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"A Note on Johnson, Minkoff and Phillips' Algorithm for the Prize-Collecting Steiner Tree Problem"

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A Note on Johnson, Minkoff and Phillips' Algorithm for the Prize-Collecting Steiner Tree Problem

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Abstract

The primal-dual scheme has been used to provide approximation algorithms for many problems. Goemans and Williamson gave a $(2-\frac{1}{n-1})$-approximation for the Prize-Collecting Steiner Tree Problem that runs in $O(n^3 \log n)$ time—it applies the primal-dual scheme once for each of the n vertices of the graph. Johnson, Minkoff and Phillips proposed a faster implementation of Goemans and Williamson's algorithm. We present a proof that the approximation ratio of this implementation is exactly 2.

1 Introduction

Consider a graph $G = (V, E)$, a function c from E into $\mathbf{Q}_{\geq}$ (non-negative rationals) and a function π from V into $\mathbf{Q}_{\geq}$. For any subset F of E and any subset W of V, let $c(F) := \sum_{e\in F} c_e$ and $\pi(W) := \sum_{w\in W} \pi_w$. The **Prize-Collecting Steiner Tree Problem** (PCST) consists of the following: given G, c, and π, find a tree T in G such that $c(E_T) + \pi(V \setminus V_T)$ is minimum. (We denote by V_H and E_H the vertex and edge sets of a graph H respectively.) The rooted variant of the problem requires T to contain a given root vertex.

Goemans and Williamson [1, 2] used a primal-dual scheme to derive a $(2-\frac{1}{n-1})$-approximation for the rooted PCST, where $n := |V|$. Trying all possible choices for the root, they obtained a $(2-\frac{1}{n-1})$-approximation for the unrooted PCST. The resulting algorithm runs in time $O(n^3 \log n)$. Johnson, Minkoff and Phillips [3] proposed a modification of the algorithm that permits running the primal-dual scheme only once, resulting in a running-time of $O(n^2 \log n)$. They claimed the modification, which we refer to as JMP, achieves the same approximation ratio as the original algorithm for the unrooted PCST. Unfortunately, their claim does not hold, as we show here.

In this note, first we present a proof that the JMP algorithm is a 2-approximation. This proof is not straightforward, since it envolves some non-trivial technical details. Second we present an example where the JMP algorithm achieves a ratio of 2, contradicting the statement of Johnson, Minkoff and Phillips [3] that their algorithm achieves a ratio of $2-\frac{1}{n-1}$. These two together settle the approximation ratio of the JMP algorithm.

2 Notation

The description of the algorithm in the next section will use a notation slightly different than the one in the seminal paper by Goemans and Williamson [1]. With this notation, we found it easier to be sure of the correctness of all the technical details in the analysis.

For any subset X of V, let $\overline{X} := V \setminus X$. For any collection $\mathcal{L}$ of subsets of V and any e in E, let

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$\mathcal{L}(e) := \{L \in \mathcal{L} : e \in \delta_G L\}$, where $\delta_G L$ stands for the set of edges of G with one end in L and the other in $\overline{L}$. Also, $\bigcup \mathcal{L}$ denotes the union of all sets in $\mathcal{L}$. For any function y from $\mathcal{L}$ into $\mathbb{Q}_{\geq}$ and any subcollection $\mathcal{M}$ of $\mathcal{L}$, let $y(\mathcal{M}) := \sum_{L \in \mathcal{M}} y(L)$.

We say that y **respects** a function c defined on E (relative to $\mathcal{L}$) if

$$y(\mathcal{L}(e)) \leq c_e \quad \text{for each } e \text{ in } E. \tag{1}$$

An edge e is **tight for** y if equality holds in (1). The inequality in (1) is the usual restriction on edge e: the sum of y_R for all R in $\mathcal{L}$ that e "crosses" does not exceed c_e.

We say y **respects** a function π defined on V (relative to $\mathcal{L}$) if

$$\textstyle\sum_{S \subseteq L} y_S \leq \pi(L) \quad \text{for each } L \text{ in } \mathcal{L}. \tag{2}$$

An element L of $\mathcal{L}$ is **tight for** y if equality holds in (2). The inequality in (2) is the usual one for PCST and says that the sum of y_S for all S in $\mathcal{L}$ contained in L does not exceed the sum of the "penalties" (π-values) of all elements in L.

An edge is **internal to** a partition $\mathcal{P}$ of V if both of its ends are in the same element of $\mathcal{P}$. All other edges are **external to** $\mathcal{P}$. For any external edge, there are two elements of $\mathcal{P}$ containing its ends. We call these two elements the **extremes** of the edge in $\mathcal{P}$.

A collection $\mathcal{L}$ of subsets of V is **laminar** if, for any two elements L_1 and L_2 of $\mathcal{L}$, either $L_1 \cap L_2 = \emptyset$ or $L_1 \subseteq L_2$ or $L_1 \supseteq L_2$. The collection of maximal elements of a laminar collection $\mathcal{L}$ will be denoted by $\mathcal{L}^*$. So, $\mathcal{L}^*$ is a collection of disjoint subsets of V.

We say a forest F in G **is connected in** a subset L of V if $F[V_F \cap L]$ is connected. For any laminar collection $\mathcal{S}$ of subsets of V, we say a tree T of G **has no bridges in** $\mathcal{S}$ if $|\delta_T S| \neq 1$ (therefore, either $\delta_T S = \emptyset$ or $|\delta_T S| \geq 2$) for all S in $\mathcal{S}$.

We denote by $\text{opt}(\text{PCST}(G, c, \pi))$ the minimum value of the expression $c(E_T) + \pi(\overline{V_T})$ when T is a tree in G.

3 Johnson, Minkoff and Phillips' algorithm

We describe the JMP algorithm and show that it is a 2-approximation for the PCST. The algorithm receives G, c, π and returns a tree T in G such that $c(E_T) + \pi(\overline{V_T}) \leq 2\,\text{opt}(\text{PCST}(G, c, \pi))$. Each iteration starts with a spanning forest F in G, a laminar collection $\mathcal{L}$ of subsets of V with $\bigcup \mathcal{L} = V$, a subcollection $\mathcal{S}$ of $\mathcal{L}$, and a function y from $\mathcal{L}$ into $\mathbb{Q}_{\geq}$ such that the following invariants hold:

(i1) all edges of F are internal to $\mathcal{L}$;

(i2) F is connected in each L in $\mathcal{L}$;

(i3) y respects c and π;

(i4) any edge of F is tight for y;

(i5) any element of $\mathcal{S}$ is tight for y;

(i6) for any vertex u and any tree T in G, if T is connected in each L in $\mathcal{L}$ and has no bridges in $\mathcal{S}$, then

$$\textstyle\sum_{e \in E_T} y(\mathcal{L}(e)) + \sum_{L \subseteq \overline{V_T}} y_L \leq 2y(\mathcal{L} \setminus \mathcal{L}_u)\,, \tag{3}$$

where $\mathcal{L}_u := \{L \in \mathcal{L} : u \in L\}$.

The first iteration starts with $F = (V, \emptyset)$, $\mathcal{L} = \{\{v\} : v \in V\}$, $\mathcal{S} = \emptyset$, and $y = 0$. Each iteration consists of the following:

Case 1: $|\mathcal{L}^\bullet \setminus \mathcal{S}| > 1$.

Let ε be the largest number in $\mathbb{Q}_\geq$ such that the function y' defined by

$$y'_L = \begin{cases} y_L + \varepsilon\,, & \text{if } L \in \mathcal{L}^\bullet \setminus \mathcal{S} \\ y_L\,, & \text{otherwise} \end{cases}$$

respects c and π.

Subcase 1A: some edge e external to $\mathcal{L}^\bullet$ is tight for y'.

Let L_1 and L_2 be the extremes of e in $\mathcal{L}^\bullet$. Set $y'_{L_1 \cup L_2} := 0$ and start a new iteration with $F + e$, $\mathcal{L} \cup \{L_1 \cup L_2\}$, $\mathcal{S}$, y' in the roles of F, $\mathcal{L}$, $\mathcal{S}$, y respectively.

Subcase 1B: some element L of $\mathcal{L}^\bullet \setminus \mathcal{S}$ is tight for y'.

Start a new iteration with F, $\mathcal{L}$, $\mathcal{S} \cup \{L\}$, y' in the roles of F, $\mathcal{L}$, $\mathcal{S}$, y respectively.

Case 2: $|\mathcal{L}^\bullet \setminus \mathcal{S}| = 1$.

Let M be the only element of $\mathcal{L}^\bullet \setminus \mathcal{S}$. Call subalgorithm GW-PRUNING with arguments $F[M]$, $\{L \in \mathcal{L} : L \subseteq M\}$, and $\{L \in \mathcal{S} : L \subseteq M\}$. The subalgorithm returns a subcollection $\mathcal{Z}$ of $\{L \in \mathcal{S} : L \subseteq M\}$. Return $T := F[M] - (\bigcup \mathcal{Z} \cap M)$ and stop. ($F[M]$ is the subgraph of F induced by M.)

Subalgorithm GW-PRUNING receives a tree T_0, a laminar collection $\mathcal{L}$ of subsets of V_{T_0}, and a subcollection $\mathcal{S}$ of $\mathcal{L}$. Each iteration begins with a subcollection $\mathcal{Z}$ of $\mathcal{S}$ such that $T := T_0 - \bigcup \mathcal{Z}$ is a tree connected in each element of $\mathcal{L}$. The first iteration begins with $\mathcal{Z} = \emptyset$.

Case A: $|\delta_T S| = 1$ for some S in $\mathcal{S}$.

Start a new iteration with $\mathcal{Z} \cup \{S\}$ in place of $\mathcal{Z}$.

Case B: $|\delta_T S| \neq 1$ for each S in $\mathcal{S}$.

Return $\mathcal{Z}$ and stop.

The subcollection $\mathcal{Z}$ of $\mathcal{S}$ that subalgorithm GW-PRUNING returns is such that the forest $T_0 - \bigcup \mathcal{Z}$ is a tree, is connected in each element of $\mathcal{L}$, and has no bridges in $\mathcal{S}$.

The following lemma and its corollary establish the correct lower bound on $\text{opt}(\text{PCST}(G, c, \pi))$.

Lemma 3.1 *Given a connected subgraph T of G, a laminar collection $\mathcal{L}$ of subsets of V and a function y from $\mathcal{L}$ into $\mathbb{Q}_\geq$ that respects c and π, then* $y(\mathcal{L}) - \sum_{L \supseteq V_T} y_L \;\leq\; c(E_T) + \pi(\overline{V_T})$.

Proof. Let $\mathcal{B} := \{L \in \mathcal{L} : \delta_T L \neq \emptyset\}$ and $\mathcal{C} := \{L \in \mathcal{L} : L \subseteq \overline{V_T}\}$. We have that

$$y(\mathcal{B}) \;=\; \textstyle\sum_{L \in \mathcal{B}} y_L \;\leq\; \sum_{L \in \mathcal{B}} |\delta_T L| y_L \;=\; \sum_{e \in E_T} y(\mathcal{L}(e)) \;\leq\; \sum_{e \in E_T} c_e \;=\; c(E_T)\,.$$

On the other hand,

$$y(\mathcal{C}) \;=\; \textstyle\sum_{L \in \mathcal{C}^\bullet} \sum_{X \subseteq L} y_X \;\leq\; \sum_{L \in \mathcal{C}^\bullet} \pi(L) \;\leq\; \pi(\overline{V_T})\,.$$

The lemma follows trivially from the two inequalities as $\mathcal{L} \setminus \{L \in \mathcal{L} : L \supseteq V_T\} = \mathcal{B} \cup \mathcal{C}$. ■

Corollary 3.2 *For any laminar collection $\mathcal{L}$ of subsets of V and any function y from $\mathcal{L}$ into $\mathbb{Q}_\geq$ that respects c and π and any optimal solution $T^\bullet$ of $\text{PCST}(G, c, \pi)$, we have that* $y(\mathcal{L}) - \sum_{L \supseteq V_{T^\bullet}} y_L \;\leq\; \text{opt}(\text{PCST}(G, c, \pi))$. ■

4 Analysis of the algorithm

Invariants (i1) to (i5) hold trivially at the beginning of each iteration. Let us verify that invariant (i6) holds as well. It is clear that it holds at the beginning of the first iteration, because $y_L = 0$ for all L in $\mathcal{L}$. Assume, by induction, that invariant (i6) holds at the beginning of an iteration where Case 1 occurs. To show that it holds at the beginning of the next iteration, it is enough to verify the following two things. First, at the end of Subcase 1A, for any vertex u and any tree T in G, if T is connected in each element of $\mathcal{L}' := \mathcal{L} \cup \{L_1 \cup L_2\}$ and has no bridges in $\mathcal{S}$, then (3) holds with $\mathcal{L}'$ and y' in the roles of $\mathcal{L}$ and y. Second, at the end of Subcase 1B, for any vertex u and any tree T in G, if T is connected in each element of $\mathcal{L}$ and has no bridges in $\mathcal{S} \cup \{L_1\}$, then (3) holds with y' in the role of y.

Assume we are at the end of Subcase 1A. If $\varepsilon = 0$ then the statement is trivial. So, we assume $\varepsilon > 0$. Since $y'_{L_1 \cup L_2} = 0$, it suffices to show that (3) holds with y' in the role of y. Clearly T is connected in each element of $\mathcal{L}$ and has no bridges in $\mathcal{S}$. Therefore, (3) holds. Let $\mathcal{A} := \mathcal{L}^*$. Since y' differs from y only in $\mathcal{A} \setminus \mathcal{S}$, the left-hand side of (3) with y' in the role of y is the sum of the left-hand side of (3) and

$$\textstyle\sum_{L \in \mathcal{A} \setminus \mathcal{S}} |\delta_T L| \varepsilon + |\{L \in \mathcal{A} \setminus \mathcal{S} : L \subseteq \overline{V_T}\}| \varepsilon .$$

Similarly, the right-hand side of (3) with y' in the role of y is the sum of the right-hand side of (3) and

$$2\,|(\mathcal{A} \setminus \mathcal{A}_u) \setminus \mathcal{S}| \varepsilon .$$

Since $|\mathcal{A}_u| \leq 1$, to prove that (3) holds with y' in the role of y suffices to prove that

$$\textstyle\sum_{L \in \mathcal{A} \setminus \mathcal{S}} |\delta_T L| + |\{L \in \mathcal{A} \setminus \mathcal{S} : L \subseteq \overline{V_T}\}| \;\leq\; 2\,|\mathcal{A} \setminus \mathcal{S}| - 2 . \qquad (4)$$

For this, consider the graph $H := (\mathcal{A}, A)$, where A is the set of edges of T external to $\mathcal{A}$ and each element of A is incident to its two extremes in $\mathcal{A}$. Note that, since T is connected in each element of $\mathcal{L}$, every component of this graph is a singleton, except for at most one, which is a tree. Thus H is a forest. Denoting by h the number of components in H, we have that $h = 1 + |\{L \in \mathcal{A} : L \subseteq \overline{V_T}\}|$ and

$$\begin{aligned}
&\textstyle\sum_{L \in \mathcal{A} \setminus \mathcal{S}} |\delta_T L| + |\{L \in \mathcal{A} \setminus \mathcal{S} : L \subseteq \overline{V_T}\}| \\
&\quad = \textstyle\sum_{L \in \mathcal{A}} |\delta_T L| - \sum_{L \in \mathcal{A} \cap \mathcal{S}} |\delta_T L| + |\{L \in \mathcal{A} \setminus \mathcal{S} : L \subseteq \overline{V_T}\}| \\
&\quad \leq 2\,|\mathcal{A}| - 2h - 2\,|\{L \in \mathcal{A} \cap \mathcal{S} : L \not\subseteq \overline{V_T}\}| + |\{L \in \mathcal{A} \setminus \mathcal{S} : L \subseteq \overline{V_T}\}| \qquad (5) \\
&\quad \leq 2\,|\mathcal{A}| - 2 - 2\,|\{L \in \mathcal{A} : L \subseteq \overline{V_T}\}| - 2\,|\mathcal{A} \cap \mathcal{S}| + 2\,|\{L \in \mathcal{A} \cap \mathcal{S} : L \subseteq \overline{V_T}\}| + |\{L \in \mathcal{A} \setminus \mathcal{S} : L \subseteq \overline{V_T}\}| \\
&\quad \leq 2\,|\mathcal{A} \setminus \mathcal{S}| - 2 ,
\end{aligned}$$

where (5) holds because H is a forest and T has no bridges in $\mathcal{S}$.

Assume now we are at the end of Subcase 1B. If $\varepsilon = 0$ then the statement is trivial, so we assume $\varepsilon > 0$. Again, let $\mathcal{A} := \mathcal{L}^*$. Since T is connected in each element of $\mathcal{L}$ and has no bridges in $\mathcal{S}$, inequality (3) holds. But y' differs from y only at elements in $\mathcal{A} \setminus \mathcal{S}$. Thus the left-hand side of (3) with y' in the role of y equals to the left-hand side of (3) plus $\sum_{L \in \mathcal{A} \setminus \mathcal{S}} |\delta_T L| \varepsilon + |\{L \in \mathcal{A} \setminus \mathcal{S} : L \subseteq \overline{V_T}\}| \varepsilon$. Similarly, the right-hand side of (3) with y' in the role of y is the sum of the right-hand side of (3) and at least $2\,|\mathcal{A} \setminus \mathcal{S}| \varepsilon - 2\varepsilon$. Therefore, the proof that (3) holds with y' in the role of y reduces again to prove (4). The proof proceeds as above and we conclude that invariant (i6) holds.

At the end of case 2, we have a subcollection $\mathcal{Z}$ of $\mathcal{S}$ such that the tree $T := F - \bigcup \mathcal{Z}$ has no bridges in $\mathcal{S}$. Observe that

$$c(E_T) \;=\; \textstyle\sum_{e \in E_T} c_e \;=\; \sum_{e \in E_T} y(\mathcal{L}(e)) \;=\; \sum_{L \in \mathcal{L}} |\delta_T L| y_L .$$

The middle equality holds because $E_T \subseteq E_F$ and because invariant (i4) assures that each edge in F is tight for y. The last equality is just a rearrangement of terms. On the other hand,

$$\begin{aligned}
\pi(\overline{V_T}) &= \pi(\cup \mathcal{Z}^*) & (6)\\
&= \textstyle\sum_{Z\in\mathcal{Z}^*} \pi(Z) \\
&= \textstyle\sum_{Z\in\mathcal{Z}^*} \sum_{L\subseteq Z} y_L & (7)\\
&= \textstyle\sum_{L\in\cup\mathcal{Z}} y_L \\
&= \textstyle\sum_{L\subseteq\overline{V_T}} y_L\,. & (8)
\end{aligned}$$

Equality (6) holds because $\overline{V_T} \subseteq \bigcup \mathcal{Z}$, while equality (7) holds because $\mathcal{Z}^*$ is a subset of $\mathcal{S}$ and, therefore, according to invariant (i5), every element of $\mathcal{Z}^*$ is tight for y. Finally, equality (8) is true because $\bigcup \mathcal{Z} = V \setminus V_T = \overline{V_T}$.

Now, let u be an arbitrary vertex of an optimal solution T^* of PCST(G, c, π). By the two equalities above, by invariant (i6) and by Corollary 3.2,

$$\begin{aligned}
c(E_T) + \pi(\overline{V_T}) &= \textstyle\sum_{L\in\mathcal{L}} |\delta_T L| y_L + \sum_{L\subseteq\overline{T}} y_L \\
&\leq 2y(\mathcal{L} \setminus \mathcal{L}_u) \\
&\leq 2(y(\mathcal{L}) - \textstyle\sum_{L\supseteq V_{T^*}} y_L) \\
&\leq 2\,\mathrm{opt}(\mathrm{PCST}(G, c, \pi))\,.
\end{aligned}$$

The applicability of invariant (i6) is assured since T has no bridges in $\mathcal{S}$ and is connected in each element of $\mathcal{L}$. Indeed, for any x and y in $L \cap V_T$, the path from x to y in T is the same as in F and uses only vertices of L because of invariant (i2).

This completes the proof of the correct version of Theorem 3.2 in [3].

Theorem 4.1 *The* JMP *algorithm is a 2-approximation for the* PCST. ■

The example in Figure 1 shows that the approximation ratio of the JMP algorithm can be arbitrarily close to 2, independent of the size of the graph. So Theorem 4.1 is tight.

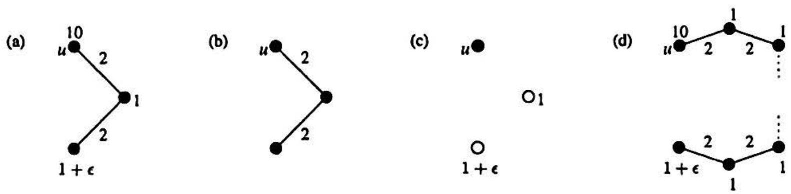

Figure 1: (a) An instance of the PCST. (b) The solution produced by the JMP algorithm when $\varepsilon > 0$. Its cost is 4. (c) The optimal solution, consisting of only vertex u, has cost $2 + \varepsilon$. (d) A similar instance of arbitrary size.

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