Omega(F) = \text{Per}(F)$, where $\Omega(F)$ denotes the set of nonwandering points of F .

This property is known to hold for interval maps (Nitecki, 1982), and it follows from an example of Kolyada (Theorem 3, Kolyada, 1992) that this result is not true for continuous triangular maps of the square.

However for smooth triangular maps of the square we have the following result (Arteaga, 1995).

Theorem. *Let $F: I^2 \rightarrow I^2$ be a C^1 triangular map of the square such that its base map has every periodic point hyperbolic and $\text{Per}(F)$ is a closed set. Then $\Omega(F) = \text{Per}(F)$. — (18 de abril de 1995).*

CONJUNTOS MINIMAIS EM SISTEMAS DINÂMICOS RANDÔNICOS

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Sejam G um grupo de Lie e g a sua álgebra de Lie. A integral multiplicativa estocástica [Anne Estrade, *Ann. Inst. H. Poincaré*, vol. 28, n. 1, pp. 107-129, 1992] de um semimartingale X_t na álgebra de Lie, fornece um exemplo de Sistema Dinâmico Randômico (L. Arnold, *Generation of Random Dynamical Systems*, preprint University Bremen, 1993] que generaliza o produto aleatório de matrizes independentes identicamente distribuídas.

Considerando $M = G/H$ um espaço homogêneo compacto de G , o sistema dinâmico randômico induzido pela integral multiplicativa é da forma

$$F_t((\omega, x)) = (\theta_x(\omega), \prod_0^t \exp(dX_u) \cdot x)$$

onde θ é um sistema dinâmico métrico ergódico no espaço de probabilidade (Ω, P) e $\prod_0^t \exp(dX_u)$ é a integral multiplicativa estocástica que é um cociclo em relação a θ . A definição de conjunto minimal utiliza somente a topologia de M . Esta noção é baseada principalmente num trabalho de H. Furstenberg [*Lecture Notes in Math.*, 842, Springer Verlag, 1981].

Um conjunto invariante de F é uma função mensurável $\Phi: \Omega \rightarrow \mathcal{H}(M)$ tal que

$$\prod_0^t \exp(dX_u)(\Phi(\omega)) = \Phi(\theta_t(\omega))P - \text{q.s.}$$

Dados dois conjuntos invariantes Φ_1 e Φ_2 definimos a ordem $\Phi_1 \leq \Phi_2$ se $\Phi_1(\omega) \subset \Phi_2(\omega)P - \text{q.s.}$

Um conjunto invariante Φ_m é minimal quando para qualquer outro conjunto invariante Φ tal que $\Phi \leq \Phi_m$ então $\Phi = \Phi_mP - \text{q.s.}$

Lema. *Como M é compacto então existe um conjunto minimal.*

No caso da dinâmica topológica temos a seguinte caracterização dos conjuntos minimais: M é minimal se e somente se a órbita por todo ponto de M é densa em M . [J.R. Brown, *Ergodic Theory and Topological Dynamics*, Academic Press, New York, 1976]. Este é o tipo de caracterização que procuramos para os conjuntos minimais de sistemas dinâmicos randômicos. O resultado que obtivemos até agora é o seguinte.

Dados V um aberto de M , e Φ um conjunto invariante de F , definimos o número $n(V, \Phi) = P\{\omega: \Phi(\omega) \cap V \neq \emptyset\}$.

Teorema. *Se Φ é minimal então para todo $A \subset \Omega: P\{A\} > 0$, todo u seleção mensurável de Φ e todo*