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COGNITIVE CONSTRUCTIVISM AND LANGUAGE

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> *"If the string is too tight it will snap,*
> *but if it is too loose it will not play."*
> Siddhartha Gautama.

Abstract

H.von Foerster characterizes the objects "known" by an autopoietic system as eigen-solutions, that is, as discrete, separable, stable and composable states of the interaction of the system with its environment. Previous articles have presented the FBST, Full Bayesian Significance Test, as a mathematical formalism specifically designed to access the support for sharp statistical hypotheses, and have shown that these hypotheses correspond, from a constructivist perspective, to systemic eigen-solutions in the practice of science. In this article several issues related to the role played by language in the emergence of eigen-solutions are analyzed. The last sections also explore possible connections with the semiotic theory of C.S.Peirce.

1 Introduction

In Stern (2005), the Full Bayesian Significance Test (FBST) formalism for evaluating the statistical support of sharp hypotheses and an extensive analysis of epistemological aspects related to the testing of such hypotheses are presented. Therein, a coherent interpretation for doing so in the context of cognitive constructivism is also given. We will refer to this setting as the Cognitive Constructivism plus FBST formalism, or CogCon+FBST framework for short.

As stated in Maturana and Varela (1980), the concept of recurrent state is the key to undestand the concept of cognitive domain in an autopoietic systems.

> *"Living systems as units of interaction specified by their conditions of being living systems cannot enter into interactions that are not specified by their organization. "The circularity of their organization continuously brings them back to the same internal state (same with respect to the cyclic process). Each internal state requires that certain conditions (interactions with the environment) be satisfied in order to proceed to the next state. Thus the circular organization implies the prediction that an interaction that took place once will take place again. If this does not happen the system maintains its integrity (identity with respect to the observer) and enters into a new prediction. In a continuously changing environment these predictions can only be successful if the environment does no change in that which is predicted. Accordingly, the predictions implied in the organization of the living system are not predictions of particular events, but of classes of inter-actions. Every interaction is a particular interaction, but every prediction is a prediction of a class of interactions that is defined by those features of its elements that will allow the living system to retain its circular organization after the interaction, and thus, to interact again. This makes living systems inferential systems, and their domain of interactions a cognitive domain."*

The epistemological importance of this circular (cyclic or recursive) regenerative processes and their eigen (auto, equilibrium, fixed, homeostatic, invariant, recurrent, recursive) -states, both in concrete and abstract autopoietic systems, are further investigated in Foerster (2003) and Segal (2001):

"The meaning of recursion is to run through one's own path again. One of its results is that under certain conditions there exist indeed solutions which, when reentered into the formalism, produce again the same solution. These are called "eigen-values", "eigen-functions", "eigen-behaviors", etc., depending on which domain this formation is applied - in the domain of numbers, in functions, in behaviors, etc."

"Objects are tokens for eigen-behaviors. Tokens stand for something else. In exchange for money (a token itself for gold held by one's government, but unfortunately no longer redeemable), tokens are used to gain admittance to the subway or to play pinball machines. In the cognitive realm, objects are the token names we give to our eigen-behavior. When you speak about a ball, you are talking about the experience arising from your recursive sensorimotor behavior when interacting with that something you call a ball. The "ball" as object becomes a token in our experience and language for that behavior which you know how to do when you handle a ball. This is the constructivist's insight into what takes place when we talk about our experience with objects."

Von Foerster also establishes several essential characteristics of these eigen-solutions, as quoted in the following paragraph from Foerster (2003, c). These essential characteristics can be translated into very specific mathematical properties, that are of prime importance when investigating several aspects of the CogCon+FBST framework.

"Eigenvalues have been found ontologically to be discrete, stable, separable and composable, while ontogenetically to arise as equilibria that determine themselves through circular processes. Ontologically, Eigenvalues and objects, and likewise, ontogenetically, stable behavior and the manifestation of a subject's "grasp" of an object cannot be distinguished."

In Stern (2005) it is shown how the eigen-solutions found in the practice of science are naturally represented by statistical sharp hypotheses. Statistical sharp hypotheses are routinely stated as natural "laws" or "invariance" principles, and most often take the form of functional equations, like $h(x) = c$.

The article also discusses why the eigen-solutions essential properties of discreteness (sharpness), stability, and composability, indicate that considering such hypotheses in the practice of science is natural and reasonable. Surprisingly, the two standard statistical theories for testing sharp hypotheses, classical (frequentist p-values) and Bayesian (orthodox Bayes factors), have well known and documented problems for handling sharp hypotheses. These problems are thoroughly reviewd, from statistical, methodological, systemic and epistemological perspectives. The FBST is a (unorthodox) Bayesian significance test specifically designed for this task. The mathematical and statistical properties of the FBST are carefully analysed. In particular, it is shown how the FBST fully supports the test and identification of eigen-solutions in the practice of science, using procedures that take into account all the essential characteristics pointed by von Foerster.

The discussion in Stern (2005) raised some interesting questions, some of which we will try to answer in the present article. The first question relates to the role and the importance of language in the emergence of eigen-solutions and is discussed in section 2. In answering it, we make extensive use of the William Rasch "two-front war" metaphor of cognitive constructivism, as exposed in Rasch (2000). As explained in section 4, this is the war against dogmatic realism at one front, and against skepticism or solipsism, at the second. To illustrate his arguments, Rasch uses some ideas of Niels Bohr concerning quantum mechanics. In section 3, we use some of the same ideas to give concrete examples of the topics under discussion. The results of the first part of the paper are summarized in section 5.

The second question, posed by Soren Brier, which asks whether the Cog-Con+FBST framework is compatible with and can benefit from the concepts of Semiotics and Peircean philosophy, is addressed in section 6. In section 7 we present our final remarks.

2 Eigen-solutions and Language

Goudsmit (1998, sec.2.3.3, Objects as warrants for eigenvalues), finds an apparent disagreement between the form in which eigen-solutions emerge, according to von Foster and Maturana:

> *"Generally, von Foersters concept of eigenvalue concerns the value of a function after a repeated (iterative) application of a particular operation. ...*

This may eventually result in a stable performance, which is an eigenvalue of the observers behavior. The emerging objects are warrants of the existence of these eigenvalues.
... contrary to von Foerster, Maturana considers the consensuality of distinctions as necessary for the bringing forth of objects. It is through the attainment of consensual distinctions that individuals are able to create objects in language. Only after an individual has attained some familiarity to the use of language, he may be able to perceive new objects without consensus with others."

Confirmation for the position attributed by Goudsmit to von Foerster can be found in several of his articles. In Foerster (2003, a), for example, one finds:

"... I propose to continue the use of the term 'self-organizing system,' whilst being aware of the fact that this term becomes meaningless, unless the system is in close contact with an environment, which possesses available energy and order, and with which our system is in a state of perpetual interaction, such that it somehow manages to 'live' on the expenses of this environment. ...
... So both the self-organizing system plus the energy and order of the environment have to be given some kind of pre-given objective reality for this view points to function."

Confirmation for the position attributed by Goudsmit to Maturana can also be found in several of his articles. In Maturana (1988), for example, one finds:

"Objectivity. Objects arise in language as consensual coordinations of actions that in a domain of consensual distinctions are tokens for more basic coordinations of actions, which they obscure. Without language and outside language there are no objects because objects only arise as consensual coordinations of actions in the recursion of consensual coordinations of actions that languaging is. For living systems that do not operate in language there are no objects; or in other words, objects are not part of their cognitive domains. ... Objects are operational relations in languaging."

The standpoints of Maturana and von Foerster are further characterized in the following paragraph from Brier (2005):

> *"The process of human knowing, is the process in which we, through languaging, create the difference between the world and ourselves; between the self and the non-self, and thereby, to some extent, create the world by creating ourselves. But we do it by relating to a common reality which is in some way before we made the difference between 'the world' and 'ourselves' make a difference, and we do it on some kind of implicit belief in a basic kind of order 'beneath it all'. I do agree that it does not make sense to claim that the world exists completely independently of us. But on the other hand it does not make sense to claim that it is a pure product of our explanations or conscious imagination."*
> *"...it is clear that we do not create the trees and the mountains through our experiencing or conversation alone. But Maturana is close to claim that this is what we do. Von Foerster is more aware of the philosophical demand that to put up a new epistemological position one has to deal with the problem of solipsism and of pure social constructivism."*
> *"The Eigenfunctions do not just come out of the blue. In some, yet only dimly viewed, way the existence of nature and its 'things' and our existence are intertwined in such a way that makes it very difficult to talk about. Von Foerster realizes that to accept the reality of the biological systems of the observer leads into further acceptance about the structure of the environment."*

In order to understand the above comments, one must realize that Maturana's viewpoints, or at least his rethoric, changed greatly over time, ranging from the very moderate and precise statements in Maturana and Varela (1980), to the most extreme positions assumed in Maturana (1991, p.36-37):

> *"Einstein said, and many other scientists have agreed with him, that scientific theories are free creations of the human mind, and he marveled that through them one could understand the universe. The criterion of validation of scientific explanation as operations in the praxis of living of the observer, however, permit us to see how it is that the first reflection of Einstein is valid, and how it is that there is nothing marvelous in that it is so.*

...the criterion of validation of scientific explanations pertain totally to the arbitrariness of the observer's mind in the sense that they arise in the flow of his or her structural determination....
What makes a scientific explanation or theory scientific is not quantification or the possibility that it creates for the observer to predict some of his or her future experiences with it, but that it is validated as it arises through the criterion of validation of scientific explanations without reference to quantification or any restriction of domain."

While the position adopted by von Foerster appears to be more Realistic or Objective, the one adopted by Maturana and Varela seems more Idealistic or (Inter) Subjective. Can these two different positions, which Goudsmit views as discrepant, be reconciled? Do we have to chose between an Idealistic or a Realistic position, or can we rather have both? This is the first question we address in this paper.

In Stern (2005) we used an example of physical eigen-solution (physical invariant) to illustrate the ideas in discussion, namely, the speed of light constant, c. Historically, this example is tied to the birth of Special Relativity theory, and the debacle of classical physics. In this article we will illustrate them with another important historical example, namely, the Einstein-Podolsky-Rosen paradox. Historically, this example is tied to questions concerning the interpretation of Quantum Mechanics. This is the main topic of the next section.

3 The Languages of Science

At the end of the 19th century, classical physics was the serene sovereign of science. Its glory was consensual and uncontroversial. However, at the beginning of the 20th century, a few experimental results challenged the explanatory power of classical physics. The problems appeared in two major fronts that, from a historical perspective, can be linked to the theories (at that time still non existent) of Special Relativity and Quantum Mechanics.

At that time, the general perception of the scientific community was that these few open problems could, should and would be accommodated in the framework of classical physics. Crafting sophisticated structural models such as those for the structure of ether (the medium in which light was supposed to propagate), and those for the atomic structure, was typical of the effort to circumvent these open problems by artfully maneuvering classical physics. But physics and engineering laboratories insisted, building up a barrage of new and challenging experimental results.

The difficulties with the explanations offered by classical physics not only persisted, but also grew in number and strength. In 1940 the consensus was that classical physics had been brutally defeated, and Relativity and Quantum Mechanics were acclaimed as the new sovereigns. Let us closely examine some facts concerning the development of Quantum Mechanics (QM).

The first step in the direction of a comprehensive QM theory was given in 1924 by Louis de Broglie, who postulated the particle-wave duality principle, which states that every moving particle has an associated "pilot wave" of wavelenght $\lambda = h/mv$, where h is planck's constant and mv is the particle's momentum, i.e., the product of its mass and velocity.

In 1926 Erwin Schrödinger stated his wave equation, capable of explaining all known quantic phenomena, and predicting several new ones that where latter confirmed by new experiments. Schrödinger theory is known as Orthodox QM, see Tomonaga (1962) for a detailed historical account. Orthodox QM uses a mathematical formalism based on a complex wave equation, and shares much of the descriptive language of de Broglie's particle-wave duality principle.

There is, however, something odd in the wave-particle descriptions of orthodox QM. When describing a model we speak of each side of a double faced wave-particle entity, as if each side existed by itself, and then inextricably fuse them together in the mathematical formalism. Quoting Cohen (1989, p.87),

> *"Notice how our language shapes our imagination. To say that a particle is moving in a straight line really means that we can set up particle detectors along the straight line and observe the signals they send. These signals would be consistent with a model of the particle as a single chunk of mass moving (back and forth) in accordance with Newtonian particle physics. It is important to emphasize that we are not claiming that we know what the particle is, but only what we would observe if we set up those particle detectors."*

From Schroedinger's equation we can derive Heisemberg's uncertainty principle, which states that we can not go around measuring everything we want until we pin down every single detail about (the classical entities in our wave-particle model of) reality. One instance of the Heisemberg uncertainty principle states that we can not simultaneously measure a particle position and moment beyond a certain accuracy. One way of interpreting this instance of the Heisemberg uncertainty principle goes as follows: In classical Newtonian physics our particles are "big enough" so that our measurement devices can obtain the information we need about the particle without disturbing it. In QM, on the other hand, the particles are so small that the measurement operation will always disturb the particle. For example, the light we have to use in order to illuminate the scene, so we can see "where" the particle is, has to be be so strong, relative to the particle size, that it "blows" the particle away changing its velocity. The consequence is that we cannot (neither in practice, nor even in principle) simultaneously measure with arbitrary precision, both the particle's position and momentum. Hence, we have to learn how to tame our imagination and constrain our language.

The need to exercise a strict discipline over what kinds of statements to use was a lesson learned by 20th century physics. A lesson that mathematics had to learn a bit earlier. A classical example from set theory of a statement that cannot be allowed is the Russell's catalog (class, set), defined in Robert (1988, p.x) as:

> *"The 'catalogue of all catalogues not mentioning themselves.' Should one include this catalogue in itself? ... Both decisions lead to a contradiction!"*

Robert (1988) indicates several ways to avoiding this paradox (or antinomy). All of them imply imposing a (very reasonable) set of rules on how to form valid statements. Under any of these rules, Russell's definition becomes an invalid or ill posed statement and, as such, should be disregarded, see also Halmos (1998, ch.1 and 2). Measure theory (of Borel, Lebesgue, Haar, etc.) was a fundamental achievement of 20th century mathematics. It defines measures (notions such as mass, volume and probability) for parts of R^n. However not all parts of R^n are included, and we must refrain of speaking about the measure of inadmissible (non-measurable) sets, see Ulam (1943) for a short article, Kolmogorov and Fomin (1982) for a standard text, and Nachbin (1965) for extensions pertinent to the FBST formalism. The main subject in Robert (1988) is Non Standard Analysis, a form of extending the languages of both Set Theory and Real Analysis, see the observations in section 6.6 and also Davis (1977, 3.4), Goldblatt (1998) and Nelson (1987).

All the preceding examples of mathematical languages have one thing in common: When crafting a specific language, one has to carefully define what kinds of statements are accepted as valid ones. Proper use of the language must be constrained to valid statements. Such constraints are necessary in order to preserve language coherence.

The issue of what kinds of statements should be accepted as valid in QM is an interesting and still subsisting issue, epitomized by the famous debate at the Brussels Solvay conference of 1930 between Niels Bohr and his friend and opponent Albert Einstein. Ruhla (1992, ch.7 and 8) and Baggott (1992, under the topic hidden variables) give very intuitive reviews of the subject, requiring minimal mathematical expertise. Without the details concerning the physics involved, one can describe the debate as: While Bohr suggested very strict rules for admissible statements in QM, Einstein advocated for more amiable ones. In 1935 Einstein, Podolsky and Rosen suggested a "gedankenexperiment", known as the EPR paradox, as a compelling argument supporting Einstein's point of view. D.Bohm, in 1952 and J.Bell, in 1964, contributed to the debate by showing that the EPR paradox could lead to concrete experiments providing a way to settle the debate on empirical grounds. It was only in 1972 that the first EPR experiment could be performed in practice. The observational evidence from these experiments seems to favor Bohr's point of view!

Today's standard formalism for QM is Abstract QM, see Hughes (1992) for a very readable text and Cohen (1989) for a concise and formal treatment. Abstract QM, which is very clean and efficient, can be stratified in two layers. In the first layer, all basic calculations are carried out using an algebra of operators in (Rigged) Hilbert spaces. In a second layer, the results of these calculations are interpreted as probabilities of obtaining specific results in physical measurements, see also Rijsbergen (2004). One advantage of using the stratified structure of abstract QM is that it naturally avoids (most of) the danger of forming invalid statements in QM language. Cohen (1989, p.vii) provides the following historical summary:

> *"Historically, ... quantum mechanics developed in tree stages. First came a collection of ad hoc assumptions and then a cookbook of equations known as (orthodox) quantum mechanics. The equations and their philosophical under pining were then collected into a model based on mathematics of Hilbert space. From the Hilbert space model came the abstraction of quantum logics."*

From the above historical comments we draw the following conclusions:

> **3.1.** Each of the QM formalisms discussed in this section, namely, de Broglie wave-particle duality principle, Schrödinger orthodox QM and Hilbert space abstract QM, operates like a "language". Maturana stated that objects arise in language. He seems to be right.
> **3.2.** It seems also that new languages must be created (or discovered) to provide us the objects corresponding to the structure of the environment, as stated by von Foerster.
> **3.3.** Exercising a strict discipline concerning what kinds of statements can be used in a given language and context, seems to be vital in many areas.
> **3.4.** It is far from trivial to create, craft, discover, find and/or use a language so that "it works", providing us the "right" objects (eigen-solutions).
> **3.5.** Even when everything looks (for the entire community) fine and well, new empirical evidence can bring our theories down as a castle of cards.

4 The Self-Reference Paradox

The conclusions established in the previous section may look reasonable. In 3.4, however, what exactly are the "right" objects? Clearly, the "right" objects are "those" objects we more or less clearly see and can point at, using as reference language the language we currently use.

There! I have just fallen, head-on, into the quicksands of the self-reference paradox. Don't worry (or do worry), but remind this: The self-reference paradox is unavoidable, at least as long we use English or any other natural human language.

Rasch has produced a very good description of the self-reference paradox and some of its consequences:

> *"having it both ways seems a necessary consequence... One cannot just have it dogmatically one way, nor skeptically the other... One oscillates, therefore, between the two positions, neither denying reality nor denying reality's essentially constructed nature. One calls this not idealism or realism, but (cognitive) constructivism."*

"What do we call this oscillation? We call it paradox. Self - reference and paradox - sort of like love and marriage, horse and carriage."

Cognitive Constructivism implies a double rejection. That of a solipsist denial of reality, and that of any dogmatic knowledge of the same reality. Rasch uses the "two front war" metaphor to describe this double rejection. Carrying the metaphor a bit further, the enemies of cognitive constructivism could be portrayed as:

- Dogmatism despotically requires us to believe in its (latest) theory. Its statements and reasons should be passively accepted with fanatic resignation as infallible truth;
- Solipsism' anarchic distrust wishes to preclude any established order in the world. Solipsism wishes to transform us into autistic skeptics, incapable of establishing any stable knowledge about the environment in which we live.

We refer to Caygill (1995, dogmatism) for a historical perspective on the Kantian use of some of the above terms.

Any military strategist will be aware of the danger in the oscillation described by Rasch, which alternately exposes a weak front. The enemy at our strong front will be subjugated, but the enemy at our weak front will hit us hard. Rasch sees a solution to this conundrum, even recognizing that this solution may be difficult to achieve:

"There is a third choice: to locate oneself directly on the invisible line that must be drawn for there to be a distinction mind / body (system / environment) in the first place. Yet when one attempts to land on that perfect center, one finds oneself oscillating wildly from side to side, perhaps preferring the mind (system) side, but over compensating to the body (environment) side - or vice versa."

So, the question is: How do we land on Rasch' fine (invisible) line, finding the perfect center and avoiding dangerous oscillations? This is the topic of the next section.

5 Objective Idealism and Pragmatism

We are now ready for a few definitions of basic epistemological terms. These definitions should help us build epistemic statements in a clear and coherent form according to the CogCon+FBST perspective.

> **5.1. Known (knowable) Object:** An actual (potential) eigen-solution of a given system's interaction with its environment. In the sequel, we may use a somewhat more friendly terminology by simply using the term Object.
> **5.2. Objective (how, less, more):** Degree of conformance of an object to the essential properties of an eigen-solution.
> **5.3. Reality:** A (maximal) set of objects, as recognized by a given system, when interacting with single objects or with compositions of objects in that set.
> **5.4. Idealism:** Belief that a system's knowledge of an object is always dependent on the systems' autopoietic relations.
> **5.5. Realism:** Belief that a system's knowledge of an object is always dependent on the environment's constraints.
> **5.6. Solipsism, Skepticism:** Idealism without Realism.
> **5.7. Dogmatic Realism, Metaphysical Realism:** Realism without Idealism.
> **5.8. Realistic Idealism, Objective Idealism:** Idealism and Realism.
> **5.9. "Ding an sich" or "Thing in itself":** These expressions are markers or labels for ill posed statements.

Cog-Con+FBST assumes an objective and idealistic epistemology. Definition 5.9 labels ill posed (dogmatic) statements. Often, these invalid statements look like:

"Something that an observer would observe if the (same) observer did not exist;" or "That which an observer could observe if he made no observations;" or "That which an observer should observe in the environment without interacting with it (or disturbing it in any way);" and many other equally nonsensical variations.

Some of the readers may not like this form of labeling invalid statements, preferring to use, instead, a more elaborate terminology, such as "object in parenthesis" (approximately) as object, "object without parenthesis" (approximately) as Ding and sich, etc. There may be good reasons for doing so, for example, this elaborate language has the advantage of automatically stressing the differences between constructivist and dogmatic epistemologies, see Maturana (1988), Maturana and Poerksen (2004) and Steier (1991). Nevertheless, we have chosen our definitions in agreement with some very pragmatic advice given in Bopry (2002):

> *" Objectivity as defined by a (dogmatic) realist epistemology may not exist within a constructivist epistemology; but, part of making that alternative epistemology acceptable is gaining general acceptance of its terminology. As long as the common use of the terms is at odds with the concepts of an epistemological position, that position is at a disadvantage. Alternative forms of inquiry need to coopt terminology in a way that is consistent with its own epistemology. I suggest that this is not so difficult. The term objective can be taken back..."*

Among the definitions 5.1 to 5.9, definition 5.2 plays a key role. It allows us to say how well an eigen-solution manifests von Foerster's essential attributes, and consequently, how good (objective) is our knowledge of it. However, the degree of objectivity can not be assessed in abstract, it must be assessed by the means and methods of a given empirical science, namely the one within which the eigen solution is presented. Hence, definition 5.2 relies on an "operational approach", and not on metaphysical arguments. Such an operational approach may be viewed with disdain by some philosophical schools. Nevertheless, for C.S.Peirce it is

"The Kernel of Pragmatism", CP 5.464-465:

> *"Suffice it to say once more that pragmatism is, in itself, no doctrine of metaphysics, no attempt to determine any truth of things. It is merely a method of ascertaining the meanings of hard words and of abstract concepts. ... All pragmatists will further agree that their method of ascertaining the meanings of words and concepts is no other than that experimental method by which all the successful sciences (in which number nobody in his senses would include metaphysics) have reached the degrees of certainty that are severally proper to them today; this experimental method being itself nothing but a particular application of an older logical rule, 'By their fruits ye shall know them'. "*

Definition 5.2 also requires a belief calculus specifically designed to measure the degree of support of empirical data to the existence of an eigen-solution. In Stern (2005) we showed why confirming the existence of an eigen-solution naturally corresponds to testing a sharp statistical hypotheses, and why the mathematical properties of FBST e-values correspond to the essential properties of an eigen-solution as stated by von Foerster, see also other of the author's previous articles in the reference list. In this sense, the FBST calculus is perfectly adequate to support the use of the term Objective and correlated terms in scientific language. Among the most important properties of the e-value mentioned in Stern (2005), we find:
Continuity: Give a measure of significance that is smooth, i.e. *continuous and differentiable*, on the hypothesis parameters and the sample statistics, under appropriate regularity conditions of the statistical model.
Consistency: Be able to provide a *consistent* test for a given sharp hypothesis, in the sense that increasing sample size should make it converge to the right accept/reject decision.

Therefore, the FBST calculus is a formalism that allow us to assess, continuously and consistently, the objectivity of an eigen-solution, by means of convergent tests, see Stern (2005). Hence, our answer to the question of how to lad on Rasch' perfect center is: Replace unstable oscillation for stable convergence!

Any dispute about objectivity (epistemic quality or value of an object of knowledge), should be critically examined and evaluated within this pragmatic program. This program (in the Luhmann' sense) includes the means and methods of the empirical science in which the object of knowledge is presented, and the FBST belief calculus, used to evaluate the empirical support of an object, given the available experimental data.

Even if over optimistic (actually hopelessly utopic), it is worth restating Leibniz' flag of *Calculemus*, as found in Gerhardt (1890, v.7,p.64,65,125,200):

> *"Quo facto, quando orientur controversiae, non magis disputatione opus erit inter duos philosophos, quam inter duos Computistas. Sufficiet enim calamos in manus sumere sedereque ad abacos, et sibi mutuo (accito si placet amico) dicere: Calculemus."*

A contemporary translation would read: *Actually, if controversies were to arise, there would be no more need for dispute between two philosophers,* *rather than between two statisticians. For them it would suffice to reach their computers and, in friendly understanding, say to each other: Let us calculate!*

6 The Philosophy of C.S.Peirce

In the previous section we presented an epistemological perspective based on a pragmatic objective idealism. Objective idealism and pragmatism are also distinctive characteristics of the philosophy of C.S.Peirce. Hence the following question, posed by Soren Brier, that we examine in this section: Is the CogCon+FBST framework compatible with and can it benefit from the concepts of Semiotics and Peircean philosophy?

In Stern (2005) we had already explored the idea that eigen-solutions, as discrete entities, can be named, i.e., become signs in a language system, as pointed by von Foerster in Segal (2001):

> *"There is an additional point I want to make, an important point. Out of an infinite continuum of possibilities, recursive operations carve out a precise set of discrete solutions. Eigen-behavior generates discrete, identifiable entities. Producing discreteness out of infinite variety has incredibly important consequences. It permits us to begin naming things. Language is the possibility of carving out of an infinite number of possible experiences those experiences which allow stable interactions of your-self with yourself."*

We believe that the process of recursively "discovering" objects of knowledge, identifying them by signs in language systems, and using these languages to "think" and structure our lives as self-concious beings, is the key for understanding concepts such as signification and meaning. These ideas are explored, in a great variety of contexts, in Bakken and Hernes (2002), Brier (1995), Ceruti (1989), Efran et al. (1990), Eibel-Eibesfeldt (1970), Ibri (1992), Piaget (1975), Wenger et al. (1999), Winograd and Flores (1987) and many others. Conceivably, the key underneath common principle is stated in Brier (2005):

> *"The key to the understanding of understanding, consciousness, and communication is that both the animals and we humans live in a self-organized signification sphere which we not only project around us but also project deep inside our systems. Von Uexküll*

calls it "Innenwelt" (Brier 2001). The organization of signs and the meaning they get through the habits of mind and body follow very much the principles of second order cybernetics in that they produce their own Eigenvalues of sign and meaning and thereby create their own internal mental organization. I call this realm of possible sign processes for the signification sphere. In humans these signs are organized into language through social self-conscious communication, and accordingly our universe is organized also as and through texts. But of course that is not an explanation of meaning."

When studying the organization of self-conscious beings and trying to understand semantic concepts such as signification and meaning, or teleological concepts such as finality, intent and purpose, we move towards domains concerning systems of increasing complexity that are organized as higher hierarchical structures, like the domains of phenomenological, psychological or sociological sciences. In so doing, we leave the domains of natural and technical sciences behind, at least for a moment, see Brent and Bruck (2006) and Muggleton (2006), in last month's issue of *Nature* (March 2006, when this article was written), for two perspectives on future developments.

As observed in Brier (2001), the perception of the objects of knowledge, changes from more objective or realistic to more idealistic or (inter) subjective as we progress to higher hierarchical levels. Nevertheless, we believe that the fundamental nature of objects of knowledge as eigen-solutions, with all the essential characteristics pointed out by von Foerster, remains just the same. Therefore, a sign, as understood in the CogCon+FBST framework, always stands for the following triad:

S-1. Some "thing" (some perceived aspects, characteristics, etc.) concerning the organization of the autopoietic system.
S-2. Some "thing" (some perceived aspects, characteristics, etc.) concerning the structure of the system's environment.
S-3. Some object (discrete, separable, stable and composable eigen-solution based on the particular aspects stated in S-1 and S-2) concerning the interaction of the autopoietic system with its environment.

This triadic character of signs bring us, once again, close to the semiotic theory of C.S.Peirce, offering many opportunities for further theoretical and applied research. For example, we are currently using statistical psychometric analyses in an applied semiotic project for the development of software user interfaces, for related examples see Fereira (2006). We defer, however, the exploration of these opportunities to forthcoming articles. In the remainder of this section we focus on a more basic investigation that, we believe, is a necessary preliminary step that must be undertaken in order to acquire a clear conceptual horizon that will assist a sound and steady progress in our future research. The purpose of this investigation is to find out whether the CogCon+FBST framework can provide a truly compatible ground in the basic concepts of Peircean philosophy. We proceed establishing a conceptual mapping of the fundamental concepts used to define the CogCon+FBST epistemological framework into analogous concepts in Peircean philosophy.

The FBST is a Continuous Statistical formalism. Our first step in constructing this conceptual mapping addresses the following questions: Is such a formalism amenable to a Perircean perspective? If so, which concepts in Peircean philosophy can support the use of such a formalism?

6.1 Probability and Statistics: The FBST is a probability theory based statistical formalism. Can the probabilistic concepts of the FBST find the necessary support in concepts of Peircean philosophy? We believe that Tychism is such a concept in Peircean philosophy, providing the first element in our conceptual mapping. In CP 6.201 Tychism is defined as:

> *"... the doctrine that absolute chance is a factor of the universe."*

6.2 Continuity: As stated in the previous section, the CogCon+FBST program pursues the stable convergence of the epistemic e-values given by the FBST formalism. The fact that FBST is a belief calculus based on continuous mathematics is essential for its consistency and convergence properties. Again we have to ask: Does the continuity concept used in the FBST formalism have an analogous concept in Peircean philosophy? We believe that the analogy can be established with the concept of Synechism, thus providing the second element in our conceptual mapping.

In CP 6.169 synechism is defined as:

> *"that tendency of philosophical thought which insists upon the idea of continuity as of prime importance in philosophy and, in particular, upon the necessity of hypotheses involving true continuity."*

6.3 Eigen-Solutions: A key epistemological concept in the CogCon +FBST perspective is the notion of eigen-solution. Although the system theoretic concept of Eigen-solution cannot possibly have an exact correspondent in Peirce philosophy, we believe that Peirce' fundamental concept of "Habit" or "Insistency" offers an adequate analog. Habit, and reality, are defined as:

"The existence of things consists in their regular behavior.", CP 1.411.

> *"Reality is insistency. That is what we mean by 'reality'. It is the brute irrational insistency that forces us to acknowledge the reality of what we experience, that gives us our conviction of any singular."*, CP 6.340.

However, the CogCon+FBST concept of eigen-solution is characterized by von Foerster by several essential properties. Consequently, in order to the conceptual mapping under construction be coherent, these characteristics have to be mapped accordingly. In the following paragraphs we show that the essential properties of sharpness (discreteness), stability and compositionality can indeed be adequately represented.

6.3a Sharpness: The first essential property of eigen-solutions stated by von Foerster is discreteness or sharpness. As stated in Stern (2005), it is important to realize that, in the sequel, the term 'discrete', used by von Foerster to qualify eigen-solutions in general, should be replaced, depending on the specific context, by terms such as lower-dimensional, precise, sharp, singular, etc. As the physical laws or physical invariants, sharp hypotheses are formulated as mathematical equations.

Can Peircean philosophy offer a good support for sharp hypotheses? Again we believe that the answer is in the affirmative. The following quotations should make that clear. The first three passages are taken from Ibri (1992, p.84-85) and the next two from CP, 1.487 and CP 1.415, see also NEM 4, p.136-137 and CP 6.203.

> *"an object (a thing) IS only in comparison with a continuum of possibilities from which it was selected."*
> *"Existence involves choice; the dice of infinite faces, from potential to actual, will have the concreteness of one of them."*
> *"...as a plane is a bi-dimensional singularity, relative to a tri-dimensional space, a line in a plane is a topic discontinuity, but each of this elements is continuous in its proper dimension."*

" Whatever is real is the law of something less real. Stuart Mill defined matter as a permanent possibility of sensation. What is a permanent possibility but a law?"
"In fact, habits, from the mode of their formation, necessarily consist in the permanence of some relation, and therefore, on this theory, each law of nature would consist in some permanence, such as the permanence of mass, momentum, and energy. In this respect, the theory suits the facts admirably."

6.3b Stability: The second essential property of eigen-solutions stated by von Foerster is stability. As stated in Stern (2005), a stable eigen-solution of an operator, defined by a fixed-point or invariance equation, can be found (built or computed) as the limit of a sequence of recursive applications of the operator. Under appropriate conditions (such as within a domain of attraction, for instance) the process convergence and its limiting eigen-solution will not depend on the starting point.

A similar notion of stability for an object-sign complex is given by Peirce. As stated in CP 1.339:

"That for which it (a sign) stands is called its object; that which it conveys, its meaning; and the idea to which it gives rise, its interpretant. The object of representation can be nothing but a representation of which the first representation is the interpretant. But an endless series of representations, each representing the one behind it, may be conceived to have an absolute object at its limit."

6.3c Compositionality: The third essential property of eigen-solutions stated by von Foerster is compositionality. As stated in Stern (2005), compositionality properties concern the relationship between the credibility, or truth value, of a complex hypothesis, H, and those of its elementary constituents, H^j, $j = 1 \dots k$. Compositionality is at the very hart of any theory of language, see Noeth (1995). As an example of compositionality, see CP 1.366 and CP 6.23. Peirce discusses the composition of forces, i.e. how the components are combined using the parallelogram law.

"If two forces are combined according to the parallelogram of forces, their resultant is a real third... Thus, intelligibility, or reason objectified, is what makes Thirdness genuine.".

"A physical law is absolute. What it requires is an exact relation. Thus, a physical force introduces into a motion a component motion to be combined with the rest by the parallelogram of forces;".

In order to establish a minimal mapping, there are two more concepts in CogCon+FBST to which we must assign adequate analogs in Peircean philosophy.

6.4 Extra variability: In Stern (2005) the importance of incorporating all sources of noise and fluctuation, i.e., all the extra variability statistically significant to the problem under study, into the statistical model is analyzed. The following excerpt from CP 1.175 indicates that Peirce's notion of falibillism may be used to express the need for allowing and embracing all relevant (and in practice inevitable) sources of extra variability. According to Peirce, falibilism is *"the doctrine that there is no absolute certainty in knowledge"*.

"There is no difficulty in conceiving existence as a matter of degree. The reality of things consists in their persistent forcing themselves upon our recognition. If a thing has no such persistence, it is a mere dream. Reality, then, is persistence, is regularity. ... as things (are) more regular, more persistent, they (are) less dreamy and more real. Fallibilism will at least provide a big pigeon-hole for facts bearing on that theory."

6.5 - Bayesian statistics: FBST is an Unorthodox Bayesian statistical formalism. Peirce has a strong and unfavorable opinion about Laplace's theory of 'inverse probabilities'.

"...the majority of mathematical treatises on probability follow Laplace in results to which a very unclear conception of probability led him. ... This is an error often appearing in the books under the head of 'inverse probabilities'." CP 2.785.

Due to his theory of 'inverse probabilities', Laplace is considered one of the earliest precursors of modern Bayesian statistics. Is there a conflict between CogCon+FBST and Peirce philosophy? We believe that a careful analysis of Peirce arguments not only dissipates potential conflicts, but also reinforces some of the arguments used in Stern (2005).

Two main arguments are presented by Peirce against Laplace's 'inverse probabilities'. In the following paragraphs we will identify these arguments and present an up-to-date analysis based on the FBST (unorthodox) Bayesian view:

6.5a - Dogmatic priors vs. Symmetry and Maximum Entropy arguments:

> " *Laplace maintains that it is possible to draw a necessary conclusion regarding the probability of a particular determination of an event based on not knowing anything at all about (it); that is, based on nothing. ... Laplace holds that for every man there is one law (and necessarily but one) of dissection of each continuum of alternatives so that all the parts shall seem to that man to be 'également possibles' in a quantitative sense, antecedently to all information.*", CP 2.764.

The dogmatic rhetoric used at the time of Laplace to justify ad hoc prior distributions can easily backfire, as it apparently did for Peirce. Contemporary arguments for the choice of prior distributions are based on MaxEnt formalism or symmetry relations, see Dugdale (1996), Eaton (1989), Kapur (1989) and Nachbin (1965). Contemporary arguments also examine the initial choice of priors by sensitivity analysis, for finite samples, and give asymptotic dissipation theorems for large samples, see DeGroot (1970). We can only hope that Peirce would be pleased with the contemporary state of the art. These powerful theories have rendered ad hoc priors unnecessary, and shed early dogmatic arguments into oblivion.

6.5b- Assignment of probabilities to (sharp) hypotheses vs. FBST possibilistic support structures:

> "*Laplace was of the opinion that the affirmative experiments impart a definite probability to the theory; and that doctrine is taught in most books on probability to this day, although it leads to the most ridiculous results, and is inherently self-contradictory. It rests on a very confused notion of what probability is. Probability applies to the question whether a specified kind of event will occur when certain predetermined conditions are fulfilled; and it is the ratio of the number of times in the long run in which that specified result would follow upon the fulfillment of those conditions to the total number of times in which those conditions were fulfilled in the course of experience.*", CP 5.169.

In the second part of the above excerpt Peirce expresses a classical (frequentist) understanding of having probability in the sample space, and not in the parameter space, i.e., he admits predictive probability statements but does not admit epistemic probability statements. The FBST is a Bayesian formalism that uses both predictive and epistemic probability statements, as explained in Stern (2005). However, when we examine the reason presented by Peirce for adopting this position, in the first part of the excerpt, we find a remarkable coincidence with the arguments presented in Stern (2005) against the orthodox Bayesian methodology for testing sharp hypotheses: The FBST e-value DOES NOT attribute a Probability to the theory (sharp hypothesis) being tested, as do orthodox Bayesian tests, but rather a Degree of Possibility. In Stern (2005) we analyze procedures that attribute a probability to a given theory, and came to the exact same conclusion as Pierce did, namely, those procedures are absurd.

6.6 Measure Theory: Let us now return to the Peircean concept of Synechism, to discuss a technical point of contention between orthodox Bayesian statistics and the FBST unorthodox Bayesian approach. The FBST formalism relies on some form of Measure theory, see comments in section 3. De Finetti, the founding father of the orthodox school of Bayesian statistics, feels very uncomfortable having to admit the existence of non-measurable sets when using measure theory in dealing with probabilities, in which valid statements are called events, see Finetti (1975, 3.11, 4.18, 6.3 and appendix). Dubins and Savage (1976, p.8) present similar objections, using the colorful gambling metaphors that are so characteristic of orthodox (decision theoretic) Bayesian statistics. In order to escape the constraint of having non-measurable sets, de Finetti readilly proposes a deal: to trade off other standard properties of a measure, like countable (σ) additivity.

> *"Events are restricted to be merely a subclass (technically a σ-ring with some further conditions) of the class of all subsets of the base space. In order to make σ-additivity possible, but without any real reason that could justify saying to one set 'you are an event', and to another 'you are not'."*

In order to proceed with our analysis, we have to search for the roots of de Finetti's argument, roots that, we believe, lay outside de Finetti's own theory, for they hinge on the perceived structure of the continuum. Bell (1998, p.2), states:

"the generally accepted set-theoretical formulation of mathematics (is one) in which all mathematical entities, being synthesized from collections of individuals, are ultimately of a discrete or punctate nature. This punctate character is possessed in particular by the set supporting the 'continuum' of real numbers - the 'arithmetical continuum'."

Among the alternatives to arithmetical punctual perspectives of the continuum, there are more geometrical perspectives. Such geometrical perspectives allow us to use an arithmetical set as a coordinate (localization) system in the continuum, but the 'ultimate parts' of the continuum, called infinitesimals, are essentially nonpunctiform, i.e. non point like. Among the proponents of infinitesimal perspectives for the continuum one should mention Leibniz, Kant, Peirce, Poincaré, Brouwer, Weyl, R.Thom, Lawvere, Robinson, Nelson, and many others. We refer to Bell (2005) for an excellent general historical review, and to Robertson (2001) for the ideas of C.S.Peirce. In the infinitesimal perspective, see Bell (1998, p.3),

"any of its (the continuum) connected parts is also a continuum and, accordingly, divisible. A point, on the other hand, is by its nature not divisible, and so (as stated by Leibniz) cannot be part of the continuum."

In Peirce doctrine of synechism, the infinitesimal geometrical structure of the continuum acts like *" the 'glue' causing points on a continuous line to lose their individual identity."*, see Bell (2005, p.211). According to Peirce, *" The very word continuity implies that the instants of time or the points of a line are everywhere welded together."*

De Finetti's argument on non-measurable sets implicitly assumes that all point subsets of R^n have equal standing, i.e., that the continuum has no structure. Under the arithmetical perspective of the continuum, de Finetti objection makes perfect sense, and we should abstain from measure theory or alternative formalisms, as does orthodox Bayesian statistics. This is how Peirce's concept of synechism helps us to overcome a major obstacle (for the FBST) presented by orthodox Bayesian philosophy, namely, the objections against the use of measure theory.

At this point it should be clear that my answer to Soren's question is emphatically affirmative. From Soren's comments and suggestions it is also clear for me now, how well he knew the answer when he asked me the question. As a maieutic teacher however, he let me look for the answers my own way. I can only thank him for the invitation that brought me for the first time into contact with the beautiful world of Semiotics and Peircean philosophy.

7 Final Remarks

The physician Rambam, Moshe ben Maimon of (the then caliphate of) Cordoba, 1135–1202, wrote Shmona Perakim, a book on psychology (medical procedures for healing the human soul) based on basic principles exposed by Aristotle in Nicomachean Ethics, see Olitzky (2000) and Rackham (1926). Rambam explains how the health of the human soul depends on always finding the 'straight path' (derech y'shara) or 'golden way' (shvil ha-zahav), at the perfect center between the two opposite extremes of excess (odef) and scarcity (choser).

> *"The straight path is the middle one, that is equidistant from both extremes.... Neither should a man be a clown or jokester, nor sad or mourning, but he should be happy all his days in serenity and pleasantness. And so with all the other qualities a man possesses. This is the way of the scholars. Every man whose virtues reflect the middle, is called a chacham... a wise man."*

Rambam explains that a (always imperfect) human soul, at a given time and situation, may be more prone to fall victim of one extreme than to its opposite, and should try to protect itself accordingly. One way of achieving this protection is to offset its position in order to (slightly over) compensate for an existing or anticipated bias.

At the dawn of the 20th century, humanity had in classical physics a paradigm of science handing out unquestionable truth, and faced the brutality of many totalitarian states. Dogmatism had the upper hand, and we had to protect ourselves accordingly.

At the beginning of the 21st century we are enjoying the comforts of an hyperactive economy that seems to be blind to the constraints imposed by our ecological environment, and our children are being threatened by autistic alienation through the virtual reality of their video games. It may be the turn of (an apathic form of) solipsism.

Finally, Rambam warns us about a common mistake: Protective offsets may be a useful precautionary tactic, or even a good therapeutic strategy, but should never be considered as a virtue per se. The virtuous path is the straight path, neither left of it nor right of it, but at the perfect center.

References

In the text, CP and NEM refer to:
- CP - C.Hartshorne, P.Weiss, A.Burks, edts. (1992). *Collected Papers of Charles Sanders Peirce.* Charlottesville: InteLex Corp.
- NEM - C.Eisele edt. (1976). *The New Elements of Mathematics of Charles S. Peirce.* The Hague: Mouton.
- J.Baggott (1992). *The Meaning of Quantum Theory.* Oxford University Press.
- T.Bakken, T.Hernes (2002). *Autopoietic Organization Theory: Drawing on Niklas Luhmann's Social Systems Perspective.* Copenhagen Business School.
- D.Barbieri (1992). Is Genetic Epistemology of Any Interest for Semiotics? *Scripta Semiotica 1, 1-6.*
- J.L.Bell (1998). *A Primer of Infinitesimal Analysis.* Cambridge Univ. Press.
- J.L.Bell (2005). *The Continuous and the Infinitesimal in Mathematics and Philosophy.* Milano: Polimetrica.
- J.Bopry (2002). Semiotics, Epistemology, and Inquiry. *Teaching & Learning,* 17, 1, 5–18.
- W.Borges, J.M.Stern (2006). Evidence and Compositionality. To appear in the proceedings of *SMPS-2006, 3th International Conference on Soft Methods in Probability and Statistics.*
- R.Brent, J.Bruck (2006). Can Computers Help to Explain Biology? *Nature,* 440/23, 416–417.
- S.Brier (1995) Cyber-Semiotics: On autopoiesis, code-duality and sign games in bio-semiotics. Cybernetics and Human Knowing, 3, 1, 3–14.
- S.Brier (2001). Cybersemiotics and Umweltlehre. *Semiotica,* Special issue on Jakob von Uexküll's Umweltsbiologie, 134 (1/4), 779-814.
- S.Brier (2005). The Construction of Information and Communication: A Cyber-Semiotic Re-Entry into Heinz von Foerster's Metaphysical Construction of Second Order Cybernetics. *Semiotica,* 154, 355–399.
- H.Caygill (1995). *A Kant Dictionary.* Oxford: Blackwell.
- M.Ceruti (1989). *La Danza che Crea.* Milano: Feltrinelli.
- D.W.Cohen (1989). *An Introduction to Hilbert Space and Quantum Logic.* NY: Springer.
- M.Davis (1977). *Applied Nonstandard Analysis.* NY: Dover.

- M.H.DeGroot (1970). *Optimal Statistical Decisions.* Mcgraw-Hill.
- J.S.Dugdale (1996). *Entropy and Its Physical Meaning.* Taylor & Francis.
- L.E.Durbins and L.J.Savage (1976). *How to Gamble if You Must: Inequalities for Stochastic Processes.* NY: Dover.
- M.Eaton (1989). *Group Invariance Applications in Statistics.* Inst. of Mathematical Statistics.
- J.S.Efran, M.D.Lukens, R,J,Lukens (1990). *Language, Structure and Change: Frameworks of Meaning in Psychotherapy.* W.W.Norton.
- I.Eibel-Eibesfeldt (1970). *Ethology, The Biology of Behaviour.* NY: Holt, Rinehart and Winston.
- J.Ferreira (2006). *Semiotic Explorations in Computer Interface Design.* Wellington: Victoria University.
- B.de Finetti (1975). *Theory of Probability. A Critical Introductory Treatment.* London: John Wiley.
- H.von Foerster (2003). *Understanding Understanding: Essays on Cybernetics and Cognition.* NY: Springer Verlag. The following articles in this anthology are of special interest: (a) On Self-Organizing Systems and their Environments; p.1–19. (b) On Constructing a Reality; p.211–227. (c) Objects: Tokens for Eigen-Behaviors; p.261–271. (d) For Niklas Luhmann: How Recursive is Communication? p.305–323.
- C.J.Gerhardt (1890). *Die philosophischen Schriften von Gottfried Wilhelm Leibniz.* Berlin: Weidmannsche Buchhandlung.
- R.Goldblatt (1998). *Lectures on the Hyperreals: An Introduction to Nonstandard Analysis.* NY: Springer.
- A.L. Goudsmit (1988). Towards a Negative Understanding of Psychotherapy. Ph.D. Thesis, Groningen University.
- P.R.Halmos (1998). *Naive Set Theory.* NY: Springer.
- R.I.G.Hughes (1992). *The Structure and Interpretation of Quantum Mechanics.* Harvard University Press.
- I.A.Ibri (1992). *Kosmos Noetos. A Arquitetura Metafísica de Charles S. Peirce.* São Paulo: Prespectiva.
- J.N.Kapur (1989). *Maximum Entropy Models in Science and Engineering.* NY: Wiley.
- A.N.Kolmogorov, S.V.Fomin (1982, Portuguese translation). *Elements of the Theory of Functions and Functional Analysis.* Moscow: MIR.
- M.Lauretto, C.A.B.Pereira, J.M.Stern, S.Zacks (2003). Comparing Parameters of Two Bivariate Normal Distributions Using the Invariant Full Bayesian Significance Test. *Brazilian Journal of Probability and Statistics*, 17, 147-168.
- M.Lauretto, J.M.Stern (2005). FBST for Mixture Model Selection. MaxEnt 2005, 25th International Workshop on Bayesian Inference and Maximum Entropy

Methods in Science and Engineering. *American Institute of Physics Conference Proceedings,* 803, 121–128.
- M.R.Madruga, L.G.Esteves, S.Wechsler (2001). On the Bayesianity of Pereira-Stern Tests. *Test,* 10, 291–299.
- M.R.Madruga, C.A.B.Pereira, J.M.Stern (2003). Bayesian Evidence Test for Precise Hypotheses. *Journal of Statistical Planning and Inference,* 117, 185–198.
- H.R.Maturana, F.J.Varela (1980). *Autopoiesis and Cognition. The Realization of the Living.* Dordrecht: Reidel.
- H.R.Maturana (1988). Ontology of Observing. The Biological Foundations of Self Consciousness and the Physical Domain of Existence. pp 18–23 in *Conference Workbook: Texts in Cybernetics.* Felton, CA: American Society for Cybernetics.
- H.R.Maturana (1991). Science and Reality in Daily Life: The Ontology of Scientific Explanations. In Seier (1991).
- H.R.Maturana, B.Poerksen (2004). Varieties of Objectivity. *Cybernetics and Human Knowing.* 11, 4, 63–71.
- S.H.Muggleton (2006). Exceeding Human Limits. *Nature,* 440/23, 409–410.
- L.Nachbin (1965). *The Haar Integral.* Van Nostrand.
- E.Nelson (1987). *Radically Elementary Probability Theory.* AM-117. Princeton University Press.
- W.Noeth (1995). *Handbook of Semiotics.* Indiana University Press.
- V.R.Nosov, V.B.Kolmanovskii (1986). *Stability of Functional Diferential Equations.* London: Academic Press.
- K.Olitzky edt. (2000). *Shemonah Perakim: A Treatise on the Soul by Moshe ben Maimon.* URJ Press.
- C.A.B.Pereira, J.M.Stern (1999). Evidence and Credibility: Full Bayesian Significance Test for Precise Hypotheses. *Entropy Journal,* 1, 69–80.
- J.Piaget (1975). *L'équilibration des Structures Cognitives: Problème Central du Développement.* Paris: PUF.
- H.Rackham (1926). *Aristotle, Nicomachean Ethics.* Harvard University Press.
- W. Rasch (2000) Niklas Luhmanns Modernity. Paradoxes of Differentiation. Stanford Univ.Press. Specially chapter 3 and 4, also published as: W.Rasch (1998). Luhmanns Widerlegung des Idealismus: Constructivism as a Two-Front War. *Soziale Systeme,* 4, 151-159; and W.Rasch (1994). In Search of Lyotard Archipelago, or: How to Live a Paradox and Learn to Like It. *New German Critique,* 61, 55-75.
- C.J.van Rijsbergen (2004). *The Geometry of Information Retrieval.* Cambridge University Press.
- A.M.Robert (2003). *Nonstandard Analysis.* Mineola: Dover.
- R.Robertson (2001). One, Two, Three, Continuity. C.S.Peirce and the Nature of the Continuum. *Cybernetics & Human Knowing,* 8, 7-24.

- C.Ruhla (1992). *The Physics of Chance: From Blaise Pascal to Niels Bohr.* Oxford University Press.
- L.Santaella (2002). *Semiótica Aplicada.* São Paulo: Thomson Learning.
- L.Segal (2001). *The Dream of Reality. Heintz von Foerster's Constructivism.* NY: Springer.
- Shin'ichiro Tomonaga (1962). *Quantum Mechanics. V.1 Old Quantum Theory; V.2, New quantum theory.* North Holland and Interscience Publishers.
- F.Steier, edt. (1991) *Research and Reflexivity.* SAGE Publications.
- J.M.Stern (2003). Significance Tests, Belief Calculi, and Burden of Proof in Legal and Scientific Discourse. 1st Bayesian Modeling Applications Workshop, at UAI'03, 19th Conference on Uncertainty in Artificial Intelligence, and Laptec'03, 4th Congress of Logic Applied to Technology. *Frontiers in Artificial Intelligence and its Applications,* 101, 139–147.
- J.M.Stern (2004). Paraconsistent Sensitivity Analysis for Bayesian Significance Tests. SBIA'04, XVII Brazilian Symposium on Artificial Intelligence. *Lecture Notes Artificial Intelligence,* LNAI 3171, 134–143.
- J.M.Stern (2005). Cognitive Constructivism, Eigen–Solutions, and Sharp Statistical Hypotheses. Preliminary version in FIS2005 - Third Conference on the Foundations of Information Science. 61, 1–23. Final version to appear in: *Cybernetics and Human Knowing.*
- J.M.Stern, S.Zacks (2002). Testing Independence of Poisson Variates under the Holgate Bivariate Distribution. The Power of a New Evidence Test. *Statistical and Probability Letters,* 60, 313–320.
- S.M.Ulam (1943). What is a Measure? *The American Mathematical Monthly.* 50, 10, 597-602.
- E.Wenger, R.Pea, J.S.Brown, C.Heath (1999). *Communities of Practice: Learning, Meaning, and Identity.* Cambridge Univ. Press.
- T.Winograd, F.Flores (1987). *Understanding Computers and Cognition: A New Foundation for Design* NY: Addison-Wesley.
- V.I.Zubov (1983). *Analytical Dynamics of Systems of Bodies.* Leningrad University.

Cognitive Constructivism, Eigen-Solutions, and Sharp Statistical Hypotheses

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Abstract

In this paper epistemological, ontological and sociological questions concerning the statistical significance of sharp hypotheses in scientific research are investigated within the framework provided by Cognitive Constructivism and the FBST - Full Bayesian Significance Test. The constructivist framework is contrasted with the traditional epistemological settings for orthodox Bayesian and frequentist statistics provided by Decision Theory and Falsificationism.

Keywords: Autopoiesis, Cognitive constructivism, Epistemology, Ontology, Scientific hypotheses, Statistical significance tests, Social systems, Systems' theory.

1 Introduction

In this paper, a few epistemological, ontological and sociological questions concerning the statistical significance of sharp hypotheses in the scientific context are investigated within the framework provided by Cognitive Constructivism, or the Constructivist Theory (ConsTh) as presented in Maturana and Varela (1980), Foerster (2003) and Luhmann (1989, 1990, 1995). Several conclusions of the study, however, remain valid, mutatis mutandis, within various other organizations and systems, see for example Bakken and Hernes (2002), Christis (2001), Mingers (2000) and Rasch (1998).

The author's interest in this research topic emerged from his involvement in the development of the Full Bayesian Significance Test (FBST), a novel Bayesian solution to the statistical problem of measuring the support of sharp hypotheses, first presented in Pereira and Stern (1999). The problem of measuring the support of sharp hypotheses poses several conceptual and methodological difficulties for traditional statistical analysis under both the frequentist (classical) and the orthodox Bayesian approaches. The solution provided by the FBST has significant advantages over traditional alternatives, in terms of its statistical and logical properties. Since these properties have already been thoroughly analyzed in previous papers, see references, the focus herein is directed exclusively to epistemological and ontological questions.

Despite the fact that the FBST is fully compatible with Decision Theory (DecTh), as shown in Madruga et al. (2001), which, in turn, provides a strong and coherent epistemological framework to orthodox Bayesian Statistics, its logical properties open the possibility of using and benefiting from alternative epistemological settings. In this article, the epistemological framework of ConsTh is counterposed to that of DecTh. The contrast, however, is limited in scope by our interest in statistics and is carried out in a rather exploratory an non exhaustive form. The epistemological framework of ConsTh is also counterposed to that of Falsificationism, the epistemological framework within which classical frequentist statistical test of hypotheses are often presented, as shown in Boyd (1991) and Popper (1959, 1963).

In section 2, the fundamental notions of Autopoiesis and Eigen-Solutions in autopoietic systems are reviewed. In section 3, the same is done with the notions of Social Systems and Functional Differentiation and in section 4, a ConsTh view of science is presented. In section 5, the material presented in sections 2, 3 and 4 is related to the statistical significance of sharp scientific hypotheses and the findings therein are counterposed to traditional interpretations such as those of DecTh. In section 6, a few sociological analyses for differentiation phenomena are reviewed. In sections 7 and 8, the final conclusions are established.

In sections 2, 3, 4, and 6, well established concepts of the ConsTh are presented. However, in order to overcome an unfortunately common scenario, an attempt is made to make them accessible to a scientist or statistician who is somewhat familiar with traditional frequentist, and decision-theoretic statistical interpretations, but unfamiliar with the constructivist approach to epistemology. Rephrasing these concepts (once again) is also avoided. Instead, quoting the primary sources is preferred whenever it can be clearly (in our context) and synthetically done. The contributions in sections 5, 7 and 8, relate mostly to the analysis of the role of quantitative methods specifically designed to measure the statistical support of sharp hypotheses. A short review of the FBST is presented in Appendix A.

2 Autopoiesis and Eigen-Solutions

The concept of autopoiesis tries to capture an essential characteristic of living organisms (auto=self, poiesis=production). Its purpose and definition are stated in Maturana and Varela (1980):

"Our aim was to propose the characterization of living systems that explains the generation of all the phenomena proper to them. We have done this by pointing at Autopoiesis in the physical space as a necessary and sufficient condition for a system to be a living one."

"An autopoietic system is organized (defined as a unity) as a network of processes of production (transformation and destruction) of components that produces the components which:

(i) through their interactions and transformations continuously regenerate and realize the network of processes (relations) that produced them; and

(ii) constitute it (the machine) as a concrete unity in the space in which they (the components) exist by specifying the topological domain of its realization as such a network."

Autopietic systems are non-equilibrium (dissipative) dynamical systems exhibiting (meta) stable structures, whose organization remains invariant over (long periods of) time, despite the frequent substitution of their components. Moreover, these components are produced by the same structures they regenerate. For example, the macromolecular population of a single cell can be renewed thousands of times during its lifetime, see Bertalanffy (1969). The investigation of these regeneration processes in the autopoietic system production network leads to the definition of cognitive domain:

"The circularity of their organization continuously brings them back to the same internal state (same with respect to the cyclic process). Each internal state requires that certain conditions (interactions with the environment) be satisfied in order to proceed to the next state. Thus the circular organization implies the prediction that an interaction that took place once will take place again. If this does not happen the system maintains its integrity (identity with respect to the observer) and enters into a new prediction. In a continuously changing environment these predictions can only be successful if the environment does no change in that which is predicted. Accordingly, the predictions implied in the organization of the living system are not predictions of particular events, but of classes of inter-actions. Every interaction is a particular interaction, but every prediction is a prediction of a class of interactions that is defined by those features of its elements that will allow the living system to retain its circular organization after the interaction, and thus, to interact again. This makes living systems inferential systems, and their domain of interactions a cognitive domain."

The characteristics of this circular (cyclic or recursive) regenerative processes and their eigen (auto, equilibrium, fixed, homeostatic, invariant, recurrent, recursive) -states, both in concrete and abstract autopoietic systems, are further investigated in Foerster (2003) and Segal (2001):

"The meaning of recursion is to run through one's own path again. One of its results is that under certain conditions there exist indeed solutions which, when reentered into the formalism, produce again the same solution. These are called "eigen-values", "eigen-functions", "eigen-behaviors", etc., depending on which domain this formation is applied - in the domain of numbers, in functions, in behaviors, etc."

The concept of eigen-solution for an autopoietic system is the key to distinguish specific objects in a cognitive domain. von Foerster also establishes several essential properties of eigen-solutions that will support the analyses conducted in this paper and conclusions established herein:

"Objects are tokens for eigen-behaviors. Tokens stand for something else. In exchange for money (a token itself for gold held by one's government, but unfortunately no longer redeemable), tokens are used to gain admittance to the subway or to play pinball machines. In the cognitive realm, objects are

the token names we give to our eigen-behavior. This is the constructivist's insight into what takes place when we talk about our experience with objects."

"Eigenvalues have been found ontologically to be discrete, stable, separable and composable, while ontogenetically to arise as equilibria that determine themselves through circular processes. Ontologically, Eigenvalues and objects, and likewise, ontogenetically, stable behavior and the manifestation of a subject's "grasp" of an object cannot be distinguished."

The arguments used in this study rely heavily on two qualitative properties of eigen-solutions, refered by von Foerster by the terms "Discrete" and "Equilibria". In what follows, the meaning of these qualifiers, as they are understood by von Foerster and used herein, are examined:

a- Discrete (or sharp):

"There is an additional point I want to make, an important point. Out of an infinite continuum of possibilities, recursive operations carve out a precise set of discrete solutions. Eigen-behavior generates discrete, identifiable entities. Producing discreteness out of infinite variety has incredibly important consequences. It permits us to begin naming things. Language is the possibility of carving out of an infinite number of possible experiences those experiences which allow stable interactions of your-self with yourself."

It is important to realize that, in the sequel, the term "discrete", used by von Foerster to qualify eigen-solutions in general, should be replaced, depending on the specific context, by terms such as lower-dimensional, precise, sharp, singular etc. Even in the familiar case of linear algebra, if we define the eigen-vectors corresponding to a singular eigen-value c of a linear transformation $T(\ )$ only by its essential property of directional invariance, $T(x) = cx$, we obtain one dimensional sub-manifolds which, in this case, are subspaces or lines trough the origin. Only if we add the usual (but non essential) normalization condition, $||x|| = 1$, do we get discrete eigen-vectors.

b- Equilibria (or stable):

A stable eigen-solution of the operator $Op(\ )$, defined by the fixed-point or invariance equation, $x_{inv} = Op(x_{inv})$, can be found, built or computed as the limit, x_∞, of the sequence $\{x_n\}$, defined by recursive application of the operator, $x_{n+1} = Op(x_n)$. Under appropriate conditions, such as within a domain of attraction, the process convergence and its limit eigen-solution will not depend on the starting point, x_0. In the linear algebra example, using almost any staring point, the sequence generated by the recursive relation $x_{n+1} = T(x_n)/||T(x_n)||$, i.e. the application of T followed by normalization, converges to the unitary eigen-vector corresponding to the largest eigen-value.

In sections 4 and 5 it is shown, for statistical analysis in a scientific context, how the property of sharpness indicates that many, and perhaps some of the most relevant, scientific hypotheses are sharp, and how the property of stability, indicates that considering these hypotheses is natural and reasonable. The statistical consequences of these findings will be discussed in sections 7 and 8. Before that, however, a few other ConsTh concepts must be introduced in sections 3 and 6.

Autopoiesis found its name in the work of Maturana and Varela (1980), together with a simple, powerful and elegant formulation using the modern language of system's theory. Nevertheless, some of the basic theoretical concepts, such as those of self-organization and autonomy of living organisms, have long historical grounds that some authors trace back to Kant. As seen in Kant (1790, sec. 65) for example, *"Self-organized being"* is characterized as one in which,

"... every part is thought as 'owing' its presence to the 'agency' of all the remaining parts, and also as existing 'for the sake of the others' and of the whole, that is as an instrument, or organ."

"Its parts must in their collective unity reciprocally produce one another alike as to form and combination, and thus by their own causality produce a whole, the conception of which, conversely, -in a being possessing the causality according to conceptions that is adequate for such a product- could in turn be the cause of the whole according to a principle, so that, consequently, the nexus of 'efficient causes' (progressive causation, nexus effectivus) might be no less estimated as an 'operation brought about by final causes' (regressive causation, nexus finalis)."

For further historical comments we refer the reader to Zelleny (1980).

3 Functional Differentiation

In order to give appropriate answers to environmental complexities, autopoietic systems can be hierarchically organized as Higher Order Autopoietic Systems. As in Maturana and Varela (1980), this notion is defined via the concept of Coupling:

"Whenever the conduct of two or more units is such that there is a domain in which the conduct of each one is a function of the conduct of the others, it is said that they are coupled in that domain."

"An autopoietic system whose autopoiesis entails the autopoiesis of the coupled autopoietic units which realize it, is an autopoietic system of higher order."

A typical example of a hierarchical system is a Beehive, a third order autopoietic system, formed by the coupling of individual Bees, the second order systems, which, in turn, are formed by the coupling of individual Cells, the first order systems.

The philosopher and sociologist Niklas Luhmann applied this notion to the study of modern human societies and its systems. Luhmann's basic abstraction is to look at social systems only at its higher hierarchical level, in which it is seen as an autopoietic communications network. In Luhmann's terminology, a communication event consists of: Utterance, the form of transmission; Information, the specific content; and Understanding, the relation to future events in the network, such as the activation or suppression of future communications.

"Social systems use communication as their particular mode of autopoietic (re)production. Their elements are communications that are recursively produced and reproduced by a network of communications that are not living units, they are not conscious units, they are not actions. Their unity requires a synthesis of three selections, namely information, utterance and understanding (including misunderstanding)."

For Luhmann, society's best strategy to deal with increasing complexity is the same as one observes in most biological organisms, namely, differentiation. Biological organisms differentiate in specialized systems, such as organs and tissues of a pluricellular life form (non-autopoietic or allopoietic systems), or specialized individuals in an insect colony (autopoietic system). In fact, societies and organisms can be characterized by the way in which they differentiate into systems. For Luhmann, modern societies are characterized by a vertical differentiation into autopoietic functional systems, where each system is characterized by its code, program and (generalized) media. The code gives a bipolar reference to the system, of what is positive, accepted, favored or valid, versus what is negative, rejected, disfavored or invalid. The program gives a specific context where the code is applied, and the media is the space in which the system operates.

Standard examples of social systems are:

- Science: with a true/false code, working in a program set by a scientific theory, and having articles in journals and proceedings as its media;
- Judicial: with a legal/illegal code, working in a program set by existing

laws and regulations, and having certified legal documents as its media;

- Religion: with a good/evil code, working in a program set by sacred and hermeneutic texts, and having study, prayer and good deeds as its media;

- Economy: with a property/lack thereof code, working in a program set by economic planning scenarios and pricing methods, and having money and money-like financial assets as its media.

Before ending this section, a notion related to the break-down of autopoiesis is introduced: Dedifferentiation (Entdifferenzierung) is the degradation of the system's internal coherence, through adulteration, disruption, or dissolution of its own autopoietic relations. One form of dedifferentiation (in either biological or social systems) is the system's penetration by external agents who try to use system's resources in a way that is not compatible with the system's autonomy. In Lumann's conception of modern society each system may be aware of events in other systems, that is, be cognitively open, but is required to maintain its differentiation, that is, be operationally closed. In Luhmann's words:

"Autopoieticists claim that the smooth functioning of modern societies depends critically on maintaining the operational autonomy of each and every one of its functional (sub)systems."

4 Eigensolutions and Scientific Hypotheses

The interpretation of scientific knowledge as an eigensolution of a research process is part of a constructivistic approach to epistemology. Figure 1 presents an idealized structure and dynamics of knowledge production. This diagram represents, on the Experiment side (left column) the laboratory or field operations of an empirical science, where experiments are designed and built, observable effects are generated and measured, and the experimental data bank is assembled. On the Theory side (right column), the diagram represents the theoretical work of statistical analysis, interpretation and (hopefully) understanding according to accepted patterns. If necessary, new hypotheses (including whole new theories) are formulated, motivating the design of new experiments. Theory and experiment constitute a double feed-back cycle making it clear that the design of experiments is guided by the existing theory and its interpretation, which, in turn, must be constantly checked, adapted or modified in order to cope with the observed experiments. The whole system constitutes an autopoietic unit, as seen in Krohn and Küppers (1990):

"The idea of knowledge as an eigensolution of an operationally closed combination between argumentative and experimental activities attempts to answer the initially posed question of how the construction of knowledge binds itself to its construction in a new way. The coherence of an eigensolution does not refer to an objectively given reality but follows from the operational closure of the construction. Still, different decisions on the selection of couplings may lead to different, equally valid eigensolutions. Between such different solutions no reasonable choice is possible unless a new operation of knowledge is constructed exactly upon the differences of the given solutions. But again, this frame of reference for explicitly relating different solutions to each other introduces new choices with respect to the coupling of operations and explanations. It does not reduce but enhances the dependence of knowledge on decisions. On the other hand, the internal restrictions imposed by each of the chosen couplings do not allow for any arbitrary construction of results. Only few are suitable to mutually serve as inputs in a circular operation of knowledge."

Experiment				Theory
Experiment design	$\Leftarrow$	Operation-alization	$\Leftarrow$	Hypotheses formulation
$\Downarrow$				$\Uparrow$
Effects observation		false / true eigen-solution		Inter-pretation
$\Downarrow$				$\Uparrow$
Data acquisition	$\Rightarrow$	Explanation	$\Rightarrow$	Statistical analysis
Sample space				Parameter space

Figure 1: Scientific production diagram.

5 Sharp Statistical Hypotheses

Statistical science is concerned with inference and application of probabilistic models. From what has been presented in the preceding sections, it becomes clear what the role of Statistics in scientific research is, at least in the ConsTh view of scientific research: Statistics has a dual task, to be performed both in the Theory and the Experiment sides of the diagram in Figure 1:

- At the Experiment side of the diagram, the task of statistics is to make probabilistic statements about the occurrence of pertinent events, i.e. describe probabilistic distributions for what, where, when or which events can occur. If the events are to occur in the future, these descriptions are called predictions, as is often the case in the natural sciences. It is also possible (more often in social sciences) to deal with observations related to past events, that may or may not be experimentally generated or repeated, imposing limitations to the quantity and/or quality of the available data. Even so, the habit of calling this type of statement "predictive probabilities" will be maintained.

- At the Theory side of the diagram, the role of statistics is to measure the statistical support of hypotheses, i.e. to measure, quantitatively, the hypothesis plausibility or possibility in the theoretical framework where they were formulated, given the observed data. From the material presented in the preceding sections, it is also clear that, in this role, statistics is primarily concerned with measuring the statistical support of sharp hypotheses, for hypotheses sharpness (precision or discreteness) is an essential characteristic of eigen-solutions.

Let us now examine how well the traditional statistical paradigms, and in contrast the FBST, are able to take care of this dual task. In order to examine this question, the first step is to distinguish what kind of probabilistic statements can be made. We make use of tree statement categories: Frequentist, Epistemic and Bayesian:

Frequentist probabilistic statements are made exclusively on the the basis of the frequency of occurrence of an event in a (potentially) infinite sequence of observations generated by a random variable.

Epistemic probabilistic statements are made on the basis of the epistemic status (degree of belief, likelihood, truthfulness, validity) of an event from the possible outcomes generated by a random variable. This generation may be actual or potential, that is, may have been realized or not, may be observable or not, may be repeated an infinite or finite number of times.

Bayesian probabilistic statements are epistemic probabilistic statements generated by the (in practice, always finite) recursive use of Bayes formula:

$$p_n(\theta) \propto p_{n-1}(\theta)p(x_n|\theta) \ .$$

In standard models, the parameter θ, a non observed random variable, and the sample x, an observed random variable, are related through their joint probability distribution, $p(x,\theta)$. The prior distribution, $p_0(\theta)$, is the starting point for the Bayesian recursion operation. It represents the initial available information about θ. In particular, the prior may represent no available information, like distributions obtained via the maximum entropy principle, see Dugdale (1996) and Kapur (1989). The posterior distribution, $p_n(\theta)$, represents the available information on the parameter after the n-th "learning step", in which Bayes formula is used to incorporate the information carried by observation x_n. Because of the recursive nature of the procedure, the posterior distribution in a given step is used as prior in the next step.

Frequentist statistics dogmatically demands that all probabilistic statements be frequentist. Therefore, any direct probabilistic statement on the parameter space is categorically forbidden. Scientific hypotheses are epistemic statements about the parameters of a statistical model. Hence, frequentist statistics can not make any direct statement about the statistical significance (truthfulness) of hypotheses. Strictly speaking it can only make statements at the Experiment side of the diagram. The frequentist way of dealing with questions on Theory side of the diagram, is to embed them some how into the Experiment side. One way of doing this is by using a construction in which the whole data acquisition process is viewed as a single outcome of an imaginary infinite meta random process, and then make a frequentist statement, on the meta process, about the frequency of unsatisfactory outcomes of some incompatibility measure of the observed data bank with the hypothesis. This is the classic (and often forgotten) rationale used when stating a p-value. So we should always speak of the p-value of the data bank (not of the hypothesis). The resulting conceptual confusion and frustration (for most working scientists) with this kind of convoluted reasoning is captured by a wonderful parody of Galileo's dialogues in Rouanet et al. (1998).

A p-value is the probability of getting a sample that is more extreme than the one we got. We should therefore specify which criterion is used to define what we mean by more extreme, i.e., how do we order the sample space, and usually there are several possible criteria to do that.

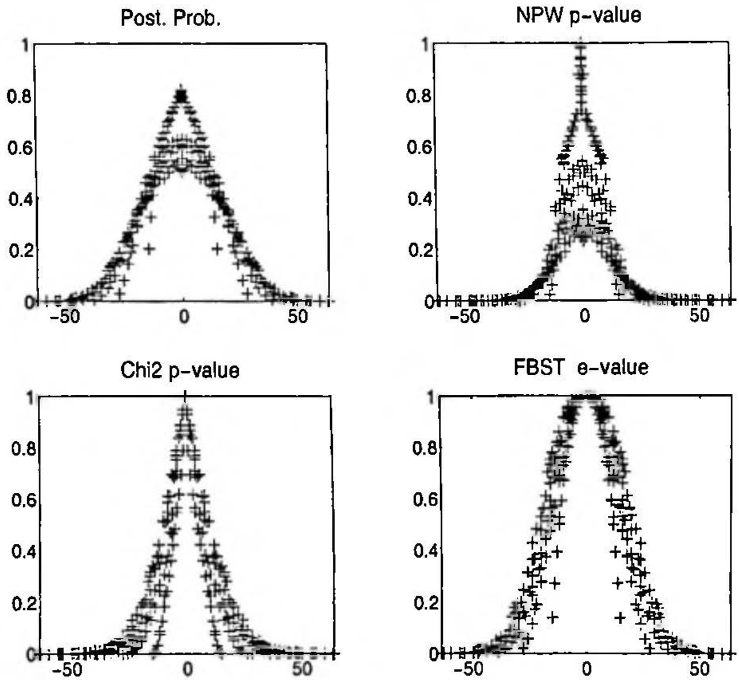

Figure 2: Independence Hypothesis, n=16

Figure 2 compares four statistics, namely, orthodox Bayesian posterior probabilities, Neyman-Pearson-Wald (NPW) p-values, Chi-square approximate p-values, and the FBST evidence value in favor of H. In this example H is the independence hypothesis in a 2×2 contingency table, for sample size $n = 16$, i.e., $y = [0,0,0,0]$ and

$$p_n(\theta \mid x) \propto \theta_{1,1}^{x_{1,1}+y_{1,1}} \theta_{1,2}^{x_{1,2}+y_{1,2}} \theta_{2,1}^{x_{2,1}+y_{2,1}} \theta_{2,2}^{x_{2,2}+y_{2,2}} \ , \quad p_0(\theta) \propto \theta_{1,1}^{y_{1,1}} \theta_{1,2}^{y_{1,2}} \theta_{2,1}^{y_{2,1}} \theta_{2,2}^{y_{2,2}} \ ,$$

$$\Theta = \{\theta \geq 0 \mid \theta_1 + \theta_2 + \theta_3 + \theta_4 = 1\} \ , \ H : \theta_{1,1} = (\theta_{1,1} + \theta_{1,2})(\theta_{1,1} + \theta_{2,1}) \ .$$

The horizontal axes shows the "diagonal asymmetry" statistics (difference between the diagonal products). The statistic D is an estimator of the unormalized Pearson correlation coefficient, ρ. For detailed explanations see Irony et al. (1995, 2000), Stern and Zacks (2002), and Madruga et al. (2003).

$$D = x_{1,1}x_{2,2} - x_{1,2}x_{2,1} \ , \quad \rho = \frac{\sigma_{1,2}}{\sigma_{1,1}\sigma_{2,2}} = \frac{\theta_{1,1}\theta_{2,2} - \theta_{1,2}\theta_{2,1}}{\sqrt{\theta_{1,1}\theta_{1,2}\theta_{2,1}\theta_{2,2}}} \ .$$

Samples that are "perfectly compatible with the hypothesis", that is, having no asymmetry, are near the center of the plot, with increasingly incompatible samples to the sides. Technically, we are taking a path in the sample space that is transversal, by the ML (maximum likelihood) operator, to the pre-image of the manifold defining the hypothesis in the parameter space. The envelope curve for the resulting FBST e-values, to be commented later in this section, is smooth (differentiable) and therefore level at its maximum, where it reaches the value 1.

In contrast the envelope curves for the p-values take the form of a cusp, i.e. a pointed curve, that is broken (non differentiable) at its maximum, where it also reaches the value one. The acuteness of the cusp also increases with increasing sample size. In the case of NPW p-values we see, at the top of the cusp, a "ladder" or "spike", with several samples with no asymmetry, but having different outcome probabilities, "competing" for the higher p-value.

This is a typical collateral effect of the artifice that converts a question about the significance of H, asking for a probability in the parameter space as an answer, into a question, conditional on H being truth, about the outcome probability of the observed sample, offering a probability in the sample space as an answer. This qualitative analysis of the p-value methodology gives us an insight on the meaning of expressions like *"increase sample size to reject"*. In the words of I.J. Good (1983):

"Very often the statistician doesn't bother to make it quite clear whether his null hypothesis is intended to be sharp or only approximately sharp. ...It is hardly surprising then that many Fisherians (and Popperians) say that - you can't get (much) evidence in favor of the null hypothesis but can only refute it."

In Bayesian statistics we are allowed to make probabilistic statements on the parameter space, and also, of course, in the sample space. Thus it seems that Bayesian statistics is the right tool for the job, and so it is! Nevertheless, we must first examine the role played by DecTh in orthodox Bayesian statistics. Since the pioneering work of de Finetti, Savage and many others, orthodox Bayesian Statistics has developed strong and coherent foundations grounded on DecTh, where many basic questions could be successfully analyzed and solved.

This foundations can be stratified in two layers:

- In the first layer, DecTh provides a coherence system for the use of probability statements, in the sense of Finetti (1974, 1981). In this context, the FBST use of probability theory is fully compatible with DecTh, as shown in Madruga et al. (2001).

- In the second layer, DecTh provides an epistemological framework for the interpretation of statistical procedures. The FBST logical properties open the possibility of using and benefiting from alternative epistemological settings such as CosTh. Hence, DecTh does not have to be "the tool for all trades".

We claim that, in the specific case of statistical procedures for measuring the support (significance tests) for sharp scientific hypotheses, ConsTh provides a more adequate epistemological framework than DecTh. This point is as important as it is subtle. In order to understand it let us first remember the orthodox paradigm, as it is concisely stated in Dubins and Savage (1965, 12.8). In a second quote, from Savage (1954, 16.3) we find that sharp hypotheses, even if important, make little sense in this paradigm, a position that is accepted throughout decision theoretic Bayesian statistics, as can also be seen in Levi (1974) and Maher et al. (1993).

"Gambling problems in which the distributions of various quantities are prominent in the description of the gambler's fortune seem to embrace the whole of theoretical statistics according to one view (which might be called the decision-theoretic Bayesian view) of the subject.

...From the point of view of decision-theoretic statistics, the gambler in this problem is a person who must ultimately act in one of two ways (the two guesses), one of which would be appropriate under one hypothesis (H_0) and the other under its negation (H_1).

...Many problems, of which this one is an instance, are roughly of the following type. A person's opinion about unknown parameters is described by a probability distribution; he is allowed successively to purchase bits of information about the parameters, at prices that may depend (perhaps randomly) upon the unknown parameters themselves, until he finally chooses a terminal action for which he receives an award that depends upon the action and parameters."

"I turn now to a different and, at least for me, delicate topic in connection with applications of the theory of testing. Much attention is given in the

literature of statistics to what purport to be tests of hypotheses, in which the null hypothesis is such that it would not really be accepted by anyone. ... extreme (sharp) hypotheses, as I shall call them...

...The unacceptability of extreme (sharp) null hypotheses is perfectly well known; it is closely related to the often heard maxim that science disproves, but never proves, hypotheses, The role of extreme (sharp) hypotheses in science and other statistical activities seems to be important but obscure. In particular, though I, like everyone who practice statistics, have often "tested" extreme (sharp) hypotheses, I cannot give a very satisfactory analysis of the process, nor say clearly how it is related to testing as defined in this chapter and other theoretical discussions."

As it is clearly seen, in the DecTh framework we speak about the betting odds for "the hypothesis wining on a gamble taking place in the parameter space". But since sharp hypotheses are zero (Lebesgue) measure sets, our betting odds must be null, i.e. sharp hypotheses must be (almost surely) false. If we accept the ConsTh view that an important class of hypotheses concern the identification of eigen-solutions, and that those are ontologically sharp, we have a paradox!

From these considerations it is not surprising that frequentist and DecTh orthodoxy consider sharp hypotheses, at best as anomalous crude approximations used when the scientist is incapable of correctly specifying error bounds, cost, loss or utility functions, etc., or then just consider them to be *"plain silly"*. In the words of D.Williams (2002):

"Bayesian significance of sharp hypothesis: a plea for sanity: ...It astonishes me therefore that some Bayesian now assign non-zero prior probability that a sharp hypothesis is exactly true to obtain results which seem to support strongly null hypotheses which frequentists would very definitely reject. (Of course, it is blindingly obvious that such results must follow)."

But no matter how many times statisticians reprehend scientist for their sloppiness and incompetence, they keep formulating sharp hypotheses, as if they where magnetically attracted to them. From the ConsTh plus FBST perspective they are, of course, just doing the right thing!

Decision theoretic statistics has also developed methods to deal with sharp hypotheses, posting sometimes a scary caveat emptor for those willing to use them. The best known of such methods are Jeffreys' tests based on Bayes Factors assigning a positive prior probability mass on the sharp hypothesis. This positive prior mass is supposed to work like a handicap system designed to balance the starting odds and make the game "fair". Out of that we only get new paradoxes, like the well documented Lindley's paradox. In opposition to its frequentist counterpart, this is an "increase sample size to accept" effect, see Shafer (1982).

The FBST e-value or evidence value supporting the hypothesis, ev(H), was specially designed to effectively evaluate the support for a sharp hypothesis, H. This support function is based on the posterior probability measure of a set called the tangential set, $\overline{T}(H)$, which is a non zero measure set (so no null probability paradoxes), see Pereira and Stern (1999), Madruga et al. (2003) and subsection A1 of the appendix.

Although ev(H) is a probability in the parameter space, it is also a possibilistic support function. The word *possibilistic* carries a heavy load, implying that ev(H) complies with a very specific logic (or algebraic) structure, as seen in Darwishe and Ginsberg (1992), Stern (2003, 2004), and subsection A3 of the appendix. Furthermore the e-value has many necessary or desirable properties for a statistical support function, such as:

1- Give an intuitive and simple measure of significance for the hypothesis in test, ideally, a *probability* defined directly in the original or *natural parameter space*.

2- Have an intrinsically geometric definition, independent of any non-geometric aspect, like the particular parameterization of the (manifold representing the) null hypothesis being tested, or the particular coordinate system chosen for the parameter space, i.e., be an *invariant* procedure.

3- Give a measure of significance that is smooth, i.e. *continuous and differentiable*, on the hypothesis parameters and sample statistics, under appropriate regularity conditions of the model.

4- Obey the *likelihood principle* , i.e., the information gathered from observations should be represented by, and only by, the likelihood function.

5- Require *no ad hoc artifice* like assigning a positive prior probability to zero measure sets, or setting an arbitrary initial belief ratio between hypotheses.

6- Be a *possibilistic* support function.

7- Be able to provide a *consistent* test for a given sharp hypothesis.

8- Be able to provide *compositionality* operations in complex models.

9- Be an *exact* procedure, i.e., make no use of "large sample" asymptotic approximations.

10- Allow the incorporation of previous experience or expert's opinion via (subjective) *prior distributions*.

For a careful and detailed explanation of the FBST definition, its computational implementation, statistical and logical properties, and several already developed applications, the reader is invited to consult some of the articles in the reference list. Appendix A provides a short review of the FBST, including its definition and main properties.

6 Semantic Degradation

In this section some constructivist analyses of dedifferentiation phenomena in social systems are reviewed. If the conclusions in the last section are correct, it is surprising how many times DecTh, sometimes with a very narrow pseudo-economic interpretation, was misused in scientific statistical analysis. The difficulties of testing sharp hypotheses in the traditional statistical paradigms are well documented, and extensively discussed in the literature, see for example the articles in Harlow et al. (1997). We hope the material in this section can help us understand these difficulties as symptoms of problems with much deeper roots. By no means the author is the first to point out the danger of analyses carried out by blind transplantation of categories between heterogeneous systems. In particular, regarding the abuse of economical analyses, Luhmann (1989) states:

"In this sense, it is meaningless to speak of "non-economic" costs. This is only a metaphorical way of speaking that transfers the specificity of the economic mode of thinking indiscriminately to other social systems."

For a sociological analysis of this phenomenon in the context of science, see for example Fuchs (1996) and DiMaggio and Powell (1991):

"...higher-status sciences may, more or less aggressively, colonize lower-status fields in an attempt at reducing them to their own First Principles. For particle physics, all is quarks and the four forces. For neurophysiology, consciousness is the aggregate outcome of the behavior of neural networks. For sociobiology, philosophy is done by ants and rats with unusual large brains that utter metaphysical nonsense according to acquired reflexes. In short, successful and credible chains or reductionism usually move from the top to the bottom of disciplinary prestige hierarchies."

"This may explain the popularity of giving an "economical understanding" to processes in functionally distinct areas even if (or perhaps because) this semantics is often hidden by statistical theory and methods based on decision theoretic analysis. This also may explain why some areas, like ecology,

sociology or psychology, are (or where) far more prone to suffer this kind of dedifferentiation by semantic degradation than others, like physics."

Once the forces pushing towards systemic degradation are clearly exposed, we hope one can understand the following corollary of von Foerster famous ethical and aesthetical imperatives:

- Theoretical imperative: Preserve systemic autopoiesis and semantic integrity, for de-differentiation is in-sanity itself.
- Operational imperative: Chose the right tool for each job: "If you only have a hammer, everything looks like a nail".

7 Competing Sharp Hypotheses

In this section we examine the concept of *Competing Sharp Hypotheses.* This concept has several variants, but the basic idea is that a good scientist should never test a single sharp hypothesis, for it would be an unfair faith of the poor sharp hypothesis standing all alone against everything else in the world. Instead, a good scientist should always confront a sharp hypothesis with a competing sharp hypotheses, making the test a fair game. As seen in Good (1983):

"Since I regard refutation and corroboration as both valid criteria for this demarcation it is convenient to use another term, Checkability, to embrace both processes. I regard checkability as a measure to which a theory is scientific, where checking is to be taken in both its positive and negative senses, confirming and disconfirming."

"...If by the truth of Newtonian mechanics we mean that it is approximately true in some appropriate well defined sense we could obtain strong evidence that it is true; but if we mean by its truth that it is exactly true then it has already been refuted."

"...I think that the initial probability is positive for every self-consistent scientific theory with consequences verifiable in a probabilistic sense. No contradiction can be inferred from this assumption since the number of statable theories is at most countably infinite (enumerable)."

"...It is very difficult to decide on numerical values for the probabilities, but it is not quite so difficult to judge the ratio of the subjective initial probabilities of two theories by comparing their complexities. This is one reason why the history of science is scientifically important."

The competing sharp hypotheses argument does not directly contradict the epistemological framework presented in this article, and it may be appropriate under certain circumstances. It may also mitigate or partially remediate the paradoxes pointed out in the previous sections when testing sharp hypotheses in the traditional frequentist or orthodox Bayesian settings. However, the author does not believe that having competing sharp hypotheses is neither a necessary condition for good science practice, nor an accurate description of science history.

Just to stay with Good's example, let us quickly examine the very first major incident in the tumultuous debacle of Newtonian mechanics. This incident was Michelson's experiment on the effect of "aethereal wind" over the speed of light, see Michelson and Morley (1887) and Lorentz et al. (1952). A clear and lively historical account to this experiment can be found in Jaffe (1960). Actually Michelson found no such effect, i.e. he found the speed of light to be constant, invariant with the relative speed of the observer. This result, a contradiction in Newtonian mechanics, is easily explained by Einstein's special theory of relativity. The fundamental difference between the two theories is their symmetry or invariance groups: Galileo's group for Newtonian mechanics, Lorentz' group for special relativity. A fundamental result of physics, Noether's Theorem, states that for every continuous symmetry in a physical theory, there must exist an invariant quantity or conservation law. For detail the reader is refered to Doncel et al. (1987), Gruber et al. (1980-98), Houtappel et al. (1965), French (1968), Landau and Lifchitz (1966), Noether (1918), Wigner (1970), Weyl (1952). Conservation laws are sharp hypotheses ideally suited for experimental checking. Hence, it seems that we are exactly in the situation of competing sharp hypotheses, and so we are today, from a far away historical perspective. But this is a post-mortem analysis of Newtonian mechanics. At the time of the experiment there was no competing theory. Instead of confirming an effect, specified only within an order of magnitude, Michelson found, for his and everybody else's astonishment, an, up to the experiment's precision, null effect.

Complex experiments like Michelson's require a careful analysis of experimental errors, identifying all significant source of measurement noise and fluctuation. This kind of analysis is usual in experimental physics, and motivates a brief comment on a secondary source of criticism on the use of sharp hypotheses. In the past, one often had to work with over simplified statistical models. This situation was usually imposed by limitations such as the lack of better or more realistic models, or the unavailability of the necessary numerical algorithms or the computer power to use them. Under these limitations, one often had to use minimalist statistical models or approximation techniques, even when these models or techniques were not recommended. These models or techniques were instrumental to provide feasible tools for statistical analysis, but made it very difficult to work (or proved very ineffective) with complex systems, scarce observations, very large data sets, etc. The need to work with complex models, and other difficult situations requiring the use of sophisticated statistical methods and techniques, is very common (and many times inescapable) in research areas dealing with complex systems like biology, medicine, social sciences, psychology, and many other fields, some of them distinguished with the mysterious appellation of "soft" science. A colleague once put it to me like this: "It seems that physics got all the easy problems...".

If there is one area where the computational techniques of Bayesian statistics have made dramatic contributions in the last decades, that is the analysis of complex models. The development of advanced statistical computational techniques like Markov Chain Monte Carlo (MCMC) methods, Bayesian and neural networks, random fields models, and many others, make us hope that most of the problems related to the use of over simplified models can now be overcome. Today good statistical practice requires all statistically significant influences to be incorporated into the model, and one seldom finds an acceptable excuse not to do so; see also Pereira and Stern (2001).

8 Final Remarks

It should once more be stressed that most of the material presented in sections 2, 3, 4, and 6 is not new in ConsTh. Unfortunately ConsTh has had a minor impact in statistics, and sometimes provoked a hostile reaction from the ill-informed. One possible explanation of this state of affairs may be found in the historical development of ConsTh. The constructivist reaction to a dogmatic objectivism (metaphysical realism) prevalent in hard sciences, specially in the XIX and the beginning of the XX century, raised a very outspoken rhetoric intended to make explicitly clear how naive and fragile the foundations of the then simplistic objectivism were. This rhetoric was extremely successful, quickly awakening and forever changing the minds of those directly interested in the fields of history and philosophy of science, and spread rapidly into many other areas. Unfortunately the same rhetoric could, in a superficial reading, make ConsTh to be perceived as either hostile or intrinsically incompatible with the use of quantitative and statistical methods, or leading to an extreme forms of subjectivism.

In ConsTh, or Idealism as presented in this article, neither does one claim to have access to a "thing in itself" or "Ding an sich" in the external environment, see Caygill (1995), as do dogmatic forms of objectivism, nor does one surrender to solipsism, as do skeptic forms of subjectivism, including some representatives of the subjectivist school of probability and statistics, as seen in Finetti (1974, 1.11, 7.5.7). In fact, it is the role of the external constraints imposed by the environment, together with the internal autopoietic relations of the system, to guide the convergence of the learning process to precise eigen-solutions, these being at the end, the ultimate or real objects of scientific knowledge. As stated by Luhmann (1990b, 1995):

"...constructivism maintains nothing more than the unapproachability of the external world "in itself" and the closure of knowing - without yielding, at any rate, to the old skeptical or "solipsistic" doubt that an external world exists at all-..."

"...at least in systems theory, they (statements) refer to the real world. Thus the concept of system refers to something that in reality is a system and thereby incurs the responsibility of testing its statements against reality."

"...both subjectivist and objectivist theories of knowledge have to be replaced by the system / environment distinction, which then makes the distinction subject / object irrelevant."

The author hopes to have shown that ConsTh not only gives a balanced and effective view of the theoretical / experimental aspects of scientific research but also that it is well suited (or even better suited) to give the necessary epistemological foundations for the use of quantitative methods of statistical analysis needed in the practice of science. It should also be stressed, according to author's interpretation of ConsTh, the importance of measuring the statistical support for sharp hypotheses. In this setting, the author believes that, due to its statistical and logical characteristics, the FBST is the right tool for the job, and hopes to have motivated the reader to find more about the FBST definition, theoretical properties, efficient computational implementation, and several of the already developed applications, in some of the articles in the reference list. This perspective opens interesting areas for further research. Among them, we mention the following two.

8.1 Noether and de Finetti Theorems

The first area for further research has to do with some similarities between Noether theorems in physics, and de Finetti type theorems in statistics. Nother theorems provide invariant physical quantities or conservation laws from symmetry transformation groups of the physical theory, and conservation laws are sharp hypotheses by excellence. In a similar way, de Finetti type theorems provide invariant distributions from symmetry transformation groups of the statistical model. Those invariant distributions can in turn provide prototypical sharp hypotheses in many application areas. Physics has its own heavy apparatus to deal with the all important issues of invariance and symmetry. Statistics, via de Finetti theorems, can provide such an apparatus for other areas, even in situations that are not naturally embedded in a heavy mathematical formalism, see Feller (1968, ch.7) and also Diaconis (1987, 1988), Eaton (1989) and Ressel (1987).

8.2 Compositionality

The second area for further research has to do with one of the properties of eigen-solutions mentioned by von Foerster that has not been directly explored in this article, namely that eigen-solutions are "composable", see Borges and Stern (2005) and section A4. Compositionality properties concern the relationship between the credibility, or truth value, of a complex hypothesis, H, and those of its elementary constituents, H^j, $j = 1 \dots k$. Compositionality questions play a central role in analytical philosophy.

According to Wittgenstein (2001, 2.0201, 5.0, 5.32):

- Every complex statement can be analyzed from its elementary constituents.

- Truth values of elementary statement are the results of those statements' truth-functions (Wahrheitsfunktionen).

- All truth-function are results of successive applications to elementary constituents of a finite number of truth-operations (Wahrheitsoperationen).

Compositionality questions also play a central role in far more concrete contexts, like that of reliability engineering, see Birnbaum et al. (1961, 1.4):

"One of the main purposes of a mathematical theory of reliability is to develop means by which one can evaluate the reliability of a structure when the reliability of its components are known. The present study will be concerned with this kind of mathematical development. It will be necessary for this

purpose to rephrase our intuitive concepts of structure, component, reliability, etc. in more formal language, to restate carefully our assumptions, and to introduce an appropriate mathematical apparatus."

In Luhmann (1989) we find the following remark on the evolution of science that directly hints the importance of this property:

"After the (science) system worked for several centuries under these conditions it became clear where it was leading. This is something that idealization, mathematization, abstraction, etc. do not describe adequately. It concerns the increase in the capacity of decomposition and recombination, a new formulation of knowledge as the product of analysis and synthesis. In this case analysis is what is most important because the further decomposition of the visible world into still further decomposable molecules and atoms, into genetic structures of life or even into the sequence human/role/action/ action-components as elementary units of systems uncovers an enormous potential for recombination."

In the author's view, the composition (or re-combination) of scientific knowledge and its use, so relevant in technology development and engineering, can give us a different perspective (perhaps a, bottom-up, as opposed to the top-down perspective in this article) on the importance of sharp hypotheses in science and technology practice. It can also provide some insight on the valid forms of iteration of science with other social systems or, in Luhmann's terminology, how science does (or should) "resonate" in human society.

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A FBST Review

The objective of this appendix is to provide a very short review of the Full Bayesian Significance Test (FBST), show a simple concrete example, and summarize the most important logical properties of the FBST support function. Several applications of the FBST, details of its efficient numerical and computational implementation, demonstrations of theoretical properties, comparison with other statistical tests for sharp hypotheses, and an extensive list of references can be found in the author's previous papers.

A.1 Evidence Value and Nuisance Parameters

Let $\theta \in \Theta \subseteq R^p$ be a vector parameter of interest, and $L(\theta \mid x)$ be the likelihood associated to the observed data x, a standard statistical model. Under the Bayesian paradigm the posterior density, $p_n(\theta)$, is proportional to the product of the likelihood and a prior density,

$$p_n(\theta) \propto L(\theta \mid X)\, p_0(\theta).$$

The (null) hypothesis H states that the parameter lies in the null set, defined by inequality and equality constraints given by vector functions g and h in the parameter space.

$$\Theta_H = \{\theta \in \Theta \mid g(\theta) \leq 0 \wedge h(\theta) = 0\}$$

From now on, we use a relaxed notation, writing H instead of Θ_H. We are particularly interested in sharp (precise) hypotheses, i.e., those in which $\dim(H) < \dim(\Theta)$, i.e. there is at least one equality constraint.

The FBST defines $\text{ev}(H)$, actually $\text{ev}(H; p_n, r)$, the e-value or evidence value supporting (in favor of) the hypothesis H, and its complement, $\overline{\text{ev}}(H)$, the evidence value against H, as

$$s(\theta) = \frac{p_n(\theta)}{r(\theta)} \; , \quad s^* = s(\theta^*) = \sup_{\theta \in H} s(\theta) \; , \quad \widehat{s} = s(\widehat{\theta}) = \sup_{\theta \in \Theta} s(\theta) \; ,$$

$$T(v) = \{\theta \in \Theta \mid s(\theta) \leq v\} \; , \quad W(v) = \int_{T(v)} p_n(\theta)\, d\theta \; , \quad \text{ev}(H) = W(s^*) \; ,$$

$$\overline{T}(v) = \Theta - T(v) \; , \quad \overline{W}(v) = 1 - W(v) \; , \quad \overline{\text{ev}}(H) = \overline{W}(s^*) = 1 - \text{ev}(H) \; .$$

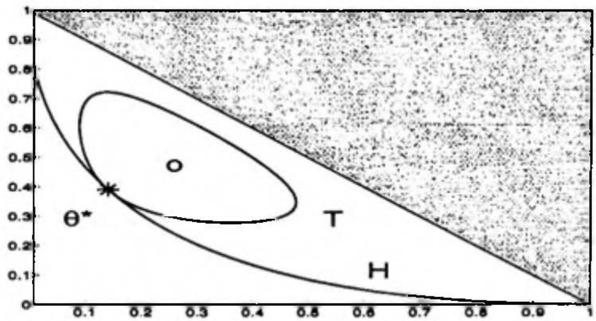

Figure A1: H-W Hypothesis and Tangential Set

The function $s(\theta)$ is known as the posterior surprise relative to a given reference density, $r(\theta)$. $W(v)$ is the cumulative surprise distribution. The surprise function was used, among other statisticians, by Good (1989), Evans (1997) and Royall (1997). Its role in the FBST is to make $\text{ev}(H)$ implicitly invariant under suitable transformations on the coordinate system of the parameter space, see next subsection.

The tangential (to the hypothesis) set $\overline{T} = \overline{T}(s^*)$, is a Highest Relative Surprise Set (HRSS). It contains the points of the parameter space with higher surprise, relative to the reference density, than any point in the null set of H. When $r(\theta) \propto 1$, the possibly improper uniform density, $\overline{T}$ is the Posterior's Highest Density Probability Set (HDPS) tangential to the null set of H. Small values of $\overline{\text{ev}}(H)$ indicate that the hypothesis traverses high density regions, favoring the hypothesis.

The evidence value, defined above, has a simple and intuitive geometric characterization. Figure A1 shows the null set of H, the tangential HRSS $\overline{T}$, and the points of constrained and unconstrained maxima, θ^* and $\widehat{\theta}$, for testing Hardy-Weinberg equilibrium law in a population genetics problem, as discussed in (Pereira and Stern 1999). In this biological application n is the sample size, x_1 and x_3 are the two homozygote sample counts and $x_2 = n - x_1 - x_3$ is heterozygote sample count. $\theta = [\theta_1, \theta_2, \theta_3]$ is the parameter vector. The posterior and maximum entropy reference densities for this trinomial model, the parameter space and the null set are:

$$p_n(\theta \mid x) \propto \theta_1^{x_1+y_1}\theta_2^{x_2+y_2}\theta_3^{x_3+y_3} \ , \quad r(\theta) \propto \theta_1^{y_1}\theta_2^{y_2}\theta_3^{y_3} \ , \quad y = [-1, -1, -1] \ ,$$

$$\Theta = \{\theta \geq 0 \mid \theta_1 + \theta_2 + \theta_3 = 1\} \ , \quad \Theta_H = \{\theta \in \Theta \mid \theta_3 = (1 - \sqrt{\theta_1}\ )^2\} \ .$$

In orthodox decision theoretic Bayesian statistics, a significance test is legitimate if and only if it can be characterized as an Acceptance (A) or Rejection (R) decision procedure defined by the minimization of the posterior expectation of a loss function, Λ. Madruga et al. (2001) gives the following family of loss functions characterizing the FBST. This loss function is based on indicator functions of θ being or not in the tangential set $\overline{T}$:

$$\Lambda(R, \theta) = a\, I(\theta \notin \overline{T}) \ , \quad \Lambda(A, \theta) = b + d\, I(\theta \in \overline{T}) \ .$$

Note that this loss function is dependent on the observed sample (via the likelihood function), on the prior, and on the reference density, stressing the important point of non-separability of utility and probability, see Kadane and Winkler (1987) and Rubin (1987).

Finally, consider the situation where the hypothesis constraint, $H : h(\theta) = h(\delta) = 0$, $\theta = [\delta, \lambda]$ is not a function of some of the parameters, λ. This situation is described by Basu (1988):

"If the inference problem at hand relates only to δ, and if information gained on λ is of no direct relevance to the problem, then we classify λ as the Nuisance Parameter. The big question in statistics is: How can we eliminate the nuisance parameter from the argument?"

Basu goes on listing at least 10 categories of procedures to achieve this goal, like using max_λ or $\int d\lambda$, the maximization or integration operators, in order to obtain a projected profile or marginal posterior function, $f(\delta \mid x)$. The FBST does not follow the nuisance parameters elimination paradigm. In fact, staying in the original parameter space, in its full dimension, explains the "Intrinsic Regularization" property of the FBST, when it is used for model selection, see Pereira and Stern (2001).

A.2 Reference and Consistency

In the FBST the role of the reference density, $r(\theta)$ is to make $\overline{\text{ev}}(H)$ implicitly invariant under suitable transformations of the coordinate system. Invariance, as used in statistics, is a metric concept. The reference density can be interpreted as a compact and interpretable representation for the reference metric in the original parameter space. This metric is given by the geodesic distance on the density surface. The natural choice of reference density is an uninformative prior, interpreted as a representation of no information in the parameter space, or the limit prior for no observations, or the neutral ground state for the Bayesian operation.

Standard (possibly improper) uninformative priors include the uniform and maximum entropy densities, for a detailed discussion the reader is refered to Dugdale (1996) and Kapur (1989). In the H-W example, using the notation above, the uniform density can be represented by $y = [0, 0, 0]$ observation counts, and the standard maximum entropy density can be represented by $y = [-1, -1, -1]$ observation counts.

Let us consider the cumulative distribution of the evidence value against the hypothesis, the confidence level function, $\overline{V}(c) = \Pr(\overline{\text{ev}} \leq c)$, given θ^0, the true value of the parameter. Under appropriate regularity conditions, for increasing sample size, $n \to \infty$, we can say the following:

- If H is false, $\theta^0 \notin H$, then $\overline{\text{ev}}$ converges (in probability) to 1, that is, $\overline{V}(0 \leq c < 1) \to 0$.

- If H is true, $\theta^0 \in H$, then $\overline{V}(c)$, converges (in distribution) to

$$QQ(t, h, c) = \text{Q}\left(t - h, \text{Q}^{-1}(t, c)\right) \ , \quad \text{where}$$

$$\text{Q}(k, x) = \frac{\Gamma(k/2, x/2)}{\Gamma(k/2, \infty)} \ , \quad \Gamma(k, x) = \int_0^x y^{k-1} e^{-y} dy \ ,$$

$t = \dim(\Theta)$, $h = \dim(H)$ and $\text{Q}(k, x)$ is the cumulative chi-square distribution with k degrees of freedom. Figure A2 portrays the function $QQ(t, h, c)$ for $t = 2 \ldots 4$ and $h = 0 \ldots t - 1$.

Under the same regularity conditions, an appropriate choice of threshold or critical level, $c(n)$, provides a consistent test, τ_c , that rejects the hypothesis if $\overline{\text{ev}}(H) > c$. The empirical power analysis developed in Stern and Zacks (2002) and Lauretto et al. (2003), provides critical levels that are consistent and also effective for small samples.

Stern (2004) presents an alternative approach, based on sensitivity analysis in the context of paraconsistent logic and bilattice structures, see also Costa et al. (1999). This analysis in based on the inconsistency induced by a set of alternative reference densities, $r, r', r''\ldots$, or a set of alternative priors, $p_0, p'_0, p''_0\ldots$, or a set of alternative likelihood power or "sample size" perturbation parameters, L^γ, $1 = \gamma > \gamma' > \gamma''\ldots > 0$.

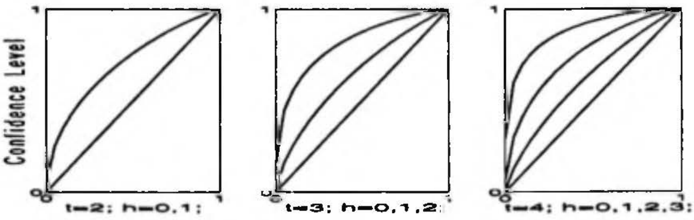

Figure A2: Test τ_c critical level vs. confidence level

A.3 Belief Calculi and Support Structures

Many standard Belief Calculi can be formalized in the context of Abstract Belief Calculus, ABC, see Darwiche and Ginsberg (1992) and Stern (2003). In a Support Structure, $\langle \Phi, \oplus, \oslash \rangle$, the first element is a Support Function, Φ, on a universe of statements, $\mathcal{U}$. Null and full support values are represented by 0 and 1. The second element is a support Summation operator, $\oplus$, and the third is a support Scaling or Conditionalization operator, $\oslash$. A Partial Support Structure, $\langle \Phi, \oplus \rangle$, lacks the scaling operation.

The Support Summation operator, $\oplus$, gives the support value of the disjunction of any two logically disjoint statements from their individual support values, i.e.,

$$\neg(A \wedge B) \Rightarrow \Phi(A \vee B) = \Phi(A) \oplus \Phi(B) \ .$$

The Support Scaling operator, $\oslash$, gives the conditional support value of B given A from the unconditional support values of A and the conjunction $C = A \wedge B$, i.e.,

$$\Phi_A(B) = \Phi(A \wedge B) \oslash \Phi(A) \ .$$

Support structures for some standard belief calculi are given in Table A1, where the support value of two statements their conjunction are given by $a = \Phi(A)$, $b = \Phi(B)$, $c = \Phi(C = A \wedge B)$.

In Table A1, the relation $a \preceq b$ indicates that the value a represents a stronger support than the value b. Darwiche and Ginsberg (1992) also gives a set of axioms defining the essential functional properties of a (partial) support function. Stern (2003) shows that the support $\Phi(H) = \text{ev}(H)$ complies with all Darwiche and Ginsberg axioms.

Table A1: Support structures for some belief calculi, $c = \Phi(C = A \wedge B)$.

$\Phi(\mathcal{U})$	$a \oplus b$	$\mathbf{0}$	$\mathbf{1}$	$a \preceq b$	$c \oslash a$	Calculus
$\{0,1\}$	$\max(a,b)$	0	1	$a \leq b$	$\min(c,a)$	Classical Logic
$[0,1]$	$a+b$	0	1	$a \leq b$	c/a	Probability
$[0,1]$	$\max(a,b)$	0	1	$a \leq b$	c/a	Possibility
$\{0\ldots\infty\}$	$\min(a,b)$	∞	0	$b \leq a$	$c-a$	Disbelief

In the FBST, the support values, $\Phi(H) = \text{ev}(H)$, are computed using standard probability calculus on Θ which has an intrinsic conditionalization operator. The computed e-values, on the other hand, have a possibilistic summation, i.e., the evidence value in favor of a composite hypothesis $H = A \vee B$, is the most favorable evidence value in favor of each of its terms, i.e., $\text{ev}(H) = \max\{\text{ev}(A), \text{ev}(B)\}$. It is impossible however to define a simple scaling operator for this possibilistic support function that is compatible with the FBST's e-value, ev( ), as it is defined.

Hence, two belief calculi are in simultaneous use in the Full Bayesian Significance Test setup: ev( ) constitutes a possibilistic partial support structure coexisting in harmony with the probabilistic support structure given by the posterior probability measure in the parameter space.

Stern (2003) comments the interpretation of this results in the juridical or legal context. In this context, the possibilistic structure corresponds to the *Onus Probandi* juridical principle, or the *In Dubito pro Reo* rule. These are "benefit of the doubt" type norms, requiring the statement presented by the defendant to be considered in most favorable manner, as seen in Gaskins (1992).

A.4 Complex Models and Compositionality

The relationship between the credibility of a complex hypothesis, H, and those of its constituent elementary hypothesis, $H^{(i,j)}$, in the independent setup, can be analyzed under the FBST, see Borges and Stern (2005) for precise definitions, and detailed interpretation.

Let us consider elementary hypotheses, $H^{(i,j)}$, in k independent constituent models, M^j, and the complex or composit hypothesis H, equivalent to a (homogeneous) logical composition (disjunction of conjunctions) of elementary hypotheses, in the composit product model, M. The following result can be established, see Borges and Stern (2005, proposition 5.1):

If H is expressed in HDNF or Homogeneous Disjunctive Normal Form,

$$H = \bigvee_{i=1}^{q} \bigwedge_{j=1}^{k} H^{(i,j)} \ , \quad M^{(i,j)} = \{\Theta^j, H^{(i,j)}, p_0^j, p_n^j, r^j\} \ ,$$

$$M = \{\Theta, H, p_0, p_n, r\} \ , \quad \Theta = \prod_{j=1}^{k} \Theta^j \ , \quad p_n = \prod_{j=1}^{k} p_n^j \ , \quad r = \prod_{j=1}^{k} r^j \ ;$$

then the e-value supporting H is

$$\mathrm{ev}(H) = \mathrm{ev}\left(\bigvee_{i=1}^{q} \bigwedge_{j=1}^{k} H^{(i,j)}\right) = W\left(\max_{i=1}^{q} \prod_{j=1}^{k} s^{*(i,j)}\right) =$$

$$W\left(\max_{i=1}^{q} s^{*i}\right) = \max_{i=1}^{q} W\left(s^{*i}\right) = \max_{i=1}^{q} \mathrm{ev}\left(\bigwedge_{j=1}^{k} H^{(i,j)}\right) = \max_{i=1}^{q} \mathrm{ev}\left(H^i\right) \ ;$$

where the cumulative surprise distribution of the composite model, $W(v)$, is given by the Mellin convolution operation, see Springer (1979), defined as

$$W = \bigotimes_{1 \le j \le k} W^j \ , \quad W^1 \otimes W^2(v) = \int_0^\infty W^1(v/y) W^2(dy) \ .$$

The probability distribution of the product of two independent positive random variables is the Mellin convolution of each of their distributions. From this interpretation, the we immediately see that $\otimes$ is a commutative and associative operator.

Mirroring Wittgenstein, in the FBST context, we can call the e-value, $\mathrm{ev}(H)$, the cumulative surprise distribution, $W(v)$, and the Mellin convolution operation, $\otimes$, respectively, truth value, truth function, and truth operation.

Finally, we observe that, in the extreme case of null-or-full support, that is, when, for $1 \le i \le q$ and $1 \le j \le k$, $s^{*(i,j)} = 0$ or $s^{*(i,j)} = \widehat{s}^j$, the evidence values (or, in this context, truth values) of the constituent elementary hypotheses are either 0 or 1, and the conjunction and disjunction composition rules of classical logic hold.

References

- T.Bakken, T.Hernes (2002). *Autopoietic Organization Theory. Drawing on Niklas Luhmann's Social Systems Perspective.* Copenhagen Business School.
- D.Basu, J.K.Ghosh (1988). Statistical Information and Likelihood. *Lecture Notes in Statistics,* 45.
- L.von Bertalanffy (1969). General System Theory. NY: George Braziller.
- Z.W.Birnbaum, J.D.Esary, S.C.Saunders (1961). Multicomponent Systems and Structures, and their Reliability. Technometrics, 3, 55-77.
- W.Borges, J.M.Stern (2005). On the Truth Value of Complex Hypothesis. *CIMCA-2005 - International Conference on Computational Intelligence for Modelling Control and Automation.* USA: IEEE.
- J.A.Bonaccini (2000). *Kant e o Problema da Coisa Em Si no Idealismo Alemão.* São Paulo: Relume Dumará.
- R.Boyd, P.Gasper, J.D.Trout, (1991). *The Philosophy of Science.* MIT Press.
- H.Caygill (1995). *A Kant Dictionary.* Oxford: Blackwell.
- J.Christis (2001). Luhmann's Theory of Knowledge: Beyond Realism and Constructivism? *Soziale Systeme,* 7, 328–349.
- M.L.Conde (1998). *Wittgenstein: Linguagem e Mundo.* São Paulo: Annablume.
- N.C.Costa, J.M.Abe, A.C.Murolo, J.I.da Silva, C.F.S.Casemiro (1999). *Lógica Paraconsistente Aplicada.* São Paulo: Atlas.
- A.Y.Darwiche (1993). *A Symbolic Generalization of Probability Theory.* Stanford Univ.
- A.Y.Darwiche, M.L.Ginsberg (1992). A Symbolic Generalization of Probability Theory. AAAI-92. 10th Conferenceof the American Association for Artificial Intelligence.
- P.Diaconis, D.Freeman (1987). A Dozen de Finetti Style Results in Search of a Theory. *Ann. Inst. Poincaré Probab. Stat.*, 23, 397–423.
- P.Diaconis (1988). *Group Representation in Probability and Statistics.* IMA.
- P.DiMaggio, W.W.Powell (1991). *New Institutionalism in Organizational Analysis.* Chicago Univ.
- M.G.Doncel, A.Hermann, L.Michel, A.Pais (1987). *Symmetries in Physics (1600-1980).* Seminari d'Història des les Ciènces. Universitat Autònoma de Barcelona.
- L.E.Dubins L.J.Savage (1965). *How to Gamble If You Must. Inequalities for Stochastic Processes.* NY: McGraw-Hill.
- J.S.Dugdale (1996). *Entropy and Its Physical Meaning.* Taylor & Francis.
- M.L.Eaton (1989). *Group Invariance Applications in Statistics.* Hayward: IMA.
- M.Evans (1997). Bayesian Inference Procedures Derived via the Concept of Relative Surprise. *Communications in Statistics,* 26, 1125–1143.

- W.Feller (1971). *An Introduction to Probability Theory and Its Applications.* NY: Wiley.
- B.de Finetti (1974). *Theory of Probability,* V1 and V2. London: Wiley.
- B.de Finetti (1981,1991). *Scritti.* V1 and V2. Padova: CEDAM
- B.de Finetti (1993). *Probabilitá e Induzione.* Bologna: CLUEB.
- H.Fleming (1979). As Simetrias como Instrumento de Obtenção de Conhecimento. *Ciência e Filosofia,* 1, 99–110.
- H.von Foerster (2003). *Understanding Understanding: Essays on Cybernetics and Cognition.* NY: Springer. The following articles (chapters) are of special interest: (1) On Self-Organizing Systems and their Environments, 1–19; (8) On Constructing a Reality, 211–227; (11) Objects: Tokens for Eigen-Behaviors, 261–271; (15) For Niklas Luhmann: How Recursive is Communication? 305–323.
- A.P.French (1968). *Special Relativity.* NY: Chapman and Hall.
- S.Fuchs (1996). The New Wars of Truth: Conflicts over science studies as differential modes of observation. *Social Science Information,* 35, 307–326.
- R.H.Gaskins (1992). *Burdens of Proof in Modern Discourse.* Yale Univ. Press.
- I.J.Good (1983). *Good Thinking.* Univ. of Minnesota.
- B.Gruber et al. edit. (1980–98). *Symmetries in Science, I–X.* NY: Plenum.
- J.Hanc, S.Tuleja, M.Hancova (2004). Symmetries and Conservation Laws: Consequenses of Noether's Theorem. *American Journal of Physics,* 72, 428–435.
- L.L.Harlow, S.A.Mulaik, J.H.Steiger (1997). *What If There Were No Significance Tests?* London: LEA - Lawrence Erlbaum Associates.
- R.Houtappel, H.van Dam, E.P.Wigner (1965). The Conceptual Basis and Use of the Geometric Invariance Principles. *Reviews of Modern Physics,* 37, 595–632.
- T.Z.Irony, C.A.B.Pereira (1995), Bayesian Hypothesis Test: Using Surface Integrals to Distribute Prior Information Among The Hypotheses, *Resenhas,* 2, 27-46.
- T.Z.Irony, C.A.B.Pereira, R.C.Tiwari (2000), Analysis of Opinion Swing: Comparison of Two Correlated Proportions, *The American Statistician,* 54, 57–62.
- T.Z.Irony, M.Lauretto, C.A.B.Pereira, J.M.Stern (2002). A Weibull Wearout Test: Full Bayesian Approach. In: Y.Hayakawa, T.Irony, M.Xie. *Systems and Bayesian Reliability,* 287–300. Singapore: World Scientific.
- B.Jaffe (1960). *Michelson and the Speed of Light.* NY: Anchor.
- J.B.Kadane, R.L.Winkler (1987). De Finetti's Methods of Elicitation. In: Viertl.
- I.Kant (1790). Critique of Teleological Judgement. In: I.Kant. *The Critique of Pure Reason; The Critique of Practical Reason; The Critique of Judgment.* Encyclopaedia Britannica Great Books of Western World, 42, 1952.
- J.N.Kapur (1989). *Maximum Entropy Models in Science and Engineering.* Wiley.
- O.Kemptorne (1980). Fundations of Statistical Thinking and Reasoning. *Australian CSIRO-DMS Newsletter.* 68, 1–5; 69, 3–7.

- W.Krohn, G.Küppers, H.Nowotny (1990). *Selforganization. Portrait of a Scientific Revolution.* Dordrecht: Kluwer.
- W.Krohn, G. Küppers (1990). The Selforganization of Science - Outline of a Theoretical Model. In: Krohn et al. (1990), 208–222.
- L.D.Landau, E. M. Lifchitz (1966). *Cours de Physique Théorique.* Moscou: MIR.
- M.Lauretto, C.A.B.Pereira, J.M.Stern, S.Zacks (2003). Comparing Parameters of Two Bivariate Normal Distributions Using the Invariant Full Bayesian Significance Test. *Brazilian Journal of Probability and Statistics,* 17, 147-168.
- M.Lauretto, J.M.Stern (2005). FBST for Mixture Model Selection. MaxEnt 2005, *American Institute of Physics Conference Proceedings,* 803, 121–128.
- I.Levi (1974). *Gambling with Truth: An Essay on Induction and the Aims of Science.* MIT Press.
- H.A.Lorentz, A.Einstein, H.Minkowski, H.Weyl (1952). *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity.* NY: Dover.
- N.Luhmann (1989). *Ecological Communication.* Chicago Univ. Press.
- N.Luhmann (1990a). The Cognitive Program of Constructivism and a Reality that Remains Unknown. In: Krohn et al. (1990), 64–86.
- N.Luhmann (1990b). *Essays on Self-Reference.* NY: Columbia Univ. Press.
- N.Luhmann (1995). *Social Systems.* Stanford Univ. Press.
- M.R.Madruga, L.G.Esteves, S.Wechsler (2001). On the Bayesianity of Pereira-Stern Tests. *Test,* 10, 291–299.
- M.R.Madruga, C.A.B.Pereira, J.M.Stern (2003). Bayesian Evidence Test for Precise Hypotheses. *Journal of Statistical Planning and Inference,* 117, 185–198.
- P.Maher, B.Skyrms, E.W.Adams, K.Binmore, J.Butterfield, P.Diaconis, W.L. Harper (1993). *Betting on Theories.* Cambridge Univ. Press.
- H.R.Maturana, F.J.Varela (1980). *Autopoiesis and Cognition. The Realization of the Living.* Dordrecht: Reidel.
- A.A.Michelson, E.W.Morley (1887). On the Relative Motion of the Earth and the Luminiferous Ether. *American Journal of Physics,* 34, 333–345.
- J.Mingers (1995) *Self-Producing Systems: Implications and Applications of Autopoiesis.* NY: Plenum.
- E.Noether (1918). Invariante Varlationsprobleme. *Nachrichten der Könighche Gesellschaft der Wissenschaften zu Göttingen.* 235–257. Translated: *Transport Theory and Statistical Physics,* 1971, 1, 183–207.
- C.A.B.Pereira, S.Wechsler (1993). On the Concept of p-value. *Brazilian Journal of Probability and Statistics,* 7, 159–177.
- C.A.B.Pereira, J.M.Stern (1999). Evidence and Credibility: Full Bayesian Significance Test for Precise Hypotheses. *Entropy Journal,* 1, 69–80.

- C.A.B.Pereira, J.M.Stern, (2001). Model Selection: Full Bayesian Approach. *Environmetrics*, 12, 559–568.
- C.A.B.Pereira, S.Wechsler, J.M.Stern (2005). Can a Significance Test be Genuinely Bayesian? Submitted.
- K.R.Popper (1959). *The Logic of Scientific Discovery*. NY: Routledge.
- K.R.Popper (1963). *Conjectures and Refutations: The Growth of Scientific Knowledge*. NY: Routledge.
- W.Rasch (1998). Luhmann's Widerlegung des Idealismus: Constructivism as a two-front war. *Soziale Systeme*, 4, 151–161.
- P.Ressel (1987). A Very General De Finetti Type Theorem. In: Viertl (1987).
- H.Rouanet, J.M.Bernard, M.C.Bert, B.Lecoutre, M.P.Lecoutre, B.Le Roux (1998). *New Ways in Statistical Methodology. From Significance Tests to Bayesian Inference*. Berne: Peter Lang.
- R.Royall (1997). *Statistical Evidence: A Likelihood Paradigm*. Chapman & Hall.
- H.Rubin (1987). A Weak System of Axioms for "Rational" Behaviour and the Non-Separability of Utility from Prior. *Statistics and Decisions*, 5, 47–58.
- L.J.Savage (1954): *The Foundations of Statistics*. Reprint 1972. NY: Dover.
- L.Segal (2001). *The Dream of Reality. Heintz von Foerster's Constructivism*. NY: Springer.
- G.Shafer (1982), Lindley's Paradox. *J. American Statistical Assoc.*, 77, 325–51.
- G.Shafer, V.Vovk (2001). *Probability and Finance, It's Only a Game!* NY: Wiley.
- M.D.Springer (1979). *The Algebra of Random Variables*. NY: Wiley.
- J.M.Stern (2003). Significance Tests, Belief Calculi, and Burden of Proof in Legal and Scientific Discourse. Laptec'03, *Frontiers in Artificial Intelligence and its Applications*, 101, 139–147.
- J.M.Stern (2004). Paraconsistent Sensitivity Analysis for Bayesian Significance Tests. SBIA'04, *Lecture Notes Artificial Intelligence*, 3171, 134–143.
- J.M.Stern, S.Zacks (2002). Testing Independence of Poisson Variates under the Holgate Bivariate Distribution. The Power of a New Evidence Test. *Statistical and Probability Letters*, 60, 313–320.
- M.Viana (2003). *Symmetry Studies, An Introduction*. Rio de Janeiro: IMPA.
- R.Viertl (1987). *Probability and Bayesian Statistics*. NY: Plenum.
- H.Weyl (1952). *Symmetry*. Princeton Univ. Press.
- E.P.Wigner (1970). *Symmetries and Reflections*. Cambride: MIT Press.
- D.Williams (2001) *Weighing the Odds*. Cambridge Univ. Press.
- R.C.Williamson (1989). *Probabilistic Arithmetic*. Univ. of Queensland.
- L.Wittgenstein (1921). *Tractatus Logico Philosophicus* (Logisch-Philosophische Abhandlung). Ed.1999, NY: Dover.
- M.Zelleny (1980). *Autopoiesis, Dissipative Structures, and Spontaneous Social Orders*. Washington: American Association for the Advancement of Science.

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